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Abstract

In this paper, we examine the convergence hypothesis using a long memory framework that allows for structural breaks and does not rely on a benchmark country using both univariate and multivariate estimates of the long memory parameter d . Using per capita GDP gaps, we confirm the findings of non-stationarity and long memory behavior that have been found previously in the literature using univariate tests. However, the support for these findings is much weaker when using a multivariate framework, in which case we find more evidence of stationary behavior. Based on these results, we also investigate club formation, something that would suggest the presence of conditional convergence. We describe a club formation methodology using the sequential testing criteria that we have employed in our analysis as the basis for forming clusters or clubs of countries with similar convergence characteristics.

JEL Classification: C32, O47.

Keywords: growth convergence, convergence clubs, long memory.

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1 Introduction

In a recent paper Stengos and Yazgan (2014a)¹ use a long memory analytical framework to examine the convergence hypothesis based on the estimation of d , the parameter that describes the underlying (long-memory) process and determines the speed of convergence of output (GDP per capita) gaps between different economies. The main finding of that paper is that although the long memory framework of analysis is much richer than a simple $I(1)/I(0)$ alternative, which produces a simple absolute divergence and rapid convergence dichotomy, the latter seems to be sufficient to capture the behavior of the gaps in per capita GDP levels and growth rates. The former produces a pattern of divergence whereas the latter produces a pattern of rapid convergence. Overall, it was found that any evidence of mean reversion and long memory was not strong enough, which is in contrast to some previous work in the literature that also uses a long-memory framework to analyze convergence, see Dufrénot et al. (2012). However, all previous research has relied on the univariate estimation of the long-memory parameter d , without accounting for possible correlations among the different output gaps (country differences), an issue that we want to address in this paper by employing a multivariate estimation and testing framework, following Shimotsu (2007). Using the latter methodology and in contrast to the results obtained using univariate estimation, we find evidence of mean reversion and slow (stationary) convergence. This evidence suggests that the overwhelming evidence in favor of divergence found in the literature may be partly explained by the use of methods that do not allow for interdependence among the persistence parameter. The latter acts as a moderating mechanism against divergence, as output pairs that appear to follow a non-stationary trajectory and non-convergence may individually be pulled back towards stationarity and convergence by their dependence on pairs that are stationary and convergent. This observation confirms the usefulness of multivariate long memory methods to address the issue of convergence, as they utilize more information than their univariate counterparts.

Using the results from the above analysis, we proceed to further examine the evidence of long memory type (absolute) convergence that we discovered. We proceed to investigate the possibility of club formation, a factor that would suggest the presence of conditional convergence. In that case, initial conditions would partly determine at least the long-run outcomes, and if

¹see also Stengos and Yazgan (2014b)

countries with similar starting points exhibit similar long-run economic behavior, one could speak of convergence clubs. Club formation has recently become a very active area of research, as there are many different ways in which one can explore their presence (or absence). We will present a methodology on club formation based on the testing criteria that we have followed in our analysis thus far, and we will employ results from graphing theory to provide evidence for the existence of such clubs in our group of countries.

The remainder of the paper is organized as follows. The next section presents the methodology. We then proceed to present the data and results of the different tests that we apply to the per capita output gaps. We then proceed to a more detailed discussion of the methodology that we employ in the convergence club analysis. The final section concludes.

2 Testing framework with long memory.

The simple univariate pair-wise difference between the log of per capita income of country i and j at time t is defined as

$$Z_t = Y_t^i - Y_t^j = \beta(t) + U_t \quad U_t \sim I(d), \quad i = 1, \dots, N, \quad i \neq j, \quad t = 1, \dots, T \quad (1)$$

The process Z_t is described as $(1 - L)^d Z_t = \varepsilon_t$, where L is the lag operator and ε_t is the disturbance term. The fractional integration parameter is given by d under the assumption that the process is invertible ($d > -0.5$). The $\beta(t)$ function is a deterministic function of the time trend t and can be linear, as in $\beta(t) = \beta_0 + \beta_1 t$. Alternately, as in Stengos and Yazgan (2014a), it can be defined in a way that admits structural breaks.

$$\beta(t) = \beta_0 + \beta_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_2 \cos\left(\frac{2\pi kt}{T}\right) \quad (2)$$

This functional form allows for the presence of (smooth) structural breaks. Note here that different values of k will have different implications for the permanent or transitory nature of the breaks. If k is an integer, temporary breaks will result, whereas fractional frequencies would imply permanent breaks because the function would not complete a full oscillation. One advantage of adopting this specification for structural breaks is that it does not require any prior knowledge of

the dates on which those breaks occur. On the contrary, it assumes that breaks happen smoothly instead of abruptly, something that would make their detection more difficult.

In a multivariate setting, the long memory process underlying Equation (1) can be expressed as

$$\begin{pmatrix} (1-L)^{d_1} & & 0 \\ & \ddots & \\ 0 & & (1-L)^{d_q} \end{pmatrix} \begin{pmatrix} Z_{1,t} \\ \\ Z_{q,t} \end{pmatrix} = \begin{pmatrix} \varepsilon_{1,t} \\ \\ \varepsilon_{q,t} \end{pmatrix}, \quad -\frac{1}{2} < d_1, \dots, d_q < \frac{1}{2}, \quad (3)$$

where $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \dots, \varepsilon_{qt})'$ is a covariance stationary process whose spectral density $f_\varepsilon(\omega_j)$ is bounded and bounded away from zero at zero frequency $\omega_j = 0$ (see Shimotsu (2007)). In the multivariate setting, the elements of the q -dimensional vector $\mathbf{Z}_t$ are interdependent and correlated with each other, in contrast with the univariate analysis.²

Following Stengos and Yazgan (2014a), one can distinguish between different convergence cases that are implied by different values of d . We follow that approach, which allows for a much richer classification of convergence types whereby one can distinguish between rapid convergence, stationary convergence and mean reverting non-stationary convergence, where initial differences either decay rapidly and play no role, or linger and have a lasting influence on the present, or fall somewhere in between them.

As in Stengos and Yazgan (2014a), we will concentrate on the estimated values of d and provide tests of convergence based on these estimates. In the next section we will elaborate on the different testing strategies that we will adopt.

