Central Bank Screening, Moral Hazard, and the Lender of Last Resort Policy

Mei Li
Department of Economics and Finance, University of Guelph
mli03@uoguelph.ca

Frank Milne
Department of Economics, Queen's University
milnef@qed.econ.queensu.ca

Junfeng Qiu
China Economics and Management Academy,
Central University of Finance and Economics
qiujunfeng@cufe.edu.cn.

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University of Guelph
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Mei Li, Frank Milne, Junfeng Qiu*

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Abstract

This paper establishes a theoretical model to examine the LOLR policy when a central bank can distinguish solvent banks from insolvent ones only imperfectly. The major results that our model produces are as follows: (1) The pooling equilibria in which, on one hand, all the banks borrow from the central bank and, on the other hand, all the banks do not borrow from the central bank could exist given certain market beliefs off the equilibrium path. However, neither equilibrium is socially efficient because insolvent banks will continue to hold their unproductive assets, rather than efficiently liquidating them. (2) Higher precision in central bank screening will improve social welfare not only by identifying insolvent banks and forcing them to efficiently liquidate their assets, but also by reducing moral hazard and deterring banks from choosing risky assets in the first place. (3) If a central bank can commit to a specific precision level before the banks choose their assets, rather than conducting a discretionary LOLR policy, it will choose a higher precision level to reduce moral hazard and will attain higher social welfare.

Keywords: Central Bank Screening, Moral Hazard, Lender of Last Resort

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*Mei Li: Department of Economics and Finance, College of Management and Economics, University of Guelph, Guelph, Ontario, Canada N1G 2W1. Email: mli03@uoguelph.ca. Frank Milne: Department of Economics, Queen's University, Kingston, Ontario, Canada K7L 3N6. Email: milnef@qed.econ.queensu.ca. Junfeng Qiu: China Economics and Management Academy, Central University of Finance and Economics, 39 College South Road, Beijing, China 100081. Email: qiujunfeng@cufe.edu.cn.

1 Introduction

The 2007-2009 subprime mortgage crisis has revealed that the lender of last resort (LOLR) policy is a crucial tool for a central bank to tackle financial crises. During the crisis, three major central banks – the Federal Reserve, the European Central Bank, and the Bank of England – all employed the LOLR policy heavily to provide liquidity to banks. Despite the importance of the LOLR policy, there have been many controversies around how the LOLR policy should be properly conducted (see, e.g., Goodhart (1999)). Although the early discussion on the LOLR policy can be dated back to Thornton (1802) and Bagehot (1873), our understanding about this issue has not become clearer as time goes by. Many economists believe that with a more developed financial system, open market operations of central banks in a well-functioning interbank loan market are enough to maintain an efficient market. As a result, the LOLR policy becomes obsolete (see, e.g., Goodfriend and King (1988)). Considering moral hazard associated with the LOLR policy, some economists even believe that we should stop using the LOLR policy.

The argument that open market operations of central banks make the LOLR policy obsolete is based on a crucial assumption that the interbank loan market functions well without any information frictions. However, the LOLR policy can be justified during a financial crisis when all the financial markets suffer most heavily from information frictions. A large body of literature has suggested that when neither the central bank nor the market can distinguish between illiquidity and insolvency, the LOLR policy can improve social welfare by preventing contagion and alleviating market freezes (see, e.g., Goodhart and Huang (1999), Freixas, Parigi, and Rochet (2004), Rochet and Vives (2004), Pritsker (2013), and Li, Milne and Qiu (2013)).

This paper examines how to conduct a proper LOLR policy when neither the central bank nor the market can distinguish between illiquidity and insolvency. In particular, our paper is inspired by Acharya and Backus (2009) who argue that the LOLR policy conducted during the subprime mortgage crisis was suboptimal and emphasize that the optimal LOLR policy has to be *conditional*. That is, when a central bank lends to banks as an LOLR, it must say no to the banks that cannot meet certain solvency conditions such as the maximum leverage ratio and minimum capital adequacy ratio. We establish rigorous theoretical models to examine how a *conditional* LOLR policy in which the central bank can screen insolvent banks from illiquid ones will affect moral hazard and social welfare. We first study a basic model in which banks are divided into two types: solvent banks and insolvent ones. Both types of banks are illiquid and need to roll over their short-term debts. The market cannot distinguish between the two types but knows the distribution of them. As an LOLR, the central bank attempts to offer loans to solvent banks, but can screen insolvent banks from solvent ones only imperfectly. Then we examine the possible equilibria and how the precision in central bank screening affects the equilibrium outcomes. Next, we extend this model to a case where banks can choose between a safe asset and a risky one and examine how the precision in central bank screening will affect banks' choice of assets in the first place.

Our model produces the following results: First, the pooling equilibria in which, on one hand, both types of banks borrow from the central bank and, on the other hand, neither type of bank borrows from the central bank could exist given certain market beliefs off the equilibrium path. However, neither equilibrium is socially efficient because insolvent banks will continue to hold their unproductive assets, rather than efficiently liquidating them. Second, higher precision in central bank screening will improve social welfare not only by identifying insolvent banks and forcing them to efficiently liquidate their assets ex post, but also by reducing moral hazard and deterring banks from choosing risky assets ex ante. Finally, we find that if a central bank can commit to a specific precision level before the banks choose their assets, rather than conducting a discretionary LOLR policy, it will choose a higher precision level to reduce moral hazard and will attain higher social welfare.

An important result produced by our paper is that moral hazard originates from imperfect information, rather than from the LOLR policy. The LOLR policy is often criticized on the ground that it induces moral hazard. Our model reveals that when neither the central bank nor the market can distinguish between illiquidity and insolvency, moral hazard will exist even in the absence of the LOLR policy. Our model demonstrates that the introduction of an LOLR policy with central bank screening will reduce moral hazard, rather than inducing it. Another important result in our paper is that a pre-committed and precise central bank screening is crucial in the LOLR policy, which can effectively reduce moral hazard and improve social welfare. This result coincides with Acharya and Backus (2009), who argue that the conditional LOLR policy can help reduce moral hazard induced by the LOLR policy.

Our paper is closely related to the literature on the LOLR policy with imperfect information. Rochet and Vives (2004) study the LOLR policy in a global game setup where depositors face both strategic complementarities and imperfect information. They find that the introduction of the LOLR policy in such a setup will improve social welfare by alleviating coordination failure. Goodhart and Huang (1999) build a model where the central bank employs the LOLR policy to prevent contagion, but has to suffer the loss caused by moral hazard when the central bank cannot perfectly distinguish between illiquid and insolvent banks. Freixas, Parigi, and Rochet (2004) also build a model to study the LOLR policy when the market cannot distinguish liquidity shocks from solvency shocks. Our model is most closely related to one particular case in their paper where insolvent banks have an incentive to gamble for resurrection and the central bank cannot distinguish insolvent banks from illiquid ones. They find that the LOLR policy is more useful in improving social welfare in this case than in other cases studied in their paper. However, they do not further examine the optimal LOLR policy. Most of the existing literature ignores the informational role played by the LOLR policy through central bank screening when insolvent banks cannot be distinguished from illiquid ones. We focus on this role and find that an LOLR policy with precise central bank screening will actually reduce moral hazard instead of inducing it.

The rest of the paper is organized as follows. Section 2 introduces a basic model where banks' asset portfolio is exogenously given and characterizes the equilibria. Section 3 extends the basic model to one in which banks can endogenously choose their asset portfolio and studies how the precision in central bank screening affects banks' portfolio choices and social welfare. Section 4 concludes.

2 The basic model

2.1 The environment

Consider a two-period model with three dates of 0, 1 and 2. There is a continuum of banks with a central bank in the economy. The initial balance sheet of each bank at $t = 0$ is exogenously given as follows:

Thus at date 0 each bank has a long-term asset with a size of A, which is financed by its

Table 1: A bank's balance sheet at $t = 0$

Long-term Assets: A	Short-term Debts : D Equity: e_0

own equity e_0 and one-period short-term debts with a size of D. As a result, $A = D + e_0$. If the long-term asset is mature at date 2, its gross return rate will be $R_H > 1$. If the asset is liquidated prematurely at date 1, a liquidation cost will be incurred. We will specify the liquidation technology later.

