Department of Economics and Finance

University of Guelph

Discussion Paper 2017-04

The Impact of Lead Time on Capital Investments

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June 5, 2017

Abstract

We study equilibrium investment strategies of firms competing in stochastic dynamic market settings and facing two types of investment structures: investment with significant lead time (or time-to-build) and investment without (or minor) lead time. We investigate how investment behavior changes when investment is subject to time-to-build versus when it is not. We characterize equilibrium investment strategies under several information structures and compare results to the social optimum. We offer some new results. The model predicts that, controlling for demand, and production and investment costs, investments and outputs can be higher in progressive industries (which often exhibit time-to-build) than in fast-paced industries (where time-to-build is insignificant). Furthermore, for both investment types (investment with or without time-to-build) we offer a novel equilibrium in which firms incrementally invest. This behavior is driven by demand uncertainty and capacity constraints. Also, expected outputs are lower than Cournot outputs as firms face uncertainty. Moreover, the amount of uncertainty has different effects over investment types.

Key Words: Capacity Investment; Capacity Constraints; Progressive Industry; Fast-paced Industry; Demand uncertainty; Time-to-build; Markov perfect Equilibrium; Open-loop Equilibrium.

JEL Codes: C73; D92; E22; G31.

1 Introduction

Some capital investment projects can be completed with alternative production technologies. First, consider a market where firms run technologies which are highly capital intensive and firm invest-

*I thank the editor Herbert Dawid, associate editor and two referees for constructive remarks and suggestions.
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ments take time to be productive, that is, there is a lead time between investment and production. In other words, investment is subject to a time-to-build constraint. Next, consider another market in which firms operate production technologies which exhibit lower or no time-to-build constraint (relative to the former market). We coin investment in this market as instantaneous investment, because firms can make their investments productive in a short time period or without a significant lag. An example for the former structure is a power market which is heavily based on nuclear or hydroelectric generators (e.g., Norway's electricity market is based on hydropower, and it is nuclear power in France). If an entrepreneur or a firm chooses to invest in this market, he/she realizes that investment can take years to be productive. For instance, the largest hydroelectric dam was constructed in China, and it took about 20 years to be completed, from construction to fully operational stages. On the other hand, if he/she decides to invest in the latter market in which main production technologies are, say, wind, solar and natural gas-fired generators (e.g., Denmark power market is predominantly wind-based supported by natural-gas generators), the investment can be productive "instantly" or with a short delay (relative to the former structure), as these production technologies can be purchased in any size from the energy technology suppliers and added to the production line. For example, a 5 MW capacity wind farm can be operational in a month from investment decision to production. To give different examples, consider cell phone industry in which firms (e.g., Samsung, Apple, and Sony) introduce their new models at least once a year. However, car producers such as Toyota, Honda, GM, and Chrysler introduce their new versions (generations) of models in about 7 years. Similarly, for passenger plane makers such as Boeing, Airbus, or Bombardier it takes several years to develop a new model airplane, and then they make lengthy flight tests before they start carrying passengers. In these examples, while car and plane sectors could be viewed as industries with significant lead times, the cell phone producers could be considered operating in an industry with a minor (or insignificant) lead time. We intend to compare the market outcomes given the investment opportunities in these two distinct markets. Specifically, we intend to address the following questions: in which market structure do firms perform better? Should an incumbent firm attempt to earn higher profits by investing in its own market, or in a new market? More generally, what are the effects of addition of time-to-build to capital investment competition?

Capacity investments which are the main subject of this paper are worth billions of dollars every year. World energy investment outlook report of International Energy Agency (IEA) (2003, 2014) has questioned capacity investments that are required in the power sector. It finds that in the OECD countries power sectors need to invest \$4 trillion between 2000 and 2030, and almost half of this is needed for power generation investment. Indeed, power generation capacity investments are projected to be 2000 GW between 1999 and 2030 in the OECD. According to this IEA report insufficient capacity investments caused market failures: price crises in New Zealand (during 4/2001-7/2001, and 4/2003-6/2003), Australia (in 1/2002-3/2002), Nord Pool (in 12/2002-03/2003), and Ontario, Canada (in 7/2002-7/2003) were attributed to delay in capacity investments, and the tight production capacities which could not catch the growth in electricity demand (IEA, 2003, Table 1, p. 26). As a result, some governments resorted to intervene into power markets to resolve the price crises by directly investing in peaking plants, or encouraging consumers to respond to the prices, or capping retail prices.[1] In relation to capital investments, the Canadian electricity sector is expected to invest about $300 billion from 2010 to 2030 to meet demand growth and maintain/refurbish existing plants, according to the Canada's electricity infrastructure report (2011). About $200 billion will be invested in generation and the rest in transmission and distribution sectors. Large scale investments are not unique to the energy sector. For example, in the automotive industry China invested over $12 billion US in 2013 to increase production capacities, while worldwide investment was nearly $18 billion. To produce more vehicles automotive assembly capacities have been increased by either building new plants or expanding existing plants (cbc.ca/news/canada/windsor/).

To shed light on capital investment issues, we initially start with a two-period imperfect competition model. There are two non-identical firms who face demand uncertainty before making their investment decisions. Firm production is constrained by capacity and capacity accumulation is endogenous. Under a time-to-build constraint, investment will be available to use in the following period. In the first period firms compete for outputs. At the same time, they make investment decisions given that demand is uncertain in the next period. After uncertainty unfolds in the second period, firms choose production quantities. Under no time-to-build constraint (i.e., instantaneous investment), firms can invest and produce simultaneously and non-cooperatively. In this structure, firms can invest in the initial and final periods while firms can only invest at the outset under the former structure (time-to-build) due to the lag between production and investment. The model incorporates uncertainty and production capacity constraints into a dynamic game-theoretic setting, as well as different information structures (i.e., equilibrium concepts) and investment types. These features are the key ingredients of our modeling aspect and add realism to the capital accumulation

[1] In a recent report published by International Energy Agency (IEA, 2016), available at https://www.iea.org/publications/freepublications/publication/REPOWERINGMARKETS.pdf in Figure 1.2, we observe that electricity markets have been restructured in most jurisdictions in the world, with different degrees of competition being introduced both in wholesale and retail sectors. See also Genc (2012) for restructuring issues.

competition. We also extend the model to three periods which will allow firms to invest consecutively. The number of possible equilibria will increase with the addition of new time stage and the equilibrium investment strategies will become an involved function of model parameters.

We compare equilibrium investment strategies under two information structures: Markov perfect and open-loop approaches. This is to compare strategic to preemptive investments. While these information structures and hence the equilibrium concepts have been commonly used in deterministic dynamic games literature (Reynolds, 1987, Cellini and Lambertini, 1998, Long et al., 1999, Dockner et al., 2000 and Figuières, 2009), we will compare their predictions in market settings under uncertainty. Each one has its own advantages in computing or characterizing equilibrium decisions. Also, this comparison is not only suitable as a technical exercise, but also becomes an interesting question for mechanism and auction designers. Furthermore, Markov perfect solution may not be feasible for some oligopolistic games under uncertainty due to curse of dimensionality, as argued in Genc et al. (2007) and Genc and Sen (2008). However, they have shown that open-loop solution for stochastic dynamic games with constraints can be found by formulating the game as a stochastic programming problem. For such large-scale games, while Markov perfect solution is out-of-reach, open-loop solution is attainable and tractable.

There is a vast literature that has examined capital investments in different optimization and market settings. They mainly utilize time-to-build constraint for general equilibrium or corporate investment models. Articles studying capacity investment games under uncertainty remain sparse. Situations involving time-to-build have rarely been at the core of such studies. Our contribution is to examine and compare market performances (prices, profits, outputs) under different investment structures (time-to-build versus no time-to-build) over various equilibrium concepts (open-loop, Markov perfect, and social optimum) in dynamic game settings under demand uncertainty to show the role of production constraints and the number of periods on investment behavior.

Some of our new findings are as follows. Controlling for demand, and production and investment costs, we determine the conditions under which investments and outputs are higher in progressive industries and the conditions under which they are higher in fast-paced industries. Also, for both investment types (investment with or without time-to-build) we offer a novel equilibrium in which firms incrementally invest. This behavior is driven by demand uncertainty and capacity constraints. Moreover, expected outputs are lower than Cournot outputs as firms face uncertainty. In addition, the amount of uncertainty has different effects on investment types.

We find that equilibrium investment strategies are multiple under both investment structures.

However, the number of equilibrium strategies is higher under instantaneous investment structure than under the time-to-build. Also, with instantaneous investment firms may prefer delaying their investments in the initial period when uncertainty increases, and then boost investment in the second period. With time-to-build, investment decisions will be made only once and before the realization of uncertain demand. Also the equilibrium investment per firm increases in demand uncertainty variable. Moreover, production capacity constraints impact investment decisions. If a firm is subject to a time-to-build constraint and undertakes investment, then its capacity constraint will always be binding in the high demand state and production equals total capacity. If a firm does not face a time lag, then it never invests when demand is "low". However, in the case of high demand realization its investment will be fully utilized and its production constraint will bind. Finally, based on the information structure (Markov perfect versus open-loop) we find that equilibrium investment predictions differ as long as firms are asymmetric in terms of capacity constraints. In particular, if one firm's capacity constraint is binding and other firm's is not then Markov perfect investment will always be higher than the open-loop investment no matter what type of investment is executed; otherwise they will yield the same investment predictions.

2 Literature Review

A common assumption in most of the investment literature is that investment at a given time is productive in the same period, that is, there is no lag between production and investment. An alternative assumption is that investment takes time to be part of capital stock. In reality, investment does not become productive instantly for many industries and there is some lead time between investment and production. As noted by Koeva (2000) time-to-build constraint is empirically observable, and it varies from industry to industry and is on average in the range of 13 to 86 months.[2] However, most studies have not paid attention to this aspect and assumed instantaneous investment in strategic capital acquisition. A justification for a no time-to-build constraint could be that firms might have already overinvested and some of their production capacities might be staying idle. Using the existing idle capacity with minor maintenance and improvement may be considered as instantaneous "investment", and hence no time-to-build constraint. Or, it could be that capacity (such as land, machinery, equipment, or buildings) can be purchased from an intermediate market

[2] While time-to-build varies over industries, the length of time-to-build is often variable within an industry and can create additional source of uncertainty in decision making process (e.g., construction of Areva?s powerful nuclear reactors called the EPR, Evolutionary Power Reactor, as pointed by a referee).

and put into production process instantly (or in a short period of time).

Most papers in the capital accumulation literature (e.g., Spence (1979), Dixit (1980), Reynolds (1987), Cellini and Lambertini (1998), Dockner et al. (2000) and Figuières (2009), among others) assumed market settings with no uncertainty and no lag between production and investment (i.e., instantaneous investment). They studied capacity investment games mainly within the linear-quadratic framework and under various assumptions including finite-time, infinite-time, and pre-commitment versus no commitment. These papers offered valuable insights on investment behavior. However, there can be many factors that create significant lag between the investment decision and the start of the production process. To our knowledge, only a few papers in game-theoretic literature (e.g., Grenadier (2000, 2002), Pacheco-de-Almeida and Zemsky (2003), Genc et al. (2007), Garcia and Shen (2010), Garcia and Stacchetti (2011), and Genc and Zaccour (2013)) implemented time-to-build constraint. They examined perfect competition and oligopoly models to characterize equilibrium investments under various information structures.

In terms of explaining the effect of time-to-build on equilibrium behavior, Pacheco-de-Almeida and Zemsky (2003) is closer to the current paper. While they do not compare the total investments and market outcomes with time-to-build, which is parameterized and is between zero and infinity, to the ones without time-to-build, we compare the market performance and investment dynamics with and without time-to-build. They mainly focus on equilibrium characterization with time-to-build and explain how the duration of time-to-build impacts market outcomes. The main conclusion of Pacheco-de-Almeida and Zemsky (2003) is that social welfare is mostly decreasing in time-to-build. However, this result can reverse when one takes into account for the effect on equilibrium existence. This is indeed similar to our finding. However, while they explain their result in terms of the duration of time-to-build, we explain our result in terms of the number of investment opportunities and initial capacity endowments. Granted that the market structure in Pacheco-de-Almeida and Zemsky (2003) is fundamentally different than our paper: they solve an infinite horizon game in which there is one time uncertainty, and assume that there is either demand or no demand for the product, and firms are symmetric and do not have any initial productive capacities, and the length of time-to-build could vary. We assume that firms always face uncertainty before they make investment decisions, there is always demand for the product, and demand will increase with some probability to justify the capital expenditure (i.e., investment). Furthermore, in our model firm productions are bounded by production capacity and firms are allowed to have non-identical capacities. The critical assumptions of their model which create some different (equilibrium) results than ours are that *i*) in their model firms can invest after the resolution of uncertainty, however in our paper firms always make investments under uncertainty; *ii*) while they solve infinite horizon game with one time uncertainty which is resolved at time 0 after which there is a known and fixed demand for a product, we solve two- and three-period models with multiple uncertainties on demand growth; *iii*) in their model the amount of uncertainty determines which equilibrium exists, in our model capacity endowments mainly determine the equilibrium type.

In terms of modeling aspect, a closely related paper to this research is Garcia and Shen (2010), who developed a dynamic duopoly model with a stochastically growing demand. Their production and investment costs are identical to our paper. Both papers (Garcia and Shen and this paper) study investment with time-to-build under uncertainty. The uncertainty modeling is the same. Demand either goes up or stays the stay with a known probability distribution function. While, Garcia and Shen do not intend to explore the effect of time-to-build, which is a modeling assumption in their paper, we explore the impact of investment lag by comparing investment outcomes with and without time-to-build. While they offer investment characterization for a given period, they do not examine multiple investment strategies and how an investment in a given period would impact the investment strategy in the following period. We offer equilibrium investment strategies made over time (in the three-period model for investment with time-to-build, and in the two-period model for instantaneous investment). Furthermore, while we specifically examine the impact of demand uncertainty on investment strategies with and without time-to-build, Garcia and Shen do not mention the effect of uncertainty. Both papers characterize the social optimum investment profiles, and find that firms with market power underinvest relative to social optimum. Also, our investment strategy characterization proposed (in Propositions 1 and 2 in section 4) for investment with time-to-build corresponds to their main result in Theorem 1. However, we also offer a different equilibrium result in our Proposition 11 (in the case of three periods) in which we characterize consecutive investment strategies in three periods. Consequently, Garcia and Shen and this current research are in similar spirit for Markov perfect equilibrium (MPE) characterization of the duopolistic competition for investment with time-to-build. The key results (such as MPE investment strategy characterization and the result that the duopolistic market fails to induce the socially optimal level of capacity) in Garcia and Shen are also available in this current paper. However, our main differences arise when i) comparing market performance with and without time-to-build; ii) extending the duopoly game to three periods to examine incremental investment dynamics; iii) including open-loop equilibrium concept and showing market outcome equivalence (and nonequivalence) between open-loop and

Markov perfect solutions.