2.1 Testing for convergence.

We will consider two types of tests on the estimated ds . The first test is based on the estimation of ds using the multivariate approach illustrated in Equation (3). In this approach, the long memory parameters $\mathbf{d} = (d_1, \dots, d_q)'$ are jointly estimated by the semiparametric estimator of Shimotsu

²As will be clear below in the present application, q is equal to the number of pair-wise differences between the log of per capita incomes of the countries in our data set.

(2007), which uses only Fourier frequencies in the neighborhood of the origin. Let $I_Z(\omega_j)$ denote the periodogram of a series Z_t based on a discrete Fourier transform $W_Z(\omega_j)$ at frequency $\omega_j = \frac{2\pi j}{T}$ for $j = 0, \dots, T-1$, such that $I_Z(\omega_j) = W_Z(\omega_j)W_Z^*(\omega_j)$ with $W_Z^*(\omega_j)$ are the complex conjugate of $W_Z(\omega_j)$, defined as $W_Z(\omega_j) = \frac{1}{\sqrt{2\pi T}} \sum_{t=1}^T Z_t e^{it\omega_j}$. As shown by (Shimotsu, 2007, p. 281), the periodogram $I_Z(\omega_j)$ can be used to define a multivariate estimator of d obtained by minimizing an appropriate likelihood function. This estimator is asymptotically normally distributed with a variance-covariance matrix that is positively related to covariances among the long memory parameters $\mathbf{d} = (d_1, \dots, d_q)'$, i.e., variances are increasing with correlations among $\mathbf{d}$.

The second type of test is based on standard univariate estimation approaches that were used in Stengos and Yazgan (2014a). Based on the estimates of the d s (either multivariate or univariate), we perform the following tests on each of them

Test 1: $H_0^1 : d = 0$ against $H_A^1 : d > 0$ (rapid convergence against long memory convergence)

Test 2: $H_0^2 : d = 0.5$ against $H_A^2 : d < 0.5$ (limit stationary long memory convergence against stationary convergence)

Test 3: $H_0^3 : d = 0.5$ against $H_A^3 : d > 0.5$ (limit stationary long memory convergence against non-stationary mean reverting convergence)

Test 4: $H_0^4 : d = 1$ against $H_A^4 : d < 1$ (non-convergence against non-stationary mean reverting convergence)

Test 5: $H_0^5 : d = 1$ against $H_A^5 : d > 1$ (non-convergence against stochastic divergence)

We calculated critical values as described below and used them to perform the tests. Then, we compare the test results obtained by the multivariate estimator with those obtained with univariate estimators, as in Stengos and Yazgan (2014a). The univariate estimators covered in this paper include the Exact Local Whittle estimator of Shimotsu and Phillips (2005, 2006), Two Stage Feasible Exact Local Whittle of Shimotsu (2010), Fully Extended Local Whittle estimator of Abadir et al. (2007) and prior de-trending versions (see Shimotsu (2010)) of the 2 latter estimators. In all, we make use of four univariate and one multivariate test.

2.2 De-trending for structural breaks

To control for structural breaks, we “de-trend” data by estimating $\beta_0, \beta_1, \beta_2$, and k in Equation (2) with the nonlinear least squares. Then, we subtract $\beta(t)$ function, estimated as such, from the data series U_t , before the estimation of $\mathbf{d}$ and applications of the tests.

2.3 Monte Carlo based critical values.

We conduct Monte Carlo simulations to compute the critical values of the statistic corresponding to each of the above tests under the null hypothesis under consideration. The test statistic is computed as

$$\frac{\sqrt{v}(\hat{d}_q - d_0)}{\sigma(\hat{d}_q)} \quad (4)$$

where v is the bandwidth parameter, d_0 is the value of d under the null hypothesis, $\hat{d}_q$ is the estimate of d , and $\sigma(\hat{d}_q)$ is its standard error defined in (Shimotsu, 2007, p. 283). For other estimates of d using for example the univariate Whittle estimators reported in Stengos and Yazgan (2014a) one applies the correspondingly appropriate standard error variance implied by the method used. For the simulations of the critical values, we consider 50,000 iterations. For each iteration, we generate a series from $U_t = Z_t \sim I(d)$ for different values of d corresponding to the different null hypotheses listed above. In the simulations, we assume that the data is already de-trended. De-trending for structural breaks after estimating the $\beta(t)$ –function avoids the problem of having to rely on specific values of the β –parameters to obtain critical values in the simulations. Hence, the test results will avoid possible misspecification due to the reliance on “incorrect” β parameter values³ In other words, we do not rely on a specific $\beta(t)$ –function with particular parametric values of the β – parameters to obtain the critical values of the various test statistics. As mentioned above, we de-trend the data by estimating $\beta_0, \beta_1, \beta_2$, and k in Equation (2) using the non-linear least square method.

In Table 1 we provide critical values at the 5 and 10 percent significance levels for $T = 100, 200$, and 500, along with those of the univariate Whittle estimators reported in Stengos and Yazgan

³Ashley and Patterson (2010) suggest isolating and separately examining both a local mean (i.e., a non-linear trend or the realization of a stochastic trend) and its deviations as a modelling strategy that would complement the estimation of a fractionally integrated model.

(2014a). These critical values are then used in the empirical analysis that follows.

Table 1

3 Data.

We update our Maddison data set that was used in Stengos and Yazgan (2014a), and also include data from the Penn World Tables (PWT) in our analysis as an additional source. The Maddison data consist of annual GDP per capita data covering the period from 1950 to 2010 for 141 countries⁴ and PWT data of annual GDP per capita for the period from 1950 to 2011 for 74 countries. The country coverage of both data sets are illustrated in Table 2.⁵ Hence our sample corresponds to $T = 60$ and $N = 141$ for Maddison and to $T = 61$ and $N = 74$ for PWT.

We first investigate the convergence of GDP per capita for all of the 141 and 74 countries taken together as a group and then separately as belonging to different groups from different continents (the Middle East and Central Asia, Europe, AsiaPacific, Sub-Saharan Africa and the Western Hemisphere, and for developing countries taken separately as a single group)⁶. These groups of countries are listed in Table 2 below.