The interest rate of short-term debts between date 0 and date 1 is exogenously given and assumed to be the riskless rate of zero for simplicity. The roll-over rate of short-term debts is determined by short-term creditors' expectations. We assume that short-term creditors are risk neutral and aim for an expected rate of zero.

We assume that at the beginning of date 1, before each bank rolls over its short-term debts, an unanticipated shock hits some banks' long-term assets. As a result, the banks are divided into two types. A proportion $\lambda \in (0, 1)$ of the banks, which we call the high type (H-type) banks, are unaffected by the shock. For the remaining banks with the proportion of $1 - \lambda$, with probability p, their long-term assets' return rate is R_H, and with probability $1 - p$, the rate is $R_L < 1$. We call these banks the low type (L-type) banks. The return rate of each L-type bank is i.i.d. Each bank knows its own type. The short-term creditors do not know the type of each bank but know the proportion of each type.

At date 1, after the shock, each bank rolls over its short-term debts through three options: borrowing from the central bank, borrowing from the market (short-term creditors), and liquidating its long-term assets. We assume that a bank rolls over its short-term debts in two stages:

In the first stage, the central bank offers to lend $L_{CB} < D$ to each bank at $r_{CB} \geq 0$. We focus on the case where r_{CB} is always lower than the prevailing market rate. We assume that each bank applying for central bank loans must agree to be inspected by the central bank. In addition, we assume that the central bank can perfectly identify an H-type bank, but can identify an L-type bank only imperfectly. More specifically, we assume that for each L-type bank, with probability $\phi < 1$, the central bank can identify it as L-type and will reject its application. With probability $1-\phi$, the central bank can not identify it as L-type and will lend to it. We believe that this assumption is realistic, because in reality a healthy bank may have safer assets, the quality of which is easier to verify, while an insolvent bank may have risker assets, the value of which could be more uncertain and, consequently, more difficult to evaluate. In addition, a healthy bank may be more cooperative, while an insolvent bank may try to hide information.[1]

Each bank determines whether to apply for central bank loans or not. When a bank applies for central bank loans, it will be inspected by the central bank and will be rejected by the central bank and forced to liquidate its assets if identified as L-type.

In the second stage, each bank determines how much to borrow on the market and how many long-term assets to liquidate. The market rate is determined as follows. For each bank, a short-term creditor decides whether to lend or not, and the interest rate if he does. The bank then decides whether to borrow or not. The creditor can not make his lending decision contingent on the quantity of debts that the bank will borrow. In addition, creditors cannot observe how many long-term assets are liquidated by a bank when determining the market rate.

The liquidation technology is as follows. For H-type and L-type banks, each unit of the assets liquidated at date 1 will yield γ_H and γ_L units of the proceeds, respectively, where $\gamma_L \leq \gamma_H < 1$.

In addition, we assume that it is socially efficient for an L-type bank to liquidate its asset at date 1 rather than continuing to hold the asset to date 2. More specifically, we assume that

$$pR_H + (1-p)R_L < \gamma_L \tag{1}$$

Moreover, we assume that

$$\gamma_L A \leq \gamma_H A < D \tag{2}$$

which means that the date 1 liquidation value of both H-type and L-type banks' assets is not enough to repay their debts. This condition will be satisfied when e_0 is sufficiently low such that the asset-debt ratio of $\frac{A}{D}$ is high or when γ_H and γ_L are sufficiently low.

[1] In a more general case, the central bank can also mistake a healthy bank for an insolvent one. Here we consider this simpler case for simplicity, which will not affect our results qualitatively.

Note that by combining conditions (1) and (2), we derive

$$pR_H + (1-p)R_L < \gamma_L < \frac{D}{A}$$

Or

$$pAR_H + (1-p)AR_L < D \tag{3}$$

Since $AR_H > D$ (because $A > D$ and $R_H > 1$), it is straightforward to see that

$$AR_L < D \tag{4}$$

This implies that an L-type bank cannot repay its debts in the down state and will default.

For simplicity in our calculations, we assume that

$$p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D} < 1 \tag{5}$$

This condition guarantees that a creditor will never roll over his debts if he knows that the bank is L-type. The proof of this condition is given in appendix A.

Each bank aims to maximize its expected equity value at date 2. A bank does not care about the loss of its creditors. As a result, an L-type bank will borrow and continue its operation until date 2 as long as it has a higher expected equity value, even when it is not socially optimal to do so. In other words, it has an incentive to gamble for resurrection.

2.2 Banks' optimal borrowing behavior

Before we characterize possible equilibria in this model, we first examine banks' optimal borrowing behavior in this model. Let r_M denote the market rate that creditors charge. Proposition 1 gives the results.

Proposition 1. *(1) For an H-type bank, if $1+r_M < \frac{R_H}{\gamma_H}$, it will borrow on the market and will never liquidate its asset. If $1+r_M \geq \frac{R_H}{\gamma_H}$, the bank will never borrow on the market and will only liquidate its asset to repay its debts. (2) An L-type bank acts in a similar way to an H-type bank, except that $\frac{R_H}{\gamma_H}$ in the above conditions is changed into $\frac{R_H}{\gamma_L}$.*

Proof: see the appendix. ■

Note that $\frac{R_H}{\gamma_L} \geq \frac{R_H}{\gamma_H}$. So when $1+r_M < \frac{R_H}{\gamma_H}$, both H-type and L-type banks will borrow on the market to meet all the liquidity need. When $1+r_M \geq \frac{R_H}{\gamma_H}$, an H-type bank will stop borrowing on the market, while an L-type bank will still want to borrow when $1 + r_M \in [\frac{R_H}{\gamma_H}, \frac{R_H}{\gamma_L}]$. However, any value of $1 + r_M$ in the range of $[\frac{R_H}{\gamma_H}, \frac{R_H}{\gamma_L}]$ can not exist in equilibrium, because creditors know that at this level of $1 + r_M$, all the borrowers must be L-type. By assumption, creditors' expected rate of return from lending to an L-type is always below 1. As a result, creditors will never offer such a market rate in equilibrium.

We also arrive at the following result from the proof of proposition 1:

Corollary 1. *An L-type bank's net asset value in the down state is negative in all situations. Thus its equity value in that state is always zero.*

Proof: see the proof of proposition 1 in the appendix. ■

This result implies that, in order to maximize the equity value, an L-type bank needs only to maximize its equity value in the up state.

2.3 The equilibrium

There are two pooling equilibria in this model, the one where all the banks apply for central bank loans and the one where all the banks do not apply for central bank loans. Here we focus on the equilibrium where all the banks apply for central bank loans. The uninteresting equilibrium where all the banks do not apply for central bank loans is not studied here.[2]

Note that separating equilibria where one type of bank apply for central bank loans and the other type do not apply can not exist in this model. Consider the equilibrium where only L-type banks apply for central bank loans. Then L-type banks will always deviate, because they can mimic H-type banks without incurring any cost by not applying for central bank loans. By doing so, an L-type bank will be identified as H-type and get the lowest possible borrowing rate on the market. Consider the equilibrium where only H-type banks apply for central bank loans. If an L-type bank follows the equilibrium strategy and does not apply, it will be identified as L-type by the market and will not be able to borrow on the market. As a result, it will be forced to liquidate all of its asset and go bankrupt at date 1. If it mimics an H-type bank and borrows from the central bank, then at least with probability $1 - \phi$, it will successfully get loans and gain positive equity

[2]In the appendix , we provide a proof about the existence of such an equilibrium.

in the up state. As a result, L-type banks will always deviate, and such an equilibrium can not exist either.