Chevalier-Roignant et al. (2011) provide an overview of strategic investment under uncertainty. They note that under uncertainty different types of investment strategies may lead to different market equilibrium outcomes. In our paper, we also show in a finite horizon discrete time stochastic game setting that investment strategies (and market outcomes) under different equilibrium concepts (open-loop and Markov perfect) may result in different investment rules. Swinney, et al. (2011) consider the timing of capacity investment decisions in monopolistic and duopolistic market settings under demand uncertainty.

While Grenadier (2000) adds time-to-build constraint into his perfectly competitive market model, Grenadier (2002) recognizes the difficulty of characterizing and computing state dependent investment strategies in imperfect competition. Grenadier (2002) studies equilibrium investment strategies of firms in a continuous time infinite horizon Cournot-Nash competition framework by using a real options approach. However, we explicitly consider strategic interactions in imperfectly competitive setting without reducing state-space to figure out subgame perfect equilibrium investment strategies with and without time-to-build. Related to capacity planning Bar Ilan et al. (2002) solve a firm's impulse control problem, assuming that the firm does not respond to changing demand conditions (nor to other firms' reactions). Note that these papers are fundamentally different than the current paper in terms of modeling and research focus.

Aid et al. (2015) employ a competitive model (a social planner problem) with time-to-build under three different assumptions of demand intercept uncertainty. They find that the impact of volatility on optimal investment could be negligible when time-to-build is present. However, in our market structure, which is imperfectly competitive market with a simple demand distribution, a firm's strategic investments with time-to-build increase in demand uncertainty variable.

Besides the game-theoretic capacity investment analyses, there is a stream of literature examining lumpy capacity investments using real options framework (e.g., Li and Wang (2010), Chen (2012), and Hahn and Kuhn (2012)). Even though time-to-build constraint is omitted in most of the game-theoretic models, it has been employed in growth literature emphasizing optimal corporate investment decisions. The corporate investment literature mainly incorporates real options view of investment and the standard net present value rule. To explain business cycles, some optimal growth models have incorporated time-to-build. Kydland and Prescott (1982) and the papers following them (such as, Majd and Pindyck (1987), Zhou (2000)) suggested that a time-to-build constraint better describes the cyclical fluctuations than do standard cost of adjustment models. Del Boca et al. (2008) generalized the Tobin's Q model of investment to accommodate multiple capital types (structure and equipment) and time-to-build.

The research in this paper is different than the previous research in several aspects. The key difference is the characterization of equilibrium investment strategies in the imperfectly competitive market with and without lead time. We also characterize and compare capacity investment strategies under two equilibrium concepts (open-loop and Markov perfect) in a dynamic game to pinpoint the strategic value of investment in a simple model which incorporates demand uncertainty and capacity constraints simultaneously. Compared to prior work, we offer some new results under time-to-build constraints.

3 Model

We examine a duopolistic non-cooperative competition model of irreversible investment under demand uncertainty. In this capital/capacity accumulation game we analyze investment behavior and compare the market outcomes in the presence of the two distinct investment structures: i) instantaneous investment which may be viewed as purchasing capacity from the competitive capacity market and making this investment productive instantly; ii) investment with lead time under which firms may choose to build their own capacities or extend the existing ones that take time to be productive. We assume that firms produce a homogeneous output (e.g., electricity).

We characterize investment strategies using Markov perfect (and open-loop Nash equilibrium presented in the extension section) solution concept(s). Both types of equilibria have been commonly used to study investment profiles of firms in non-cooperative competition settings without uncertainty. These concepts generally lead to different market predictions and investment profiles, and hence we will compare and contrast the market outcomes under both frameworks.

For any market setting time evolves discretely and state variables (demand and capacity states) have a continuous support. For a given demand distribution and capacity state vectors, each firm may prefer to increase its production capacity before demand uncertainty unfolds. To keep the model manageable, we start with a two-period game $t \in \{0, 1\}$, where there are two possible demand states at time 1 (in theoretical framework two-stage models are common and it is assumed that they provide a good approximation for investment projects). We will extend it to three periods, in which there will be four additional demand states at time 2, to track the evolution of investment strategies.

Following Garcia and Shen (2010) the inverse market demand is $P_t(Q_t) = \bar{p} - \sigma_t Q_t$, which is the price that consumers face for the total consumption quantity Q_t, where the slope term σ_t has the following distribution.

$$\sigma_{t+1} = \begin{cases} \sigma_t/(1+g) & \text{with probability } \theta \\ \sigma_t & \text{with probability } (1-\theta) \end{cases} \tag{1}$$

At the initial period ($t = 0$) assume $\sigma_0 = 1$, without loss of generality. The demand has a variable slope and shifts pivotally around the maximum willingness to pay price $\bar{p}$. Demand either increases with a growth rate $g > 0$ into the high demand state (called "upstate") or stays the same (called "intermediate state"). We essentially take demand growth into account so that firms have an incentive to invest. These demand scenarios intrinsically represent growth and stagnation states in the economy. Another relevant demand scenario could be a downstate in which demand would go down and correspond to a recession state; however for technical reasons we omit this state as the number of periods would increase the inclusion of downstate could theoretically cause negative price.

For simplicity, the production of q units of homogeneous product costs $C(q) = cq$ with $c \geq 0$. In general firms could have different marginal costs and the analysis will hold even if $c = 0$, as can be seen from the expressions (2) and (4) below. Similar to Pacheco-de-Almeida and Zemsky (2003) and Garcia and Shen (2010), investment in production capacity costs $F(I) = fI$ for investment level $I \in \Re_+$, where $f \geq 0$. It is common to assume away a fixed cost component in the investment cost function, as the fixed cost creates non-convexity and hence the possibility of lumpy investments. There is no innovation so that capacity expansion does not impact the marginal production cost. The endogenous capacity states evolve as follows. In the instantaneous investment case, the production capacity at time $t+1$ will be equal to the production capacity at time t plus the investment made at the time $t+1$, that is $K_{t+1} = K_t + I_{t+1}$. For the investment with time-to-build, the production capacity at time $t+1$ is $K_{t+1} = K_t + I_t$ in which the investment made at time t will be available for production at time $t+1$. We assume away capacity depreciation as there are only a few periods in the game. To include capacity depreciation into the model K_t should be replaced by $(1-\tau)K_t$, where τ would be depreciation rate between 0 and 1. We will examine the impact of each investment type on market outcomes.

There are two firms i and j, and $i \neq j$, and each firm is risk-neutral and maximizes its expected sum of profits independently of its rival. We assume an ongoing competition so that each firm has K_{k0} units of production capacity, $k = i, j$, and has the option of increasing its capacity through investment I_{kt}. Investment is irreversible and perfectly divisible. The model can easily be extended to include more than two firms, if we would assume identical firms in terms of initial capacities, as can be seen from the expressions (4) and (5) below.

It will be useful to present the results in connection with Cournot equilibrium (interior solution) outputs. For firm k let q^c_{ku} and q^c_{kd} denote interior outputs in period 1 (u denotes upstate and d denotes intermediate) demand states when no capacity constraints bind, and q^c_{k0} be the corresponding quantity in period 0;

$$q^c_{k0} \equiv \frac{\bar{p}-c}{3}, \qquad q^c_{kd} \equiv \frac{\bar{p}-c}{3}, \qquad q^c_{ku} \equiv \frac{(\bar{p}-c)(1+g)}{3}. \tag{2}$$

4 Investment with Lead Time

Although businesses often prefer a shorter lead time, which is the required amount of time between starting and finishing a process (including pre-processing, processing and post processing), it is an important constraint in supply chain management.

This section examines investment behavior when there is one period lead time between investment and production. Firms i and j start with initial capacities $K_{i0} \geq 0$ and $K_{j0} \geq 0$ which can be different.

Producer i maximizes its expected sum of profits to choose its investment and output strategy denoted $\psi^i = (I_{it}, q_{it})$ and solves

$$\max_{\psi^i} E_0 \sum_{t \in \{0,u,d\}} [P_t(Q)q_{it} - cq_{it} - fI_{it}] \tag{3}$$

subject to

$$0 \leq q_{it} \leq K_{it},$$

$$K_{it+1} = K_{it} + I_{it}, \qquad I_{it} \geq 0.$$

, where E_0 is the expectation operator at time zero. There are two states at time 1, namely upstate (u) and intermediate state (d).[3] Because there are two periods, we ignore a discount rate, which can be easily embedded into the above expected profit function, does not change our results qualitatively. It will enter the equilibrium investment function linearly. Our first result describes firms' investment policies under Markov perfect solution.

[3]Instead of writing $1u$ ($1d$) to represent the upstate (downstate) in period 1, we simply write u (d).

Proposition 1: For firms $k = i,\ j,\ i \neq j$ let $K_{i0}, K_{j0} \geq 0$ be their initial capacities at $t = 0$. Assume that investment exhibits one period lead time and there are two demand states at $t = 1$, namely u and d. Then the Markov perfect equilibrium investment strategies at $t = 0$ are:

$$I_{k0} = \begin{cases} 0 & \text{if } q^c_{ku} \leq K_{k0} \\ \dfrac{(1+g)(\bar{p} - c - f/\theta)}{3} - K_{k0} & \text{if } q^c_{kd} \leq K_{k0} < q^c_{ku} \\ \dfrac{(1+g)(\bar{p} - c - f)}{3(1+g-\theta g)} - K_{k0} & \text{if } 0 \leq K_{k0} < q^c_{kd} \end{cases} \tag{4}$$

Proof: See the Appendix.

Observe that investment decisions of the firms change with respect to the model parameters and initial capacities. Firm investments would be identical if their initial capacities were the same. This, in turn, implies that there is no asymmetric equilibrium investment strategy profile as long as firms start with identical initial capacities. Note that in a similar capital accumulation model Garcia and Shen (2010) assumed symmetric firms and symmetric Markov perfect equilibrium. Under these assumptions our investment policies, when both firms invest, boil down to their main result in Theorem 1.[4]

In Pacheco-de-Almeida and Zemsky (2003) time-to-build creates “initial price premium”. The price premium is the ratio of difference of initial price and final price to final price. Firms initially produce from ex ante investment which is lower than Cournot output and therefore the initial price is high. Ex post investments come online and price declines. On the other hand, in our setting all investments with time-to-build are made under uncertainty. However, the initial price premium can occur in our model, when demand does not go up in the next period. The total output expands (price drops) due to investment made before uncertainty. However, there is no initial price premium, if high demand unfolds.

It is clear that equilibrium investment quantity under the low initial capacity must be higher than the investment under the high initial capacity. That is, when the initial capacity is low, satisfying $0 \leq K_{k0} < q^c_{kd}$, even it is smaller than Cournot output at the initial period, firms will invest to profit from future demand states. When the initial capacity is high, satisfying $q^c_{kd} \leq K_{k0} < q^c_{ku}$, the equilibrium investment will be lower as firms accumulate sufficient amount of capacity to meet

[4]The Figures 1, 3, and 4 for regions in capacity states and equilibrium investment strategies in Garcia and Shen (2010) will apply to our analysis of Markov perfect equilibrium with time-to-build.

the highest possible level of future demand. The capacity utilization rate at the final period will be 100%.

When investment is positive, we find a "bang-bang" solution for production. That is, production constraints bind and firms operate at the capacity. This finding is also consistent with Abel and Blanchard (1986), and Zhou (2000) who emphasize that the product markets are often not perfect and hence firms may face binding output constraints. During high demand periods some firms often operate near their capacities in markets such as electricity production and hot spot industries. Note that even in low demand periods some generators can operate at the capacity. For example, nuclear plants generally operate near capacity, and wind turbines produce electricity at the maximum utilization rate during windy fall season.

When the initial capacity is low, that is $0 \leq K_{k0} < q^c_{kd}$ holds, the expected output in the final period is $E_{1b}(q_1) = (1+g)(\bar{p} - c - f)/3(1+g-\theta g)$ for both firms. Although firms start with different initial capacities and invest different quantities, their expected outputs will be identical because production and investment costs are the same for both firms. When the initial capacity is high, that is $q^c_{kd} \leq K_{k0} < q^c_{ku}$ holds, the expected output in the final period becomes $E_{1a}(q_1) = [(1+\theta g)(\bar{p} - c) - (1+g)f]/3$ for both firms, because capacity constraint binds in the high demand state and it is interior in the intermediate demand state. In this case, again expected outputs and actual outputs in the final period will be the same for both firms, as they have the identical production and investment costs and both firm investments only differ in terms of initial capacities.

The following result examines asymmetric equilibrium investment case.

Proposition 2: Let firm i be small capacity firm whose initial capacity holds $0 \leq K_{i0} \leq q^c_{iu}$ and firm j be large capacity firm with initial capacity satisfying $K_{j0} \geq ((\bar{p} - c)(1+g) - \beta)/2 = (3q^c_{ju} - \beta)/2$, where $\beta = \dfrac{(1+g)(\bar{p} - c - 2f)}{2(1+g-g\theta)}$. Then firm j does not invest and firm i invests (i.e., $I_{j0} = 0 \leq I_{i0}$). Markov perfect equilibrium investment strategies for firm i are:

$$I_{i0} = \begin{cases} 0 & \text{if } q^c_{iu} \leq K_{i0} \\ \dfrac{(1+g)[(\bar{p} - c) - 2f/\theta]}{2} - K_{i0} & \text{if } q^c_{id} \leq K_{k0} < q^c_{iu} \\ \dfrac{(1+g)(\bar{p} - c - 2f)}{2(1+g-g\theta)} - K_{i0} & \text{if } 0 \leq K_{i0} < q^c_{id} \end{cases} \tag{5}$$

Proof: See the Appendix.

Under the conditions of Proposition 2 firm j has a high initial capacity, and the small firm i increases its capacity based on the level of its initial capacity and the market parameters, but independently of firm j's capacity.

Observe that when firm i's initial capacity is low, that is $0 \leq K_{i0} < q^c_{id}$, it invests more relative to the investment it undertakes when he starts with high initial capacity, that is $q^c_{id} \leq K_{i0} < q^c_{iu}$. But it does not invest (following its competitor), when its initial capacity is very high, that is $q^c_{iu} \leq K_{i0}$. The investment made in the low initial capacity case will be totally used in the second period whether demand turns out to be low or high. However, the investment made in the case of a high initial capacity will only benefit the high demand scenario, if it unfolds, otherwise this investment will be futile. This is a risky investment because low demand market is possible in the end. However, the firm invests as if he would face the high demand market.