Table 2

In addition to these geographical groups, we will also consider other categories based on levels of economic development, such as emerging markets, the Group of Seven (G7) and the OECD. Emerging markets are grouped according to both FTSE and S& P classifications. Moreover, we also use groupings based on data availability. Countries whose data are available from 1830, 1850, 1860, 1900, and 1930 onwards are taken as a group in the Maddison data set. These country groups are presented in Table 3 below.

Table 3

⁴The data come from the Maddison Project (Bolt and van Zanden, 2013). Some countries are missing observations at the end of the period in the final two years. The data are available at <http://www.ggdgc.net/maddison/maddison-project/home.htm>, and they include all possible countries available.

⁵The PWT data come from Feenstra et al. (2013). We use PPP converted GDP per Capita G-K Methods in USD Dollars. Some countries have some missing observations at the beginning of the period, and 53 of these 74 countries are missing no observations. The remaining countries have some missing observations, but no country is missing more than 9. The data are available at <http://www.rug.nl/research/ggdgc/data/penn-world-table>

⁶This classification is based on the usual classification made by the International Monetary Fund's regional economic outlook documents.

4 Empirical Findings.

Following Pesaran (2007), to analyze output per capita convergence across 141 and 74 countries, we apply the five tests discussed above (each corresponding to a convergence classification) to all possible pairs of $Z_t = Y_t^i - Y_t^j, i = 1, 2, \dots, N - 1$, and $j = i + 1, 2, \dots, N$ in a sequential manner. Hence, we examine all $N(N - 1)/2 = 9,870$ and $N(N - 1)/2 = 2,701$ output gaps for Maddison and PWT data sets, respectively. For each test, to obtain evidence on the specific type of convergence of which the rejection (non-rejection) of the test indicates, we expect the fraction of output gap pairs for which the null hypothesis is rejected to be greater (smaller) than the size of the test applied to the individual output gap pairs. For example, if one tests for mean reverting convergence by using Test 4 above for a group of N countries, Pesaran's approach requires all $N(N - 1)/2$ pairs are subjected to unit root testing and a fraction of rejection among them to exceed the size of tests.

In particular, Pesaran (2007) showed that, if a group of N countries are non-convergent, the rejection rate of the null hypothesis of unit root ($H_0 : Z_t \sim I(d = 1)$) calculated by $N(N - 1)/2$ tests is equal to the nominal size of the individual tests, i.e. the probability of Type 1 error. More specifically, it is shown that under the null hypothesis of N countries being non-convergent, the rejection rate of individual tests converges to the nominal size, α as N and $T \rightarrow \infty$, even though individual tests are not independent cross-sectionally. Similarly, any weak correlation among the different pairs over time would not matter asymptotically as in the case of approach in (Shimotsu, 2007, p. 283). Thus, in order to reject convergence of N countries, it is enough to show that the proportion of rejections over $N(N - 1)/2$ tests is larger than the significance level of individual tests. In that case for example, if the significance level is 5%, the proportion of rejections must exceed 0.05⁷. Even though this approximation may not yield an accurate estimate of the true size of the test, reporting the rejection rates would give us an idea of the evidence in favour or against the particular test in question when comparing the different estimators of d used. To summarize, rejection rates higher than a given significance level in a given application would imply evidence against the convergence hypothesis. On the other hand, rejection rates lower or

⁷Clearly in small samples there will be significant size distortions and in applications, the power of the tests used relative to size distortions should be given attention. The above results rest on asymptotic arguments that the rejection rate would converge to α in the limit and that of course would not be the case if N and T are relatively small in a given application.

close to the employed significance level will provide evidence for the non-rejection (validity) of the convergence hypothesis. Hence, in Table 4 below, rejection frequencies that greatly exceed a nominal size of 0.05 would be taken as evidence against the null. Conversely, rejection frequencies that are below the nominal size value will be taken as evidence in favor of the null.

The five tests are applied in sequential order in the sense that we continue to apply them until we find evidence in favour of some type of convergence, if there is any. The column denoted by ALL of Tables 4 and 5 summarizes the results of the four tests applied to all 9,870 (Maddison) and 2,701 (PWT) GDP per capita gap pairs at the 5 significance level based on critical values computed for $T = 100$. The table shows the rejection frequencies of the five tests defined above that are obtained from de-trended series using multivariate and univariate estimators, as described above.

Table 4 and Table 5

As shown in Table 4, all of the test results belonging to Test 1 report strong rejection of the null hypothesis of rapid convergence against the alternative of long memory. The evidence from Test 2, however, suggests that all of the tests calculated based on multivariate and univariate estimators find evidence in favor of the null hypothesis of a limit stationary long memory process. Test 3 registers very high rejection rates that conclusively indicate evidence in favor of limit stationary convergence and non-convergence. The fact that the rejection rates of Test 4, obtained from univariate estimators, are slightly above the 5 percent significance level, constitutes weak evidence in favor of the alternative hypothesis of mean reverting non-stationary convergence, given possible size distortions due to the sequential nature of our testing procedure (although the different tests are assumed to be independent, there may still be size distortions).⁸ However, the results associated with multivariate estimator of Shimotsu (2007) conclusively indicate mean reverting convergence with a much larger rejection rate. These results also hold for all of the grouping varieties considered in Table 4.

The evidence presented in Table 5, obtained from the smaller dataset of PWT, generally confirms the results obtained from the Maddison dataset, although some evidence on stationary convergence is visible when Test 2 and multivariate estimators are considered for the Western

⁸The results obtained in Stengos and Yazgan (2014a) are similar but weaker with slightly smaller rejection ratios.

hemisphere in particular. The test based on multivariate estimators also shows that the European and Middle-East and Asian countries also display weak evidence of stationary convergence.⁹

We present the group results in Table 6 for the data from the Maddison data set only. Although the country groups whose data are available from 1830 and 1850 onwards provide evidence on stationary convergence, for the remaining groups, the evidence is somewhat weaker.