Now let us examine the equilibrium where all the banks apply for central bank loans. All the banks aim to maximize their expected equity value. In order to find out whether such an equilibrium exists or not, we need to find each type of banks' expected equity value when following the equilibrium strategy and when deviating. The no-deviation condition will be the gap between the two values being positive. An equilibrium exists if and only if the no-deviation condition holds for both types of banks.

In order to find each type of bank's no-deviation condition, we need to find the market rate when a bank does and does not deviate. First, we find the equilibrium market rate when banks follow the equilibrium strategy. In this case, the banks identified as L-type by the central bank will be forced to liquidate their assets at date 1 and disappear. Creditors' belief about the remaining banks that successfully receive central bank loans is as follows. The conditional probability to get central bank loans is 1 for an H-type bank, and $1-\phi$ for an L-type one. Let g denote creditors' ex post belief that a bank is H-type. Thus

$$g = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)} > \lambda \tag{6}$$

Note that g is higher than λ, the prior belief.

The equilibrium market rate is determined based on this belief. We focus on the case where an equilibrium market rate exists. In this case, creditors' expected return rate from an H-type bank is $1 + r_M$. Similarly, their expected return rate from an L-type bank in the up state is also $1 + r_M$. Creditors' expected return rate from an L-type bank in the down state is $\frac{AR_L}{D}$. This is because, in the down state, an L-type bank goes bankrupt as we proved before. As a result, its asset is allocated between the central bank and creditors proportionally to their principals.[3]

Thus the equilibrium market rate is decided by

$$1 = g(1 + r_M) + (1-g)\left(p(1 + r_M) + (1-p)\frac{AR_L}{D}\right)$$

Or

$$r_M = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{g + (1-g)p} - 1\right) \tag{7}$$

[3]For simplicity, we assume that when a bank's asset value is below the principals of its debts, its asset will be allocated among creditors proportional to their principals.

In order for an equilibrium market rate to exist, $1 + r^*_M \leq min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}, \frac{R_H}{\gamma_H}\}$.[4] We focus on this case and assume that this condition always holds. Note that the quantity that an H-type or L-type bank will borrow is the same so that any information about the type of each bank is not revealed.

Next, we find the market rate if a bank deviates to not applying for central bank loans. Then we need to specify creditors' belief off the equilibrium path first. Let $\hat{\lambda} \in [0, 1]$ be the probability that a creditor assigns to a bank being H-type when observing it not borrow from the central bank. When $\hat{\lambda}$ is sufficiently low, the market freezes and a bank will gain the minimum payoff of zero equity. As a result, banks will have no incentive to deviate. We focus on the case where $\hat{\lambda}$ is high enough such that the market rate exists. The market rate off the equilibrium path, denoted by $\hat{r}_M$, is determined as follows:

$$\hat{r}_M = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\hat{\lambda} + (1 - \hat{\lambda})p} - 1\right) \tag{8}$$

Proposition 2 summarizes the results.

Proposition 2. *The pooling equilibrium where both types of banks apply for central bank loans exists if and only if*

$$(1 - \phi)\left[AR_H - D(1 + r_M) + L_{CB}(r_M - r_{CB})\right] - \left[AR_H - D(1 + \hat{r}_M)\right] > 0 \tag{9}$$

It implies that this equilibrium exists if and only if $\hat{\lambda}$ is lower than a threshold level, $\hat{\lambda}_{th,L}$, which is determined by

$$(1 - \phi)\left[AR_H - D(1 + r_M) + L_{CB}(r_M - r_{CB})\right] - \left[AR_H - D(1 + \hat{r}_M(\hat{\lambda}_{th,L}))\right] = 0 \tag{10}$$

Proof: see the appendix. ■

The intuition behind proposition 2 is as follows. H-type banks have a less strict no-deviation condition than L-type banks, because H-type banks's payoff from following the equilibrium strategy of applying for central bank loans is higher than L-type banks: they will be identified as H-type for sure, while L-type banks may be identified as L-type with probability ϕ. Meanwhile, two types of banks have the same payoffs from deviating to not applying for central bank loans. In addition, with a higher belief off the equilibrium

[4] The maximum market rate that an H-type bank can pay is $\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}$. Thus any $1 + r^*_M > \frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}$ could not be an equilibrium market rate. $1 + r^*_M > \frac{R_H}{\gamma_H}$ could not be an equilibrium rate, as we explained previously.

path (a higher $\hat{\lambda}$), the creditors will charge a lower $\hat{r}_M$ when a bank deviates, inducing a stronger incentive for banks to deviate. When $\hat{\lambda} < \hat{\lambda}_{th,L}$, condition (9) holds such that L-type banks will not deviate. It implies that H-type banks will not deviate either since they have a less strict no-deviation condition. Because an equilibrium exists if and only if both types of banks do not deviate, we have a threshold level of $\hat{\lambda}$, $\hat{\lambda}_{th,L}$, below which the equilibrium exists.

2.4 A numerical example

Here we provide a simple numerical example to illustrate the intuition of our analytical results. The parameter values are given as follows: $\lambda = 0.7$, $A = 1$, $R_H = 1.2$, $R_L = 0.4$, $p = 0.25$, $D = 0.9$, $e_0 = 0.1$, $L_{CB} = 0.25$, $r_{CB} = 0$, $\gamma_H = 0.8$, $\gamma_L = 0.7$, and $\phi = 0.25$. We will use these baseline parameter values in the rest of the paper. Note that these values satisfy the assumptions in the model. First, the expected value of an L-type bank's asset is given by

$$pAR_H + (1-p)AR_L = 0.6 \tag{11}$$

which is lower than $\gamma_L A = 0.7$. So it is socially optimal for an L-type bank to liquidate its asset at date 1. Second, asset liquidation values at date 1 for an H-type and L-type banks are 0.8 and 0.7 respectively, both of which are smaller than $D = 0.9$. Third, creditors will not lend to an L-type bank if they know its type certainly. This is because $\frac{R_H}{\gamma_L} = 1.7143$, and we have

$$p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D} = 0.7619 < D = 0.9 \tag{12}$$

In addition, the maximum rate that can be paid by the bank in the up state is $\frac{AR_H - L_{CB}}{D - L_{CB}} = 1.4615 < \frac{R_H}{\gamma_H} = 1.5$. So the bank will stop borrowing even before the rate reaches $\frac{R_H}{\gamma_H}$.

We find that the updated market belief is given by $g = 0.7568 > \lambda = 0.7$, and the equilibrium market rate is given by $r_M = 0.1240$. In addition, the threshold of $\hat{r}_M$ below which an L-type bank will deviate is 0.1707, and the corresponding $\hat{\lambda}_{th,L}$ below which the equilibrium exists is 0.6866.

3 Central bank screening and banks' ex ante choices

In this section, we extend our basic model to study moral hazard associated with the LOLR policy. More specifically, we allow banks to choose between a safe and risky assets at date 0. We will examine how the LOLR policy with central bank screening may affect banks' ex ante choices. Our model reveals that: (1) An LOLR policy with a higher precision level in central bank screening can reduce moral hazard more. (2) If a central bank can commit to a specific screening precision level before the banks choose their assets, instead of conducting a discretionary LOLR policy, the central bank will attain higher social welfare by choosing a higher screening precision level and reducing moral hazard more.

3.1 A model where ϕ is exogenously given

We first study the case where ϕ is exogenously given. Later we will examine the case where the central bank optimally chooses ϕ.

We assume that there is a continuum of banks with mass 1. A typical bank can choose between a safe and a risky long-term asset at date 0. The safe asset will mature at date 2 always with a return of R_H. The return of the risky asset is uncertain with two states: an up state state which occurs with probability p and a down state which occurs with probability $1-p$. The realized return in each state depends on the incidence of a financial crisis. More specifically, with a probability of π, a financial crisis occurs. In this case, the risky asset's return at date 2 is R_H in the up state, and R_L in the down state. With probability $1-\pi$, no financial crisis occurs. In this case, its return is R_H in the up state, and $R_M \in (1, R_H)$ in the down state. For simplicity, we assume that the return of all the risky assets are perfectly correlated. Following the basic model, we assume that each asset has a fixed size of A, and each bank finances its asset by its equity of e_0 and its short-term debts of D. Other assumptions in the basic model about the two types of banks remain unchanged here.