The equilibrium characterized in Proposition 2 is akin to Commitment equilibria of Pacheco-de-Almeida and Zemsky (2003) in which the leader firm invests and the follower does not invest under uncertainty. This type of equilibrium emerges as Stackelberg equilibrium outcome in their model. However, from the outset, we assume Cournot-Nash equilibrium approach in our model, and Commitment-like equilibrium emerges just due to very asymmetric initial capacities of the firms.

5 Instantaneous Investment

This section explores investment behavior when firms can instantly increase their production capacities by either acquiring new production technologies or expanding capacities of the existing units via refurbishing/maintenance. Investment is productive instantaneously and there is no (or significant) lag between production and investment as opposed to the investment with lead time structure. That is, time-to-build constraint is either negligible or non-existent. In this structure, firms have an option to postpone their investments in the first period, which is absent from the former structure. However, for technical reasons (shown in the proofs) equilibrium computations can become harder under this investment type.[5]

As an example, in the electricity generation industry some firms can instantly expand their production capacities via purchasing (small-scale) gas-fired generators, which are available in various capacity sizes. Alternatively, a photo-voltaic technology (which converts sun lights into thermal energy and then into kinetic energy) or a wind turbine (comprised of blades, gearbox, generator, and

[5] In a different model Pacheco-de-Almeida and Zemsky, the case of instantaneous investment, which occurs when the investment lag parameter is zero (T=0), is a special case of investment with time-to-build. Their model reduces to two-period model, in which firms can invest in two periods (before and after uncertainty) and production and sales occur in the second period only.

tower) can easily be purchased from the energy equipment suppliers with various sizes of production capacities (a modern wind generator can produce upto 5 MWh electricity). Investment into these technologies can be regarded as instantaneous investment in which these technologies can become productive instantly after they are installed or can take a minor lead time from investment stage (e.g., purchasing equipment) to the production process compared to other technologies such as nuclear and hydro plants, which exhibit significant lead times from investment to production stages.

Because today's decisions could impact future decisions and profitability we characterize Markov perfect Nash equilibrium investment strategies under various market conditions. Therefore, we solve the equilibrium problem backwards so that decisions are subgame perfect.

To examine the impact of instantaneous investment we first take a close look at one of the periods, say initial period, to see how instantaneous investing is affecting the market outcomes. The objective function of firm i to be maximized at the initial period is

$$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0}),$$

where λ_{i0} is a multiplier. If both firms are constrained, that is their capacity constraints bind, then equilibrium investment strategies satisfy $I_{k0} = (\bar{p} - c - f)/3 - K_{k0}$ for $k = i, j, i \neq j$. If firm i is constrained and firm j's production is interior (and hence firm j does not invest), then firm i's investment is equal to $I_{i0} = (\bar{p} - c - 2f)/3 - K_{i0}$. If firm i is constrained, and firm j is also constrained but does not invest, then firm i's investment satisfies $I_{i0} = (\bar{p} - c - f - K_{j0})/2 - K_{i0}$. If both firms' productions are interior, then they do not invest and produce at Cournot output $q_{i0} = (\bar{p} - c)/3 = q_{j0}$. This simple analysis indicates that if we increase the number of time stages, the possible number of equilibrium investment strategies is going to increase.

Next we analyze the investment behavior when firms compete in two periods in the market.

Let producer i's strategy be $s^i = (I_{it}, q_{it})$, which is chosen as a solution to the problem:

$$\max_{s^i} E_0 \sum_{t \in \{0,u,d\}} [P_t(Q)q_{it} - cq_{it} - fI_{it}] \tag{6}$$

subject to

$$0 \leq q_{it} \leq K_{it},$$

$$K_{it} = K_{it-1} + I_{it}, \qquad I_{it} \geq 0.$$

Although there are multiple MPE investment quantities over the capacity regions, the equilibrium will be unique for a given parameter region. Namely, because we have three demand states (initial demand in period 0, and up and intermediate state demands in period 1) and two possible solutions in each state (interior output or constrained production) we have eight scenarios to consider for equilibrium investment. Four types of investment profiles can emerge as equilibria: a) Firm k invests only in the initial period, $I_{k0} > 0$, $I_{ku} = 0 = I_{kd}$; b) Firm k invests in the first period and the second period upstate demand, $I_{k0} > 0$, $I_{ku} > 0$, $I_{kd} = 0$; c) Firm k invests only in the upstate node, $I_{k0} = 0$, $I_{ku} > 0$, $I_{kd} = 0$. d) Firm k does not invest at all, $I_{k0} = 0$, $I_{ku} = 0$, $I_{kd} = 0$. In any investment profile, firms never invest in the second period intermediate demand state. We will examine all investment profiles: firms invest at the beginning or invest in the second period or invest in both periods. We will analyze symmetric and asymmetric MPE strategies.

5.1 Committing to Initial Investment

This subsection analyzes equilibrium investment solutions for duopolists when they invest at time zero only. Before examining investment profiles, we ask the following question: why do firms invest now and utilize their investments now and in the future given that investment is instantaneous? We argue that this behavior could be related to, for instance, time constraints imposed by firms' other projects, instant growth incentives, managerial/shareholder pressure, and/or macroeconomic conditions. Alternatively, it might be due to strategic reasons. For example, whether firms are symmetric or asymmetric in initial capacities, one firm may emerge and invest right away to deter the rival firm's incentive to invest. Also, a firm with small capacity may wish to invest right away to be able to compete with a large rival firm in the future high demand market. By investing earlier small firm can increase its size and send a signal to its competitor that it is a contender in the high demand market (if unfolds) and will be able to increase its sales. If, however, intermediate market materializes in the second period, the small firm can expand its market share and increase its cash flow by investing earlier.

Among the reasons of why invest now, mentioned above, the current model encompasses macroeconomic conditions through demand growth and strategic reasons through commitment incentives. Other reasons are not directly observable in the model setting. If the initial capacity at the outset is low, firms have to invest in the initial period. In addition, they consider the future before they choose the investment quantities in the beginning as it is likely that demand would go up in the next period. Then one firm could pre-commit to invest more at the beginning to be able to benefit from the current and the future states and would potentially become a market leader. But the other firm has the same objective and hence would follow the same suit. Therefore, both firms would play commit-commit strategies. This is what is captured in Proposition 3. On the other hand, one of the firms may choose not to commit any investment strategy and play stay put, because of its massive initial capacity endowment. This business strategy is captured in Proposition 4.

Proposition 3: For firms $k = i,\ j,\ i \neq j$ let $K_{i0},\ K_{j0}$ be their initial capacities at $t = 0$. Assume that there are two states at $t = 1$ namely u and d. Then, the Markov perfect Nash equilibrium initial investment strategies at $t = 0$ are:

$$I_{k0} = \begin{cases} 0 & \text{if } q^c_{ku} \leq K_{k0} \\ \dfrac{(1+g)(\bar{p} - c - f/\theta)}{3} - K_{k0} & \text{if } q^c_{kd} \leq K_{k0} < q^c_{ku} \\ \dfrac{(1+g)(2(\bar{p} - c) - f)}{6(1+g) - 3\theta g} - K_{k0} & \text{if } 0 \leq K_{k0} < q^c_{kd} \end{cases} \tag{7}$$

Proof: See the Appendix.

Since we have three demand states $(0, u, d)$ and there are two possibilities (binding or interior) for each state, eight scenarios emerge. We cover three of them in the above proposition and show that these three scenarios are part of the MPE. Other cases are not part of the equilibrium as we explain next. The case of binding capacity in the initial state and interior solutions in the up and intermediate states is not possible. To have this case hold, $K_{k0} + I_{k0} < q^c_{k0}$ and $K_{k0} + I_{k0} > q^c_{ku}$ and $K_{k0} + I_{k0} > q^c_{kd}$ must be satisfied simultaneously. Obviously, it is a contradiction to have these inequalities held because $q^c_{k0} = q^c_{kd}$. Another case to be considered is the interior solution at time zero, and binding capacity constraints in the up and intermediate states. But, $K_{k0} + I_{k0} > q^c_{k0}$ and $K_{k0} + I_{k0} < q^c_{ku}$ and $K_{k0} + I_{k0} < q^c_{kd}$ do not hold simultaneously for any admissible initial capacity. There are other possibilities to consider: a) It is interior solution in the upstate and the constraints are binding in the initial and intermediate states; b) It is only binding in the intermediate state, and interior in other states; c) Constraints are not binding for all states under which the investment quantity is clearly zero. Under these circumstances, it is not possible to have a positive investment as the inequalities do not hold simultaneously.

Thus far we have examined symmetric MPE, the following proposition covers asymmetric equilibrium investment strategies.

Proposition 4: Assume that investment is instantaneous. Let firm i be small capacity firm whose initial capacity holds $0 \leq K_{i0} \leq q^c_{iu}$ and firm j be large capacity firm whose initial capacity satisfies $K_{j0} \geq ((\bar{p}-c)(1+g)-\gamma)/2 = (3q^c_{ju}-\gamma)/2$, where $\gamma = \dfrac{2(1+g)(\bar{p}-c-f)}{5(1+g)-2g\theta}$. Then firm j does not invest and firm i invests (i.e., $I_{j0} = 0 \leq I_{i0}$), and the Markov perfect equilibrium investment strategies for firm i satisfy:

$$I_{i0} = \begin{cases} 0 & \text{if } q^c_{iu} \leq K_{i0} \\ \dfrac{(1+g)[(\bar{p}-c)-2f/\theta]}{2} - K_{i0} & \text{if } q^c_{id} \leq K_{k0} < q^c_{iu} \\ \dfrac{2(1+g)(\bar{p}-c-f)}{5(1+g)-2g\theta} - K_{i0} & \text{if } 0 \leq K_{i0} < q^c_{id} \end{cases} \tag{8}$$

Proof: See the Appendix.

In this proposition firm j does not invest due to its larger initial capacity. Comparing the investment strategies when there is a lead time versus there is no lead time, we find that the outcomes in Proposition 2 is same as the ones in Proposition 4, if the capacity constraints are binding only in the demand growth (upstate) case. However, the investment policies are different if the constraints bind in all relevant states (as explained in Section 7 in detail).

5.2 Delaying and Investing Later

This section examines equilibrium investment policies of competitors when they do not invest at the outset but invest only at the second period which embeds the demand growth scenario. Note that this investment scenario was not possible under time-to-build.

Proposition 5: Let firms $k = i,\ j,\ i \neq j$ start with initial capacities $K_{i0},\ K_{j0} \geq 0$ at $t = 0$. Then Markov perfect Nash equilibrium investment strategies at $t = 1$ are:

$$I_{ku} = \begin{cases} \dfrac{(1+g)(\bar{p}-c-f)}{3} - K_{k0} & \text{if } q^c_{kd} \leq K_{k0} < q^c_{ku} \\ 0 & \text{if } q^c_{ku} \leq K_{k0} \end{cases} \tag{9}$$

Proof: See the Appendix.

Note that in equilibrium both firms produce the same level of output which is $(1+g)(\bar{p}-c-f)/3$ no matter what their initial capacities or investment levels are.

5.3 Investing in All Periods

When investment is instantaneous, it is possible that firms invest in all periods. Although this never happens under time-to-build, this case has to be examined for the sake of completeness.[6]

[6] This issue has been raised by an anonymous referee.

In this subsection we will show that firms will invest in both periods only if the initial capacity is "very low". Otherwise, they will invest once, either at the beginning or at the end of the period such that they will follow the investment rules described in subsections 5.1 and 5.2 above.

Assume that initial capacity is very low such that it cannot even provide Cournot output in the initial period. That is, it satisfies $K_{k0} < q^c_{k0}$. In this case firms may choose to invest in both periods. When they invest in both periods, their investments provide benefit to all demand states. The profit function that will be maximized, for firm i, becomes

$$\pi_i(.) = (K_{i0} + I_{i0})(\bar{p} - c - (K_{i0} + I_{i0} + K_{j0} + I_{j0})) - fI_{i0} + \theta\pi_{iu}(.) + (1 - \theta)\pi_{id}(.),$$

where the profit in the upstate is $\pi_{iu}(.) = (K_{i0}+I_{i0}+I_{iu})(\bar{p}-c-(K_{i0}+I_{i0}+I_{iu}+K_{j0}+I_{j0}+I_{ju})/(1+g)) - fI_{iu}$ and the profit in the intermediate state is $\pi_{id}(.) = (K_{i0}+I_{i0})(\bar{p}-c-(K_{i0}+I_{i0}+K_{j0}+I_{j0}))$. The maximization of the total profit function with respect to the choice of upstate investment implies $\partial\pi_i(.)/\partial I_{iu} = 0$ for firm i. Solving the same problem for firm j, and then solving the investment best response functions of both firms simultaneously result in

$I_{ku}(I_{k0}) = \dfrac{(1+g)(\bar{p}-c-f)}{3} - K_{k0} - I_{k0}$ for firm $k = i, j$.

Next firm i optimizes its total profit function with respect to its initial investment, that is $\partial\pi_i(.)/\partial I_{i0} = 0$. This leads to $(\bar{p} - c - f) - 2(K_{i0} + I_{i0}) - (K_{j0} + I_{j0}) + \theta\partial\pi_{iu}(.)/\partial I_{i0} + (1 - \theta)\partial\pi_{id}(.)/\partial I_{i0} = 0$, where $\partial\pi_{iu}(.)/\partial I_{i0} = [(\bar{p}-c)(1+g)-2(K_{i0}+I_{i0}+I_{iu})-(K_{j0}+I_{j0}+I_{ju})]/(1+g)$, and $\partial\pi_{id}(.)/\partial I_{i0} = [(\bar{p} - c) - 2(K_{i0} + I_{i0}) - (K_{j0} + I_{j0})]$. Solving the same problem for firm j, and solving the best response investment functions simultaneously, and then inserting the value of $I_{ku}(I_{k0})$ lead to

$I_{k0} = \dfrac{(\bar{p}-c)(2-\theta) - f(1-\theta)}{3(2-\theta)} - K_{k0}$ for firm $k = i, j$.

Inserting this expression into the investment function above, the second period optimal investment for firm k becomes

$I_{ku} = \dfrac{(2-\theta)[g(\bar{p}-c) - f(1+g)] + f(1-\theta)}{3(2-\theta)}$.

Different than the investment at the initial period, the second period investment is irrelevant of initial capacity level, but depends on the demand growth rate as well as other model parameters.

