Speculative Constraints on Oligopoly

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Abstract: We examine an infinite horizon game in which producers' output can be purchased by speculators for resale in a future period. The existence of speculators serves to constrain the feasible set of prices that can result from producers' output game in each period. Absent speculation, producers play a repeated Cournot game with random demand. With speculative inventories possible, the game becomes a dynamic one in which speculative stocks are a state variable which firms can control via their influence on price. We employ collocation methods to find the unknown expected price and value functions required for computation of equilibrium quantities. We demonstrate that strategic considerations result in an incentive to sell to speculators that is non-monotonic in the number of producers: speculation has the largest effect on equilibrium prices and welfare for market structures intermediate between monopoly and perfect competition. Using a computed example, we demonstrate that the effect of speculative storage on the average price level can be substantial, even though the effects on social welfare can be ambiguous.

Keywords: Inventory, speculation, oligopoly, commodity markets

JEL classification: L13, D43, Q03

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1 Introduction

The impact of speculative storage on prices, profits and welfare, has recently received a surge of interest in the public debate, mostly due to substantial primary commodity price increases combined with the difficulty of consumers in developing countries to access some of these products. For example, the world oil market has been the object of recent political interest due to the sudden increase in speculation of the early 2000s. As pointed out in Smith (2009), this is an oligopolistic market dominated by OPEC with both "commercial" and "non-commercial" speculators active. Similarly, the impact of speculation on price, the importance of inventories, the access to important resources for developing countries, and the overall economic performance of commodity markets have been the subject of several recent debates. For example, the U.S. Senate committee on Homeland Security and Governmental Affairs pointed out in U.S. Senate (2006) that inventories of crude oil and natural gas have increased in the U.S. and in OECD countries due to an overall increase in speculation that sustained high prices and gave financial incentives to agents to store. According to this report, the inventory-price relationship has been perturbed compared to the usual negative correlation historically observed.[1] Likewise, the European Commission (2011) lists 14 critical raw materials[2] for which production is concentrated in the hands of few firms or a small number of countries. Finally, the formation of speculative bubbles on markets of vital or strategic importance for the development of emerging countries has attracted the attention of the United Nations Conference on Trade and Development (Gilbert (2010)), for their crucial consequences on economic development and on the risks populations face. These questions have triggered substantial academic interest investigating the relationship between inventories and speculative trading on commodity markets from an econometric point of view (Frankel and Rose (2010), Kilian (2008), Kilian (2009), and Kilian and Murphy (2014)).

The effects of speculative storage when production is perfectly competitive is fairly well understood, with important contributions made in Newbery and Stiglitz (1981), Newbery (1984), Williams and Wright (1991), Deaton and Laroque (1992), Deaton and Laroque (1996), and McLaren (1999). The focus in these papers is on the effects of storage on the distribution of prices caused by the movement of production across periods due to random production (harvest) shocks. As aggregate inventories cannot be negative, speculators smooth prices across periods only when positive inventories exist. Unexpectedly large prices result in stock-outs which leads to a breakdown of the price smoothing role of speculative storage. These occasional stock-outs lead to a skewed distribution of price. Market power has been considered by examining imperfect competition in the storage function (Newbery (1984), Williams and Wright (1991), McLaren (1999)), but production itself remains perfectly competitive in these papers, hence there is no scope for strategic considerations on the part of producers. While

[1]U.S. Senate (2006), p.15, figure 6

[2]Antimony, Beryllium, Cobalt, Fluorspar, Gallium, Germanium, Graphite, Indium, Magnesium, Niobium, Platinum Group Metals, Rare earths, Tantalum, and Tungsten.

this approach is reasonable for modelling many agricultural commodities, where market power is often exhibited by intermediaries instead of primary producers, for many other commodities, such as the mineral and energy commodities discussed above, models with market power at the producer level are more appropriate.

The effects of speculation when there is imperfect competition at the producer level has been examined in Mitraille and Thille (2009) for monopoly production, and in Mitraille and Thille (2014) for oligopoly production,[3] although in a finite horizon setting. In Mitraille and Thille (2014) a two-period model of oligopoly production is used to demonstrate that speculative sales can result in a rich set of equilibria, including i) stockouts, ii) deterrence of speculative holdings, iii) speculative holdings along with consumer purchases, iv) speculative purchases of the entire output, and v) zero production.

Our contribution in this paper is to extend the analysis of Mitraille and Thille (2014) to an infinite horizon setting and to explore the implications of speculative storage on the price distribution under oligopolistic production. We do this by analyzing the Feedback equilibrium to an infinite horizon game in which oligopolists produce a commodity which can be purchased and stored for future sale by competitive speculators. We demonstrate that speculative storage can have significant effects on the distribution of prices and profits of an oligopoly compared to what would happen in the absence of storage. We find that for every market structure but monopoly, mean prices are lower or equal to the mean equilibrium price that occurs in the absence of speculative storage. Moreover the distribution of prices differences with and without speculative storage is asymmetric: prices below those of the Cournot equilibrium occur relatively frequently which means that speculative storage has a pro-competitive effect. When the number of firms increases, the price distribution converges to the Cournot one, but from below. Higher prices than those of the Cournot equilibrium are nonetheless possible: when the number of firms is low enough the equilibrium price may be high enough to exclude consumers from purchasing or to deter speculators from purchasing. This is particularly true when the market is monopolized, in which case the mean price is strictly higher when speculative storage is possible than when it is not.

We confirm these findings by studying the average profit deviation from Cournot competition absent competitive speculation: profit is the smallest compared to Cournot when the number of firms is intermediate, while profits converge to Cournot from below when the number of firms increases. Despite the gains to an oligopoly due to price and cost smoothing, the presence of competitive speculators increases competition and lowers profits compared to Cournot. Similar results can be found when comparing consumers surplus and total welfare to Cournot competition: the average gain in consumers surplus is the largest for an intermediate market structure.

