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Emission Taxes and Damage Thresholds in the Presence of Pre-existing Regulations

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EMISSION TAXES AND DAMAGE THRESHOLDS IN THE PRESENCE OF PRE-EXISTING REGULATIONS

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Abstract: This paper makes two contributions to the economics of pollution policy. First, many studies have looked at the effects of emission taxes in the absence of regulations and vice versa, but the implications for optimal tax design when one is layered on top of the other have been ignored, even though the practice is commonly observed. I develop a model of multiple polluting sectors capable of providing a tractable characterization of this case. Second, numerical modeling has shown that tax interactions can yield a positive damage threshold below which any emission tax is welfare-reducing even if marginal damages are positive, but this has largely been ignored in both the theoretical and policy literatures. I show that a positive damage threshold occurs when the policy is not revenue-raising and/or the rest of the tax system is not optimized, but can also occur in a second-best context with optimal taxes and full revenue-recycling, a result not previously shown. Introducing a pollution tax when one firm is already subject to an emissions constraint yields a positive damage threshold that goes up, the more the regulation distorts the income tax base. Hence, under more general conditions than have previously been realized, pollution taxes are not guaranteed to raise welfare even when marginal damages are positive and revenues are fully recycled.

Key words: emissions taxes, tax interactions, second-best, carbon taxes

JEL Codes: H21, H23, Q54, Q58

1 INTRODUCTION

A large literature has shown that the classical Pigovian prescription of an emissions tax τ equal to the marginal damages (MD) of pollution emissions holds only under very limited conditions and otherwise must be modified to take into account interactions with the rest of the tax system and other real-world considerations. The variations are of the form $\tau = aMD - b$ where a and b are coefficients that depend on the characteristics of the rest of the economy. Sandmo (1975) analyzed a consumption-based externality in a second-best economy in which the government revenue requirement and the no lump-sum tax rule necessitates positive ad valorem commodity and income taxes. He showed that, after certain simplifications, $b = 0$ and a equals the inverse of the marginal cost of public funds ($MCPF$). Bovenberg and Goulder (1996) derived the same result in a model where a polluting intermediate input is taxed. Numerous other authors have explored these issues under similar specifications with comparisons to Command-and Control (CAC) regulations or tradable quotas (Bovenberg & de Mooij 1994, Fullerton & Metcalfe 1997, Parry 1997, Parry et al. 1999, Goulder 1998, Metcalfe 2003, Schöb 2003). A common finding is that the welfare gain from reducing a pollution externality is at least partly offset by welfare losses from increased distortions in the tax system so the pollution tax should be less than the Pigovian level ($a < 1$). Revenue-raising instruments (taxes and auctioned permits) allow for reductions in other taxes, making non revenue-raising policies like tradable quotas and CAC regulations more costly by comparison.

Theoretical treatments, such as Sandmo (1975), Bovenberg and Goulder (1996) and Schöb (2003), tend to yield results characterized by the combination ($a < 1, b = 0$), in which the tax interaction and revenue-recycling effects go to zero when the emission tax is zero so that a small emission tax must improve welfare as long as $MD > 0$. Bovenberg and Goulder (1996) also presented numerical simulations of carbon taxes in the US economy and found cases where $b > 0$, implying the existence of a positive net distortion at the unregulated emission level, such that even if MD were positive but below the threshold value $Z = b/a$ the optimal carbon tax would be zero. This arose in cases in which the rest of the tax system is assumed not to be optimal and/or the emissions policy is non-revenue raising, such as with tradable quotas. In these cases an optimal tax perturbation may involve increasing factor taxes and subsidizing the polluting good. Adding a non-negativity constraint on the emissions tax then creates the discontinuity at the unregulated emissions level, which implies the first unit of abatement has a strictly positive welfare cost. The size of the threshold was policy-dependent, but in the case of tradable quotas was large enough to dwarf many estimates of the current social cost of carbon (SCC) (Inter-Agency Working Group 2013), implying that any non-revenue raising carbon dioxide emission reduction policy is welfare-reducing. Parry et al. (1999), Goulder (1998), and Goulder (2013) discussed this point further, emphasizing that it arises under non-revenue-raising policies because the tax interaction costs are not offset by benefits from revenue-recycling.

The various outcomes are summarized in Figures 1 and 2. In Figure 1 the MD line is assumed to be horizontal, the line labeled MAC_p denotes *private* marginal abatement costs, or marginal profits of emissions, and the horizontal intercept $\bar{E}$ is the unregulated emissions level. The classical Pigovian emissions tax is τ_P, and the associated optimal emissions level, where $MD = MAC_P$, is denoted by E_P. The Sandmo rule is shown as the tax rate $\tau_1 = aMD$ where $a = MCPF^{-1}$ and the associated emissions level is E_1. An alternative representation of this case would involve, instead of deflating the MD line, rotating the MAC_p line up to MAC_s, which denotes *social* marginal abatement costs, namely MAC_p plus the welfare costs of tax interaction effects net of revenue-recycling benefits. The rotation implies that the social costs of the policy go to zero as the emission fee goes to zero. The optimal emissions level is where $MD = MAC_s$ which is at E_1, and since firms respond to a tax according to MAC_p, the corresponding price is τ_1.

Figure 2 illustrates the case if the rest of the tax system is non-optimal and/or the emissions policy is non-revenue raising. Then MAC_p undergoes both a rotation and a translation out to the new MAC_s. If negative pollution taxes are ruled out then the maximum emissions level is $\bar{E}$ and the first unit of emission reductions has a social cost of $Z > 0$. (Solving for the value of MD consistent with an optimal emissions tax of zero yields $Z = b/a$.) As drawn, $Z > MD$ so the emissions policy is unambiguously welfare-reducing even though MD is positive. If $MD > Z$ policy is still warranted but the effect of the threshold on the gap between MAC_p and MAC_s needs to be taken into account.