We assume that banks can derive private benefits from investing in the risky asset, but cannot derive any private benefit from investing in the safe asset.[5] Banks are hetero-

[5] We introduce private benefits by following Holmstrom and Tirole (1997). The private benefit can be thought of as actual benefits that a manager can derive or as the costs reduced by adopting a less strict

geneous in terms of private benefits that they derive. More specifically, a bank derives a private benefit of $B_i = a_0 + a_1 h_i$ from investing in risky assets, where $a_0 \geq 0$ and $a_1 > 0$ are constant. Each bank has a different h_i that we assume is uniformly distributed between 0 and 1 among all the banks. As a result, a bank with a higher h_i will have a stronger incentive to choose the risky asset. We denote a bank with a private benefit of B_i by h_i.

Note that given the assumptions above, once a financial crisis occurs, banks holding the safe asset are H-type, and banks holding the risky asset are L-type.

3.1.1 Equilibrium characterization

There may be multiple pooling equilibria in this model as we found in the basic model. Here we assume that the pooling equilibrium where all the banks apply for central bank loans exists, and the central bank can always coordinate all the banks toward this equilibrium.[6] In addition, for simplicity we confine our attention to the case where an equilibrium market rate exists.

Proposition 3 characterizes the symmetric trigger strategy equilibrium in this model.

Proposition 3. *With reasonable parameter values, there exists a symmetric trigger strategy Nash equilibrium in which all the banks with h_i below a threshold level of $\bar{h}$ will choose the safe asset. All the banks with h_i above $\bar{h}$ will choose the risky asset. Here $\bar{h}$ is determined by*

$$\begin{aligned} &\pi(1-p)A(R_H - R_M) + (1-\pi)\left[1 - p(1-\phi)\right] \\ &\left[AR_H - D(1 + r_{M,g_1}) + L_{CB}(r_{M,g_1} - r_{CB})\right] = a_0 + a_1 h \end{aligned} \tag{13}$$

where

$$r_{M,g_1} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{g_1 + (1-g_1)p} - 1\right) \tag{14}$$

and

$$g_1 = \frac{\bar{h}}{\bar{h} + (1-\bar{h})(1-\phi)} \tag{15}$$

risk management procedure.

[6] We believe that this assumption is realistic. The central bank has an incentive to coordinate all the banks toward this equilibrium, because its screening will improve social welfare only in this equilibrium. Since the no-deviation condition for this equilibrium depends crucially on the central bank's lending conditions of L_{CB} and r_{CB}, the central bank can ensure that this equilibrium exists by setting L_{CB} high and r_{CB} low. In addition, a higher ϕ can also make this equilibrium more likely.

Proof: see the appendix. ■

The intuition behind proposition 3 is as follows. In this equilibrium, bank $\bar{h}$ must be indifferent between the safe and risky assets. Equation (13) gives bank $\bar{h}$'s indifference condition. The LHS of equation (13) is the gap of bank $\bar{h}$'s expected equity values between the safe and risky assets. The RHS is the private benefit that bank $\bar{h}$ derives from choosing the risky asset. Thus bank $\bar{h}$ is indifferent if and only if equation (13) holds. Any bank with $h_i > \bar{h}$ has the LHS less than the RHS and will prefer the risky asset. Any bank with $h_i < \bar{h}$ has the LHS greater than the RHS and will prefer the safe asset.

3.1.2 Numerical examples

Here we give a numerical example to illustrate how the equilibrium threshold level of h_i, $\bar{h}$, is determined. Note that since h_i is uniformly distributed, the equilibrium proportion of H-type banks, λ, equals exactly $\bar{h}$. Let $\pi = 0.9$, meaning that, with a probability of 0.9, no financial crisis occurs. Let $a_0 = 0$, $a_1 = 0.08$, and $R_M = 1.14$. Parameter values are chosen such that a unique solution to $\bar{h} \in [0, 1]$ exists without a market freeze. The values of all the other parameters follow the baseline parameter values in the previous numerical example.

Figure 1 illustrates the intuition about how $\bar{h}$, or equivalently the equilibrium λ, is determined when $\phi = 0.5$. Note that given λ, the proportion of banks choosing the safe asset, there exists a threshold level of h, $\bar{h}$, such that a bank chooses the risky asset if and only if $h_i > \bar{h}$. That is, $\bar{h}$ is a function of λ. At equilibrium, $\bar{h}(\lambda) = \lambda$, which is illustrated by the intersection of the $\bar{h}(\lambda)$ curve and the 45 degree line in figure 1. With parameter values chosen in our numerical example, we find that starting from any value of λ that excludes a market freeze, the economy will converge to the unique equilibrium where $\bar{h}(\lambda) = \lambda$.

Figure 2 shows the values of equilibrium λ, or $\bar{h}$ at different levels of ϕ. We can see that higher values of ϕ will induce more banks to choose the safe asset. This is mainly because with higher values of ϕ, insolvent banks will be more likely to be identified and fail to get central bank loans, inducing a lower expected payoff for the risky asset. Thus we can see that the LOLR policy with central bank screening can effectively reduce moral hazard.

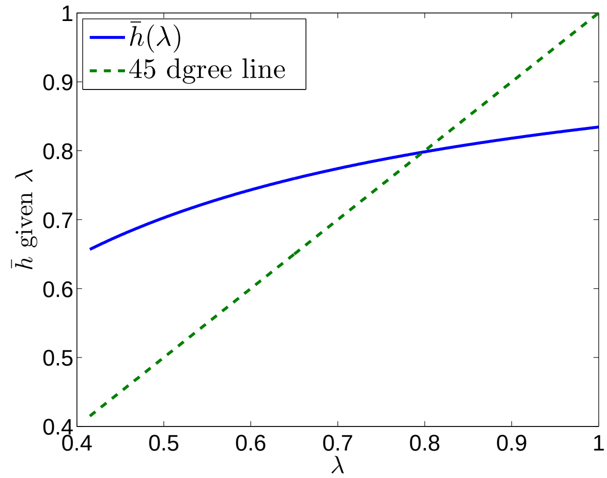

Figure 1: The determination of equilibrium λ given $\phi = 0.5$.

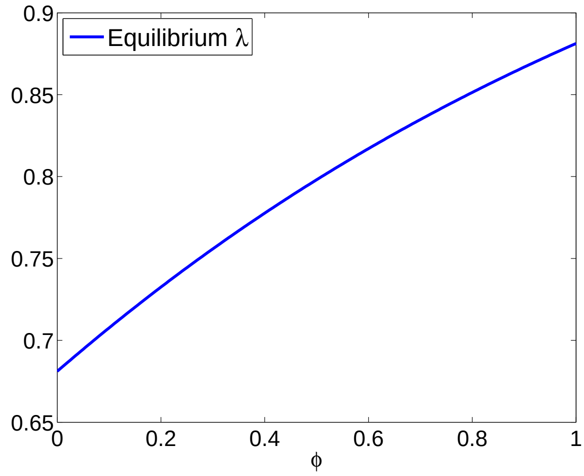

Figure 2: Equilibrium λ at different levels of ϕ.

3.2 A model where ϕ is endogenously chosen

In this section, we extend our model to a case where the central bank is allowed to optimally choose the precision of screening, ϕ. We find this case interesting because "constructive ambiguity" is often discussed to prevent moral hazard associated with the LOLR policy. However, in our model we find that if the central bank can commit to a specific screening precision level at date 0, then it will choose a higher precision level than if it cannot commit at date 0. As a result, fewer banks will invest in the risky asset at date 0, and moral hazard will be reduced. Thus, our model reveals that when the central bank can screen insolvent banks as an LOLR, a clearly specified pre-committed LOLR policy in the first place can actually reduce moral hazard.