Table 6

The evidence is contrary to the previous findings in Stengos and Yazgan (2014a), where using only univariate statistics provided strong evidence in favor of (absolute) non-convergence, which is also confirmed here but not for the multivariate statistic. The difference in the evidence found using the latter as opposed to the former can be explained by the fact that correlations among the estimates of the d 's result in the standardized multivariate test statistics to account for the interdependence among the different pairs. In that case, the variances of high d 's are mitigated by the presence of negative covariances with other pairs that result in smaller variances overall for the test statistics. Hence, pairs that appear to suggest non-stationary behavior and non-convergence on their own may be pulled back towards stationarity and convergence by their dependence on pairs that are stationary and convergent. In that case, the test statistics of the multivariate test may take "larger" absolute values on average than their univariate counterparts. This evidence in favor of the convergence hypothesis is all the more remarkable in that it is obtained without relying on a benchmark country and allowing for the presence of structural breaks. To summarize, contrary to previous evidence, such as in Stengos and Yazgan (2014a) and Dufrénot et al. (2012), which relied on univariate statistics, using a multivariate approach to estimate the long memory coefficients results in evidence that points towards a mean reverting process for per capita output gaps. These results hold for all different groups of countries, the Middle East and Central Asia, G7, S&P, FTSE, and OECD groups. For Europe and two small country groups whose data dated back to 1830 and 1850, considerable evidence on stationary convergence is present. For Asia and the Pacific, the Western Hemisphere and for three relatively small groups of countries whose data is available from 1860, 1900 and 1930, the evidence on stationary convergence is present but weaker, leaving mean reverting convergence as a second possibility.

⁹Although we stopped the sequence of testing at Test 2, relying on this evidence for the Western hemisphere, we continued for the remaining two groups to guard against possible size distortions in these tests.

It is worth noting that as in Stengos and Yazgan (2014a), the results are based on pair wise comparisons for all possible pairs within a group, as opposed to relying on a benchmark or group leader, as in Dufrénot et al. (2012). Using a benchmark results in differences in output gaps are to be expected, whereas these differences are smoothed out if gaps are only constructed as a difference of individual countries from the leader in the group. This is certainly true for the univariate tests, all of which point towards a long memory non-stationary behavior in the transitional dynamics of the output gaps. The evidence from the multivariate test, however, points towards mean reverting convergence irrespective of the absence of a benchmark leader country due to the greater interdependence between the different pairs captured by this test, which was ignored entirely by its univariate counterparts. We will now proceed to further analyze the evidence found above by exploring the possibility of conditional convergence and club formation.

5 Convergence Clubs: A Maximal Clique Method

The above analysis largely implies that the dominant form convergence is of a non-stationary mean reverting nature, which seems to hold unconditionally for all countries. However, the analysis also indicates that stationary convergence, another stronger form of convergence, is also present in smaller group of countries forming convergence clubs. These results indicate the presence of conditional convergence because the differences among some groups of countries show high persistence that can only be corrected in the very long run, indicating the presence of cross-country structural heterogeneity. If initial conditions determine, at least partly, long-run outcomes, and countries with similar initial conditions exhibit similar long-run outcomes, then one can speak of convergence clubs (Durlauf et al. (2005)).

The issues of the definition of convergence clubs and their clustering have been widely discussed in the economic growth literature where the evidence on convergence clubs is usually provided on the basis of the convergence of various a-priori defined homogeneous country groups, which were assumed to share the same initial conditions.¹⁰

¹⁰Baumol (1986) for example grouped countries with respect to political regimes (OECD membership, command economies and middle income countries), Chatterji (1992) allowed for clustering that based on initial income per capita levels and tested convergence cross-sectionally, while Durlauf and Johnson (1995) grouped countries using a regression tree method based on different variables such as initial income levels and literacy rates that determined the different "nodes" of the regression tree that defined the country clubs with the common initial conditions and literacy characteristics. Hausmann et al. (2005), by considering a priori grouping criteria such as initial incomes, found some evidence on convergence clubs by using time series methods.

In this sense, the above pair-wise method can also be used for testing convergence among a-priorily determined a group of N countries. In fact, Pesaran (2007) also considered different initial set of countries based on geographic characteristics for his pairwise method, but found no evidence on convergence clubs. However, Phillips and Sul (2007) developed an algorithm that classifies groups endogenously rather than using a-priori criteria. Similar to Phillips and Sul (2007), in this section, we also attempt to develop an endogenous clustering algorithm to determine the formation of convergence clubs using the pair-wise framework. As stated by Pesaran, "in principle, the convergence results from the analysis of pair-wise output gaps can be used to form "convergence clubs", but special care must be taken in addressing the specification search bias that such a strategy would entail." (Pesaran, 2007, p. 314).

Our approach attempts to pin down the convergence clubs for each type of convergence considered above. Among all pair-wise test results, we search for sets of countries that would yield the desired test result when subjected to the given test. For example, to obtain rapidly converging clubs, we search for sets of countries that would provide a rejection rate below 5 percent for Test 1 considered above. In other words, sets indicating non-rejection of the null of $H_0^1 : d = 0$ form rapidly converging clubs. To begin this search, we use all the pair-wise test results obtained from the all-country analysis. This problem can be solved by using an algorithm designed to find the maximal complete subgraph, or the maximal clique in graph theory terminology.

Consider the above four tests in terms of their implications for convergence types in terms of our sequential procedure. While non-rejection of the null of $H_0^1 : d = 0$ implies unconditionally rapid convergence, the non-rejection of $H_0^3 : d = 0.5$ implies the possibility of mean reverting convergence, provided that the non-rejection of the null of Test 2 has already been obtained in favor of limit stationarity. However, the rejection of the null of Test 2 and 4 provides evidence in favor of the alternatives, $H_A^2 : d < 0.5$ and $H_A^4 : d < 1$ implying stationary and mean reverting convergence. The maximal clique method that we present in this subsection, combines the maximal clique algorithm of graph theory with the previously described pairwise convergence tests of $H_0 : Z_t \sim I(d)$. Rather than testing a priori grouped country clusters, the method explores all convergent groups in a list of N countries that was previously subjected to pairwise convergence tests. In this sense, the method is an endogenous extension of Pesaran (2007) similar to the one by Hobijn and Franses (2000)

The method consists of two steps. First, all possible pairwise differences of N countries, included in the initial set of countries, are subjected to one of the tests outlined above. If the rejection rate obtained from $N(N-1)/2$ tests falls below (above) the significance level, that would be evidence in favor of the convergence type implied the alternative (null) hypothesis and the list of N countries will be taken to form a convergence group. If this club involves all examined countries, then all countries are said to be convergent and we do not go any further in seeking out the presence of convergence clubs. However, as shown above, and as in Pesaran (2007), Dufrénot et al. (2012) and Stengos and Yazgan (2014a) it is very unlikely to examine all countries as a group and find evidence for a stronger form of convergence for all. Nevertheless, if a subgroup of countries is found convergent via pairwise method, then it can be said that this subgroup constitutes a convergence club. The main challenge, as indicated above, is to find a method to determine this subgroup rather than relying on a-priori classifications. In the second step we undertake this task.