The first contribution of this paper is to develop and solve a general model of emission taxes that relaxes some of the separability and linearity assumptions of previous models while remaining tractable and reasonably transparent, yielding an expression for the optimal emission tax in the form $\tau = aMD - b$ where the signs and magnitudes of the coefficients can be traced to the underlying structure of the economy and the policy environment. Standard results from earlier literature are reproduced, and some additional insights are obtained. In general I find a<1 and the sign of b is ambiguous but likely greater than zero. If the emission policy is not revenue raising then the damage threshold Z is always positive—an important departure from the classical case as typically presented in introductory textbooks. Another important result shown herein is that even in a first-best setting in which the emission tax can fully fund the government, the optimal tax is less than MD because of price effects throughout the economy. The reason this has not been demonstrated in earlier treatments is discussed.

The model then allows us to look at an important case that has hitherto been overlooked in the second-best emissions tax literature. Many studies have compared the outcomes in tax-*vs*-CAC experiments, but have not looked at tax-plus-CAC. In real-world settings, however, especially regarding carbon taxes, layering emission charges on top of pre-existing regulations has been the rule rather than the exception. While a few commentators[1] have pointed out that economic theory calls for a policy swap rather than a combination, many others explicitly call for carbon taxes to be used in combination with other regulatory measures (for example, Stavins 2010, Fankhauser 2012).[2] In practice the latter approach has been ubiquitous when emission pricing is introduced. For example, the Government of Canada recently announced[3] a minimum national carbon price of ten dollars per tonne in 2018, rising to $50 per tonne by 2022, which provincial governments must implement either as a tax or a tradable permit system. But Canadian federal and provincial governments have already implemented numerous sectoral carbon regulations such as a phaseout of coal-fired electricity generation in Ontario, a hard cap on carbon dioxide emissions from the oil sands sector in Alberta, national ethanol blending requirements in gasoline, etc., none of which are targeted for repeal as a result of the introduction of the pricing requirement. Similarly, carbon permit trading systems in California, the European Union and elsewhere operate in addition to, not

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[1] For example Clemens and Green (2017), Taylor (2015).

[2] Fischer et al. (2017) look at a sectoral model of electricity where there are multiple distinct market failures, including knowledge spillovers and CO_2 emissions, hence multiple policy instruments are required. This differs from the present case in which multiple instruments are applied on a single market failure.

[3] See Government of Canada website http://news.gc+.ca/web/article-en.do?nid=1132149 accessed May 4, 2017.

instead of, motor vehicle fuel efficiency standards, renewable power requirements in electricity production and other regulations that aim to limit carbon dioxide emissions.

Previous analyses consider a competitive equilibrium with a second-best optimal tax system and a single unregulated polluting sector, then examine the welfare effects of perturbations in the pollution quantity or price. I begin with that analysis, but in light of the above examples I then look at how a pre-existing restriction on some (but not all) emissions affects the optimal direction of the emissions tax reform. The analytical models referred to above all contain only a single polluting sector, so none are capable of analyzing this case. I study an economy with two polluting firms when one of them is subject to a binding emission constraint and then an emission tax is introduced. As expected, welfare is unambiguously lower compared to the optimal tax case without partial regulations. The regulations create a positive damage threshold which rises with the stringency of the emission cap. This potentially changes the optimal direction of the pollution tax reform since if marginal damages are below the threshold the optimal tax becomes negative. In light of this result, and the extensive literature preceding it, some care needs to be taken in policy discussions around emission pricing, especially carbon taxes. Economic theory does not generally show that an optimal emissions tax should be set equal to marginal damages, or that pricing an uncontrolled externality will always raise social welfare. Such outcomes are only assured under restrictive conditions that are often not observed in real-world economies. Under more general conditions, marginal damages (or the Social Cost of Carbon (SCC) as it is called in climate policy) is never the correct rate to use for the optimal emission tax, and using it as such may or may not raise welfare even if marginal damages are positive and revenues are fully recycled. This point has largely been overlooked not only in the academic literature but also in the popular and applied discussions of carbon pricing.[4]

The next section outlines the structure of the model. The starting point of the analysis is a second-best economy in general competitive equilibrium with a labour tax, a positive government spending requirement and an unregulated pollution externality. I examine a directional tax reform using a pollution tax and characterize the welfare effects, then in Section 3 I identify the form of the optimal tax. Section 3 also contrasts it with non revenue-raising and sub-optimal cases. Section 4 then looks at an alternative case in which the externality is partially regulated prior to introducing the emission tax. Section 5 presents a discussion and conclusions.

2 MODEL SET-UP

There are N identical households, two goods each produced by a separate firm, a labour market and a government. Household-specific consumption is denoted x_i, $i = (1,2)$, the corresponding prices are p_i, and aggregate demand is denoted $X_i = Nx_i$. Households each have a time endowment t which can be allocated to labour l or leisure h, so the aggregate labour supply is $L = Nl$, aggregate leisure is $H = Nh$ and the aggregate time endowment is $T = Nt$. The before-tax nominal wage rate

[4] For instance, Gregory Mankiw's famous 2006 blog post calling for higher gasoline taxes http://gregmankiw.blogspot.ca/2006/10/alternatives-to-pigou-club.html lists (and rejects) what he sees as the four reasons why an economist might oppose immediately raising externality taxes, ignoring the possibility of a positive damage threshold. Metcalfe and Weisbach (2009 pp. 11-13) assert that the optimal carbon tax should be set equal to MD, raising the issues of tax interactions only to dismiss them as second-order and unimportant.

is ω and the real wage is $w = \omega/\iota(p)$ where the denominator is a price index. We will set ω as the numeraire so it is constant and equal to unity but for notational clarity I retain it in the derivations.