Assume that the central bank has a screening technology as follows: to attain a precision of ϕ, the central bank will incur a cost of $aA\phi^2$, where $a > 0$ is a constant. Note that the cost is convex, meaning that the cost for additional precision is higher when the precision increases. Here we assume that $r_{CB} = 0$ for simplicity. Again, we assume that the pooling equilibrium where both types of banks apply for central bank loans exists, and the central bank can always coordinate all the banks toward this equilibrium. The central bank's ex ante expected loss function is defined as follows:

$$\begin{aligned} EL^{ex\ ante} &= \pi\left[(1-\lambda)A(1-p)(R_H - R_M)\right] + (1-\pi) \\ &\quad [(1-\lambda)A[R_H - \phi\gamma_L - (1-\phi)(pR_H + (1-p)R_L)] + aA\phi^2 + \\ &\quad b(1-\phi)(1-p)(1-\lambda)L_{CB}(1 - \frac{AR_L}{D})] \end{aligned} \tag{16}$$

The first term, $\pi\left[(1-\lambda)A(1-p)(R_H - R_M)\right]$, denotes the expected social loss in the absence of a financial crisis. It is caused by banks investing in the risky asset rather than the safe one. In particular, $[(1-\lambda)A(1-p)(R_H - R_M)]$ is the output gap between the safe and risky assets without a financial crisis. The second term denotes the expected social loss when a financial crisis occurs. It consists of three components: (1) the social loss caused by banks investing in the risky asset rather than the safe one. In particular, $(1-\lambda)A[R_H - \phi\gamma_L - (1-\phi)(pR_H + (1-p)R_L)]$ is the output gap between safe and risky assets with a financial crisis. To see it, note that a safe asset always produces a proceed of AR_H. While a risky asset produces a proceed of $A\gamma_L$ with a probability of ϕ when it is identified by the central bank and produces an expected proceed of $A[pR_H + (1-p)R_L]$ with a probability of $1-\phi$ if it is not identified by the central bank. (2) the screening costs, $aA\phi^2$; and (3) the loss from lending to L-type banks, $b(1-\phi)(1-p)(1-\lambda)L_{CB}(1-\frac{AR_L}{D})$. In particular, $b > 0$ is the weight the central bank assigns to the third type of loss. A higher b means that a central bank is more reluctant to use taxpayers' money to finance insolvent banks. $(1-\phi)(1-p)(1-\lambda)L_{CB}(1-\frac{AR_L}{D})$ is the expected losses caused by lending to L-type banks, because they may fail to repay their full debts. The probability that the central bank lends to an L-type bank that fails to repay the loans is $(1-\phi)(1-p)$. There is a proportion of $1-\lambda$ L-type banks. The central bank loan loss for each of them is $L_{CB}(1-\frac{AR_L}{D})$ where $\frac{AR_L}{D}$ is the recovery rate for the defaulted loans. It is obvious that the second component is decreasing in ϕ, and the first and third components are

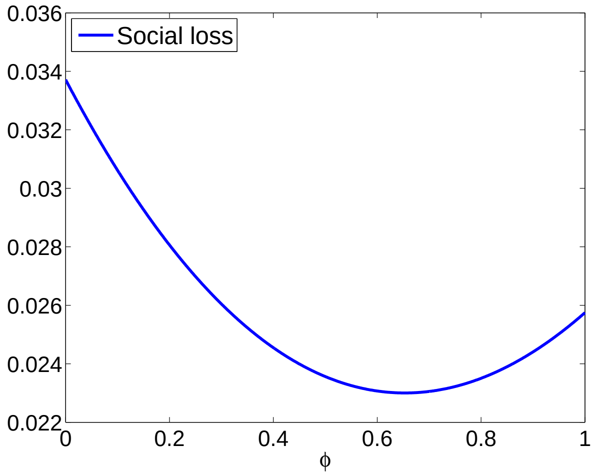

Figure 3: Ex ante social losses given different levels of ϕ.

increasing in ϕ. Note that by assumption, $\gamma_L > pR_H + (1-p)R_L$.

3.2.1 The optimal ϕ under pre-commitment policy

Now suppose the central bank can commit to a specific level of ϕ at date 0. The central bank minimizes its expected social loss by choosing an optimal level of ϕ. Note that since the central bank can commit to a specific level of ϕ, λ is now a function of ϕ that is given by equation (13). The closed-form solution to the central bank's loss minimization problem is not available. Here we give a numerical example with $a = 0.15$ and $b = 0.5$ to illustrate the intuition. The rest parameter values follow the baseline ones. Figure 3 shows the result. The social loss reaches its minimum value of 0.023 when $\phi^* = 0.653$. The resultant equilibrium λ is $\lambda^*_{commit} = 0.8266$. That is, with this pre-committed LOLR policy, 82.66% of banks will choose the safe asset at date 0.

3.2.2 The optimal ϕ under discretionary policy

Now we consider the case where the central bank cannot commit to a specific level of ϕ at date 0, that is, the central bank conducts discretionary policy. Backward induction is used to find the equilibrium. First, at date 1, the central bank will minimize its loss function, taking λ as given. Note that the central bank's ex post loss function is now given by

$$EL^{ex\ post} = (1-\lambda)(1-\phi)A[\gamma_L - pR_H - (1-p)R_L] + aA\phi^2 + b(1-\phi)(1-\lambda)(1-p)L_{CB}(1-\frac{AR_L}{D}) \quad (17)$$

Its first component differs from the one in the commitment case. This is because now the central bank takes λ as given, and the first best allocation occurs when all the L-type banks liquidate at date 1. Thus the central bank's expected output gap between the actual and first best allocation for each L-type bank is $\gamma_L A - \phi\gamma_L A - (1-\phi)(pR_H + (1-p)R_L)A = (1-\phi)A[\gamma_L - pR_H - (1-p)R_L]$.

The first order condition gives us $\phi^* = min\{\frac{1-\lambda}{aA}[\gamma_L A - pAR_H - (1-p)AR_L + b(1-p)L_{CB}(1-\frac{AR_L}{D})], 1\}$.[7] It is straightforward to see that the central bank will choose a higher ϕ if the proportion of L-type banks is large ($1-\lambda$ is high), the screening technology cost is cheap (a is low), the social cost of an unliquidated asset of L-type banks is high ($\gamma_L A - pAR_H - (1-p)AR_L$ is high), and the expected loss from lending to L-type banks is high ($(1-p)L_{CB}(1-\frac{AR_L}{D})$ is high).

Now we move back to date 0. Rational banks will take into account the optimal choice of the central bank at each level of λ and choose between the safe and risky assets. Therefore there exists an equilibrium level for both λ and ϕ, $\lambda^*_{discretion}$ and $\phi^*_{discretion}$ such that given that $\phi = \phi^*_{discretion}$, the trigger strategy $\bar{h} = \lambda^*_{discretion}$ is optimal for all the banks. On the other hand, given that $\lambda = \lambda^*_{discretion}$, the central bank will optimally choose $\phi = \phi^*_{discretion}$ to minimize its ex post loss function.

The closed-form solutions for $\lambda^*_{discretion}$ and $\phi^*_{discretion}$ are not available. Here we give a numerical example with the same parameter values as in the commitment case to illustrate the result. We find the solutions numerically as follows. First, at each given level of ϕ, we find the corresponding equilibrium trigger strategy $\bar{h}$ of banks at date 0. Thus $\bar{h}$ is a function of ϕ that we denote by $\bar{h} = \Gamma(\phi)$. Second, at each given level of $\lambda = \bar{h}$, we find the corresponding optimal ϕ that the central bank chooses. Thus ϕ is a function of $\bar{h}$. The equilibrium occurs at the intersection of the two functions.