Suppose $\mathcal{W}$ denotes the set of all countries. Hence by definition, the cardinality of $\mathcal{W}$ is equal to N ; mathematically if $\#()$ denotes the cardinality, we have $\#(\mathcal{W}) = N$. Further, suppose that $\mathcal{E}$ is a subset of $\mathcal{W}$. In this case, in order $\mathcal{E}$ to be a convergence club, all (or a certain percentage determined by the significance level) binary combinations obtained with elements of $\mathcal{E}$ should satisfy the pairwise tests. Hence, since $\#(\mathcal{E}) = M < N$ the rejection rate obtained via all $M(M-1)/2$ pairs should fall below (or above) the significance level. Therefore in the second step, from the $N(N-1)/2$ test results, the objective is to find a class of subsets $\mathcal{G}$ for which all subsets, e.g. $\mathcal{E}$ satisfy pairwise convergence property. Let $\mathcal{G}$ denotes the class of all subsets satisfying the desired pairwise property. Then,

$$\mathcal{G} := \{\mathcal{E} : \text{for } \gamma \text{ (or less) percent } i, j \in \mathcal{E}, i \neq j, t(Z^{ij}) = 1\}$$

where $Z^{ij} = Y_i - Y_j$, $t(\cdot)$ is the test result of the series in the bracelet and takes the value of 1 for convergent pair, i, j and 0 otherwise. γ is the desired rejection rate.¹¹

¹¹The description above represents the type of convergence by the null hypothesis. In that case, γ denotes the acceptable number of proportions of rejections, under the null. Asymptotically γ should be equal to the significance level applied to all individual tests. Similarly, one could express the type of convergence by the validity of the alternative. In that case, a high proportions of rejections of the null would correspond as evidence against the null in favour of the alternative, depending on the power of the test employed with γ asymptotically approaching unity. However, below we will adopt somewhat different γ levels, taking into considerations all possible size distortions as well as weaknesses in the power of the tests under consideration.

Hence, the problem can be expressed as

$$\arg \max_{\mathcal{G}} \{ \#\mathcal{E} : \mathcal{E} \in \mathcal{G}. \}$$

In graph theory terms, countries become vertices, the test result (rejecting or not rejecting pairs) of a pair become edges, and as such the set of all vertices and edges constitutes an undirected graph. If an undirected graph has edges between all vertices then the graph is said to be complete. If there is a subset of an undirected graph having all properties of a complete graph, the subset is so called a clique. Therefore, in our case, all convergence clubs of a country list can be expressed as cliques. Solving the problem defined above is known as finding maximal cliques.

Pairwise test results form an undirected graph and accordingly, countries and test results determine the vertices and edges respectively. Hence, the problem becomes to find a subgraph that has edges between each vertices, or in other words, a maximal clique. Figures 1 and 2 present a graphical presentation of the concepts introduced above.

Finding a maximal clique can be too hard from a computational point of view. The computational complexity of solution to maximal clique problem is known as NP-Complete whose brute-force solution requires $2^N - \binom{N}{2} - N - 1$ trials. First, Bron and Kerbosch (1973) developed an algorithm to solve the problem in exponential time. In the recent literature, various planar graph algorithms have been developed that enables the problem to be solved in polynomial time. In this study, we will employ the branch and bound algorithm proposed by Konc and Janezic (2007) which is an improved branch-and-bound algorithm that ends in polynomial time.

We should note that, the maximal clique method is not a conclusive technique. In other words, it does not cluster the country list into subgroups, but finds club(s) having a maximum number of elements. Hence we offer the following clustering algorithm to detect convergence clubs.

1. Apply the desired test to all Z_{ij} such that $i, j \in \mathcal{W}$ and $i \neq j$
2. Test the null hypothesis. If the desired convergence type embedded in the null (alternative) hypothesis, the resulting variable takes the value of 0 (1) or 0 otherwise.
3. Construct adjacency matrix from the resulting variable values obtained in (2).

4. Find maximal clique(s) from the adjacency matrix via the algorithm proposed by Konc and Janezic (2007). If more than one clique is detected, proceed to the next step. If only one clique is detected jump to (6)
5. Select one of the detected cliques randomly and proceed to the next step.
6. The group of countries in the clique is labeled as a convergence club. Eliminate respective rows and columns of the countries from the adjacency matrix. And step back to (5). Stop if all the rows and columns are eliminated from adjacency matrix.

To illustrate our approach, we applied our procedure to the results we have already obtained. Because the approach requires a considerably large amount of computational time, we restrict our universe $\mathcal{W}$ to a pre-selected group. Also note that as will be clear below, this does not place any restrictions on the main message presented below. The pre-selected groups are illustrated in Table 7, which presents the results of the search for different type of convergences.

Table 7

In Table 7, in the search for convergence clubs, when the type of convergence depends on the non-rejection of the null hypothesis (rapid convergence), we took a 10 percent rejection rate as the benchmark (we hereby attempt to compensate for possible over-rejection displayed by the different tests that we use). In other words, any club producing a 10 percent rejection rate or less is taken as evidence for the validity of the null of rapid convergence. However, we set a 50 percent benchmark rejection rate for the cases where the type of convergence depends on the rejection of the null hypothesis (stationary and mean reverting convergence), accounting for all the possible size distortions due to the sequential nature of our testing procedure.¹²

For the case of rapid convergence, there is no evidence for convergence clubs because the set of 7 and 6 convergent pairs in Europe or in the G7 extended by the group of emerging countries (represented by S&P) are not able to form a convergence club with at least three countries. For the same country groups, 2 convergence pairs are able to form a convergent club of 3 in the case

¹²Because the significance level of individual tests is adopted as 5 %, in principle, these benchmarks should be equal asymptotically to 5 % and 100 % respectively, if the tests suffer from no size distortions and are consistent. As mentioned above, taking into account the effect of all possible size distortions on the Type I error and allowing for weak test power we made the choices of 10% and 50% respectively. Clearly, these choices are arbitrary and need to be further investigated within the context of a full analysis of the clustering method. We leave this issue for further research.

of stationary convergence. However, as shown in Table 7, there are many convergence clubs with different numbers of countries for the case of mean reverting convergence. Note also that these clubs are not required to be disjoint sets. The maximum size of these clubs is 12, and there are 6 different convergent clubs, each containing 12 countries. Figure 3 below illustrates one of these clubs with its associated universe.