In what follows we first describe the model and then explore the implications of speculative storage for the nature of equilibria that we expect to find. We then describe

[3]The effects of producer storage on the equilibrium in a Cournot duopoly is examined in Thille (2006), in which, rather than speculators engaging in storage, producers themselves store in the face of random variations in demand and cost.

the computational approach that we take to finding the Feedback equilibrium to the game and finish with a description of the equilibrium for a computed example.

2 The Model

The model that we present here is an infinite horizon version of that in Mitraille and Thille (2014). We consider a discrete time model with an infinite number of periods in which risk-neutral consumers, producers, and speculators interact on the market for a homogeneous non-perishable product. We assume that consumers and speculators are price takers and behave competitively, while a finite number n of producers behave as an oligopoly. Speculators are able to store the product while producers and consumers cannot.

In every period t, consumers have a demand, D_t, which they can buy on a spot market. Consumers' demand in period t, D_t, is a decreasing function of the spot price p_t, and is an increasing function of a random state a_t. We assume that consumer's demand is a linear function of p_t and a_t, given by

$$D(a_t, p_t) = \max\{a_t - p_t, 0\} \tag{1}$$

where the random state a_t is drawn by Nature at the beginning of period t and known to every participant of the spot market before decisions are made. We assume that random states $\{a_t\}$ are independently and identically drawn from period to period as the random variable $\tilde{a}$, distributed over the support $[0, A]$ with a continuous cumulative distribution function $F(a)$, with $f(a)$ the associated density function[4]. We denote the mean of $\widetilde{a}$ by

$$E(a) = \int_0^A a\, dF(a). \tag{2}$$

Random changes in a_t may be interpreted as random shocks affecting the distribution of income in the population of consumers from period to period, modifying in turn the willingness to pay for the product sold by firms and stored by speculators.

In every period t, speculators are able to buy or sell on the spot market, and are able to store the product. Let x_t denote the position of speculators on the spot market of period t: if x_t is positive, then speculators are selling the product, while if x_t is negative speculators are buying the product. Speculators are able to store the product and we denote by S_t the amount of available inventories at the beginning of period t. This amount S_t is observable to all market participants. We assume that the rate of depreciation of inventories is constant and equal to γ; the transition equation for inventories is then

$$S_{t+1} = (1 - \gamma)\ (S_t - x_t). \tag{3}$$

[4]In Mitraille and Thille (2014) a uniform demand is considered.

Negative inventories are not allowed and initial inventories are equal to 0, $S_0 = 0$. Consequently in every period t aggregate speculative sales must satisfy

$$x_t \in (-\infty, S_t]. \tag{4}$$

We assume that the cost of storage of speculators paid in every period is a linear function of the level of initial inventories held in that period, and equal to

$$W(S_t) = wS_t \tag{5}$$

with $w \geq 0$. Let the discount factor be $\delta \in (0, 1]$, and let E_t denote the expectation operator conditional on the information available in period t.

In every period t, there are n producers in Cournot competition, each of which chooses the quantity it wants to produce, $q_t^i \in \mathbb{R}^+$, $i = 1, ..., n$. All firms produce their output using the same technology which results in the cost function

$$C(q_t^i) = \frac{c}{2}(q_t^i)^2. \tag{6}$$

with $c > 0$.

Firms cannot store their production: the quantity they produce in any period is equal to the quantity they sell on the market. We denote the aggregate quantity produced in period t by Q_t, and the aggregate quantity produced by all firms but i by Q_t^{-i}, where $Q_t = \sum_{i=1}^n q_t^i$ and $Q_t^{-i} = \sum_{j=1, j\neq i}^n q_t^j$. The vector of individual producer outputs will be denoted $q_t = (q_t^1, q_t^2, ..., q_t^n)$. Let p_t denote the market price, then producer i's instantaneous profit in period t is equal to

$$\pi_t^i = p_t q_t^i - C(q_t^i) \tag{7}$$

and the total expected profit discounted in period 0, Π_0^i, is

$$\Pi_0^i = E_0 \sum_{t=0}^{\infty} \delta^t \pi_t^i \text{ for all } i = 1, ..., n, \tag{8}$$

where E_t denotes the expectation operator conditional to the information available to all agents in period t.

The timing of the game adapts the Cournot timing to our dynamic setting where long-lived speculators have rational expectations over future prices. We assume that speculative inventories, S_t, and the demand state, a_t, are observed by all agents at the beginning of period t. Consequently, information is symmetric across agents. In period t, the timing of the interaction is therefore:

1. Demand level, a_t, is realized and observed by all agents. Aggregate inventory holdings, S_t, is observed by all agents.

2. Producers choose q_t^i, $i = 1, 2, ..., n$.

3. Speculators choose x_t.
4. Auctioneer sets p_t such that $D_t - x_t = q_t$.
5. Transactions occur and stage payoffs are realized.

Finally we assume that producers play stationary Feedback, or Markov, strategies,[5] depending only on the current state, (a_t, S_t). Producer i's strategy, σ^i, is a mapping from the set of states (a_t, S_t) to the set of quantities, $\sigma^i : [0, \overline{A}] \times \mathbb{R}^+ \to \mathbb{R}^+$. Given a strategy for each producer, $\sigma \equiv (\sigma^1, ..., \sigma^n)$, define $V^i(\sigma) = E_0 \sum_{t=0}^{+\infty} \delta^t \pi^i(\sigma, a_t, S_t)$ be the payoff to producer i under the strategy profile σ. Then,

Definition 1 *A Feedback equilibrium with rational expectations is a n-tuple of strategies* $\sigma^* \equiv (\sigma^{1*}, ..., \sigma^{n*})$ *such that*

$$V^i(\sigma^*) \geq V^i((\sigma^{1*}, ..., \sigma^{(i-1)*}, \sigma^i, \sigma^{(i+1)*}, ..., \sigma^{n*})) \ \forall \ \sigma^i \text{ for all } i = 1, ..., n \tag{9}$$

with, for every period t, inventories in period $t+1$ follow

$$S^*_{t+1} = (1-\gamma)\left(S^*_t - X^*(\sum \sigma^i(a_t, S^*_t), a_t, S^*_t)\right), \tag{10}$$

the market price p^*_t clears the market:

$$D^*_t - X^*(\sum \sigma^i(a_t, S^*_t), a_t, S^*_t) = \sum \sigma^i(a_t, S^*_t), \tag{11}$$

and the future market price $E_t[p^*_{t+1}]$ is rationally expected by all agents.