Similar to Fullerton and Metcalfe (1997) we consider emissions as a productive input. Each firm has a single unit of fixed capital K_i equal to unity and a production function $F^i(L_i, E_i)K_i$ where the first argument denotes labour usage and the second denotes emissions, since the right to dispose of pollution in the environment is a productive input for the firm. Assume that F^i has decreasing returns to scale in L_i and E_i. The profit function for firm *i* is

$$\pi_i = p_i F^i(L_i, E_i) - \omega L_i - \tau_E E_i \tag{1}$$

where τ_E is the tax on emissions, $F_L^i > 0$ and $F_E^i > 0$. The first-order conditions imply

$$F_L^i = \omega/p_i \tag{2}$$

and

$$F_E^i = \tau_E/p_i. \tag{3}$$

Note that $\frac{d\pi_i(L_i^*, E_i^*)}{d\tau_E} = -E_i$ by the envelope theorem. Note also that (3) implies

$$\tau_E = p_i F_E^i \tag{4}$$

which is the private marginal abatement costs for firm i or, equivalently, the private marginal profits of emissions. Decreasing returns to scale imply that profits are positive and represent the rate of return to capital for each firm. We assume shares in firms are distributed equally among all households.

Firms pay the nominal wage but the household labour supply decision depends on the real wage w. We will assume that prices are initially normalized so that $\iota(p) = \omega = w = 1$. The tax rate on household income is τ_Y and net income is

$$y' = \left(\frac{\pi_1 + \pi_2}{N} + wt\right)(1 - \tau_Y). \qquad (5)$$

Note that $'$ denotes a tax-inclusive term. The household budget constraint is

$$p_1 x_1 + p_2 x_2 + w'h = y' \qquad (6)$$

where $w' = w(1 - \tau_Y)$. The corresponding national budget constraint (NBC) is

$$p_1 X_1 + p_2 X_2 + w'H = Y' \qquad (7)$$

where $Y' = (\pi_1 + \pi_2 + wT)(1 - \tau_Y)$.

The government purchases some of the available production of good 2 and gives it to households in equal shares. It finances this through taxes on total emissions $E = E_1 + E_2$ and on nominal profits and labour income. Hence the Government Budget Constraint (GBC) is

$$p_2G = \tau_Y B + \tau_E E \tag{8}$$

where the income tax base B equals $\pi + \omega L$ and $\pi = \pi_1 + \pi_2$.

Goods Market Equilibrium (GME) occurs where $X_1 = F_1$ and $X_2 + G = F_2$. Labour Market Equilibrium (LME) occurs where $L_1 + L_2 = T - H$. I confirm in the Appendix that imposing LME and GME on the NBC implies the GBC holds; likewise any three implies the fourth.

We assume that tax rates are adjusted to hold G constant. Consequently we are imposing a G-neutrality condition, rather than a p_2G-neutrality condition, or in other words real revenue-neutrality rather than nominal revenue-neutrality (as did Parry et al. 1999, in contrast to Bovenberg and Goulder 1996, Sandmo 1975 and others). Differentiating Equation (8) yields

$$Gdp_2 = \tau_Y dB + Bd\tau_Y + \tau_E dE + Ed\tau_E.$$

This rearranges to the G-neutrality condition

$$\frac{d\tau_Y}{d\tau_E} = \frac{1}{B}\left(G\frac{dp_2}{d\tau_E} - \tau_Y\frac{dB}{d\tau_E} - \tau_E\frac{dE}{d\tau_E} - E\right). \tag{9}$$

Since the emissions tax raises the cost of providing the public good and causes the income tax base and emissions to decline, the first three terms in the brackets must sum to a positive number. We will assume that we are operating in a region of the economy for which the new tax revenue (represented by the fourth term) is sufficiently large as to make whole derivative negative, meaning that an increase in emission taxes permits a reduction in the income tax.

Household utility is $u(x_1, x_2, h) + \frac{\alpha G}{N} - \delta E$ where α is the positive welfare weight on the public good G and δ is the marginal welfare cost of each unit of emissions. We will use the indirect utility function $v(p_1, p_2, w', y')$ to define the national social welfare function

$$W = Nv(p_1, p_2, w', y') + \alpha G - \delta NE. \tag{10}$$

The planner's problem is to choose τ_E to maximize (10). Since G is assumed fixed and given, τ_Y is then determined by equation (8).

3 Emission Taxes Without Regulations

3.1 Derivation of Optimal Taxes

The first derivative of equation (10) with respect to τ_E is

$$\frac{dW}{d\tau_E} = N\left(v_1 \frac{dp_1}{d\tau_E} + v_2 \frac{dp_2}{d\tau_E} + v_w \frac{dw'}{d\tau_E} + v_y \frac{dy'}{d\tau_E}\right) - \delta N \frac{dE}{d\tau_E} \tag{11}$$

where first derivatives of v are subscripted in order of the arguments. Divide equation (11) by v_y and apply Roy's theorem to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1 \frac{dp_1}{d\tau_E} - X_2 \frac{dp_2}{d\tau_E} - H \frac{dw'}{d\tau_E} + \frac{dY'}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}. \tag{12}$$

Note that $\frac{dY'}{d\tau_e} = (1-\tau_Y)\frac{d\pi}{d\tau_E} - \pi\frac{d\tau_Y}{d\tau_E} + H\frac{dw'}{d\tau_E} + L\frac{dw'}{d\tau_E}$. In the Appendix I show that this expression combined with equations (9) and (12) yield