Figure 4 shows the equilibrium. $\phi^*_{discretion} = 0.2574$, and the corresponding $\lambda^*_{discretion} = 0.7461$. Recall that under the pre-commitment policy, $\phi^*_{commit} = 0.653$ and $\lambda^*_{commit} = 0.8266$. Then we can tell that both equilibrium ϕ and λ are higher under the pre-

[7] The second order condition is satisfied to guarantee a minimum solution.

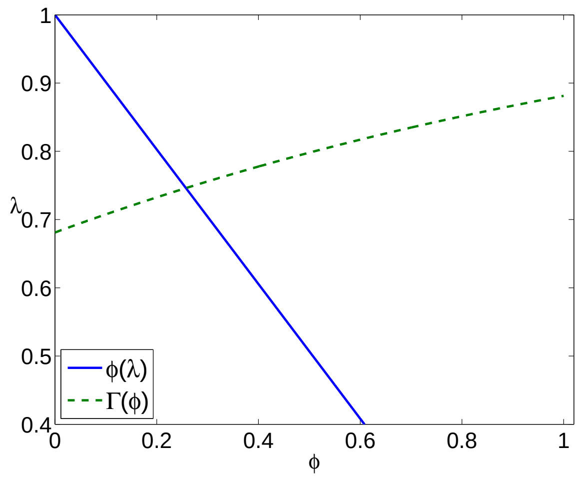

Figure 4: Equilibrium λ and ϕ under the discretionary policy

commitment policy than under the discretionary policy. Moreover, we find that the ex ante social welfare loss defined by equation (16) under the discretionary policy is 0.0268, which is higher than the one of 0.023 under the pre-commitment policy. Thus, we find that if the central bank can commit to an LOLR policy with an optimal screening precision level, it will attain higher social welfare and reduce moral hazard. The reason that the central bank will choose a higher ϕ under the pre-commitment policy is that the central bank has an incentive to use the LOLR policy to affect banks' choices at date 0 if it can commit. If it cannot commit, this incentive disappears. Thus the central bank will choose a higher ϕ to deter more banks from choosing the risky asset at date 0 if it can commit.

4 Conclusions

This paper studies the LOLR policy when insolvent banks have an incentive to gamble for resurrection, and neither the central bank nor the market can distinguish between illiquidity and insolvency. We find that provided that the central bank can screen insolvent banks imperfectly, both the pooling equilibria in which, on one hand, all the banks borrow from the central bank and, on the other hand, all the banks do not borrow from the central bank could exist, conditional on creditors' beliefs off the equilibrium path. However, neither equilibrium is socially efficient because insolvent banks will inefficiently continue to operate. In addition, we find that if we allow banks to choose their assets at the beginning of the model, the precision in central bank screening will greatly affect banks' decisions. An LOLR policy with high precision in central bank screening can greatly improve social welfare not only by singling out insolvent banks and forcing them to liquidate early, but also by deterring banks from choosing risky assets in the first place. Thus, the precision of central bank screening is the key to reduce moral hazard. Finally, we find that if a central bank can commit to a specific precision level before the banks choose their assets, rather than conducting a discretionary LOLR policy, it will choose a higher precision level and attain higher social welfare.

A Proof of condition 5

Let r_M denote the market rate that creditors offer to banks. We know that a bank will never borrow on the market if $1 + r_M > \frac{R_H}{\gamma}$, where $\gamma = \gamma_H$ for H-type banks and $\gamma = \gamma_L$ for L-type banks. This is because with such a high market rate, the bank is always better off by liquidating its own assets (please see appendix B for a rigorous proof). As a result, the highest possible return rate that a creditor can gain from lending to an L-type bank is $1 + r_M = \frac{R_H}{\gamma_L}$, and the highest possible expected return that a creditor can gain from lending to an L-type bank is $p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D}$. By assuming $p\frac{R_H}{\gamma_L} + (1-p)\frac{AR_L}{D} < 1$, creditors' expected return rate from lending to an L-type bank can never exceed 1. Thus they will never lend to an L-type bank. ■

B Proof of proposition 1

We first prove the optimal choices for H-type banks. More specifically, we prove that when $1 + r_M < \frac{R_H}{\gamma_H}$, an H-type bank will always roll over all of its debts by borrowing on the market and will never liquidate its asset. On the other hand, when $1 + r_M > \frac{R_H}{\gamma_H}$, an H-type bank will never borrow on the market and will always liquidate its asset until its debts are repaid.

Suppose that an H-type bank chooses to liquidate l_H of its asset, where $0 \leq l_H \leq A$. When an H-type bank does not borrow from the central bank, its net asset value is given by

$$NV_H = (A - l_H)R_H - (D - \gamma_H l_H)(1 + r_M) \tag{18}$$

When an H-type bank borrows from the central bank, its net asset value is given by

$$NV_H = (A - l_H)R_H - (D - \gamma_H l_H - L_{CB})(1 + r_M) - L_{CB}(1 + r_{CB}) \tag{19}$$

In both cases, the first-order derivative of NV_H with respect to l_H is given by $\frac{\partial NV_H}{\partial l_H} = -R_H + \gamma_H(1+r_M)$. It is straightforward to see that when $1+r_M < \frac{R_H}{\gamma_H}$, $\frac{\partial NV_H}{\partial l_H} < 0$. Thus $l_H^* = 0$. That is, an H-type bank will never liquidate its asset when $1 + r_M < \frac{R_H}{\gamma_H}$. When $1 + r_M > \frac{R_H}{\gamma_H}$, $\frac{\partial NV_H}{\partial l_H} > 0$. Thus $l_H^* = A$. That is, an H-type bank will liquidate all of its asset when $1+r_M > \frac{R_H}{\gamma_H}$. Note that the bank will have to liquidate all of its asset and go bankrupt in this case because we assume $\gamma_H A < D$.

Now we prove the optimal choices for L-type banks. More specifically, we prove that when $1 + r_M < \frac{R_H}{\gamma_L}$, an L-type bank will always roll over all of its debts by borrowing on the market and will never liquidate its asset. On the other hand, when $1 + r_M > \frac{R_H}{\gamma_L}$, an L-type bank will never borrow on the market and will liquidate its asset until its debts are repaid.

In order to prove the above results, we first prove that an L-type bank's net asset value is always negative such that its equity value is always zero in the down state. The proof is as follows. No matter whether an L-type bank borrows from the central bank or not, the maximum payoff that an L-type bank can gain from its asset in the down state is the one when it liquidates all the asset at date 1, $A\gamma_L$. This is because $\gamma_L > R_L$ by assumption. No matter whether an L-type bank borrows from the central bank or not, the minimum repayment for an L-type bank's debt is D when it is charged a zero interest rate. Thus, the maximum net asset value for an L-type bank is $A\gamma_L - D$. However, by assumption, $A\gamma_L < D$. Thus, an L-type bank's net asset value in the down state is always negative. As a result, an L-type bank's equity value in the down state is always zero. This result implies that an L-type bank aims only at maximizing its equity value in the up state.

An L-type bank's net asset value in the up state is the same as that of an H-type bank except that now γ_L replaces γ_H. Thus we prove the results. ■

C Proof of proposition 2

When an H-type bank follows the equilibrium strategy, its equity value is given by

$$\begin{aligned} e_H &= AR_H - L_{CB}(1+r_{CB}) - (D - L_{CB})(1+r_M) \\ &= AR_H - D(1+r_M) + L_{CB}(r_M - r_{CB}) \end{aligned} \tag{20}$$

where r_M is given by equation (7).

If an L-type bank applies for central bank loans, it will be rejected with a probability of ϕ and will be accepted with a probability of $1-\phi$. Using subscripts A and R to denote the cases where the loan application is accepted and rejected, respectively, we have

$$e^u_{L,A} = AR_H - L_{CB}(1+r_{CB}) - (D - L_{CB})(1+r_M) \tag{21}$$

$$e^d_{L,A} = 0 \tag{22}$$

$$e_{L,R} = 0 \tag{23}$$

The equity value of an individual bank deviating to not applying for central bank loans is as follows. An H-type bank will borrow a debt of D at the market rate of $\hat{r}_M$. Thus its equity value is:

$$\hat{e}_{H,NCB} = AR_H - D(1+\hat{r}_M) \tag{24}$$

where $\hat{r}_M$ is given by equation (8).