Figure 3

We repeat the same analysis using the country groups based data availability as universes in which to search for convergence clubs. The results are illustrated in Table 8. We observe a similar pattern here. While there is little evidence for convergence clubs in rapid convergence, there is ample evidence for convergent clubs in the case of stationary convergence. We also provide a sample from rapidly converging clubs in Figure 4.

Table 8 and Figure 4

6 Conclusions.

In this paper, we use a long memory framework of analysis that does not rely on a benchmark country but allows for the presence of structural breaks to estimate the time series properties of output gaps for counties in the post-World War II period and, as such, provide evidence on the convergence hypothesis. The focus of the paper is first the estimation of d , the parameter that determines the speed of convergence between different economies. We estimate d using a multivariate estimator from Shimotsu (2007) and a number of other univariate methods found in recent studies. The main finding of our paper is that for per capita GDP gaps, the parameter d takes values in the range from $0.5 < d < 1$ (mean reverting convergence), and the range from $0 < d < 0.5$ (stationary convergence) seems be a possibility, especially for some country groups when using the multivariate approach.

The difference between univariate and multivariate procedures in these findings can be explained by the fact that correlations among the estimates of the d 's in the multivariate test statistics account for the interdependence among the different pairs. In that case, any divergent behavior is mitigated by the presence of convergent pairs that act as stabilizing factors for the group as a whole.

Using the results from the above analysis, we proceed to further examine the evidence we found for long memory type (absolute) convergence. We then investigate the possibility of club formation, something that would suggest the presence of conditional convergence and offer a club formation methodology based on maximal cliques in conjunction with sequential testing criteria. An important issue in this context that we plan to explore in future research is the examination of the properties of the proposed maximal clique club formation method. This would make for a very fruitful area of new research as to the best of our knowledge the properties of club formation mechanisms have not been explored yet in the literature. This research will shed light on issues regarding possible mis-classifications and incorrect club membership which need to be addressed in order to guide practitioners who will be applying these methods to explore convergence and club formation.

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Tables

Table 1: Empirical critical values of Test1, 2, 3, and 4 for $T = 100, 200$, and 500 .

	Test 1			Test 2			Test 3			Test 4			Test 5		
CV	95%			05%			95%			05%			95%		
T	100	200	500	100	200	500	100	200	500	100	200	500	100	200	500
FELW	2.207	1.990	1.812	-2.216	-2.076	-1.930	2.254	2.071	2.084	-2.253	-2.153	-2.000	2.180	1.974	1.798
FELWd	1.945	1.765	1.624	-3.079	-2.668	-2.355	2.149	1.939	1.938	-2.647	-2.235	-2.042	2.175	1.975	1.795
2FELW	2.206	1.990	1.812	-2.216	-2.076	-1.930	2.253	2.018	1.795	-2.312	-2.153	-2.000	2.180	1.974	1.798
2FELWd	1.944	1.765	1.624	-3.079	-2.668	-2.354	2.153	1.878	1.670	-2.534	-2.235	-2.042	2.175	1.975	1.795
MLW	2.02	1.985	1.924	-2.879	-2.268	-2.082	2.013	1.912	1.785	-2.267	-2.125	-1.942	2.004	1.854	1.536

Notes: FELW: Fully Extended Local Whittle, 2FELW:2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out assuming $v = T^{0.6}$ for all Whittle estimators.

Table 2: Countries and group of countries belonging to Maddison and PWT datasets.

	Maddison and PWT	Only PWT	Only Maddison
Middle-East and Central Asia	Egypt, Iran, Islamic Republic of, Jordan, Morocco, Pakistan		Afghanistan, Bahrain, Iraq, Kuwait, Lebanon, Oman, Qatar, Saudi Arabia, Syrian Arab Republic (Syria), United Arab Emirates, Yemen, Palestinian Territory, Occupied, Algeria, Djibouti, Libya, Mauritania, Somalia, Sudan, Tunisia
Europe	Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Sweden, Turkey	Cyprus, Iceland, Luxembourg, Malta	Albania, Bulgaria, Czechoslovakia, Hungary, Poland, Romania, Yugoslavia, Croatia, Macedonia, Republic of, Slovenia
Asia and Pacific	Australia, Bangladesh, China, India, Japan, Korea, Republic of, Sri Lanka, Malaysia, New Zealand, Philippines, Thailand, Taiwan, Republic of China		Indonesia, Myanmar, Hong Kong, Special Administrative Region of China, Nepal, Singapore, Cambodia, Lao PDR, Mongolia, Korea, Democratic People's Republic of, Viet Nam
Sub-Saharan Africa	Benin, Burkina Faso, Congo, Democratic Republic of the, Ethiopia, Ghana, Guinea, Kenya, Mauritius, Malawi, Nigeria, Uganda, South Africa, Zambia	Zimbabwe	Angola, Botswana, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo (Brazzaville), Côte d'Ivoire, Equatorial Guinea, Gabon, Gambia, Guinea-Bissau, Lesotho, Liberia, Madagascar, Mali, Mozambique, Namibia, Niger, Rwanda, Sao Tome and Principe, Senegal, Seychelles, Sierra Leone, Swaziland, Tanzania, United Republic of, Togo
Western Hemisphere	Argentina, Bolivia, Brazil, Canada, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, Guatemala, Honduras, Jamaica, Mexico, Panama, Peru, Paraguay, El Salvador, Trinidad and Tobago, Uruguay, United States of America, Venezuela (Bolivarian Republic of)		Cuba, Haiti, Nicaragua, Puerto Rico