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = \frac{dE}{d\tau_E}\left(\tau_E - \frac{\delta N}{v_y}\right) - Q + \tau_Y R \tag{13}$$

where $Q = \left(F^1\frac{dp_1}{d\tau_E} + F^2\frac{dp_2}{d\tau_E} - L\frac{dw}{d\tau_E}\right)$ and $R = \left(\omega\frac{dL}{d\tau_E} - L\frac{dw}{d\tau_E}\right)$. We can decompose this expression into some standard components (compare with, e.g., Parry et al 1999, equation 2.13). The first term on the right hand side of equation (13) is the primary welfare effect of the emissions tax, represented by the change in emissions times the difference between the tax τ_E and $\frac{\delta N}{v_y}$, which is the marginal external cost of emissions, or *MD*. Q represents the tax interaction effect, namely the welfare losses accompanying the increases in prices and reduction in real wages that result from increased emissions taxes. Note $Q > 0$ because the emissions tax raises output prices and reduces the real wage rate. $\tau_Y R$ represents the marginal revenue-recycling benefit that arises via the labour market. It incorporates the change in the labour tax revenue at the margin due to an emission tax-induced change in employment, as well as the tax component of the labour term in Q which nets against its cost to households. The first term has an ambiguous sign. Expand the derivative of L as follows:

$$\frac{dL}{d\tau_E} = \frac{\partial L}{\partial\tau_E} + \frac{\partial L}{\partial w}\frac{\partial w}{\partial\tau_E} + \frac{\partial L}{\partial\tau_Y}\frac{\partial\tau_Y}{\partial\tau_E}. \tag{14}$$

An increase in τ_E has three distinct effects. The first term represents the combination of a substitution effect on the demand side of the labour market as the cost of E relative to L rises, and a scale effect as the increased operating cost of the firm causes input demands to decline. Although it is reasonable to suppose that a large-enough emissions tax will lead to decreased employment we are focusing on infinitesimal changes. A small increase in τ_E is like a small increase in the cost of one input relative to another and the substitution effect along the firm's isoquant implies that $\frac{\partial L}{\partial \tau_E} > 0$. The second term is negative since an increase in the emissions tax reduces real wages which reduces labour supply. The third term is positive since a decrease in τ_Y increases the labour supply and we are assuming revenue recycling takes place ($\frac{\partial \tau_Y}{\partial \tau_E} < 0$). We will adopt the assumption that at low levels of the tax the first and third terms dominate making the derivative in equation (14) positive. Since $\frac{dw}{d\tau_E} < 0$ we conclude $R > 0$.

Equation (13) allows us to contrast the optimal policy that emerges in the present framework with that which emerges in other models that employ different assumptions. In the Sandmo (1975) framework, for instance, if the government revenue requirement is low enough to be fully satisfied by the externality tax, the optimal policy would entail a tax on the dirty good equal to marginal social damages and no other tax. That outcome does not emerge here, however. If we set $G = \tau_E E/p_2$ and $\tau_Y = 0$ then setting equation (13) equal to zero yields

$$\tau_E = \frac{\delta N}{v_y} + \frac{Q}{dE/d\tau_E}$$

which is strictly less than MD. The reason for the difference is that the model herein allows producer prices to change whereas in the Sandmo framework they are fully determined by fixed input-output coefficients. If prices were similarly fixed herein, Q would be zero and the classical solution would emerge. Otherwise, even if we can fully fund the government with the emissions tax, the tax interaction effect through prices means the classical Pigovian formula ($\tau_E = MD$) never emerges as an optimal solution.

If we imposed constant returns to scale, as in Fullerton and Metcalfe (1997), Parry et al. (1999), and others, some important differences would arise. The price derivatives would take the form $\frac{dp_i}{d\tau_E} = \frac{E_i}{F^i}$ (see derivation in the Appendix A, Parry et al. 1999) and profits would be identically zero. Hence the income tax base B would consist only of labour income, so the tax optimization problem would reduce to a choice of relative tax burdens on each of the two inputs to production. Since neither tax base would encompass the other there would be no unambiguous efficiency differences and the tax interaction and revenue recycling effects would follow from arbitrary assumptions about market demand elasticities. Parry et al. (1999) include three production inputs (labour, a clean good and a dirty good) so the labour tax has a broader base than that on the dirty good, which creates a difference in relative distortions independent of the market elasticities.

We can derive the optimal emissions tax implied by the current model by rearranging the GBC (equation 8) to get an expression for τ_Y and substituting it into equation (13), then solving for $\frac{dW}{d\tau_E}\frac{1}{v_y} = 0$. In the Appendix I show that this yields an expression of the form $\tau = aMD - b$, namely

$$\tau_E^* = \gamma_1 MD - \gamma_2 \left(R \frac{p_2 G}{B} - Q \right) \qquad (15)$$

where

$$\gamma_1 = \frac{\frac{dE}{d\tau_E}}{\frac{dE}{d\tau} - \frac{RE}{B}}$$

and

$$\gamma_2 = \frac{1}{\frac{dE}{d\tau} - \frac{RE}{B}}.$$

Since $R > 0, \gamma_1$ is positive and less than unity. Its interpretation as an inverse-MCPF term is somewhat more complex than in the case of a simple revenue-raising tax because a charge on marginal damages is justified independently of the need to fund government spending. We can interpret it more clearly by multiplying the top and bottom by τ_E to yield

$$\gamma_1 = \frac{\tau_E \frac{dE}{d\tau_E}}{\tau_E \frac{dE}{d\tau} - \tau_E \frac{E}{B} R}.$$

The numerator is the change in emissions resulting from an increase in the emissions tax, evaluated at the private marginal profits of emissions, which is the direct loss in private welfare due to the emission tax increase. The denominator is the same term combined with a second term that represents the cost of funding government spending. From Equation (8), if government spending were zero but marginal damages were positive (hence $\tau_E > 0$) we could use the emission tax revenue to subsidize labour at the rate $-\tau_E E/B$. This represents the opportunity cost of needing to fund G. The second term is therefore the total opportunity cost of government spending valued at the marginal revenue-recycling benefit ($\tau_Y R$) per dollar of the income tax rate (ωt_Y). Consequently the denominator of γ_1 is the marginal (with respect to τ_E) opportunity cost of financing government spending through τ_E, and the inverse of γ_1 is this amount relative to the direct economic cost of the emission tax increase. Thus γ_1 has an interpretation similar to the inverse-MCPF weights found in previous models (Sandmo 1975, Bovenberg and Goulder 1996, Parry et al. 1999).