In a Feedback equilibrium, conditioning strategies to past prices or quantities is ruled out, so strategies allowing firms to implement tacit collusion are not considered.

3 Speculators' behaviour and firms' strategies

As in standard commodity storage models, speculators' behaviour is driven by the relationship between current and expected future prices. Speculators maximize their profit taking the current price as given and expecting the future price that results from the quantity of inventory carried into the next period, S_{t+1}. It will be useful to introduce the notation $p^e(S_{t+1}) = E_t[p_{t+1}|S_{t+1}]$ to represent the expected future price conditional on the level of stocks carried into $t+1$. As the behaviour of speculators determines the demand that will be faced by producers, we need to determine speculative sales as a function of producers' output. As the derivation is the same as for the finite horizon case, here we present a brief description of it. For a more detailed derivation see Mitraille and Thille (2014).

The aggregate behaviour of these speculators ensures that $p_t \geq \delta(1-\gamma)(p^e(S_{t+1})-w)$ with a stock-out occurring if the inequality is strict. The non-negativity constraint on

[5] See Başar and Olsder (1995).

aggregate speculative inventories implies that speculators' aggregate behaviour satisfies the complementarity condition

$$(S_t - x_t)(p_t - \delta(1-\gamma)(p^e(S_{t+1}) - w)) = 0,$$
$$S_t - x_t \geq 0, \quad p_t - \delta(1-\gamma)(p^e(S_{t+1}) - w) \geq 0 \quad (12)$$

Either no inventories are carried ($S_{t+1} = 0$) and the return to storage is negative, or inventories are carried ($S_{t+1} > 0$) and the return to storage is zero. Using $X^*(Q_t, a_t, S_t)$ to denote the equilibrium storage undertaken when producers sell Q_t in aggregate and the state is (a_t, S_t), the market clearing price, $P(Q_t, a_t, S_t)$, must be such that the total of consumer and speculative purchases satisfy

$$a_t - P(Q_t, a_t, S_t) - X^*(Q_t, a_t, S_t) = Q_t. \quad (13)$$

From (12), there is a threshold level of aggregate output, which we denote Q_t^L, below which $p_t > \delta(1-\gamma)(p_t^e(0) - w)$, as speculators cannot carry negative inventories. This threshold is the level of output which leads to zero return to speculation when there is a stockout:

$$a_t - S_t - Q_t^L = \delta(1-\gamma)(p_t^e(0) - w). \quad (14)$$

It is also possible that speculators purchase the entire production of firms, resulting in zero consumer purchases. This exclusion of consumers will occur if speculators value producers' output more highly than consumers do: $\delta(1-\gamma)(p_t^e(S_t + Q_t) - w) > a_t$. Consequently, there is another threshold output, which we denote $\widehat{Q}_t$, for which only speculators buy if $Q_t < \widehat{Q}_t$ and consumers buy and speculators carry inventories if $Q_t > \widehat{Q}_t$. This threshold is determined by the level of aggregate output that just extinguishes consumer demand when that output is purchased entirely by speculators:

$$a_t = \delta(1-\gamma)(p_t^e(S_t + \widehat{Q}_t) - w). \quad (15)$$

As with Q_t^L, the slope of the demand faced by producers changes discontinuously at $\widehat{Q}_t$. It is important to note that only one of Q_t^L and $\widehat{Q}_t$ can be positive as long as $p_t^e()$ is a decreasing function.[6]

Finally, when aggregate output exceeds the relevant threshold, Q_t^L or $\widehat{Q}_t$, speculative sales are determined implicitly by the relationship between p_t and $p_t^e(S_{t+1})$ that must hold. We denote speculative sales in this case as $\widetilde{X}(Q_t, a_t, S_t)$, which is the solution in X to

$$a_t - X - Q_t = \delta(1-\gamma)\left(p_t^e(S_t - X) - w\right). \quad (16)$$

Summarizing, speculative sales are given by

$$X^*(Q_t, a_t, S_t) = \begin{cases} S_t & \text{if} \quad Q_t \leq Q_t^L \text{ and } Q_t^L > 0 \\ -Q_t & \text{if} \quad Q_t \leq \widehat{Q}_t \text{ and } \widehat{Q}_t > 0 \\ \widetilde{X}(Q_t, a_t, S_t) & \text{otherwise.} \end{cases} \quad (17)$$

[6]If both thresholds were positive, both (14) and (15) must hold. The left hand side of (15) is clearly higher than that of (14) while the right hand side of (15) is lower than that of (14) if the expected price function is decreasing in future stocks, so both (14) and (15) cannot hold simultaneously.

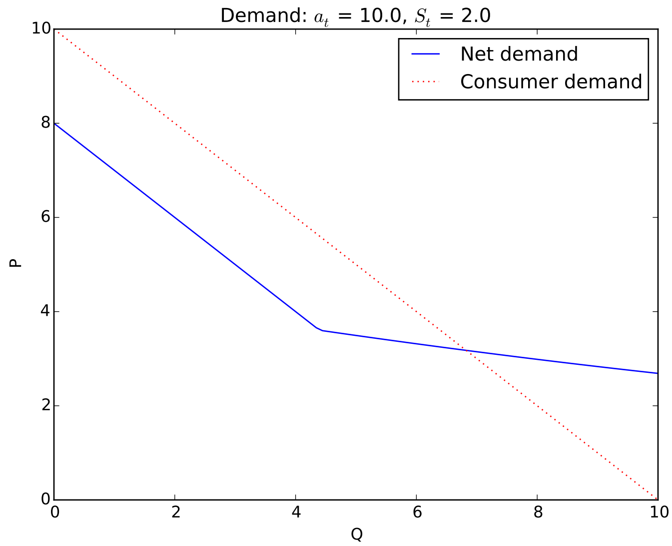

Figure 1: Net demand and consumer demand when $Q_t^L > 0$. Net demand is generated using the same parameters as in the numerical solution reported below with the number of firms set at three.