An L-type bank will also borrow a debt of D at the market rate of $\hat{r}_M$, and its date 2 equity will be

$$\hat{e}^u_{L,NCB} = AR_H - D(1+\hat{r}_M) = \hat{e}_{H,NCB} \tag{25}$$

$$\hat{e}^d_{L,NCB} = 0 \tag{26}$$

An H-type bank's no-deviation condition is $e_H > \hat{e}_{H,NCB}$. We have

$$\begin{aligned} & e_H - \hat{e}_{H,NCB} \\ = \; & [AR_H - D(1+r_M) + L_{CB}(r_M - r_{CB})] - [AR_H - D(1+\hat{r}_M)] \\ = \; & L_{CB}(r_M - r_{CB}) - D(r_M - \hat{r}_M) > 0 \end{aligned}$$

An L-type bank's no-deviation condition is $(1-\phi)pe^u_{L,A} > p\hat{e}^u_{L,NCB}$, or $(1-\phi)e^u_{L,A} > \hat{e}^u_{L,NCB}$. We have

$$\begin{aligned} & (1-\phi)e^u_{L,A} - \hat{e}^u_{L,NCB} \\ = \; & (1-\phi)\left[AR_H - D(1+r_M) + L_{CB}(r_M - r_{CB})\right] - \left[AR_H - D(1+\hat{r}_M)\right] > 0 \end{aligned} \tag{27}$$

which is condition (9). Comparing these two conditions, we can see that $e_H - \hat{e}_{H,NCB} > (1-\phi)e^u_{L,A} - \hat{e}^u_{L,NCB}$ such that an H-type bank's no-deviation condition is easier to hold. The intuition behind this result is that an L-type bank cares only about its equity value in the up state. An H-type bank has a higher equity value from applying for central bank loans than an L-type bank in the up state because it will always be identified as H-type. On the other hand, the equity values of H-type banks and L-type banks in the up state are the same when they deviate.

Note that the LHS of both types of banks' no-deviation condition is strictly increasing in $\hat{r}_M$. Since $\hat{r}_M$ is strictly decreasing in $\hat{\lambda}$, there exists a threshold level of $\hat{\lambda}$, say $\hat{\lambda}_{th,H}$, above which an H-type bank will always deviate. Meanwhile, there exists a threshold level of $\hat{\lambda}$, say $\hat{\lambda}_{th,L}$, above which an L-type bank will always deviate. Since $e_H - \hat{e}_{H,NCB} > (1-\phi)e^u_{L,A} - \hat{e}^u_{L,NCB}$, we have $\hat{\lambda}_{th,H} > \hat{\lambda}_{th,L}$.

An equilibrium exists if and only if both types of banks have no incentive to deviate. Thus we find that a pooling equilibrium in which both types of banks apply for central bank loans exist if and only if $\hat{\lambda} < \hat{\lambda}_{th,L}$. ■

D Proof of proposition 3

First, we find the threshold level of h_i, $\bar{h}$. A bank with a private benefit of $B(\bar{h})$ must be indifferent between investing in safe and risky assets. Its expected date 2 equity value from investing in the safe asset is given by:

$$Ee^s = \pi e^s_{noshock} + (1-\pi)e^s_{shock} \tag{28}$$

Here $e^s_{noshock} = AR_H - D$ denotes the bank's equity value at date 2 in the absence of a crisis. In this case, all the banks will be solvent and roll over their debts at the riskless rate of zero. e^s_{shock} denotes the bank's equity value at date 2 when a crisis occurs. In this case, a proportion $\lambda = \bar{h}$ of banks will be H-type, and a proportion $1-\lambda = 1-\bar{h}$ of banks will be L-type. The equity value at date 2 for the bank to choose the safe asset is given by equation (20), which we replicate here:

$$\begin{aligned} e^s_{shock} &= AR_H - L_{CB}(1+r_{CB}) - (D - L_{CB})(1+r_{M,g_1}) \\ &= AR_H - D(1+r_{M,g_1}) + L_{CB}(r_{M,g_1} - r_{CB}) \end{aligned}$$

where r_{M,g_1} is given by equation (7), which we replicate here:

$$r_{M,g_1} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{g_1 + (1-g_1)p} - 1\right)$$

Here $g_1 = \frac{\lambda}{\lambda+(1-\lambda)(1-\phi)}$ and $\lambda = \bar{h}$. Note that here we focus on the case with no market freeze.

As a result, the bank's expected equity from choosing the safe asset is

$$Ee^s = \pi(AR_H - D) + (1-\pi)(AR_H - D(1+r_{M,g_1}) + L_{CB}(r_{M,g_1} - r_{CB})) \tag{29}$$

If the bank chooses the risky asset, its expected equity value at date 2 is given by:

$$Ee^r = \pi Ee^r_{noshock} + (1-\pi)Ee^r_{shock} \tag{30}$$

Here $Ee^r_{noshock} = p(AR_H - D) + (1-p)(AR_M - D)$ denotes the bank's expected equity value at date 2 in the absence of a crisis. Ee^r_{shock} denotes the bank's expected equity value at date 2 when a crisis occurs. We have

$$Ee^r_{shock} = (1-p) \times 0 + p\phi \times 0 + p(1-\phi)e^u_{L,A} \tag{31}$$

where $e^u_{L,A} = e^s_{shock}$ is given by equation (20). We have the above equation because when a crisis occurs, with a probability of $1-p$, the down state is realized and the bank's equity is zero. With a probability of $p\phi$, the up state is realized and the bank's application is rejected by the central bank. In this case, the bank's equity is zero too. With a probability of $p(1-\phi)$, the up state is realized and the bank's application is accepted by the central bank. In this case, the bank's equity is given by $e^u_{L,A}$.

The bank with $h_i = \bar{h}$ must be indifferent between the two choices. Thus we have

$$Ee^s = Ee^r + (a_0 + a_1\bar{h}) \tag{32}$$

As a result, we have

$$\pi(1-p)A[R_H - R_M] + (1-\pi)\left[1 - p(1-\phi)\right]\left[AR_H - D(1+r_{M,g_1}) + L_{CB}(r_{M,g_1} - r_{CB})\right] = a_0 + a_1\bar{h}$$

Next we prove that this trigger strategy $\bar{h}$ is optimal for each bank. It is straightforward to see that given that each bank follows this trigger strategy, any bank with $h_i > \bar{h}$ will have the LHS of the above equation unchanged, and the RHS higher. Thus it is indeed optimal for it to choose the risky asset. On the other hand, any bank with $h_i < \bar{h}$ will have the LHS unchanged and the RHS lower. Thus it is indeed optimal for it to choose the safe asset. Note that e^s_{shock} is strictly increasing in $\bar{h}$. We assume that the values of a_0 and a_1 are so that a unique solution between 0 and 1 to equation (13) is guaranteed when there is no market freeze. ■

E Proof of the existence of a pooling equilibrium where all the banks do not apply for central bank loans

The market rate when a bank follows the equilibrium strategy, r_M, is given by

$$r^*_M = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\lambda + (1-\lambda)p} - 1\right) \tag{33}$$

When a bank deviates, the market rate depends on creditors' belief off the equilibrium path. Let $\tilde{\lambda}$ denote creditors' belief off the equilibrium path that a bank is H-type when observing it deviate to applying for central bank loans. If a bank's application is rejected, then creditors will know that the bank is L-type, and the bank will not be able to get loans from the market either. If a bank's application is accepted, then creditors' ex post belief that the bank is H-type will become

$$\tilde{g} = \frac{\tilde{\lambda}}{\tilde{\lambda} + (1-\tilde{\lambda})(1-\phi)} > \tilde{\lambda} \tag{34}$$