The interpretation of γ_2 is also assisted by multiplying the top and bottom by τ_E. Now the denominator is the same as that of γ_1 and the numerator is τ_E, so this coefficient is the (negative) inverse of the marginal cost of funding G using a tax on E, per dollar of the emissions tax. This weight applies directly to Q and, since it is negative, the combination would have a negative effect on the optimal emissions. The other term in the brackets would vanish if $G = 0$. Hence the combined term captures the other aspect of the opportunity cost of funding G, namely a revenue-raising requirement. A positive level of government spending requires the emission tax rate to be higher than would otherwise be the case, where the additional amount is the nominal cost of government spending per dollar of the income tax base, weighted by the marginal revenue recycling effect, and scaled down by γ_2.

We need to sign the term in the brackets, $R\frac{p_2 G}{B} - Q$. Since emission taxes are not a large revenue source, it will be convenient to replace $p_2 G/B$ with the labour tax rate τ_Y (though exact equivalence is not necessary for this argument). Thus we have the revenue-recycling effect times the labour tax rate, $\tau_Y R$, minus the tax interaction effect Q. The output terms in Q add up to real GDP $(F^1 + F^2)$ weighted by the emission tax-induced price changes, which we will denote Fdp. We can thus expand out $\tau_Y R - Q = \omega\tau_Y \frac{dL}{d\tau_E} - Fdp + L\frac{dw}{d\tau_E}(1-\tau_Y)$. Suppose for a moment that $\tau_Y = 1$, so $R - Q = \omega\frac{dL}{d\tau_E} - Fdp$. If we are starting at the unregulated emissions level ($\tau_E = 0$) and firms are operating at a profit maximizing point ($\pi' = 0$) the change in output and prices induced by the first unit of the emission tax will follow $d\pi = Fdp + pdF - Ed\tau_E - \omega dL = 0$ which implies $Fdp = -pdF + Ed\tau_E + \omega dL$. Since each term on the right hand side is positive it must be that $Fdp > \omega dL$, making $R - Q$ negative. As τ_Y drops below unity, $\tau_Y R$ gets smaller but Q remains unchanged, so $\tau_Y R - Q$ remains negative. Thus the term in the brackets is likely (though not unambiguously) negative and since $\gamma_2 < 0$ the combined term will tend to reduce τ_E.

Equation (15) can be rendered into the same terms as Figures 1 and 2 by noting that MAC_p corresponds to τ_E and the optimum occurs where $MAC_s = MD$, so rearranging yields

$$MAC_s = \frac{1}{\gamma_1}MAC_p + \frac{\gamma_2}{\gamma_1}\left(R\frac{p_2 G}{B} - Q\right)$$

making clear the difference between the two can involve both a rotation and a translation.

3.2 Benefits Threshold

If $Z > 0$ under a specific policy then there exists a positive marginal damage threshold below which the optimal emissions tax is zero, implying that the first unit of abatement from the unregulated emissions level has a discrete positive welfare cost. We can derive an expression for the threshold value by solving equation (15) for $\delta N/v_y$ assuming $\tau_E = 0$. We obtain

$$Z = \frac{\left(R\frac{p_2 G}{B} - Q\right)}{dE/d\tau_E}. \tag{16}$$

For the reasons explained above the numerator is likely negative, and the denominator is definitely negative, so we expect $Z > 0$. The larger is Fdp, meaning the larger the effect on prices of the emissions tax, and the smaller the increase in employment resulting from the emission tax, the greater the tendency for Z to be positive. Note that, unlike previous derivations cited above, this result occurs even in a second-best optimal setting with a revenue-raising instrument. Numerical simulations in Bovenberg and Goulder (1996) for the US economy found that Z is non-negative but the magnitude strongly depends on the form of the policy.

Non-revenue raising policy

Suppose that the policy is non revenue-raising, such as in the case of CAC or tradable quotas. Total emissions are restricted to $\hat{E}$ which has an associated shadow price which we will call $\hat{\tau}_E$, which corresponds to the marginal rental value to the firm of being allowed to increase emissions by one unit. We will assume this price is the same for each firm, as would happen under tradable quotas. Hence $\hat{\tau}_E = d\pi/d\hat{E}$, corresponding to the firms' private marginal abatement costs MAC_p. The planner's problem can be re-stated as an optimization of W by choice of $\hat{E}$. In the Appendix I show that the solution occurs at

$$MAC_p = MD + \hat{Q} - \tau_Y \hat{R} \tag{17}$$

where $\hat{Q} = F^1 \frac{dp_1}{d\hat{E}} + F^2 \frac{dp_2}{d\hat{E}} - L \frac{dw}{d\hat{E}}(1-\tau_Y)$ and $\hat{R} = \omega \frac{dL}{d\hat{E}}$. Note the signs of derivatives here are different. There is now only one tax in the economy (τ_Y), the derivative of $\hat{E}$ with respect to τ_E does not appear and there is no weighting coefficient on MD. An increase in $\hat{E}$ reduces costs for firms and raises real wages so $\hat{Q} < 0$. Since an increase in allowed emissions (as opposed to a change in the relative cost of labour and emissions) raises employment we have $\hat{R} > 0$. Hence the solution occurs where $MAC_p < MD$.