As only one of Q_t^L and $\widehat{Q}_t$ can be positive, only one of the first two conditions in (17) is possible for a given (S_t, a_t).

Given the behaviour of speculators in (17), we can now state the inverse demand function faced by producers:

$$P(Q_t, a_t, S_t) = \begin{cases} a_t - S_t - Q_t & \text{if} \quad Q_t \leq Q_t^L \text{ and } Q_t^L > 0 \\ \delta(1-\gamma)(p_t^e(S_t + Q_t) - w) & \text{if} \quad Q_t \leq \widehat{Q}_t \text{ and } \widehat{Q}_t > 0 \\ a_t - \widetilde{X}(Q_t, a_t, S_t) - Q_t & \text{otherwise.} \end{cases} \tag{18}$$

We plot this inverse demand for alternative demand levels in Figures 1 and 2. Figure 1 illustrates a situation with $Q_t^L > 0$, in which speculators sell their entire stock of inventories when aggregate production is low, shifting down demand in a parallel fashion. Once aggregate production exceeds Q_t^L, a stockout no longer occurs and price is linked to the expected future price, which is less steeply sloped than consumer demand. Figure 2 illustrates demand for the same level of speculative inventories but with a lower demand state. Here, for low levels of aggregate production consumers do not buy any output as price exceeds their maximal willingness to pay of 2.5 and the entire production is purchased and stored by speculators. Once aggregate production exceeds $\widehat{Q}_t$ price is low enough to induce consumer purchases on top of speculators demand.

With the behaviour of speculators determined by (17) and (18), payoffs to producers can be specified entirely in terms of output. The marginal payoff to a firm in any period will be discontinuous at an output that results in aggregate output of Q_t^L or $\widehat{Q}_t$, so the nature of the equilibrium in period t depends on where aggregate output falls in relation

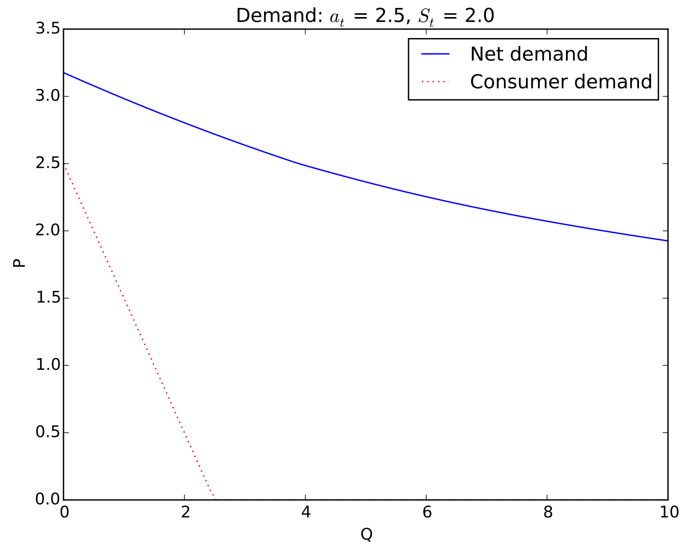

Figure 2: Net demand and consumer demand when $\widehat{Q}_t > 0$. Net demand is generated using the same parameters as in the numerical solution reported below with the number of firms set at three.

to these thresholds. First, a stockout may occur in which speculators sell their entire stocks and consumers buy the total of speculative and producer sales. We will denote this type of equilibrium with a C. Second, consumers may buy nothing and speculators purchase the entire output of firms, which we will denote with an S. Third, consumers may make some purchases and speculators carry inventory into the following period, denoted with a CS. Finally, there is the possibility that producers deter speculators by producing exactly Q_t^L in aggregate, which we will denote with an L.

With the effects of speculation on the demand faced by producers determined, we can write the profit received by a producer in period t as

$$\pi^i(q_t, a_t, S_t) = P(Q_t, a_t, S_t)q_t^i - \frac{c}{2}(q_t^i)^2 \tag{19}$$

and the producer's payoff as

$$\Pi_0^i = E_0 \sum_{t=0}^{+\infty} \delta^t \pi^i(q_t, a_t, S_t) \text{ for } i = 1, ..., n. \tag{20}$$

Given the behaviour of speculators determined above, we can express the dynamics of speculative inventory as depending on production choices:

$$S_{t+1} = (1 - \gamma)(S_t - X^*(Q_t, a_t, S_t)). \tag{21}$$

Consequently, the game played by producers has payoffs given by (20) and state dynamics given by (21).

As in Mitraille and Thille (2014), producer payoffs, while continuous, are non-differentiable at the thresholds Q_t^L and $\widehat{Q}_t$. As proven in that paper, in the context of the two-period game starting in period T-1, the fact that payoffs are not differentiable at threshold output levels Q_t^L and $\widehat{Q}_t$ generate discontinuities in the marginal profits, which results in upward jumps at $\widehat{Q}_t$, as well as upward or downward jumps at Q_t^L. This implies that profit comparisons must be performed in order to determine which of the different potential equilibria exist. For example when $\widehat{Q}_t > 0$, the equilibrium can be either the one with consumer exclusion (S), or the one with consumer and speculative purchases (CS), and firms profits must be compared to determine which one occurs, with potentially a multiplicity of outcomes when none of the local equilibrium candidates can be ruled out by a global unilateral deviation. Similarly, when $Q_t^L > 0$, profit comparisons must be performed to determine which of the candidates, between a stock-out equilibrium (C) and an equilibrium with consumers and speculative purchases (CS), is the equilibrium to the game. Moreover, in this case, the discontinuities in marginal profits also implies that an equilibrium with aggregate output equal to Q_t^L may exist. These forces, demonstrated in Mitraille and Thille (2014) for the two period game, still exist in the infinite horizon game. The determination of the equilibrium of the game for a given set of parameters requires verification of which of the potential candidates, C, S, CS, or L, exist.