Table 3: Country Groups based on Economic Characteristics and Data Availability

1830	Italy, Sweden, UK, USA, Denmark, France, Netherlands, Norway, Australia
1850	1830 + Belgium, Germany, Greece, Spain
1860	1850 + Finland, Switzerland
1900	1860 + Austria, Portugal, New Zealand, Canada, Brazil, Chile, Colombia, Peru, Uruguay, Venezuela, Japan, Sri Lanka, Argentina, Mexico, Ecuador, India
1930	1900 + Ireland, Turkey, Costa Rica, Guatemala, South Africa
G7	France, Germany, Italy, UK, Canada, USA, Japan
FTSE	Hungary, Poland, Brazil, Mexico, Thailand, Taiwan, Malaysia, Turkey, South Africa
S&P	Brazil, Hungary, Poland, Chile, Colombia, Mexico, Peru, China, India, Philippines, Thailand, Taiwan, Malaysia, Turkey, Egypt, Morocco, South Africa
OECD	Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, UK, Ireland, Greece, Portugal, Spain, Australia, New Zealand, Canada, USA, Hungary, Poland, Chile, Mexico, Japan, South Korea, Singapore, Israel, Turkey

Table 4: Rejection frequencies of Test 1, Test 2, Test 3, and Test 4 for Maddison Data.

		ALL	EUR	WHE	MEA	AAP	SSA	G7	S&P	FTSE	OECD
Test 1	FELW	0.998	1.000	0.997	0.996	1.000	0.997	1.000	1.000	1.000	1.000
	FELWd	0.998	1.000	0.997	0.996	1.000	0.998	1.000	1.000	1.000	1.000
	2FELW	0.998	1.000	0.997	0.996	1.000	0.997	1.000	1.000	1.000	1.000
	2FELWd	0.998	1.000	0.997	0.996	1.000	0.998	1.000	1.000	1.000	1.000
	MLW	0.996	1.000	1.000	1.000	0.978	0.998	1.000	1.000	1.000	1.000
Test 2	FELW	0.015	0.011	0.007	0.018	0.039	0.017	0.000	0.000	0.000	0.045
	FELWd	0.014	0.011	0.007	0.014	0.035	0.014	0.000	0.000	0.000	0.045
	2FELW	0.014	0.011	0.013	0.014	0.039	0.016	0.000	0.000	0.000	0.045
	2FELWd	0.013	0.011	0.013	0.011	0.035	0.013	0.000	0.000	0.000	0.045
	MLW	0.034	0.032	0.007	0.014	0.056	0.020	0.000	0.000	0.000	0.000
Test 3	FELW	0.974	0.981	0.993	0.971	0.944	0.973	0.857	1.000	1.000	0.955
	FELWd	0.974	0.981	0.993	0.967	0.944	0.974	0.857	1.000	1.000	0.955
	2FELW	0.974	0.979	0.987	0.975	0.944	0.973	0.857	1.000	1.000	0.955
	2FELWd	0.975	0.979	0.987	0.975	0.944	0.974	0.857	1.000	1.000	0.955
	MLW	0.903	0.921	0.940	0.899	0.905	0.950	1.000	0.938	0.875	1.000
Test 4	FELW	0.140	0.119	0.100	0.174	0.121	0.121	0.286	0.188	0.375	0.273
	FELWd	0.132	0.108	0.090	0.174	0.117	0.114	0.286	0.188	0.375	0.227
	2FELW	0.140	0.122	0.097	0.174	0.117	0.121	0.286	0.188	0.375	0.273
	2FELWd	0.135	0.116	0.093	0.174	0.117	0.115	0.286	0.188	0.375	0.227
	MLW	0.473	0.418	0.503	0.576	0.468	0.362	0.286	0.625	0.750	0.409

Notes: The abbreviations used in the table are as follows: ALL (All countries), AAP (Asian and Pacific countries), MEA (Middle-East and Asian countries), EUR (European countries), SSA (Sub-Saharan countries), WHE (Western-hemisphere countries), G7: Group of 7 countries, OECD: OECD countries, FTSE: Financial Times emerging market country group, S&P: Standart and Poors country group, FELW: Fully Extended Local Whittle, 2FELW: 2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out by assuming $v = T^{0.6}$ for all Whittle estimators..

Table 5: Rejection frequencies of Tests 1, 2, 3, and 4 for PWT data.

		ALL	EUR	WHE	MEA	AAP	SSA	G7	S&P	FTSE	OECD
Test 1	FELW	0.995	0.996	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000
	FELWd	0.995	0.996	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000
	2FELW	0.996	0.996	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000
	2FELWd	0.996	0.996	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000
	MLW	0.973	0.926	1.000	0.970	1.000	1.000	1.000	1.000	1.000	1.000
Test 2	FELW	0.024	0.035	0.000	0.061	0.022	0.019	0.000	0.000	0.000	0.000
	FELWd	0.024	0.035	0.000	0.045	0.022	0.014	0.000	0.000	0.000	0.000
	2FELW	0.023	0.035	0.000	0.061	0.022	0.019	0.000	0.000	0.000	0.000
	2FELWd	0.023	0.035	0.000	0.045	0.022	0.014	0.000	0.000	0.000	0.000
	MLW	0.076	0.108	0.200	0.106	0.011	0.052	0.000	0.000	0.000	0.000
Test 3	FELW	0.937	0.931	-	0.864	0.945	0.919	1.000	1.000	1.000	1.000
	FELWd	0.941	0.939	-	0.864	0.934	0.924	1.000	1.000	1.000	1.000
	2FELW	0.940	0.935	-	0.864	0.945	0.933	1.000	1.000	1.000	1.000
	2FELWd	0.942	0.939	-	0.864	0.934	0.933	1.000	1.000	1.000	1.000
	MLW	0.807	0.814	-	0.758	0.868	0.810	0.800	0.667	0.750	0.810
Test 4	FELW	0.216	0.216	-	0.288	0.187	0.195	0.000	0.250	0.250	0.190
	FELWd	0.207	0.212	-	0.258	0.176	0.176	0.000	0.250	0.250	0.190
	2FELW	0.212	0.203	-	0.273	0.187	0.186	0.000	0.333	0.500	0.238
	2FELWd	0.209	0.203	-	0.258	0.187	0.181	0.000	0.333	0.500	0.238
	MLW	0.721	0.714	-	0.773	0.868	0.733	0.800	0.750	0.750	0.810

Notes: The abbreviations used in the table are as follows: ALL (All countries), AAP (Asian and Pacific countries), MEA (Middle-East and Asian countries), EUR (European countries), SSA (Sub-Saharan countries), WHE (Western-hemisphere countries), G7: Group of 7 countries, OECD: OECD countries, FTSE: Financial Times emerging market country group, S&P: Standart and Poors country group, FELW: Fully Extended Local Whittle, 2FELW: 2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out by assuming $\nu = T^{0.6}$ for all Whittle estimators.