Note that firms never invest in the intermediate state, as they already invested in the initial state, which is identical to the intermediate state in terms of demand. The total investment quantity will be $I_{k0} + I_{ku} = (1 + g)(\bar{p} - c - f)/3 - K_{k0}$.

We learn from the above analysis that if the initial capacity is too low (i.e., $K_{k0} < q_{k0}$), firms can invest in both periods. In this case all capacity constraints in all demand states will bind. This opens the following question: do firms invest in both periods if they start with somewhat intermediate level of capacity (neither high nor low)? This corresponds to the capacity level $K_{k0} \geq q^c_{k0}$ and $K_{k0} < q^c_{ku}$. When we solve the problem expressed in (6) with this initial capacity level we find that firms will not invest in the initial period, but invest only in the second period upstate, if it unfolds. Algebraically, we find that the KKT condition for firm i is $(\partial \Pi_i / \partial I_{i0}) I_{i0} = (-f + \rho_{i0} + \rho_{iu} + \rho_{id}) I_{i0} = 0$, where $\rho's$ are the Lagrange multipliers for the states described in the subscripts. It will follow that $\rho_{i0} = 0 = \rho_{id}$ because the investment made in the initial period will not be used neither in the initial state nor in the intermediate state. However, $\rho_{iu} > 0$ will hold because investment made in the upstate will be productive, and the total output will be equal to total capacity. Therefore, $I_{i0} = 0$ will hold. Consequently, this case will boil down to the investment behavior in subsection 5.2, where optimal investment in the final period will be the one expressed in (9).

This analysis indicates that positive investments in both periods only occur when the initial capacity is too low. Otherwise, firms will either invest in period 0 (subsection 5.1) or invest in period 1 (subsection 5.2).

In the analysis of Pacheco-de-Almeida and Zemsky (2003) (AZ) for instantaneous investment case, they state (in their Proposition 1) that "Firms never make incremental investments". In our analysis above, we show that this type of equilibrium is possible. This is because in their model there is a stage in which firms are allowed to make investment ex post. In our setting firms can make investment ex ante (which is the investment quantity at time zero) and this investment can benefit initial period demand as well as future demand which is uncertain at the time of decision making. On the other hand, there is no demand at the initial period in AZ; therefore investment will be carried out after it is certain that demand is positive.

Furthermore, in AZ there are two types of equilibria (in their Proposition 1) when investment is instantaneous: Delay equilibrium and Commit-delay equilibrium. Their Delay equilibrium (i.e., both firms wait and invest after the resolution of uncertainty) is same as our investment characterization in Proposition 5, in which firms invest right after they figure out that they are at the high demand state. Their Commit-delay equilibrium (in which the leader firm invests at the outset and the follower firm delays and invests in the final period) occurs when uncertainty is not too great (demand uncertainty parameter is sufficiently high). In this case cost of investing early is less than the benefit of committing. In our setting, by modeling choice (i.e., Cournot-Nash equilibrium concept) we do not have Commit-delay type equilibrium. In AZ, firms neither make incremental investments nor carry out ex-ante investments simultaneously. However, these types of equilibria are possible in our setting (in our incremental investment analysis above and Commitment type investment in our Proposition 4).

6 Socially Optimal Investment

In this section we investigate whether duopoly investment strategies would be different than social optimum, which is obtained by solving the social planner's problem through maximizing expected sum of welfare (consumer surplus less production and investment costs). We will analyze the investment behavior with and without time-to-build constraint.

When the investment is subject to time-to-build, the planner chooses outputs and investment to solve the following problem:

$$\begin{aligned} \max \quad & E_0 \sum_t \left[\int_0^{q_t} P_t(q)\mathtt{d}q - cq_t - fI_t \right] \\ s.t. \quad & 0 \leq q_t \leq K_t \\ & I_t \geq 0 \\ & K_{t+1} = K_t + I_t \end{aligned}$$

In the two-period planning, the optimality conditions are

$q_0 = \bar{p} - c$ for the initial output, $-f + \lambda_u + \lambda_d = 0$ for the investment, $\theta(\bar{p} - c - q_u/(1 + g)) = \lambda_u$ for upstate output, and $(1 - \theta)(\bar{p} - c - q_d) = \lambda_d$ for intermediate state output. If the production constraints are binding in the upstate only, then the optimal investment will be $I_0 = (1 + g)(\bar{p} - c - f/\theta) - K_0$. If the constraint is binding in both states, then the investment will be $I_0 = (1+g)(\bar{p}-c-f)/(1+g(1-\theta)) - K_0$. Otherwise, there is no investment. When we compare these investment profiles to the ones in Proposition 1, it is clear that duopolists underinvest relative to social optimum, independent of status of the capacity constraints. Because, i) under the MPE the total investment in Proposition 1 is $2(1 + g)(\bar{p} - c - f/\theta)/3 - K_{i0} - K_{j0}$, which is less than $I_0 = (1+g)(\bar{p}-c-f/\theta)-K_0$, where $K_0 = K_{i0}+K_{j0}$, when the investment is benefiting the upstate demand only; ii) the total MPE investment is $2(1+g)(\bar{p}-c-f)/3(1+g-\theta g) - K_{i0} - K_{j0}$, which is less than the efficient investment $(1+g)(\bar{p}-c-f)/(1+g(1-\theta)) - K_0$ when investment provides benefits to both up and intermediate demand states in period 1.

On the other hand, when the investment is instantaneous or does not exhibit a significant lag between investment and production the analysis of optimum investment is as follows. The planner chooses all outputs and investment to maximize the total welfare similar to the above analysis.

When investment is carried out in the initial stage only, similar to the time-to-build investment analysis above, the optimality conditions satisfy:

$(\bar{p}-c-q_0) = \lambda_0$ for the initial output, $-f+\lambda_u+\lambda_d+\lambda_0 = 0$ for the investment, $\theta(\bar{p}-c-q_u/(1+g)) = \lambda_u$ for the upstate output and $(1-\theta)(\bar{p}-c-q_d) = \lambda_d$ for the intermediate state output. If the production constraints are binding in the upstate only, then the optimal investment will be $I_0 = (1+g)(\bar{p}-c-f/\theta)-K_0$. If the constraint is binding in both up and intermediate states at time 1, then the efficient investment will be $I_0 = (1+g)(2(\bar{p}-c)-f)/(2(1+g)-\theta g)-K_0$. Otherwise, there is no investment. Comparing these optimum investment strategies to the ones under the MPE in Proposition 3, it is clear that duopolists underinvest.

Then we obtain the following result:

Proposition 6: Whether investment is instantaneous or subject to a lead time, firms' total investments under the MPE are lower than socially optimum investment.

7 The Role of Investment Types

This section compares the investment outcomes based on the investment types (instantaneous vs. time to build). As the propositions 1-5 show, depending on the initial capacity levels of firms, investments made under uncertainty will be productive either in upstate only, or in both initial state and upstate, or in all demand states. One of these investment strategies will unfold based on the parameter regions defined in the previous sections. In each case, we observe some common investment characteristics: investment is decreasing in initial capacity, production and investment costs, and increasing in demand uncertainty. If firms start with large capacity endowments, they may not invest at all or invest little, depending on demand conditions. In the limit, if the initial capacity is infinity, there is no need for investment. In all propositions the rate of change of investment strategy with respect to initial capacity is negative. Also, in all propositions the rate of change of investment with respect to investment cost or production cost is negative. Firms tend to invest less when investment costs increase. As the investment quantity is going to be used for production, production cost will also negatively impact the investment. Therefore, higher production and investment costs will lead to lower investments. Also, as explained in detail in Section 8, the derivative of investment strategy with respect to uncertainty (represented by θ, the probability of demand growth) is always positive. This is because favorable resolution of uncertainty implies higher θ which implies higher expected demand and therefore higher investment.

When we rank these investment quantities we find that the highest level of investments occurs when investment provides benefit to all demand states $(0, u, d)$, and the lowest level occurs when it only provides benefit to the upstate. The intuition for this result is that for a given level of demand, lower initial capacity entails into higher investment quantity. Although the investment quantity is the largest when the capacity constraints are binding in all states, the outputs are lower than Cournot outputs. Similarly, if the capacity constraint is binding only in the upstate then initial and intermediate state outputs are at their Cournot output levels. Hence, prices (in all states) will be the lowest when the investment is positive and at its lowest level. This is due to the fact that the initial capacity is high and investment provides benefit to upstate only, hence consumers enjoy lower prices because of high initial capacities. An implication of this finding in electricity markets context is that excess reserve capacity or installed capacity becomes necessary to depress prices and smooth price hikes.

When investment is subject to a time-to-build, we have seen that investment occurs only once in the two-period model. Depending on the initial capacity level of a firm, the investment will be productive either at the high demand state or at all states in the second period (Proposition 1). However, when investment is instantaneous investment choices are rich such that firms can either invest at the initial period (Proposition 3), or in the final period (Proposition 5), or in both periods (subsection 5.3).

We find in subsection 5.3 that firms invest in both periods only if their initial capacity is too low (i.e., lower than Cournot output). Otherwise, firms invest only once (either in the initial period or in the final period). When firms invest in both periods the total investment for firm k, as characterized in subsection 5.3, is $(1+g)(\bar{p}-c-f)/3-K_{k0}$. On the other hand, the investment with time-to-build is $(1+g)(\bar{p}-c-f)/3(1+g-\theta g)-K_{k0}$ in Proposition 1 when investment benefits all demand states in the second period. Comparing these investment quantities, clearly the total investment when there is no time-to-build is higher than the investment when there is time-to-build. Note that this comparison holds for a small parameter region in which the initial capacity is too low $(0 \leq K_{k0} < q^c_{k0})$. This is not a surprising result because of the technological advantage of having the ability of investing instantly. Since firms start with low initial capacities, they invest at the beginning so as to meet the current and the future demand. In the second period they will keep investing to further benefit from the demand growth.

Nevertheless, when the investments made just once under both investment types, we compare Proposition 1 (time-to-build) to Proposition 3 (instantaneous). We observe that equilibrium investments are equal if it is benefiting upstate demand only. This investment quantity is $(1+g)(\bar{p}-c-f/\theta)/3-K_{k0}$. However, when investment made only once and investment provides benefit to all (relevant) demand states, (up and intermediate demand states) in Proposition 1 we have $(1+g)(\bar{p}-c-f)/3(1+g-\theta g)-K_{k0}$, which is higher than the instantaneous investment in Proposition 3 which equals $(1+g)(2(\bar{p}-c)-f)/(6(1+g)-3\theta g)-K_{k0}$, if the marginal cost of investment is lower than a bound: $f<(\theta g(\bar{p}-c))/(1+g)$.

Consequently we obtain the following result.

Theorem 1: The ranking of investment quantities with respect to investment types mainly depends on the initial capacity levels. In particular, when investment is instantaneous and firms start with low initial capacities, they can invest every period and their total investments will be higher than the ones observed under time-to-build constraint. Firm investments will be identical under time-to-build and no time-to-build, if firms invest once and this investment only benefits the upstate demand. On the other hand, instantaneous investment quantity could be lower than time-to-build investment, if the marginal cost of investment is very low. Also, the ranking of outputs and consumer surplus will follow from the ranking of investment quantities.

Note that instantaneous investment encompasses richer set of equilibrium investment strategies. Investment can be postponed and made after uncertainty unfolds. Firms can invest at the beginning, at the end, or in both periods. Also, it can benefit more demand states. Therefore, the number of equilibria is higher under instantaneous investment than under investment with time lag.

When firms are asymmetric in investment choices, that is while one firm invests the other does not, we find (by comparing investment policies in Propositions 2 and 4) that under time-to-build constraint the firm's equilibrium investment (weakly) exceeds the investment under no time-to-build.

It is notable that investment decisions under time-to-build (Propositions 1 and 2) or instantaneous investment (Propositions 3 and 4) are both subject to the same demand uncertainty. Investment decision in the initial period I_{i0} under both structures is made before the realization of demand states in the following period. In the former investment will be used in the following period, while in the latter investment will be used both at the initial and final periods.

Theorem 1 is also consistent with Proposition 7 in Pacheco-de-Almeida and Zemsky (2003) who find that social welfare falls in time-to-build, because firms cannot respond fast to the arrival of information about demand. But this result is not general and does not hold for some equilibria and parameter configuration. In particular, the effect of time-to-build on welfare reverses "If an increase in time-to-build creates Commitment equilibrium, then welfare can increase" for certain model parameter combination (see their p.177). We have a similar finding of market performance ranking with and without time-to-build changing with respect to parameter regions (initial capacity levels and investment costs).

8 The Impact of Uncertainty

We observe from demand uncertainty formulation in expression (1) that at time 0 the initial demand is $Q_0 = \bar{p} - P_0$. At time 1 demand will be either $Q_1 = (\bar{p} - P_0)(1 + g)$ with probability θ or $Q_1 = (\bar{p} - P_0)$ with probability $(1 - \theta)$. The expected demand at time 1 is $(\bar{p} - P_0)(1 + \theta g)$ which is increasing in uncertainty θ. The variance of demand is $(\bar{p} - P_0)^2 g^2 \theta(1 - \theta)$ which is increasing in θ for $\theta < 1/2$ and decreasing in θ for $\theta > 1/2$. Clearly demand variability becomes zero at $\theta = 0$, and expected demand is the highest as θ approaches one. Favorable resolution of uncertainty implies higher θ which implies higher expected demand and therefore should imply higher investment. Alternatively, increase in θ makes the upstate demand more likely, and therefore investment made under uncertainty becomes less risky.

Based on the analyses of equilibrium investments with and without investment lag above, we will examine the effect of uncertainty on investment. In the model θ, the probability of demand growth, captures demand uncertainty.[7]

When investment is subject to time-to-build, from Proposition 1 we observe that θ impacts the investment strategies differently, depending on the initial capacity levels. The derivative of the investment function with respect to θ, when the initial capacity is low, is $g(1+g)(\overline{p}-c-f)/3(1+g(1-\theta))^2 > 0$, and it is, when the initial capacity is high, $(1 + g)f/\theta^2 > 0$. Clearly, these derivatives are positive and implying that investment increases as upstate demand becomes more likely. Moreover, the rate of change of investment with respect to uncertainty has different magnitudes, depending on the initial capacity level. In particular, when the initial capacity is low so that investment benefits all demand states in the future period, the rate of change of investment is impacted by all model parameters, including the price cap, and marginal costs of production and investment. However, when it is high so that only high demand state benefits from the investment, the rate of change of investment is impacted by only the growth rate, the marginal cost of investment, and the likelihood of high demand state. These findings are also valid when firms are very asymmetric in terms of

[7] In Pacheco-de-Almeida and Zemsky (2003), the probability of positive demand (their parameter a) represents uncertainty.

initial capacities (in Proposition 2).