In order to solve the game, we need to determine the expected price function and the value function associated with the equilibrium production strategies of producers. The expected price function is required to determine the thresholds Q_t^L and $\widehat{Q}_t$ as well as speculative sales when prices are smoothed (from (16)) and is computed as

$$p_t^e(S_{t+1}) = \int_0^A p^*(a, S_{t+1})dF(a) \tag{22}$$

where $p^*(a, S)$ is the Feedback equilibrium price when producers play equilibrium strategies $\sigma^*(a, S)$. The value function associates the expected payoff to a firm under the equilibrium strategies when the current state is (a_t, S_t):

$$V(a_t, S_t) = p^*(a_t, S_t)\sigma^*(a_t, S_t) - \frac{c}{2}\sigma^*(a_t, S_t)^2 + \delta E_t[V(a_{t+1}, S_{t+1})], \tag{23}$$

with $S_{t+1} = (1-\gamma)(S_t - X^*(n\sigma^*(a_t, S_t), a_t, S_t))$. We next turn to describing the method we use to compute these functions.

4 Numerical approach

Following the strategy used by Williams and Wright (1991), who compute approximations to the smooth $p^e(S_{t+1})$ rather than $p^*(a_t, S_t)$ for the competitive case, we approximate $p^e(S_{t+1})$ as well as the expected value function:

$$V^e(S_{t+1}) = \int_0^A V(a, S_{t+1})dF(a). \tag{24}$$

We apply the collocation method,[7] using cubic splines to approximate the expected price and value functions, denoting these approximations ρ and ν. We start with the period T solution[8] from Mitraille and Thille (2014) and iterate until convergence of the expected price and value function approximations. Given a vector of m values for S_{t+1}, $\bar{S} \equiv (0, S_1, ..., S_{max})$, in the jth iteration we compute the quantities $\bar{p}^e_{jk}$ and $\bar{V}^e_{jk}$ which are the expected price and value associated with future stocks given by each element of $\bar{S}$, i.e., for $k = 1, 2, ..., m$,

$$\bar{p}^e_{jk} = \int_0^A p^*_{j-1}(a, \bar{S}_k) dF(a) \tag{25}$$

and

$$\bar{V}^e_{tk} = \int_0^A V_{j-1}(a, \bar{S}_k) dF(a) \tag{26}$$

with $p^*_{j-1}(a, S)$ and $V_{j-1}(a, S)$ the equilibrium price and value functions found in iteration $j - 1$ using the iteration $j - 1$ approximations ρ_{j-1} and ν_{j-1}. Our approximation to $p^e_j(S)$, $\rho_j(S)$, is found by fitting a cubic spline to the $\bar{S}$ and $\bar{p}^e_j$ vectors. Similarly, we fit a cubic spline to $\bar{S}$ and $\bar{V}^e_j$ to generate our approximation to V^e_j, $\nu_j(S)$.

In summary, to find the solution in any period t_0:

Step 0 Compute the iteration 0 equilibrium price and value, $p^*_0(a, \bar{S}_k)$ and $V_j(a, \bar{S}_k)$, $k = 1, ...m$, using (22) and (23). Set $j = 1$.

Step 1 For each $k = 1, ..., m$ compute

$$\bar{p}^e_{jk} = \int_0^A p^*_{j-1}(a, \bar{S}_k | \rho_{j-1}, \nu_{j-1}) dF(a), \tag{27}$$

$$\bar{V}^e_{jk} = \int_0^A V^*_{j-1}(a, \bar{S}_k | \rho_{j-1}, \nu_{j-1}) dF(a) \tag{28}$$

Step 2 Fit a cubic spline to ($\bar{S}$, $\bar{p}^e_j$) and ($\bar{S}$, $\bar{V}^e_j$) to form $\rho_j(S)$ and $\nu_j(S)$.

Step 3 Return to Step 1 until $\rho_j(S)$ and $\nu_j(S)$ have not changed appreciably from the previous iteration.

When computing the equilibrium for any iteration an equilibrium selection is required for the cases in which multiple equilibria occur. We assume producers play C when both C and CS are possible and CS when both CS and S are possible.[9]

[7] Judd (1998), Ch. 11 provides a description of the method, which he applies to the competitive storage problem in Ch. 17.4

[8] In order to facilitate convergence of the expected value function we replace the terminal value of zero in Mitraille and Thille (2014) with the value of an infinite stream of Cournot profit following some "terminal time" beyond which speculation is not possible. In consequence, the initial condition for the value function is that which occurs in a period in which speculators are forced to sell their inventory and unable to replenish it again.

[9] The code used to compute the solution uses numerical routines from NumPy and SciPy (Jones et al. (2001–)). The code is available from the authors on request.