The benefits threshold is found by setting Equation (17) equal to zero and solving for MD, yielding $\hat{Z} = -\hat{Q} + \hat{R}\tau_Y$. This is unambiguously positive, unlike in the revenue-raising case. Consequently, even if MD is positive, a reduction in emissions below the unregulated amount through a non-revenue raising policy is not guaranteed to improve welfare unless MD exceeds the threshold. Even when it does exceed the threshold the optimal emissions level occurs at $MAC_s = MD$, where $MAC_s = MAC_p - \hat{Q} + \tau_Y \hat{R}$, which raises the optimal emission level relative to the conventional textbook case of $MAC_p = MD$.

Non-optimal tax system

Suppose we use a pricing instrument but the rest of the tax system is sub-optimal. The motivation for this case comes from the numerical simulations in Bovenberg and Goulder (1996) in which the optimal pollution tax only corresponded to the Sandmo result ($MD/MCPF$) when the tax system was (second-best) optimized. Away from that point the optimal pollution tax was lower, and could go negative, although that was not an outcome shown in their theoretical model. It can be demonstrated in this context by returning to equation (13) but instead of setting it equal to zero, set it equal to some other value ρ:

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = \frac{dE}{d\tau_E}\left(\tau_E - \frac{\delta N}{v_y}\right) - Q + \tau_Y R = \rho. \tag{18}$$

The non-optimality we are interested in involves an overly-high income tax τ_Y, which under the GBC implies a lower emissions tax. The solution to equation (18) is

$$\tau_E^{\rho} = \gamma_1 MD - \gamma_2\left(R\frac{p_2 G}{B} - Q\right) + \gamma_2\rho\,. \tag{19}$$

Comparing this to equation (15) we see that the difference between τ_E^{ρ} and τ_E^{*} is the term $\gamma_2\rho$. Since the welfare function is convex, $\tau_E^{\rho} < \tau_E^{*}$ implies a positive value of ρ (which implies $\gamma_2\rho < 0$) and vice-versa. The benefits threshold is now

$$Z^{\rho} = \left(\frac{dE}{d\tau_E}\right)^{-1}\left(R\frac{p_2 G}{B} - Q\right) - \frac{\rho}{dE/d\tau_E}. \tag{20}$$

Since the second term is positive we have $Z^{\rho} > Z$, or in other words when the tax system is not optimized such that labour is over-taxed, the benefits threshold is unambiguously raised. Even if the benefits threshold in the optimized case is zero, so that the first unit of emission reduction under an emission pricing policy is welfare-enhancing, the existence of a non-optimal labour tax is sufficient to change this such that a positive threshold for MD now must be exceeded for the first unit of emission reduction to be welfare-enhancing. The steeper is the MAC the smaller is $dE/d\tau_E$ and hence the larger is the benefits threshold. Even if sufficient forms of linearity and separability are imposed on the model that $Q = R = 0$,[5] equation (20) would still yield a positive benefits threshold.

4 EMISSION TAXES UNDER PRE-EXISTING REGULATIONS

We now turn to the case in which pre-existing command-and-control regulations cover some, but not all, emissions. The outcome under this case is denoted with $\sim$. Suppose that firm 2 is required to reduce emissions to a fixed target level $\tilde{E}_2$ which is below the level associated with any proposed emissions tax rate.[6] The shadow price associated with the emissions constraint is denoted $\tilde{\tau}_E$, which must lie above the emissions tax rate τ_E by assumption. Both firms pay the tax but only firm 1 freely chooses its emissions level. Consequently the first order conditions remain the same for firm 1, but for firm 2 only that related to labour remains the same. The profit function for firm 2 is now

$$\pi_2 = p_2 F^2\left(L_2, \tilde{E}_2\right) - \omega L_2 - \tau_E \tilde{E}_2.$$

The emissions charge is a lump-sum fee for firm 2, but the regulation restricts the production function so it results in an upward shift of its supply curve, by an amount exceeding what would have been experienced under the emission tax. The price charged under the regulation is denoted

[5] For example, this outcome could be obtained by assuming Leontief production technology and a fixed labour supply.

[6] If $\tilde{E}_2$ exceeded this level then the constraint would not bind and we would simply be back to the case of a uniform emissions tax across both sectors.

$\tilde{p}_2$ and is strictly greater than p_2. Firm 2 profits $\tilde{\pi}_2$ are also lower than in the previous case. The regulation does not change the relative price of the inputs instead it forces down output and profitability so it has a negative effect on labour demand, leading to $\tilde{L}_2 < L_2$. Since profits are reduced and consumer prices are increased we expect $\tilde{y}' < y'$. These changes make it unambiguous that $v(p_1, \tilde{p}_2, \widetilde{w}', \tilde{y}') < v(p_1, p_2, w', y')$. Also the tax base shrinks, i.e. $\tilde{B} < B$. The regulation increases p_2 therefore $\widetilde{w}' < \omega$ even when the emission tax is zero. This implies, in turn, that the nominal tax base at the initial point $\tilde{B} = \omega\tilde{L} + \tilde{\pi} > \widetilde{w}\tilde{L} + \tilde{\pi} \equiv \tilde{B}_r$ where the r subscript denotes the real tax base, or the purchasing power of the nominal tax base. Denote the ratio $r \equiv \tilde{B}_r/\tilde{B} < 1$ as the fractional shrinkage in the real tax base due to the emission regulation, or alternatively the inverse of the effective real increase in the income tax rate.