When investment is instantaneous, we obtain the same qualitative results as in time-to-build case. When investment is made only in the initial period (Proposition 3) or in the both periods (subsection 5.3), the (initial) investment increases in uncertainty and its rate of change with respect to uncertainty varies depending on the level of initial capacity. The only difference is what happens in the final period when investment is made in both periods. The uncertainty will not directly impact value of the second period investment. After the initial investment is made, firms will invest in the second period as soon as the high demand state unfolds. This is clear from the equilibrium relation $I_{ku}(I_{k0}) = (1+g)(\bar{p}-c-f)/3 - K_{k0} - I_{k0}$ in subsection 5.3. However, as the value of the random variable impacts the initial investment, it will indirectly impact the quantity of second period investment. Consequently, the higher uncertainty (θ) will increase the quantity of initial investment, which in turn will decrease the quantity of final period investment.

Next we compare the effect of uncertainty on investment with and without investment lag. When we compare the investment strategies in Propositions 1 and 3, we observe that the investment quantities are the same when the initial capacity is high, and they are different when the initial capacity is low. Therefore, the impact of uncertainty would be same whether investment exhibits time-to-build or not, if the initial capacity is high. However, the uncertainty will impact the investment decisions differently when the initial capacity is low. Specifically, the explicit impact of demand uncertainty is as follows. The derivative of investment quantity with respect to θ, under time-to-build (superscript t2b), is

$$\partial I_{k0}^{t2b}/\partial\theta = (1+g)g(\overline{p}-f-c)/3(1+g-g\theta))^2 > 0$$

and it is

$$\partial I_{k0}^{inst}/\partial\theta = (1+g)g(2(\overline{p}-c)-f)/3(2(1+g)-\theta g)^2 > 0$$

under instantaneous investment (superscript inst). The ratio of these derivatives is $(\partial I_{k0}^{t2b}/\partial\theta)/(\partial I_{k0}^{inst}/\partial\theta) = (\overline{p}-f-c)(2(1+g)-\theta g)^2/(2(\overline{p}-c)-f)(1+g-g\theta)^2$. Denote this ratio ε, which is higher than 1 if and only if $(1+g)^2[2(\overline{p}-c)-3f] - \theta^2 g^2[\overline{p}-c] + (1+g)\theta g[2f] > 0$. Clearly, the coefficient of the term in the first bracket is always higher than the coefficient of the second one, that is $(1+g)^2 > \theta^2 g^2$. The third term is always positive. Then it is sufficient to have the relation $\overline{p}-c > 3f$ between the marginal costs and the price cap so that the summation of all terms in the above inequality becomes always positive. Therefore, the increase in uncertainty can have a larger impact on investment with time-to-build than investment without time-to-build.

The following theorem summarizes the impact of uncertainty on investments with and without time-to-build.

Theorem 2: The equilibrium investment with or without time-to-build increases in uncertainty. Depending on the initial capacity, the rate of change of investment with respect to uncertainty can be the same or different under both types of investments. Specifically, if the initial capacity is high (so that investment will become productive in upstate demand only), then the impact of uncertainty on investment strategies is identical for both investment types. If the initial capacity is low (so that investment will provide benefit to all relevant demand states), the effect of uncertainty on investment with time-to-build can be higher than the one with instantaneous investment.

For a given value of θ, we can also measure the impact of demand growth rate g on investment. The rate of change of investment strategy with respect to demand growth rate is always positive for investments with and without lag. However, their explicit impacts could be different. When initial capacity is high the rate of change of investment with respect to g is equal to $(\overline{p} - c - f/\theta)/3 > 0$ whether investment is subject to time-to-build (Proposition 1) or it is instantaneous (Proposition 3). When initial capacity is low, the derivative of investment with respect to growth rate equals $\theta(\overline{p} - c - f)/3(1 + g(1 - \theta))^2 > 0$ with time-to-build, and it is $\theta(2(\overline{p} - c) - f)/3(2(1 + g) - \theta g)^2 > 0$ without time-to-build. Clearly, the impacts of growth rate are different and one can be greater than the other for a given parameter range.

9 Extensions

Hitherto we have examined two-period competition setting with and without lead time using Markov perfect equilibrium concept. In this section as a robustness check we will extend the competition setting to three periods and analyze investments under open-loop Nash equilibrium (OLNE) concept, which is commonly used in the deterministic dynamic games literature. The difference between OLNE and MPE will give the strategic value of investment. Specifically in subsection 9.1 we will compare and contrast Markov perfect investment strategies to open-loop Nash equilibrium outcomes. We will show that Markov perfect investment solution coincides with open-loop counterpart under certain conditions, independent of the investment type (instantaneous or not). However, we also pinpoint the conditions under which they predict different investment profiles in subsection 9.2. In subsection 9.3, we will extend the time stages to three and examine implications of investment opportunities over time.

9.1 Open-loop Nash Equilibrium (OLNE) Investment Solution

This section examines OLNE investment outcomes under uncertainty with and without time-to-build constraint. Open-loop approach is generally employed as a benchmark case to differentiate the strategic investment. Although OLNE may not be subgame perfect in general, equilibrium computations can be tractable and simpler with appropriate reformulation.[8]

We show that open-loop solution can be equal to Markov perfect one for some capacity levels that we identify. In addition, we note that open-loop equilibria can be used in a moving-horizon approach to approximate a Markov perfect (or closed-loop) equilibrium (see van den Broek (2002)). The principle is simple: At each period t, the players determine the open-loop Nash-equilibrium strategies for a given T-period planning horizon. However, only the initial control action is implemented. At period $t+1$, the players again compute the equilibrium strategies for the next T periods, implement the (new) initial action, and so on. The resulting moving-horizon equilibrium trajectories constitute an approximation of the closed-loop equilibrium trajectories that would have been obtained at the outset of the dynamic game. Moreover, preemptive investment (open-loop concept) could be an optimum strategy for a firm if its rival chooses the investment strategy at the outset of the game. Similar to the open-loop concept, in the wholesale electricity markets the traders regularly employ fixed-mix investment strategies for power portfolio optimization (Sen et al., 2006). Further, some studies found that open-loop equilibria have some empirical support. For instance, Haurie and Zaccour (2004) compared the predicted open-loop equilibrium strategies to the realizations in the European gas market, and found that they are close.

[8] As noted by an anonymous referee that the notion of open-loop strategy could be more challenging when one introduces uncertainty. Haurie and Zaccour (2004) formulated open loop solution under uncertainty. They named this equilibrium as S-adapted (sample adapted) open-loop Nash equilibrium. With S-adapted open-loop information, at any time each player's information set includes the current calendar time, the current demand state, the distribution of future demand, and the initial values of capacity states. Genc et al. (2007) extended this equilibrium concept using stochastic programming approach and Genc and Sen (2008) applied it to oligopolistic Ontario wholesale electricity market model to solve for power firms' investment and production decisions under uncertainty using actual market data. Genc and Zaccour (2013) examined the market performance of open-loop behavior relative to closed-loop and Markov perfect solutions in a duopolistic market under uncertainty.

9.1.1 OLNE with Lead Time

Proposition 7: When the capacity investment is subject to one period time-to-build constraint, under the conditions of Propositions 1 the investment policies with Markov perfect information are identical to those with open-loop information.

Proof: See the Appendix.

For various initial capacity combinations the OLNE investment policies coincide with the MPE outcomes in this two period game, because there is one time investment opportunity. In fact, there is no impact of initial investment on the future investment, and the strategic impact of one player's investment on the other (namely $\partial I_{ju}/\partial I_{i0}$) is zero under both information structures. This is because the capacity constraints of both players are binding which creates corner solutions of the outputs, and the productions will be equal to the available capacities of the players. Therefore, a firm will not be able to affect its rival's output through its investments.[9]

9.1.2 OLNE without Lead Time

The following proposition describes the OLNE investment solution when firms are able to increase their production capacities without a (significant) delay.

Proposition 8: When the capacity investment is instantaneous, under the conditions of Proposition 3 open-loop investments are equal to Markov perfect investments.

Proof: See the Appendix.

In the following proposition we will cover the market condition such that in equilibrium firms choose to investment in the final period only.

Proposition 9: Assume that investment is instantaneous and both firms invest at the high demand state u. Then open-loop investments are equal to Markov perfect investments.

Proof: See the Appendix.

The intuition for this result is that there is no future after the second period and the initial period decisions have no impact on the future profits because no investment is carried out at the outset of the game.

[9]As pointed out by an anonymous referee considering risk aversion in firm's objective function may lead to interesting differences between the open-loop and Markov perfect equilibrium results. This could be an interesting future research direction.

9.2 Asymmetric Outcomes

This section shows how Markov perfect predictions differ from the open-loop counterpart. The following proposition deals with asymmetric capacity constraints which will result in an investment rule that exhibits differences between equilibrium predictions.

Proposition 10: Whether investment is instantaneous or subject to a lead time, if the firms' initial capacities are asymmetric (as in the Propositions 2 and 4), then Markov perfect investment is higher than the open-loop investment.

Proof: See the Appendix.

Based on the information structure (Markov perfect versus open-loop) the market equilibrium investment predictions show differences as long as firms are asymmetric in terms of their capacity status. In particular, if one firm's capacity constraint is binding (because of small capacity) and other firm's is not (due to large capacity) then Markov perfect investment will always be higher than open-loop investment. This result will hold for either type of investment (instantaneous or not).

With time-to-build, the difference between the OLNE and MPE investments stems from the impact of initial investment on the next period rival output. Under Markov perfect structure, firm i invests more (than open-loop) to reduce firm j's next period output. In the open-loop equilibrium firm i has no such incentive; it makes investment just to benefit from high demand state in the second period. Similar reasoning follows with instantaneous investment.

If we were to extend the game to three stages, the difference between these equilibrium investment strategies would spring from the effect of first stage investment on the second stage one. This impact, $\partial I_{iu}/\partial I_{i0}$, is zero in the OLNE and it is minus one in the MPE, which will generate the differences in equilibrium predictions. The source of this difference can be traced by backward solution of the MPE.

9.3 Extension to Three Periods

In this section we extend the basic investment model with lead time to three periods and characterize MPE investment strategies. Incremental investments under time-to-build is a novel equilibrium (see Pacheco-de-Almeida and Zemsky, 2003). We will show that our main results in the two-stages can be extended to three-stage version of the game for certain parameter regions.

The extension of demand function to three stages ($t = 0, 1, 2$) produces more demand states and slope of the inverse demand in the last period will be

$$\sigma_{t=2} = \begin{cases} \sigma_0/(1+g)^2, & \text{with probability } \theta^2 \\ \sigma_0/(1+g), & \text{with probability } 2\theta(1-\theta) \\ \sigma_0, & \text{with probability } (1-\theta)^2 \end{cases} \tag{10}$$

The four stages in the final period are up-up, up-intermediate, intermediate-up, and intermediate-intermediate states.

Different than Garcia and Shen (2010) who focus on one time investment strategy profile (which they call "stationary Markovian investment strategies", on page 33, which is the outcome that is obtained in our two-period model formulation), we offer a dynamic investment strategy profile for firms who invest more than once under time-to-build constraint. Therefore, we will be able to trace the impact of strategic investment in a given period on both the firm's and its rival's investments in the following period.

In the two period version of the game we obtained multiple equilibria depending on the initial capacity. Increasing the number of time stages to three will further raise the possible number of equilibria. At time t, the number of demand states is 2^t, and there are 3 possible capacity status at each demand state (both players' capacities are binding, capacities are not binding, and one player's capacity is binding and it is non-binding for the other). Then at the final period ($t = 2$) there are 81 possible equilibria (3^{2^t}), and at time 1 there are 9 possible equilibria. Hence, in the entire game the total possible number of equilibria (including symmetric and asymmetric ones) is 2187 ($= 3 * 3^2 * 3^4$). Characterization of all these equilibria is beyond the scope of this paper. Even if we would concentrate on symmetric outcomes only, equilibrium investments will be high degree polynomials of model parameters, and hence the equilibrium comparisons would be non-tractable. A source of the complexity is that second period investment will be a function of the first period investment and the capacity constraints might be binding both at the first and second periods. With its current investment choice firm i is able to affect its future investment levels as well as the rival firm's current and future investment levels. These strategic interactions along with the binding production constraints have lingering effects which would complicate equilibrium predictions. However, to be able to compare the two period results to the three period ones, we will focus on a particular symmetric equilibrium strategy in which firms will only invest if they expect to see the highest demand scenario to be unfolded in each period (this scenario is similar to the one analyzed in Proposition 1). This corresponds to the equilibrium behavior such that firms invest at the initial node in period 0 and the upstate node in period 1. That is, we will characterize equilibrium at which capacity constraints will be binding only at the highest demand scenarios in each time period so that I_{k0} and I_{ku} is positive. The equilibrium analysis for other possible equilibria would be similar to the proofs of the following propositions.

Proposition 11: Assume a time-to-build constraint between investment and production. When the competition is extended to three periods the MPE and OLNE investments coincide, if production constraints bind only at the up-up (time 2) and the up (time 1) demand states for both firms.

Proof: See the Appendix.

With time-to-build constraint we obtain the equilibrium investment equivalence result under both types of information structures. However, we argue that the same result will apply under no time-to-build. For the sake of briefness we skip the proof, as the mechanics of it will be the same. Note that we have already proved the investment equivalence result in Propositions 1 and 3 when the production constraints were binding only at the up-state.

As shown in the Appendix, the total investments under both information structures will be identical. Hence the players will produce at the same amounts at the highest possible demand scenario where production constraints will bind and they produce at the capacity. Firms will totally utilize their initial investments as well as the final investments in each production stage. Moreover, while the equilibrium initial investment will be a function of the initial capacity, the equilibrium final investment will be independent of the initial capacity.

In Pacheco-de-Almeida and Zemsky (2003) it is endogeneity of the price premium that leads to incremental investment (p. 172). The price premium is the ratio of difference of initial price and final price to final price. The initial price is a function of the investment made under uncertainty. Investment is low and hence initial price is high. The final price is a function of total investment which is the summation of investments made before and after uncertainty. The total investment, which is equal to total output, is higher than initial investment therefore the final price is lower. Consequently, the price premium is positive. On the other hand, in the current paper it is the uncertainty in demand growth that leads to incremental investment. Firms split their investments across periods because of demand uncertainty. Prices are higher than Cournot prices due to lower investments and binding capacity constraints.

For the social planner's problem, the planner has several investment opportunities under time-to-build. There will be multiple equilibria (which will have similar characteristics to the two-period planning), and hence below we will only examine optimum investments which will benefit both up and up-up states. A sketch of proof of the following result is presented in the Appendix.