When $\tilde{\lambda}$ and subsequently $\tilde{g}$ is sufficiently high, the equilibrium market rate, now denoted by $\tilde{r}_{M,g}$, exists and is given by

$$\tilde{r}_{M,g} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\tilde{g} + (1-\tilde{g})p} - 1\right) \tag{35}$$

When $\tilde{\lambda}$ is sufficiently low, or more specifically, when $1+\tilde{r}^*_{M,g} > min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}, \frac{R_H}{\gamma_H}\}$, the market freezes. $\frac{AR_H - L_{CB}(1+r_{CB})}{D - L_{CB}}$ is the maximum interest rate that an H-type bank can pay to the creditors when it deviates to borrowing from the central bank. Thus an equilibrium market rate cannot exceed it. We also proved previously that any $1 + \tilde{r}^*_{M,g} > \frac{R_H}{\gamma_H}$ could not be an equilibrium market rate. In this case, banks will liquidate their assets to repay their debts of $D - L_{CB}$ at date 1. Since $\tilde{r}_{M,g}$ is strictly decreasing in $\tilde{\lambda}$, there exists a level of $\tilde{\lambda}$, $\tilde{\lambda}_{freeze}$, below which $1+\tilde{r}^*_{M,g} > min\{\frac{AR_H - L_{CB}(1+r_{CB})}{D-L_{CB}}, \frac{R_H}{\gamma_H}\}$, and a market freeze occurs.

We first consider the case without market freeze ($\tilde{\lambda} > \tilde{\lambda}_{freeze}$). In this case, the equilibrium market rate, now denoted by $\tilde{r}_{M,g}$, exists. The market rate is given by

$$\tilde{r}_{M,g} = \left(1 - \frac{AR_L}{D}\right)\left(\frac{1}{\tilde{g} + (1-\tilde{g})p} - 1\right) \tag{36}$$

When banks follow the equilibrium strategy and borrow only on the market, two types of banks' payoffs are given by

$$\begin{aligned} e_{H,NCB} &= AR_H - D(1 + r_M) \\ e^u_{L,NCB} &= AR_H - D(1 + r_M) \\ e^d_{L,NCB} &= 0 \end{aligned}$$

Now consider banks' payoffs when they deviate. If an H-type bank deviates, its date 2 equity will be

$$\begin{aligned} \tilde{e}_H &= AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + \tilde{r}_{M,g}) \\ &= AR_H - D(1 + \tilde{r}_{M,g}) + L_{CB}(\tilde{r}_{M,g} - r_{CB}) \end{aligned} \tag{37}$$

If an L-type bank deviates, it will be identified as L-type and gain zero equity with probability ϕ. With probability $1 - \phi$, it can successfully get central bank loans and then borrow on the market at the rate of $\tilde{r}_{M,g}$. Its equity values in different cases are specified as follows.

$$\tilde{e}_{L,Rej} = 0 \tag{38}$$

$$\tilde{e}^u_{L,Acc} = AR_H - L_{CB}(1 + r_{CB}) - (D - L_{CB})(1 + \tilde{r}_{M,g}) = \tilde{e}_H \tag{39}$$

$$\tilde{e}^d_{L,Acc} = 0 \tag{40}$$

An H-type bank's no-deviation condition is $e_{H,NCB} > \tilde{e}_H$, or

$$\begin{aligned} & [AR_H - D(1 + r_M)] - [AR_H - D(1 + \tilde{r}_{M,g}) + L_{CB}(\tilde{r}_{M,g} - r_{CB})] \\ = \; & D(\tilde{r}_{M,g} - r_M) - L_{CB}\left(\tilde{r}_{M,g} - r_{CB}\right) > 0 \end{aligned} \tag{41}$$

An L-type bank's no-deviation condition is $pe^u_{L,NCB} > (1-\phi)p\tilde{e}^u_{L,Acc}$, or

$$[AR_H - D(1+r_M)] - (1-\phi)[AR_H - D(1+\tilde{r}_{M,g}) + L_{CB}(\tilde{r}_{M,g} - r_{CB})] > 0 \quad (42)$$

It is obvious to see that an L-type bank's deviation payoff is lower than an H-type bank. As a result, an H-type bank has a stronger incentive to deviate than an L-type bank.

Next, we consider the case with a market freeze ($\tilde{\lambda} < \tilde{\lambda}_{freeze}$). In this case, banks attain the same payoff when following the equilibrium strategy. However, their payoffs when deviating are different now. A deviating bank faces a market freeze and has to liquidate its assets. For an H-type bank,

$$\gamma_H l_H = D - L_{CB} \Rightarrow l_H = \frac{D - L_{CB}}{\gamma_H} \quad (43)$$

and

$$\begin{aligned} \tilde{e}_{H,freeze} &= (A - l_H)R_H - L_{CB}(1 + r_{CB}) \\ &= AR_H - \frac{D}{\gamma_H}R_H + L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB}) \end{aligned} \quad (44)$$

Similarly, for an L-type bank

$$\gamma_L l_L = D - L_{CB} \Rightarrow l_L = \frac{D - L_{CB}}{\gamma_L} \quad (45)$$

and

$$\tilde{e}^u_{L,freeze} = (A - l_L)R_H - L_{CB}(1 + r_{CB}) \quad (46)$$

$$\tilde{e}^d_{L,freeze} = 0 \quad (47)$$

As a result, provided that there is a market freeze, H-type banks' no-deviation condition is:

$$e_{H,NCB} - \tilde{e}_{H,freeze} = D\left[\frac{R_H}{\gamma_H} - 1 - r_M\right] - L_{CB}(\frac{R_H}{\gamma_H} - 1 - r_{CB}) > 0 \quad (48)$$

If an L-type bank deviates, its expected equity value is given by:

$$\begin{aligned} \tilde{e}_{L,devi} &= 0 \times (1-p) + p(1-\phi)\tilde{e}^u_{L,freeze} + p\phi \times 0 \\ &= p(1-\phi)\tilde{e}^u_{L,freeze} \end{aligned} \quad (49)$$

As a result, its no-deviation condition changes into $pe^u_{L,NCB} > \tilde{e}_{L,devi}$, or

$$e^u_{L,NCB} - (1-\phi)\tilde{e}^u_{L,freeze} = e_{H,NCB} - (1-\phi)\tilde{e}^u_{L,freeze} > 0 \tag{50}$$

Recall that $\tilde{e}^u_{L,freeze} \leq \tilde{e}_{H,freeze}$ because H-type banks have a higher liquidation rate ($\gamma_H \geq \gamma_L$). Meanwhile, $0 < \phi < 1$. As a result, H-type banks have a stricter no-deviation condition. That is, H-type banks are more likely to deviate.

The above analysis reveals that in both cases with and without market freeze, H-type banks have a stricter no-deviation condition. Since an equilibrium exists if and only if both types of banks have no incentive to deviate, in both cases with and without market freeze, the actual no-deviation condition is given by the no-deviation condition of H-type banks.

Note that once condition (41) is satisfied, condition (48) will always be satisfied, because $\frac{R_H}{\gamma_H} - 1 > \tilde{r}_{M,g}$ and $D > L_{CB}$. Thus, a necessary condition for this equilibrium to exist is that when $\tilde{\lambda} < \tilde{\lambda}_{freeze}$, condition (48) is satisfied. If condition (48) is not satisfied, such an equilibrium can never exist.

In sum, when $\tilde{\lambda} > \tilde{\lambda}_{freeze}$, a deviating bank does not face a market freeze. In this case, a pooling equilibrium where neither type of bank applies for central bank loans exists if and only if condition (41) holds.

When $\tilde{\lambda} < \tilde{\lambda}_{freeze}$, a deviating bank faces a market freeze. In this case, this pooling equilibrium exists if and only if condition (48) holds. This pooling equilibrium will never exist if condition (48) does not hold. ■

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