Denote the total level of emissions in this case as $\tilde{E} = E_1 + \tilde{E}_2$. Since the restriction binds on firm 2, $\tilde{E} < E$. Because both firms pay the emissions tax we have $\frac{d\tilde{\pi}}{d\tau} = -E_1 - \tilde{E}_2$. The NBC is $p_1X_1 + \tilde{p}_2X_2 + \widetilde{w}'H = (\pi_1 + \tilde{\pi}_2 + \widetilde{w}T)(1 - \tau_Y)$. The GBC is $\tau_E\tilde{E} + \tau_Y\tilde{B} = \tilde{p}_2G$. Since the emissions level is fixed for firm 2, $\frac{d\tilde{p}_2}{d\tau_E} = 0$ and the G-neutrality condition is written

$$-\tilde{B}\frac{d\tau_Y}{d\tau_E} = \tau_E\frac{dE_1}{d\tau_E} + \tilde{E} + \tau_Y\frac{d\tilde{B}}{d\tau_E}.$$

The derivative of W with respect to τ_E looks like Equation (12) with E_1 replacing E:

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} - H\frac{d\widetilde{w}'}{d\tau_E} + \frac{d\tilde{Y}'}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}.$$

The form of $\frac{d\tilde{Y}'}{d\tau_E}$ is unchanged from before and the derivation proceeds in the same way. In the Appendix I show that the optimal tax rate is now

$$\tilde{\tau}_E = \tilde{\gamma}_1 \frac{\delta N}{v_Y} - \tilde{\gamma}_2 \left(\frac{\tilde{p}_2 G}{\tilde{B}} \tilde{R} - \tilde{Q} - \tilde{E}(1-r) \right) \tag{21}$$

where

$$\tilde{\gamma}_1 = \frac{\frac{dE_1}{d\tau_E}}{r\frac{dE_1}{d\tau_E} - \frac{\tilde{R}\tilde{E}}{\tilde{B}}},$$

$$\tilde{\gamma}_2 = \frac{1}{r\frac{dE_1}{d\tau_E} - \frac{\tilde{R}\tilde{E}}{\tilde{B}}},$$

$$\tilde{Z} = \left(F^1 \frac{dp_1}{d\tau_E} - L \frac{d\tilde{w}}{d\tau_E} \right)$$

and

$$\tilde{R} = \left(r\omega \frac{d\tilde{L}}{d\tau_E} - \tilde{L} \frac{d\tilde{w}}{d\tau_E} + \tilde{E}(1-r) \right).$$

It is not possible to compare RE/B to $\tilde{R}\tilde{E}/\tilde{B}$ since the changes to the individual terms go in different directions or are ambiguous. However we can examine the new damage threshold

$$\tilde{Z} = \left(\frac{1}{\frac{dE_1}{d\tau_E}} \right) \left(\frac{\tilde{p}_2 G}{\tilde{B}} \tilde{R} - \tilde{Q} - \tilde{E}(1-r) \right) \tag{22}$$

as follows. Since emission taxes are a relatively small component of government revenue we denote $\tilde{p}_2 G/\tilde{B}$ as approximately equal to the labour tax rate $\tilde{\tau}_Y$ (though again the argument doesn't require exactness). Then

$$\tilde{\tau}_Y \tilde{R} - \tilde{Q} - \tilde{E}(1-r) = \left\{ r\tilde{\tau}_Y \omega \frac{dL_1}{d\tau_E} - F^1 \frac{dp_1}{d\tau_E} \right\} + \left\{ L \frac{d\tilde{w}}{d\tau_E}(1-\tilde{\tau}_Y) \right\} + \{\tilde{E}(r-1)(1-\tilde{\tau}_Y)\}.$$

If $r = 1$, meaning the regulation on sector 2 does not shrink the real tax base, this reduces to the bracketed expression in equation (15). For the reasons spelled out in the previous section regarding the sign of $R\frac{p_2 G}{B} - Q$ the term in the first set of curly braces would likely be negative even without $r\tilde{\tau}_Y$ multiplying into the labour derivative, but since $r\tilde{\tau}_Y < 1$ it makes the term even larger negative. The other two terms in curly braces are also unambiguously negative. Hence $\tilde{Z}$ is strictly positive.

The distortion of the partial emission regulation operates through the parameter *r*. Its effect on the damage threshold is

$$\frac{d\tilde{Z}}{dr} = \left(\frac{1}{\frac{dE_1}{d\tau_E}} \right) \left(\tilde{\tau}_Y \frac{d\tilde{R}}{dr} + \tilde{E} \right)$$

$$= \left(\frac{1}{\frac{dE_1}{d\tau_E}} \right) \left(\tilde{\tau}_Y \left(\omega \frac{d\tilde{L}}{d\tau_E} - \tilde{E} \right) + \tilde{E} \right)$$

$$= \left(\frac{1}{\frac{dE_1}{d\tau_E}} \right) \left(\tilde{\tau}_Y \omega \frac{d\tilde{L}}{d\tau_E} + \tilde{E}(1-\tilde{\tau}_Y) \right) < 0.$$

A reduction in r (an increase in the distorting effect of the partial emission regulation) thus raises the damage threshold $\tilde{Z}$, making it less likely that the first unit of an emissions tax will be welfare-improving.

5 DISCUSSION AND CONCLUSIONS

This paper has examined the design of an optimal emission tax in an economy where pollution generates positive marginal damages. Previous literature has shown the optimal emission fee is of the form $\tau_E = aMD - b$ where $a < 1$ is a commonly-derived result and $b > 0$ is possible but not guaranteed, and often excluded by construction. The model developed herein is somewhat more general than that of earlier literature. For instance there are two polluting sectors rather than one, production exhibits decreasing returns to scale and separability between leisure and goods is not imposed. One novel result is that even in a first-best world where the government budget can be fully funded with an emissions tax, $a < 1$ due to the welfare costs of increased prices. In line with earlier results I show that when the pollution policy is non revenue-raising and/or the rest of the tax system is not optimized, the damage threshold Z is strictly positive implying that the first unit of abatement has a non-zero cost.