Proposition 12: The social planner invests more than the duopolists in the three period version of the game.

10 Concluding Remarks

While traditional investment models have mainly focused on instantaneous investment decisions from a single firm's perspective, we examine strategic investment decisions in competition settings. We characterize and analyze investment strategies of firms in markets with different investment structures, investment with lead time (or investment under time-to-build) and investment without lead time (or instantaneous investment), in the presence of capacity constraints and uncertainty. In broader terms, comparison of these investment types boils down to comparing "progressive industries" in which time-to-build is long and significant relative to the "fast-paced industries" where time-to-innovate or -build (and then making the products available to the customers) is short. In these terms, a more indirect instance could be that time-to-build would concern industries with heavy R&D process such as designing and making efficient computer chips, operating systems, or airplanes, and the instantaneous investment model would encompass high-tech industries such as cell-phone or personal computer producers.

We compare firms' capital investment behavior based on the investment types and information structures. We find that for a given investment type equilibrium investment predictions differ across the information structures as long as firms are asymmetric in terms of capacity constraints. In particular, if one firm's capacity constraint is binding and other firm's is not then Markov perfect investment will always be higher than the open-loop investment. These results will follow when the game is extended to three periods.

We offer some new results. The impact of lead time on capital investments is that controlling for demand, and production and investment costs, we determine the conditions under which investments and outputs are higher in progressive industries and the conditions under which they are higher in fast-paced industries. Also, for both investment types (investment with or without time-to-build) we offer a novel equilibrium in which firms incrementally invest. This behavior is driven by demand uncertainty and capacity constraints. Moreover, in contrast to previous findings, expected outputs are lower than Cournot outputs as firms face uncertainty. In addition, the amount of uncertainty has different effects on investment types.

We examine both two- and three-period versions of the model and find the same impact of time-to-build constraint on social welfare. Therefore, we argue that this finding would generalize if we were to extend the model to T finite periods. However, due to the tree structure of demand uncertainty and the status of the capacity constraints (binding or not), the number of equilibria will explode, as we explain in the extensions section.

There are several possible future research directions. It would be interesting to explore the dynamics of equilibrium investments when firms employ technologies with different lengths of time-to-build. This is because the length of time-to-build may create additional source of uncertainty for decision makers. Also, risk consideration could be an important aspect for capital investments. One could assume risk-averse firms instead of risk-neutral firms in the market. Considering risk aversion in firm's objective function may lead to interesting differences between the open-loop and Markov perfect equilibrium results.

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Appendix-Proofs

Proof of Proposition 1:

To characterize Markov perfect Nash equilibrium investment strategies we start with the final period and solve the game backwards. The profit at the upstate (denoted "u") for firm i is $\pi_{iu} = (\bar{p} - (q_{iu} + q_{ju})/(1+g) - c)q_{iu}$, and it is $\pi_{id} = (\bar{p} - (q_{id} + q_{jd}) - c)q_{id}$ when demand stays the same (denoted "d"). At the initial period firm i chooses the initial investment and output to maximize current and expected future profits, $\pi_i(.) = (\bar{p} - (q_{i0} + q_{j0}) - c)q_{i0} - fI_{i0} + \theta\pi_{iu}(.) + (1-\theta)\pi_{id}(.) + \lambda_{i0}(K_{i0} - q_{i0})$. There are several MPE investment strategies depending on whether constraints are binding or not in the final period.

i) When initial capacity can meet demand in the intermediate (and initial) state(s) but comes short to meet high demand in the upstate, that is $q^c_{kd} \leq K_{k0} < q^c_{ku}$ holds, firms find it profitable to invest at the outset to benefit from high demand in the following period. Once investment is made at time 0 and becomes productive at time 1, the upstate production constraint will bind for both players. That is, investment will be fully utilized and hence no idle capacity will be left out.

The first order condition for output choice in the upstate is $(\bar{p} - (2q_{iu} + q_{ju})/(1+g) - c) = 0$ for an interior solution. However, the output will be equal to the capacity as the constraint binds, hence $q_{iu} = K_{i0} + I_{i0}$.

The objective function to be maximized at the initial period will be a function of the state variables,

$\pi_i(.) = (\bar{p} - \sigma_0(q_{i0} + q_{j0}) - c)q_{i0} - fI_{i0} + \theta[(\bar{p} - (K_{i0} + I_{i0} + K_{ju} + I_{j0})/(1+g) - c)(K_{i0} + I_{i0})] + (1-\theta)[(\bar{p} - (q_{id} + q_{jd}) - c)q_{id} + \lambda_{i0}(K_{i0} - q_{i0})$.

The optimal investment at time 0 will satisfy

$\frac{\partial \pi_i}{\partial I_{i0}} = -f + \theta[(\bar{p} - (2K_{i0} + 2I_{i0} + K_{j0} + I_{j0})/(1+g) - c)] = 0$, which results in

(1a) $\quad I_{i0} = (1+g)(\bar{p} - c - f/\theta)/3 - K_{i0}$,

Let $\alpha = (1+g)(\bar{p} - c - f/\theta)$ be a constant, then $I_{i0} = \alpha/3 - K_{i0}$.

ii) When the initial capacity falls into the interval $0 \leq K_{k0} < q^c_{kd}$ the investment made at time 0 will benefit both up and intermediate demand states at time 1. In this low initial capacity case, both up and intermediate state capacity constraints will bind for both players.

With the binding constraints the profit in the upstate will be $v_{iu}(.) = (\bar{p} - (K_{i0} + I_{i0} + K_{ju} + I_{j0})/(1+g) - c)(K_{i0} + I_{i0})$, and the profit in the downstate will be $v_{id}(.) = (\bar{p} - (K_{i0} + I_{i0} + K_{ju} + I_{j0}) - c)(K_{i0} + I_{i0})$.

The objective function to be maximized in the initial period as a function of state variables is $\pi_i(.) = (\bar{p} - \sigma_0(q_{i0} + q_{j0}) - c)q_{i0} - fI_{i0} + \theta v_{iu}(.) + (1-\theta)v_{id}(.) + \lambda_{i0}(K_{i0} - q_{i0})$.

The derivative with respect to the optimal investment is

$\frac{\partial \pi_i}{\partial I_{i0}} = -f + \theta[(\bar{p} - (2K_{i0} + 2I_{i0} + K_{ju} + I_{j0})/(1+g) - c)] + (1-\theta)[(\bar{p} - (2K_{i0} + 2I_{i0} + K_{ju} + I_{j0}) - c)] = 0$, which results in

(1b) $\quad I_{i0} = \frac{(1+g)(\bar{p} - c - f)}{3(1+g-\theta g)} - K_{i0}$. Similarly we obtain the investment strategy for firm j, $I_{j0} = \frac{(1+g)(\bar{p} - c - f)}{3(1+g-\theta g)} - K_{j0}$.

iii) On the other hand, if the initial capacity is high, that is $q^c_{ku} \leq K_{k0}$ then firms do not invest in equilibrium: the capacity is sufficient to meet the maximum (Cournot) output, and any incremental investment will be idle at a positive cost. □

Proof of Proposition 2:

Given firm j's production capacity firm i's investment will benefit either upstate demand only or all demand states in period 1. Firm i's equilibrium investment quantities are characterized in a) and b).

a) Investment benefits upstate demand and hence only upstate production constraint binds for firm i .

The value function to be maximized in the initial period will be

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(I_{i0}, I_{j0}) + (1-\theta)v_{id}(I_{i0}, I_{j0}) + \lambda_{i0}(K_{i0} - q_{i0})$,

where $v_{iu}(I_{i0}) = (K_{i0} + I_{i0})(\bar{p} - c - (K_{i0} + I_{i0} + q_{ju})/(1+g))$ is the profit in the upstate, and the production quantities are $q_{ju} = ((\bar{p} - c)(1+g) - q_{iu})/2$, and $q_{iu} = K_{i0} + I_{i0}$.

Then $v_{iu}(I_{i0}) = (K_{i0} + I_{i0})[\bar{p} - c - (K_{i0} + I_{i0} + ((\bar{p} - c)(1+g) - K_{i0} - I_{i0})/2)/(1+g)]$

Also the profit expression in the intermediate state is

$v_{id}(I_{i0}, I_{j0}) = q_{id}(\bar{p} - q_{id} - q_{jd} - c)$, and the interior outputs hold $q_{id} < K_{i0} + I_{i0}$, and $q_{jd} < K_{j0}$.

The FOC $dv_{i0}/dI_{i0} = 0$ yields,

$-f + \theta[(\bar{p} - c)(1+g) - 2(K_{i0} + I_{i0})]/2(1+g) = 0$. Then the solution will be

(2a) $\qquad I_{i0} = \dfrac{(1+g)(\bar{p} - c) - 2f(1+g)/\theta}{2} - K_{i0}.$

b) As its initial capacity is "low", firm i's production is constrained, that is both up and intermediate state constraints bind. Since firm j's initial capacity is "large" (defined below) it plays its best response strategy. We optimize the investment choice for firm i.

The objective function in the upstate becomes

$v_{iu}(I_{i0}) = (K_{i0} + I_{i0})[(\bar{p} - c)(1+g) - (K_{i0} + I_{i0})]/2(1+g)$

In the intermediate state it becomes

$v_{id}(I_{i0}) = (K_{i0} + I_{i0})[(\bar{p} - c) - (K_{i0} + I_{i0})]/2$. Then the maximization of the objective function for firm i yields

$\dfrac{\partial \pi_i}{\partial I_{i0}} = -f + \theta[(\bar{p} - c)(1+g) - 2(K_{i0} + I_{i0})]/2(1+g) + (1-\theta)[(\bar{p} - c) - 2(K_{i0} + I_{i0})]/2 = 0$, and $q_{iu} = K_{i0} + I_{i0} = q_{id}$, and $q_{ju} = ((\bar{p} - c)(1+g) - q_{iu})/2$ and $q_{jd} = (\bar{p} - c - q_{id})/2$. Then the MPE investment for firm i becomes

(2b) $\qquad I_{i0} = \dfrac{(1+g)(\bar{p} - c - 2f)}{2(1+g-g\theta)} - K_{i0}.$

Note that the output of firm j in the upstate will be $q_{ju} = ((\bar{p} - c)(1+g) - q_{iu})/2$, where $q_{iu} = K_{i0} + I_{i0}$. The highest level of output for firm j in the upstate will be obtained when q_{iu} is the lowest, which happens when both up and intermediate state capacity constraints bind for firm i. This corresponds to $q_{iu} = (1+g)(\bar{p} - c - 2f)/2(1+g-g\theta) = \beta$. We then obtain the upper bound of the firm j's initial capacity $K_{j0} \geq ((\bar{p} - c)(1+g) - \beta)/2 = (3q^c_{ju} - \beta)/2$, which ensures that firm j never invests and produces at the interior output level. $\square$

Proof of Proposition 3:

The MPE solution involves several equilibrium possibilities due to the capacity constraints: At each decision stage (node on the demand tree) there are two possible outcomes; either production is interior or constrained by the capacity. Since we have three demand states in the two periods, we have eight output possibilities. However, only several of them are feasible and part of the equilibrium behavior. The feasible investment scenarios will emerge under the following conditions: *i*) All capacity constraints bind in the two periods; *ii*) The production constraints bind only in the upstate demand; *iii*) The constraints never bind so that production is interior and there is no investment. These cases are mutually exclusive and will result in different investment strategies. Other scenarios such as binding constraints in the initial and the second period upstate demands are ruled out because of contradicting capacity constraints.

The objective function to be maximized in the second period upstate demand for firm i will be,

$v_{iu} = q_{iu}(\bar{p} - (q_{iu} - q_{ju})/(1+g) - c) + \lambda_{iu}(K_{i0} + I_{i0} - q_{iu})$.

In the second period intermediate state it will be,

$v_{id} = q_{id}(\bar{p} - q_{id} - q_{jd} - c) + \lambda_{id}(K_{i0} + I_{i0} - q_{id})$.

The conditions under which firms only invest at the outset will be derived below. We start with examining three cases separately and characterize equilibrium investment strategies.

i) When the initial capacity for any firm, say firm i, is low enough, that is $K_{i0}+I_{i0} \leq q^c_{id}$ satisfied, the investment will benefit the initial node as well as up and intermediate states in the second period. That is, initial investment will benefit all demand states and the production constraints will be binding.

The objective function to be maximized for firm i in the initial period will be,

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(I_{i0},\, I_{j0}) + (1-\theta)v_{id}(I_{i0},\, I_{j0}) + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0})$.

where $i \neq j$ and $v_{iu}(I_{i0}, I_{j0}) = [(K_{i0} + I_{i0})(\bar{p} - (K_{i0} + I_{i0} + K_{j0} + I_{j0})/(1+g) - c)]$, and

$v_{id}(I_{i0}, I_{j0}) = [(K_{i0} + I_{i0})(\bar{p} - K_{i0} - I_{i0} - K_{j0} - I_{j0} - c)]$.

The optimality condition for investment choice is $dv_{i0}/dI_{i0} = 0$ which yields

(3a) $-f+\lambda_{i0}+\theta(\bar{p}-(2(K_{i0}+I_{i0})+K_{j0}+I_{j0})/(1+g)-c)+(1-\theta)(\bar{p}-2(K_{i0}+I_{i0})-K_{j0}-I_{j0}-c) = 0$.

When the constraint at the initial stage binds, $(K_{i0} + I_{i0} - q_{i0}) = 0$, we will have $\lambda_{i0} \geq 0$, which can be obtained by solving,

$dv_{i0}/dq_{i0} = 0$: $\bar{p} - c - 2(K_{i0} + I_{i0}) - K_{j0} - I_{j0} - \lambda_{i0} = 0$.

Inserting this into (3a) and solving the same problem for player j, we obtain

$-f + (\bar{p} - c - 3(K_{i0} + I_{i0})) + \theta(\bar{p} - c - 3(K_{i0} + I_{i0})/(1+g)) + (1-\theta)(\bar{p} - c - 3(K_{i0} + I_{i0})) = 0$.

This yields the optimal investment function for firm $k = i,\, j,\, i \neq j$

$I_{k0} = \dfrac{(1+g)(2(\bar{p}-c) - f)}{6(1+g) - 3\theta g} - K_{k0}$.

This is the investment strategy when the production constraints bind in all demand states. To have this investment scenario hold, $K_{i0} + I_{i0} \leq q^c_{i0}$ must be satisfied.

ii) Since the highest demand level is reached in the second period upstate, one should consider whether it is optimum to invest in the initial period and have the capacity fully utilized in the upstate. This case occurs when the condition $q^c_{kd} \leq K_{k0} < q^c_{ku}$ holds, that is when the initial capacity is high enough so that it does not benefit the intermediate or initial state but low enough so that initial investment provides benefit in the upstate only.