5 Results

We solve the infinite horizon game for the same values of the model parameters used in Mitraille and Thille (2014), namely $\delta = 0.95$, $w = 0.2$, $\gamma = 0$, and $c = 0.6$. The random demand parameter is distributed uniform on [0, 20]. For the cubic spline interpolations used in approximating the expected price and value functions, we use a grid of 25 values for the level of speculative stocks on a range between 0 and 60. We present results from the solution obtained after 50 iterations, by which time the maximal change in the expected value function is of the order of 0.1%.[10]

In order to demonstrate the effects of speculation on oligopoly, we present statistics gathered from running simulations of 1,000 periods for each n and computing statistics of interest. A short sample of a simulated time series with three producers is presented in Figure 3 in which we see instances of the alternative equilibria. Large realizations of a_t are often associated with relatively high price and a stockout. For example, periods 11 and 15 are ones in which the C equilibrium occurs. It is interesting to note that a large a_t is not sufficient to generate a stockout: in periods 9 and 10 there are relatively high realizations of a_t but a stockout has not occurred due to the rather high level of inventory that was built up during a sequence of below average realizations of a_t in periods 3–8. We also see examples of zero consumer purchases in Figure 3. For example, periods 18 and 19 see $p_t > a_t$ which implies zero consumption. Producers are selling solely to speculators at this time resulting in a rapid accumulation of speculative stocks. The overall frequencies of the alternative equilibria from this simulation with three firms are 26.2% C, 64.1% CS, and 9.7% S, with no occurrences of the L equilibrium.[11]

We now summarize the effects of speculation by examining deviations of the quantities of interest in the game with speculation from that which occurs in the absence of speculation (the repeated Cournot game with random demand[12]). First, in order to see what effect speculators have on price levels, we plot the average deviation from the Cournot price in Figure 4. Consistent with Mitraille and Thille (2009), speculation has an anti-competitive effect on prices in the case of monopoly. The increase in average price for $n = 1$ in Figure 4 is roughly 17% of the mean price in the absence of speculation. In this case, a monopolist's desire to price in such as way as to limit the building up of speculative inventories results in substantially higher prices on average. However, for more than one firm, speculation has a pro-competitive effect. In the oligopoly setting, firms compete to sell to speculators (effectively competing to supply

[10]The expected price function converges much more quickly than the expected value function.

[11]It is demonstrated in Mitraille and Thille (2014) that the L equilibrium is unlikely to occur when there are few, but more than one, firms.

[12]In using this benchmark, we are examining the effects of adding speculators with a storage technology to a model in which no storage technology exists. To examine the effects of speculation alone, we would need to allow producers to store the good, which would be a substantial complication. However, Thille (2006) has shown that, in the absence of speculators, the average price level was the same in the equilibrium in which producers can store the commodity as in the equilibrium in which they could not store. Consequently, we are confident that the results we present below are predominantly due to the presence of speculation and not simply due to the addition of a storage technology.

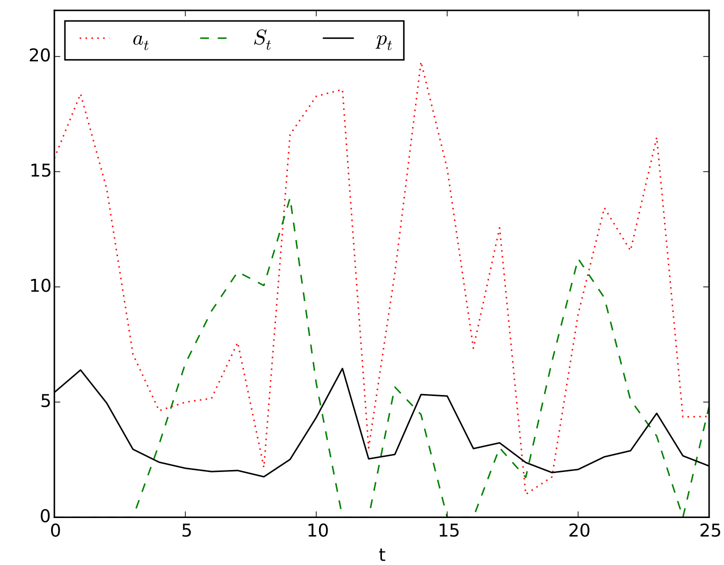

Figure 3: Sample time series plot of the demand state, a_t, beginning of period stocks, S_t, and price, p_t, for the case $n = 3$.

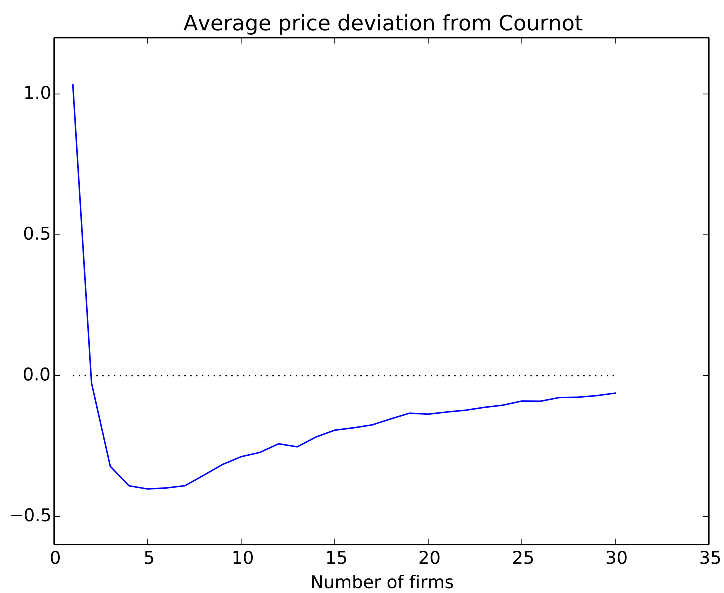

Figure 4: Average difference between price with speculators and that without speculators over a 1,000 period simulation.