Table 6: Rejection frequencies of Tests 1 and 2 for group of countries having available data since some selected years between 1830 and 1930 according to Maddison's data.

		1830	1850	1860	1900	1930
Test 1	FELW	0.916	0.846	0.876	0.940	0.946
	FELWd	0.972	0.846	0.876	0.942	0.949
	2FELW	1.000	0.846	0.876	0.940	0.946
	2FELWd	1.000	0.846	0.876	0.942	0.950
	MLW	1.000	0.987	0.952	0.953	0.954
Test 2	FELW	0.222	0.372	0.210	0.120	0.127
	FELWd	0.222	0.372	0.181	0.110	0.121
	2FELW	0.111	0.308	0.210	0.120	0.135
	2FELWd	0.111	0.308	0.181	0.110	0.124
	MLW	0.222	0.294	0.133	0.140	0.141

Notes: FELW: Fully Extended Local Whittle, 2FELW:2-Stage Feasible Exact Local Whittle estimator, 2FELWd: 2-Stage Feasible Exact Local Whittle estimator with detrending, FELWd: Fully Extended Local Whittle with detrending; MLW: Multivariate Local Whittle Estimator. Simulations are carried out by assuming $v = T^{0.6}$ for all Whittle estimators.

Table 7: Convergence Clubs

Test Type	Country Groups			Club Sizes											
				# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10	# 11	# 12	
Rapid Conv. ($H_0^1 : d = 0$)	G7	+	Europe	1	-	-	-	-	-	-	-	-	-	-	
	Europe	+	Emerging	7	-	-	-	-	-	-	-	-	-	-	
	G7	+	Emerging	6	-	-	-	-	-	-	-	-	-	-	
Stationary Conv. ($H_A^2 : d < 0.5$)	G7	+	Europe	-	-	-	-	-	-	-	-	-	-	-	
	Europe	+	Emerging	2	1	-	-	-	-	-	-	-	-	-	
	G7	+	Emerging	2	1	-	-	-	-	-	-	-	-	-	
Mean Reverting Conv. ($H_A^4 : d < 1$)	G7	+	Europe	38	726	3246	3737	4577	6185	3639	731	67	-	-	
	Europe	+	Emerging	96	2971	18474	25220	41378	84168	72565	24080	6925	1397	6	
	G7	+	Emerging	66	310	1739	3353	2533	2586	3265	1605	243	23	-	

Table 8: Convergence Clubs

Club Type	Years	Club Sizes			
		# 2	# 3	# 4	# 5
Rapid Conv.	1830	-	-	-	-
	1850	12	2	-	-
	1860	14	2	-	-
	1900	29	3	-	-
	1930	37	3	-	-
Stationary Conv.	1830	-	-	-	-
	1850	12	23	49	25
	1860	14	30	72	28
	1900	29	90	381	208
	1930	38	139	649	299

Figure 1: A sample undirected graph

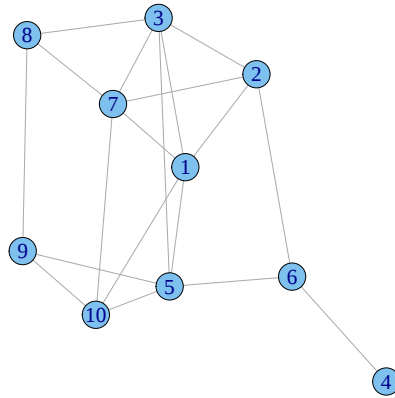


Figure 2: A sample maximum clique

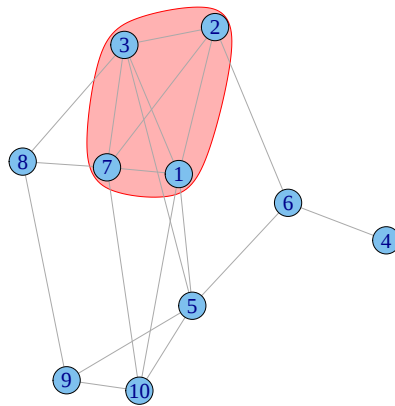


Figure 3: A Club of Mean Reverting Convergence (Europe + Emerging Markets)

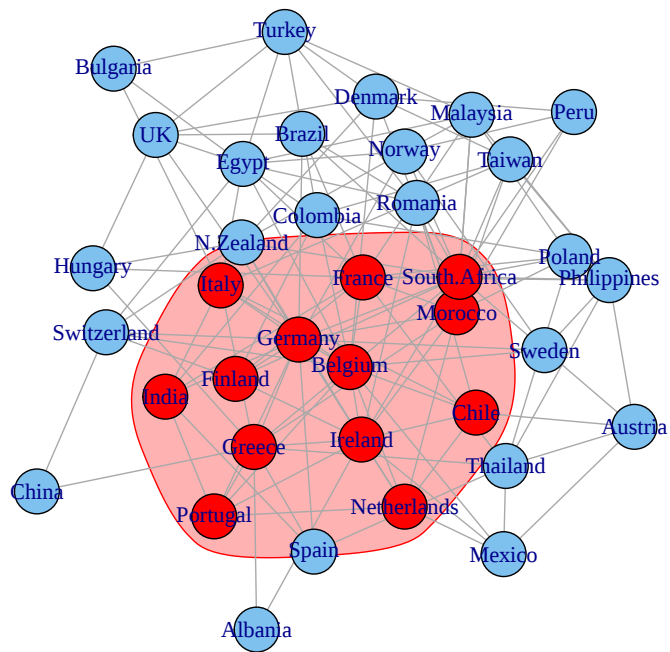


Figure 4: A Rapidly Converging Club (1930)