For firm i the value function to be maximized in the initial period will be

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(I_{i0}, I_{j0}) + (1-\theta)v_{id}(I_{i0}, I_{j0}) + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0})$,

where $v_{iu}(I_{i0}, I_{j0}) = (K_{i0} + I_{i0})(\bar{p} - (K_{i0} + I_{i0} + K_{j0} + I_{j0})/(1+g) - c)$, and $v_{id}(I_{i0}, I_{j0}) = q_{id}(\bar{p} - q_{id} - q_{jd} - c)$.

The necessary condition $dv_{i0}/dI_{i0} = 0$ yields,

(3b) $\quad -f + \lambda_{i0} + \delta\theta(\bar{p} - (2(K_{i0} + I_{i0}) + K + I_{j0})/(1+g) - c) = 0$, where $\lambda_{i0} = 0$ holds. Then the solution for both firms are,

$I^{MP}_{k0} = \dfrac{(1+g)(\bar{p} - c - f/\theta)}{3} - K_{k0}$, $k = i, j, i \neq j$.

This is the investment quantity when the production constraint binds only in the upstate.

In the final case *iii*), the investment quantity is zero in equilibrium because production constraints never bind. This is due to high initial capacities. □

Proof of Proposition 4:

If firm j's production capacity is high so that $K_{j0} \geq ((\bar{p} - c)(1+g) - \gamma)/2$ is satisfied then firm i's investment will hold either a) or b) below.

a) Firm i's production constraint binds upstate only.

The value function to be maximized in the initial period will be

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(I_{i0}, I_{j0}) + (1-\theta)v_{id}(I_{i0}, I_{j0}) + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0})$,

where $v_{iu}(I_{i0}, I_{j0}) = (K_{i0} + I_{i0})(\bar{p} - (K_{i0} + I_{i0} + q_{ju})/(1+g) - c)$, and $q_{ju} = ((\bar{p} - c)(1+g) - q_{iu})/2$, and $q_{iu} = K_{i0} + I_{i0}$. Also

$v_{id}(I_{i0}, I_{j0}) = q_{id}(\bar{p} - q_{id} - q_{jd} - c)$, where $q_{id} < K_{i0} + I_{i0}$, and $q_{jd} < K_{j0}$.

The FOC $dv_{i0}/dI_{i0} = 0$ yields,

$-f + \lambda_{i0} + \theta[(\bar{p} - c)(1+g) - 2(K_{i0} + I_{i0})]/2(1+g) = 0$, where $\lambda_{i0} = 0$.

The solution will be

(4a) $\qquad I_{i0} = \dfrac{(1+g)(\bar{p} - c) - 2f(1+g)/\theta}{2} - K_{i0}$.

b) Capacity constraints bind in all states.

The value function to be maximized in the initial period will be

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(I_{i0}, I_{j0}) + (1-\theta)v_{id}(I_{i0}, I_{j0}) + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0})$,

where $v_{iu}(I_{i0}, I_{j0}) = (K_{i0}+I_{i0})(\bar{p}-(K_{i0}+I_{i0}+q_{ju})/(1+g)-c)$, and $q_{ju} = ((\bar{p}-c)(1+g)-q_{iu})/2$, and $q_{iu} = K_{i0} + I_{i0}$. Then, $v_{iu}(I_{i0}, I_{j0}) = (K_{i0} + I_{i0})((\bar{p} - c)(1 + g) - (K_{i0} + I_{i0}))/2(1 + g)$,Also

$v_{id}(I_{i0}, I_{j0}) = q_{id}(\bar{p} - q_{id} - q_{jd} - c) = (K_{i0} + I_{i0})(\bar{p} - c - K_{i0} - I_{i0} - (\bar{p} - c - K_{i0} - I_{i0})/2)$, and

The optimization $dv_{i0}/dI_{i0} = 0$ yields,

The maximization of the objective function for firm i yields

$\frac{\partial \pi_i}{\partial I_{i0}} = (\bar{p} - c - 2q_{i0} - q_{j0})\partial q_{i0}/\partial I_{i0} - f + \theta[(\bar{p} - c)(1 + g) - 2(K_{i0} + I_{i0})]/2(1 + g) + (1-\theta)[(\bar{p} - c) - 2(K_{i0} + I_{i0})]/2 = 0$. Also,

$q_{iu} = K_{i0} + I_{i0} = q_{id}$, and $q_{ju} = ((\bar{p} - c)(1 + g) - q_{iu})/2$, and $q_{jd} = (\bar{p} - c - q_{id})/2$, and $q_{i0} = K_{i0} + I_{i0}$, and $q_{j0} = (\bar{p} - c - q_{i0})/2$. Then the MPE investment for firm i boils down to

(4b) $$I_{i0} = \frac{2(1+g)(\bar{p} - c - f)}{5(1+g) - 2g\theta} - K_{i0}.$$

Note that the output of firm j in the upstate will be $q_{ju} = ((\bar{p} - c)(1 + g) - q_{iu})/2$, where $q_{iu} = K_{i0} + I_{i0}$. The highest level of output for firm j in the upstate will be obtained when q_{iu} is the lowest, which happens when both up and intermediate state constraints for firm i bind at the second stage. This corresponds to $q_{iu} = 2(1+g)(\bar{p}-c-f)/5(1+g) - 2g\theta \equiv \gamma$. Then we obtain the upper bound of firm j's initial capacity $K_{j0} \geq ((\bar{p} - c)(1 + g) - \gamma)/2 = (3q^c_{ju} - \gamma)/2$, which ensures that firm j's production is interior and never invests. $\square$

Proof of Proposition 5:

If firms are investing at the second period, then they are facing the high demand market that is the upstate demand. If both firms invest then their production constraints will be binding in the upstate because firms' investments will be fully utilized and there will be no idle capacity left.

In the case of investment benefiting the upstate only, the value function to be maximized for firm i will be

$v_{iu}(I_{i0}, I_{j0}) = (K_{i0} + I_{i0})(\bar{p} - c - (K_{i0} + I_{i0} + K_{j0} + I_{j0})/(1 + g)) - fI_{iu}$.

The necessary condition $dv_{iu}/dI_{iu} = 0$ yields,

$[\bar{p} - c - (2(K_{i0} + I_{iu}) + K_{j0} + I_{ju})/(1 + g) - f] = 0$.

Then MPE investment solution for both firms is

$I_{ku} = \frac{(1+g)(\bar{p} - c - f)}{3} - K_{k0}$, $k = i,\, j,\, i \neq j$. $\square$

Proof of Proposition 7:

In characterizing the OLNE investments, we will examine the same cases we studied for MPE characterization.

The profit function to be maximized for player i is

$\pi_i(.) = (\bar{p} - \sigma_0(q_{i0} + q_{j0}) - c)q_{i0} - fI_{i0} + \theta[(\bar{p} - (q_{iu} + q_{ju})/(1+g) - c)q_{iu}] + (1-\theta)[(\bar{p} - (q_{id} + q_{jd}) - c)q_{id} + \Lambda$, where Λ is a function of the Lagrange multipliers and equals

$\Lambda = \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{iu}(K_{i0} + I_{i0} - q_{iu}) + \lambda_{id}(K_{i0} + I_{i0} - q_{id})$.

Case 1: Upstate constraint is binding for both players.

The first order conditions for player i are

$\dfrac{\partial \pi_i}{\partial I_{i0}} = -f + \lambda_{iu} = 0$, and $\dfrac{\partial \pi_i}{\partial q_{iu}} = \theta[\bar{p} - (2q_{iu} + q_{ju})/(1+g) - c] - \lambda_{iu} = 0$ and $q_{iu} = K_{i0} + I_{i0}$. Solving them simultaneously yields

$I_{i0} = (\alpha - 2K_{i0} - K_{j0} - I_{j0})/2$, for $i, j = 1, 2$, where $\alpha = (1+g)(\bar{p} - c - f/\theta)$

Solving investment expressions for both players, the equilibrium investment function will be

(7a) $\quad I_{k0} = \alpha/3 - K_{k0},\ k = i,\ j,\ i \neq j.$

Clearly, when investment is benefiting the upstate only the investment rules are identical under both open-loop and Markov perfect information structures.

Case 2: Both up and intermediate demand states bind for both players.

The maximization of the objective function for firm i yields

$\dfrac{\partial \pi_i}{\partial I_{i0}} = -f + \lambda_{iu} + \lambda_{id} = 0$, and $\dfrac{\partial \pi_i}{\partial q_{iu}} = \theta[\bar{p} - (2q_{iu} + q_{ju})/(1+g) - c] - \lambda_{iu} = 0$ and $\dfrac{\partial \pi_i}{\partial q_{id}} = (1-\theta)[\bar{p} - (2q_{id} + q_{jd}) - c] - \lambda_{id} = 0$, and

$q_{iu} = K_{i0} + I_{i0} = q_{id}$. The investment expression for firm i becomes,

$I_{i0} = -K_{i0} - \dfrac{(K_{j0} + I_{j0})}{2} + \dfrac{(1+g)(\bar{p} - c - f)}{2(1+g-g\theta)}$. Similarly we can obtain the investment expression for firm j. Solving them simultaneously yields the optimal OLNE investments,

(7b) $\quad I_{i0} = \dfrac{(1+g)(\bar{p} - c - f)}{3(1+g-\theta g)} - K_{i0}$, and $I_{j0} = \dfrac{(1+g)(\bar{p} - c - f)}{3(1+g-\theta g)} - K_{j0}$.

Note that investments will be identical for both firms as long as initial capacities are the same.

Proof of Proposition 8:

Case A: This is the case in which all capacity constraints bind in all periods.

The objective function to be maximized in the initial period is,

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(.) + (1-\theta)v_{id}(.) + \sum_t \lambda_{it}(K_{it} + I_{it} - q_{it})$,

where $v_{iu}(I_{i0}, I_{j0}) = [(K_{i0} + I_{i0})(\bar{p} - c - (K_{i0} + I_{i0} + K_{j0} + I_{j0})/(1+g))]$, and

$v_{id}(I_{i0}, I_{j0}) = [(K_{i0} + I_{i0})(\bar{p} - K_{i0} - I_{i0} - K_{j0} - I_{j0} - c)]$.

The first order necessary condition $dv_{i0}/dI_{i0} = 0$ yields

$-f + \lambda_{i0} + \lambda_{iu} + \lambda_{id} = 0, \quad i, j = 1, 2, \quad i \neq j$, where

$\lambda_{iu} = \theta(\bar{p} - (2(K_{i0} + I_{i0}) + K_{j0} + I_{j0})/(1+g) - c)$, and $\lambda_{id} = (1-\theta)(\bar{p} - 2(K_{i0} + I_{i0}) - K_{j0} - I_{j0} - c)$, $\lambda_{i0} = (\bar{p} - 2(K_{i0} + I_{i0}) - K_{j0} - I_{j0} - c)$.

When the constraints bind, we will have λ_{i0}, λ_{iu}, $\lambda_{id} \geq 0$.

Inserting these into the above first order condition and solving the same problem for player j, we obtain

$$I_{k0}^{OL} = \frac{(1+g)(2(\bar{p} - c) - f)}{6(1+g) - 3\theta g} - K_{k0},\ k = i,\ j.$$

Observe that the open-loop investment strategy is exactly same as the Markov perfect investment strategy characterized earlier. This investment will hold for the same parameter region as defined for the Markov perfect investment.

Case B: Since the highest demand level is reached in the second period upstate, one should consider whether it is optimal to invest in the initial period and have the capacity constraint binding in the upstate demand alone, given that firms employ open-loop investment strategies.

The first order necessary condition $dv_{i0}/dI_{i0} = 0$ yields

$(B.2) \quad -f + \lambda_{i0} + \lambda_{iu} + \lambda_{id} = 0, \quad i, j = 1, 2, \quad i \neq j.$

$\lambda_{iu} = \theta(\bar{p} - (2(K_{i0} + I_{i0}) + K + I_{j0})/(1+g) - c) \geq 0$, and $\lambda_{id} = 0$, $\lambda_{i0} = 0$.

Inserting them into (B.2) and solving the same problem for player j, we obtain

$$I_{i0}^{OL} = \frac{(1+g)(\bar{p} - c - f/\theta)}{3} - K_{i0}, \quad i, j = 1, 2, \quad i \neq j.$$

Clearly this investment expression is identical to the MPE investments.□

Proof of Proposition 9:

Open-loop Solution

All capacity constraints bind in the second period upstate

The objective function to be maximized in the initial period will be,

$$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) + \theta v_{iu}(.) + (1-\theta) v_{id}(.) + \sum_t \lambda_{it}(K_{it} - q_{it}).$$

where $v_{iu}(I_{i0}, I_{j0}) = [q_{iu}(\bar{p} - c - (q_{iu} + q_{ju})/(1+g)) - f I_{iu}]$, and

$v_{id}(I_{i0}, I_{j0}) = [q_{id}(\bar{p} - q_{i0} - q_{j0} - c)]$.

The first order necessary condition $dv_{i0}/dI_{iu} = 0$ yields $\lambda_{iu} - \theta f = 0, \quad i, j = 1, 2, \quad i \neq j$, where

$\lambda_{iu} = \theta(\bar{p} - (2(K_{i0} + I_{i0}) + K_{i0} + I_{j0})/(1+g) - c)$.

Solving for the optimal investment leads to

$I_{iu}^{OL} = \dfrac{(1+g)(\bar{p}-c-f)}{3} - K_{i0}, \quad i,j = 1,2, \quad i \neq j$. This result is equivalent to the MPE solution characterized in Proposition 5. □

Proof of Proposition 10:

We first start the equilibrium investment analysis with time-to-build. We will prove this when only upstate production constraint binds for firm i. The proof when they bind in other states is similar (see the proofs of Propositions 2 and 4) and hence it is omitted.

With time-to-build investment

Under *Markov perfect equilibrium*, the investment decisions are made as follows. The objective function to be maximized in the initial period will be a function of state variables,

$\pi_i(.) = (\bar{p} - \sigma_0(q_{i0}+q_{j0}) - c)q_{i0} - fI_{i0} + \theta[(\bar{p} - c - (K_{i0}+I_{i0}+q_{ju})/(1+g))(K_{i0}+I_{i0})] + (1-\theta)[(\bar{p} - (q_{id}+q_{jd}) - c)q_{id} + \lambda_{i0}(K_{i0} - q_{i0})$, where $q_{ju} = ((\bar{p}-c)(1+g) - K_{i0} - I_{i0})/2$ which is the best response function of firm j at the upstate.