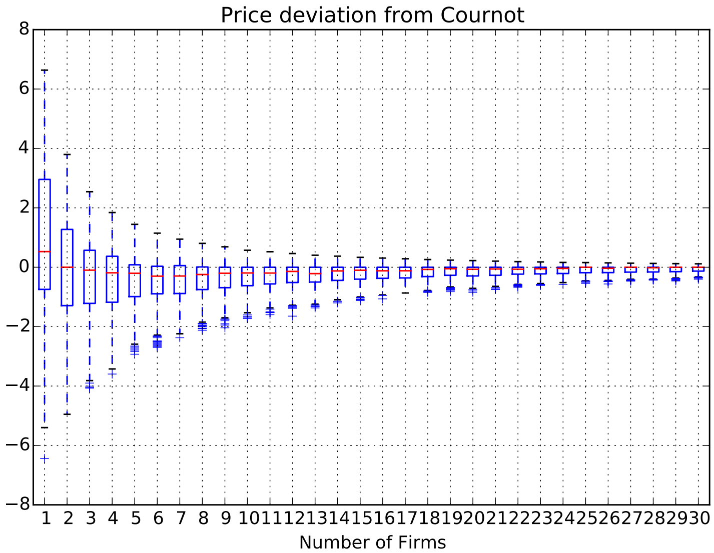

Figure 5: Box and whisker plot for the difference in price between the model with and without speculators. The box extends from the lower to the upper quartile, while the whiskers are set at 1.5 times the inter-quartile range.

future demand) resulting in prices lower than in the absence of speculation. Again, this effect is not trivial, the gap between prices with and without speculation in Figure 4 is more than 20% in some cases. Hence, simply by decoupling sales to consumers from production over time, speculation has a substantial effect on the average level of price in an oligopoly. It is important to note that in a similar setting in which there are no speculators, but producers themselves have the ability to store, Thille (2006) demonstrates that the average level of price is not affected by the addition of a storage technology for producers. Consequently, we attribute the effects on average price in Figure 4 to the presence of speculators with a storage technology, and not to the addition of the storage technology alone.

In order to illustrate the distribution of the deviation of prices from the Cournot equilibrium, box-plots of price are plotted for each n in Figure 5. Price deviations from Cournot tend to be asymmetrically distributed, negative values being more frequent than positive ones for $n > 1$. The opposite occurs when production is monopolized, due to the fact that a monopoly selects more often the limit equilibrium compared to a more competitive market structure. As the number of firms in competition increases, the price distribution converges to that of the unique Cournot equilibrium price, but largely from below. Figure 5 also illustrates that the magnitude of the effect of speculation on price can be quite large for relatively concentrated market structures.

The effects of speculation on mean profits are presented in Figure 6. Not surprisingly, the pro-competitive effect of speculation that results in lower average prices in

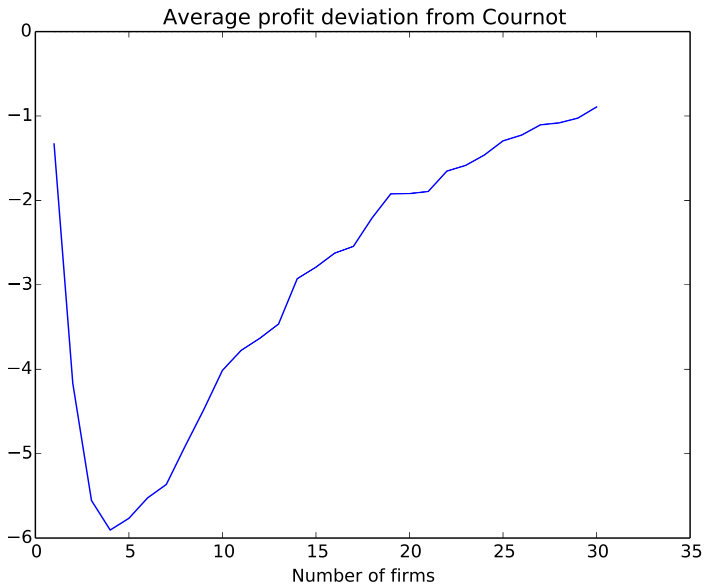

Figure 6: Average difference between profit with speculators and that without speculators over a 1,000 period simulation.

an oligopoly also reduces producer profits relative to the case in which speculation is absent. This is true even for monopoly. By smoothing prices over time, speculators restrict to some extent the ability of the monopolist to realize maximal profit. This is easiest to see in the limit equilibrium, where the monopolist chooses a level of output that is lower than the one that maximizes profit in the absence of speculation in order to deter speculative purchases. Although the monopolist does limit speculation this way, it still earns lower profit than it would if speculators were absent.

Given these non-monotonic effects of speculation on price and profit, it is interesting to examine the net effect on consumer surplus and welfare. These are plotted in Figures 7 and 8. The average consumer surplus, depicted in Figure 7, is positive for any number of firms. The gain is relatively low for $n = 1$, rises to a maximum at $n = 3$, and then declines slowly as the number of firms increases. It is interesting to note that for $n = 1$, consumers benefit from speculation even though average price is higher. This is due to the variation in the effects of speculation as shown in Figure 5 and the the fact that price affects consumer surplus in a non-linear manner. Intuitively, in periods of high demand (large a_t) prices tend to be higher, causing speculators to sell their stocks. Hence, speculators dampen price when it has the largest effect on consumer surplus. The limit equilibrium does not exist at high levels of demand, so the situations in which price is increased due to speculation occur in states where demand is lower, having a smaller effect on consumer surplus. For $n > 1$ this effect is complemented by the average reduction in price due to speculation.

Combining the effects of speculation for both consumers and producers, we see from

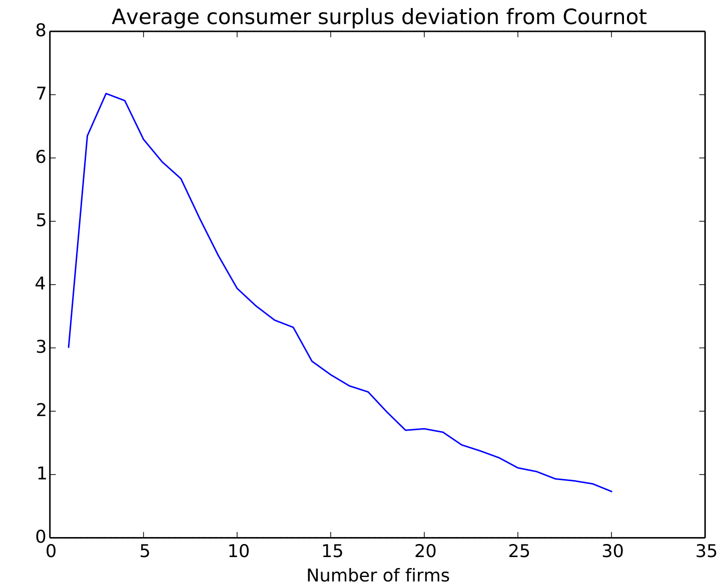

Figure 7: Average difference between consumer surplus with speculators and that without speculators over a 1,000 period simulation.