The derivative with respect to the investment is

$\dfrac{\partial \pi_i}{\partial I_{i0}} = -f + \theta[\bar{p} - c - (K_{i0}+I_{i0}+(\bar{p}-c)(1+g))/2(1+g) - (K_{i0}+I_{i0})/2(1+g)] = 0$. Solving for investment quantity leads to

$I_{i0}^{M} = \dfrac{(\alpha - f(1+g)/\theta)}{2} - K_{i0}$.

Next we analyze *open-loop equilibrium* investment strategy. The first order conditions for player i are

$\dfrac{\partial \pi_i}{\partial I_{i0}} = -f + \lambda_{iu} = 0$, and $\dfrac{\partial \pi_i}{\partial q_{iu}} = \theta[\bar{p} - (2q_{iu}+q_{ju})/(1+g) - c] - \lambda_{iu} = 0$ where $q_{iu} = K_{i0} + I_{i0}$, and $q_{ju} = ((\bar{p}-c)(1+g) - K_{i0} - I_{i0})/2$. The equilibrium investment will be

$I_{i0}^{O} = \dfrac{(\alpha - f(1+g)/\theta)}{3} - K_{i0}$, where $\alpha = (1+g)(\bar{p} - c - f/\theta)$.

Clearly $I_{i0}^{M} > I_{i0}^{O}$. That is, MPE investment exceeds OLNE one.

Note that we have already analyzed the case in which both constraints bind and one of the firms invests in the earlier section.

With instantaneous investment

The corresponding scenario in the instantaneous investment would be the case in which firm i invests at $t = 1$ high demand state only and its capacity constraint binds in that state.

Under *Markov perfect equilibrium*, the investment strategy is obtained as follows. The value function to be maximized in the initial period is

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) + \theta v_{iu}(.) + (1-\theta)v_{id}(.) + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0}),$

where $v_{iu}(I_{i0}) = (K_{i0} + I_{iu})(\bar{p} - c - (K_{i0} + I_{iu} + q_{ju})/(1+g)) - fI_{iu}$, and $q_{ju} = ((\bar{p}-c)(1+g) - K_{i0} - I_{iu})/2$, also $v_{id}(.) = q_{id}(\bar{p} - q_{id} - q_{jd} - c)$.

The FOC $dv_{i0}/dI_{iu} = 0$ yields,

$\theta[\bar{p} - c - (K_{i0} + I_{iu} + (\bar{p}-c)(1+g))/2(1+g) - (K_{i0} + I_{iu})/2(1+g) - f] = 0.$

The solution is

$I_{iu}^{M} = \dfrac{(1+g)(\bar{p} - c - 2f)}{2} - K_{i0}.$

Under *open-loop equilibrium,* the investment profile is characterized as follows. Note that all capacity constraints bind in the second period upstate.

The objective function to be maximized in the initial period will be,

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) + \theta v_{iu}(.) + (1-\theta)v_{id}(.) + \sum_t \lambda_{it}(K_{it} - q_{it}).$

where $v_{iu}(I_{i0}, I_{j0}) = [q_{iu}(\bar{p} - c - (q_{iu} + q_{ju})/(1+g)) - fI_{iu}]$, and $v_{id}(I_{i0}, I_{j0}) = [q_{id}(\bar{p} - q_{i0} - q_{j0} - c)]$. The first order necessary conditions $dv_{i0}/dI_{iu} = 0$ yield

$\lambda_{iu} - \theta f = 0,$

and $\lambda_{iu} = \theta(\bar{p} - (2q_{iu} + q_{ju})/(1+g) - c)$, $q_{iu} = K_{i0} + I_{iu}$, and $q_{ju} = ((\bar{p}-c)(1+g) - q_{iu})/2$

Solving for the equilibrium investment results in

$I_{iu}^{O} = \dfrac{(1+g)(\bar{p} - c - 2f)}{3} - K_{i0}.$

An alternative scenario can also emerge. In the instantaneous investment case it is likely that a firm invests at $t = 0$ and its capacity constraint binds in the upstate demand alone, because the highest demand level is reached in the second period upstate.

The corresponding *Markov perfect equilibrium* investment strategy has the following property.

The value function to be maximized in the initial period will be

$v_{i0} = q_{i0}(\bar{p} - q_{i0} - q_{j0} - c) - fI_{i0} + \theta v_{iu}(.) + (1-\theta)v_{id}(.) + \lambda_{i0}(K_{i0} + I_{i0} - q_{i0}),$

where $v_{iu}(I_{i0}, I_{j0}) = (K_{i0} + I_{i0})(\bar{p} - c - (K_{i0} + I_{i0} + q_{ju})/(1+g))$, and $v_{id}(.) = q_{id}(\bar{p} - q_{id} - q_{jd} - c)$.

The FOC $dv_{i0}/dI_{i0} = 0$ yields,

$-f + \lambda_{i0} + \theta[\bar{p} - c - (K_{i0} + I_{i0} + (\bar{p}-c)(1+g))/2(1+g) - (K_{i0} + I_{i0})/2(1+g)] = 0$, where assume interior initial output without loss of generality and hence $\lambda_{i0} = 0$.

The equilibrium is

$I_{i0}^{M} = \dfrac{(1+g)(\bar{p} - c - 2f/\theta)}{2} - K_{i0}.$

Also, the corresponding *open-loop equilibrium* investment strategy is obtained as follows.

The first order necessary condition $dv_{i0}/dI_{i0} = 0$ yields

$-f + \lambda_{i0} + \lambda_{iu} = 0$, where

$\lambda_{iu} = \theta(\bar{p} - c - (2(K_{i0} + I_{i0}) + ((\bar{p} - c)(1 + g) - (K_{i0} + I_{i0}))/2)/(1 + g)) = 0$, and $\lambda_{i0} = 0$.

The equilibrium investment will be

$I^O_{i0} = \dfrac{(1+g)(\bar{p} - c - 2f/\theta)}{3} - K_{i0}$.

Clearly, for all cases examined the MPE investments are higher than the OLNE investments. □

Proof of Proposition 11:

Markov Perfect Equilibrium (MPE) Solution

Since the production constraints at the up-up (at time 2) and up (at time 1) states will be binding for both players, the profit expression in the final period up-up state is $\pi_{iuu} = (K_{i0} + I_{i0} + I_{iu})(\bar{p} - (K_{i0} + I_{i0} + I_{iu} + K_{j0} + I_{j0} + I_{ju})/(1 + g)^2 - c)$ and the expected profit at the first period up-state is $\pi_{iu} = (K_{i0} + I_{i0})(\bar{p} - (K_{i0} + I_{i0} + K_{j0} + I_{j0})/(1 + g) - c) - fI_{iu} + \theta\pi_{iuu} + (1 - \theta)\pi_{iud}$, where π_{iud} is the profit in the up-intermediate state that does not include any investment term in it as the constraints do not bind in this state. The profit function to be maximized at the initial period is, $\pi_{i0}(.) = q_{i0}(\bar{p} - (q_{i0} + q_{j0}) - c) - fI_{i0} + \theta\pi_{iu}(.) + (1 - \theta)\pi_{id}(.) + \lambda_{i0}(K_{i0} - q_{i0})$, where the expression π_{id} is the profit function at the intermediate state which is independent of the investment terms as the constraints in the intermediate state do not bind.

The optimality condition for the upstate investment is, $\dfrac{\partial \pi_{iu}}{\partial I_{iu}} = -f + \theta[(\bar{p} - (2K_{i1} + 2I_{iu} + K_{j1} + I_{ju})/(1 + g)^2 - c)] = 0$, where $K_{i1} = K_{i0} + I_{i0}$. This results in $I_{iu} = \dfrac{(m - f/\theta)(1 + g)^2}{2} - \dfrac{(2K_{i1} + K_{j1} + I_{ju})}{2}$. For firm j $I_{ju} = \dfrac{(m - f/\theta)(1 + g)^2}{2} - \dfrac{(2K_{j1} + K_{i1} + I_{iu})}{2}$. Solving them together leads to $I_{iu}(I_{i0}) = (m - f/\theta)(1 + g)^2/3 - K_{i1}(I_{i0})$ for $i, j = 1, 2,\ i \neq j$.

Then $\dfrac{\partial I_{iu}}{\partial I_{i0}} = -1$, and $\dfrac{\partial I_{ju}}{\partial I_{i0}} = 0$.

The optimality condition for the initial period investment is, $\dfrac{d\pi_{i0}}{dI_{i0}} = \dfrac{\partial \pi_{i0}}{\partial I_{i0}} + \theta\dfrac{\partial \pi_{iu}}{\partial I_{i0}} = 0$, as the production constraints bind only in the up (u) and up-up (uu) states.

$\dfrac{\partial \pi_{iu}}{\partial I_{i0}} = -f\dfrac{\partial I_{iu}}{\partial I_{i0}} + (m - (2(K_{i0} + I_{i0}) + K_{j0} + I_{j0})/(1 + g) + \theta\dfrac{\partial \pi_{iuu}}{\partial I_{i0}}$

Given I_{j0} we optimize with respect to I_{i0}: $\dfrac{\partial \pi_{iuu}}{\partial I_{i0}} = (1 + \dfrac{\partial I_{iu}}{\partial I_{i0}})(m - (K_{i0} + I_{i0} + I_{iu} + K_{j0} + I_{j0} + I_{ju})/(1 + g)^2) + (K_{i0} + I_{i0} + I_{iu})(0 - (1 + \dfrac{\partial I_{iu}}{\partial I_{i0}})/(1 + g)^2) = 0$ due to $\dfrac{\partial I_{iu}}{\partial I_{i0}} = -1$.

Then $\dfrac{d\pi_{i0}}{dI_{i0}} = -f + \theta[(m - (2(K_{i0} + I_{i0}) + K_{j0} + I_{j0})/(1+g) + f] + \theta^2[0] = 0$. Inserting $I_{iu}(I_{i0})$, and solving for I_{i0} results in the MPE initial investment,

$I_{i0} = \dfrac{(1+g)}{3\theta}(\theta(m+f) - f) - K_{i0} = \alpha/3 - K_{i0} + f(1+g)/3$. The investment in the upstate will be

$I_{iu} = \dfrac{(m - f/\theta)(1+g)^2}{3} - \dfrac{(1+g)}{3\theta}(\theta(m+f) - f) = [g(\alpha - f) - f]/3.$

Open-loop Equilibrium Solution

The profit expressions are similar to the ones in the MPE analysis. The difference is the solution procedure. MPE uses backward solution, whereas OLNE uses forward solution where decisions are made at the outset.

The profit function at the period one upstate is,

$v_{iu} = (\bar{p} - (q_{iu} + q_{ju})/(1+g) - c)q_{iu} - fI_{iu} + \theta[(\bar{p} - (q_{iuu} + q_{juu})/(1+g)^2 - c)q_{iuu} + (1-\theta)v_{iud}(.)$

subject to

$q_{iu} \leqslant K_{iu}$, and $q_{iuu} < K_{iu} + I_{iu}$, and $q_{iud} \leqslant K_{iu} + I_{iu}$.

In this profit function, the term $v_{iud}(.)$is the profit at the state up-intermediate (ud) and it is not a function of investment level as the production constraint is interior.

The profit function at the period one intermediate state is,

$v_{id} = (\bar{p} - (q_{id} + q_{jd}) - c)q_{id} - fI_{id} + \theta[(\bar{p} - (q_{idu} + q_{jdu})/(1+g) - c)q_{idu} + (1-\theta)v_{idd}(.)$

subject to

$q_{id} \leqslant K_{id}$, and $q_{idu} < K_{id} + I_{id}$, and $q_{idu} \leqslant K_{id} + I_{id}$,

where $v_{idd}(.)$ is the profit at the intermediate-intermediate state.

The profit function to be maximized at the outset of the game is,

$\pi_{i0}(.) = (\bar{p} - (q_{i0} + q_{j0}) - c)q_{i0} - fI_{i0} + \theta v_{iu}(.) + (1-\theta)v_{id}(.) + \lambda_{i0}(K_{i0} - q_{i0})$.

The first order profit maximizing conditions for firm i, with the binding up and up-up states, will be

$\dfrac{\partial\pi_{i0}}{\partial I_{i0}} = -f + \lambda_{iuu} + \lambda_{iu} = 0$, and $\dfrac{\partial\pi_{i0}}{\partial q_{iu}} = \theta[\bar{p} - (2q_{iu} + q_{ju})/(1+g) - c] - \lambda_{iu} = 0$ and $\dfrac{\partial\pi_{i0}}{\partial q_{iuu}} = \theta^2[\bar{p} - (2q_{id} + q_{jd})/(1+g)^2 - c] - \lambda_{iuu} = 0$, and

$\dfrac{\partial\pi_{i0}}{\partial I_{iu}} = -\theta f + \lambda_{iuu} = 0$, where the λ terms are the multipliers of the production constraints at the corresponding states. Also

$q_{iu} = K_{i0} + I_{i0} = q_{id}$, and $q_{iuu} = K_{i0} + I_{i0} + I_{iu} = q_{iud}$.

The solution is

$I_{i0} = (m\theta - f + f\theta)(1+g)/3\theta - K_{i0} = \alpha/3 - K_{i0} + f(1+g)/3$, where $\alpha = (1+g)(m - f/\theta)$, where $m = \bar{p} - c$ and

$I_{iu} = [g(\alpha - f) - f]/3$. The investment in the upstate is independent of the initial capacity.□

Proof of Proposition 12:

In the three period planning the optimality conditions are,

$q_0 = \bar{p} - c$ for the initial output, $-f + \lambda_u + \lambda_{uu} = 0$ for the initial investment, and $\theta(\bar{p} - c - q_u/(1+g)) = \lambda_u$ and $\theta^2(\bar{p} - c - q_{uu}/(1+g)^2) = \lambda_{uu}$ for the upstate and the up-upstate outputs, resp. Also, the optimality condition for the upstate investment satisfies $-\theta f + \lambda_{uu} = 0$. When the production constraints is binding in the upstate and up-upstate, the optimal investments will be $I_0 = (1+g)(\bar{p} - c) - f(1-\theta)(1+g)/\theta - K_0$ at the initial period and $I_u = (1+g)^2(\bar{p} - c - f/\theta) - (1+g)(\bar{p} - c - f(1-\theta)/\theta)$. Hence, the total capacity at the final period will be $K_0 + I_0 + I_u = (1+g)^2(\bar{p} - c - f/\theta)$. Note that second period investment is independent of the initial capacity, and the total capacity at the final period is also irrelevant of the initial capacity.□