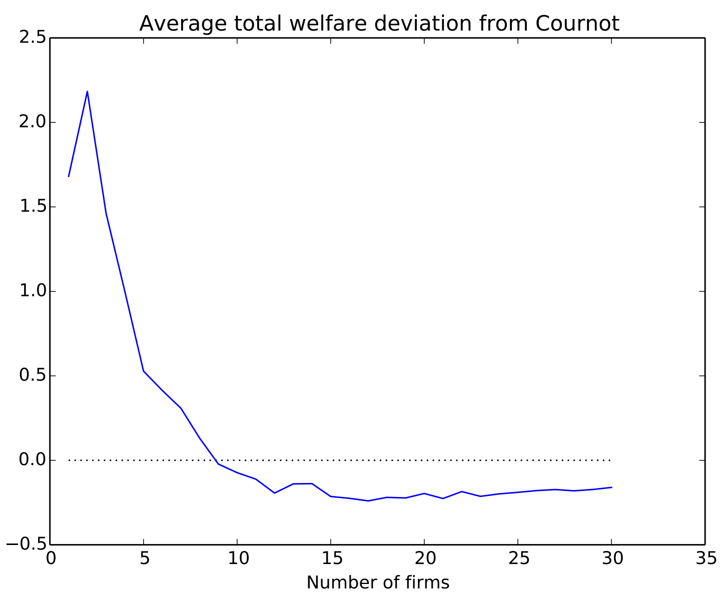

Figure 8: Average difference between welfare with speculators and that without speculators over a 1,000 period simulation.

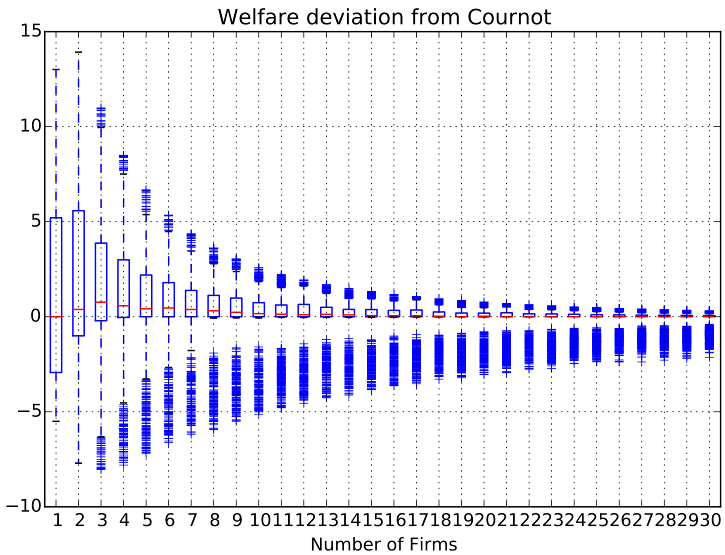

Figure 9: Box and whisker plot for the difference in welfare between the model with and without speculators. The box extends from the lower to the upper quartile, while the whiskers are set at 1.5 times the inter-quartile range.

Figure 8 that the average effect of speculation for social welfare is ambiguous. For relatively concentrated market structures ($n < 9$), the large gain in consumer surplus offsets the loss in profit resulting in a net welfare gain. However, for $n \geq 9$, the smaller gain in consumer surplus no longer offsets the loss in profits and there is a net loss in welfare. This is a rather counter-intuitive result: even though speculation is "pro-competitive" on average for large n in the sense that average price is lower, average welfare is lower than it would be in the absence of speculation. A box-and-whisker plot of the welfare effect is plotted in Figure 9, where the skewness of the effects of speculation on welfare is evident: although the median change in welfare due to speculation is positive, there are relatively few periods with large welfare losses due to speculation. These welfare losses tend to occur in periods in which a stockout occurs (the C equilibrium). Given the state of demand, a_t, price is lower in these periods relative to the Cournot outcome. Although this generates benefits to consumers, much of this lower price is due to speculative sales, not due to an increase in output by producers. As producers see a lower price at the same time that they are lowering output, the loss in profits they see are larger than the gain that flows to consumers, resulting in a net welfare loss. In a sense, this welfare loss is due to storage costs as the "excess" loss of producer profit is compensating speculators for their storage costs incurred when they carry stocks. Even though speculators earn zero profit on average, their storage costs are essentially coming out of producer profit in periods in which stocks are sold.

6 Conclusion

By examining a dynamic game in which oligopolistic producers are faced with competitive speculators who can purchase, store and resell their output, we have seen that predictions of oligopoly theory can be substantially affected. In particular, the presence of speculators leads to more competitive behaviour by producers resulting in a reduction in the average price as compared to what occurs in the absence of speculators. In a computed example, we see that this effect can be quite large, on the order of 20%. Conversely, speculators can have a substantial anti-competitive effect in the case of monopoly, where the attempts to deter speculative purchases leads to higher prices.

In contrast to studies of speculation in markets with competitive production, we find that the welfare effects of speculation are ambiguous in the oligopoly setting. Both consumer gains and producer profits are affected in a non-monotonic manner by speculative activity with the net effect either positive or negative. In our computed example, speculation improves mean social welfare when production is relatively concentrated, but reduces mean social welfare for less concentrated market structures.

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