I then consider the case in which emissions are partially regulated prior to the emission tax being introduced. While absent from previous analyses, it is ubiquitous in practice. The partial regulation reduces the welfare of the starting equilibrium relative to the pure pricing case. The stringency of the regulation affects consumer purchasing power, parameterized herein as a ratio r between the real and nominal values of the income tax base. The damage threshold is larger as a result of the partial regulation, implying that marginal damages need to be higher in order for the first dollar of the pollution tax to yield a welfare improvement. The more stringent the pre-existing regulation, the higher is the damage threshold.

These findings have direct implications on the many ongoing debates about climate policy. At a time when carbon taxes and emission pricing have risen very high in public discussions it is unfortunate how little work has been done to identify the conditions that give rise to a damage threshold and how little empirical information has been generated about its magnitude. Parry et al. (1999) provided a rudimentary estimate, placing it at around US$18 per tonne of carbon, which would be about $25 in 2017 dollars. Bovenberg and Goulder (1996) used a large-scale general equilibrium model of the economy and estimated the threshold for non revenue-raising policies (like tradable quotas) was about US$55, which would exceed $75 per tonne in 2017 dollars. Mainstream estimates of the SCC are below this latter amount (Interagency Working Group 2013)[7] making problematic the US Administration's (now-rescinded) instructions to use the SCC as a criterion for determining the benefits of new regulations. Though empirical evidence is scant, in light of the few estimates that do exist the threshold issue cannot be dismissed as a mere second-order subtlety (as in Metcalfe and Weisbach 2009) but should be the focus of new empirical work.

There is likewise no empirical evidence regarding the effect on the threshold of the partial regulations implemented since Bovenberg and Goulder (1996) and Parry et al. (1999). The results herein show, however, that they have likely raised it.

The analysis presented above, and the literature on which it is based, raises important questions about the optimal design of climate policy. Even if the SCC is as high as recent estimates

[7] They may even be below the Parry et al. threshold estimate: see Dayaratna et al. (2017)

suggest, neither regulatory nor price-based policies are likely to be welfare-enhancing if the implementation occurs under conditions that currently appear to hold in developed economies. It also illustrates that standard treatments of optimal emission policy design in introductory environmental economics textbooks, which assume an equivalence between private and social marginal abatement costs, are materially inaccurate. Real-world divergences between these two measures change the analysis in fundamental ways.

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7 FIGURES

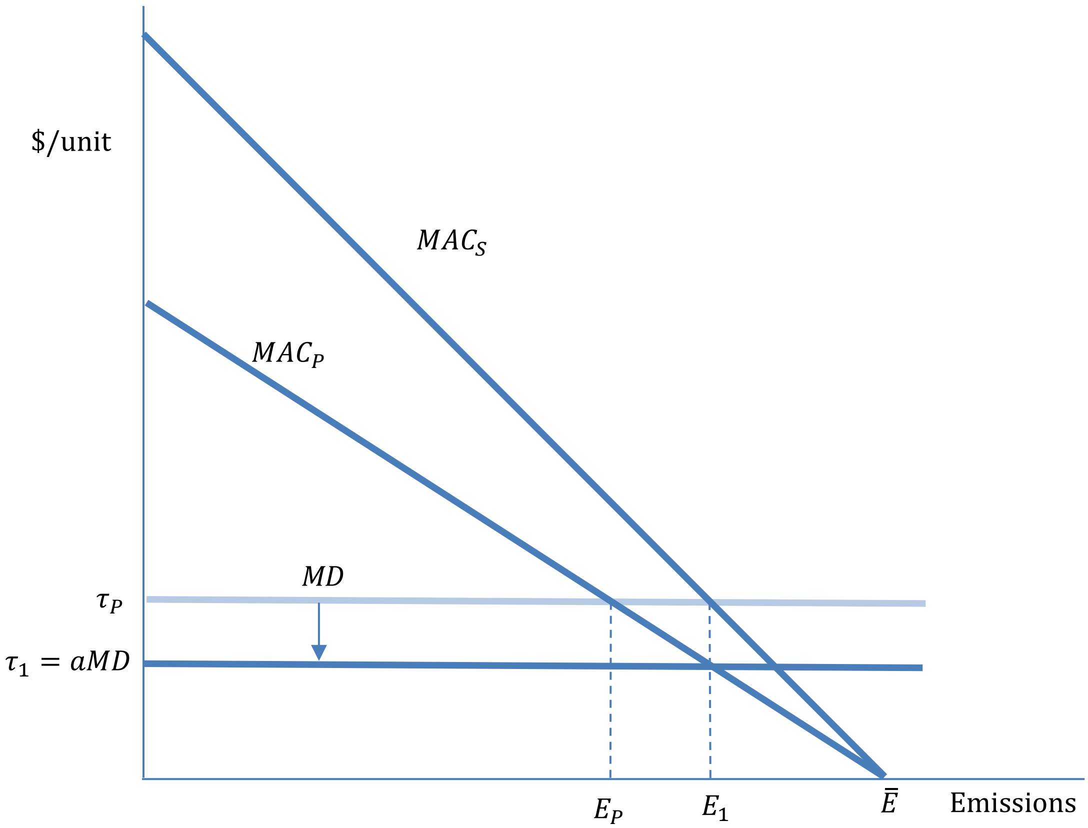

FIGURE 1: The classical Pigovian tax τ_P and the Sandmo result τ_1.

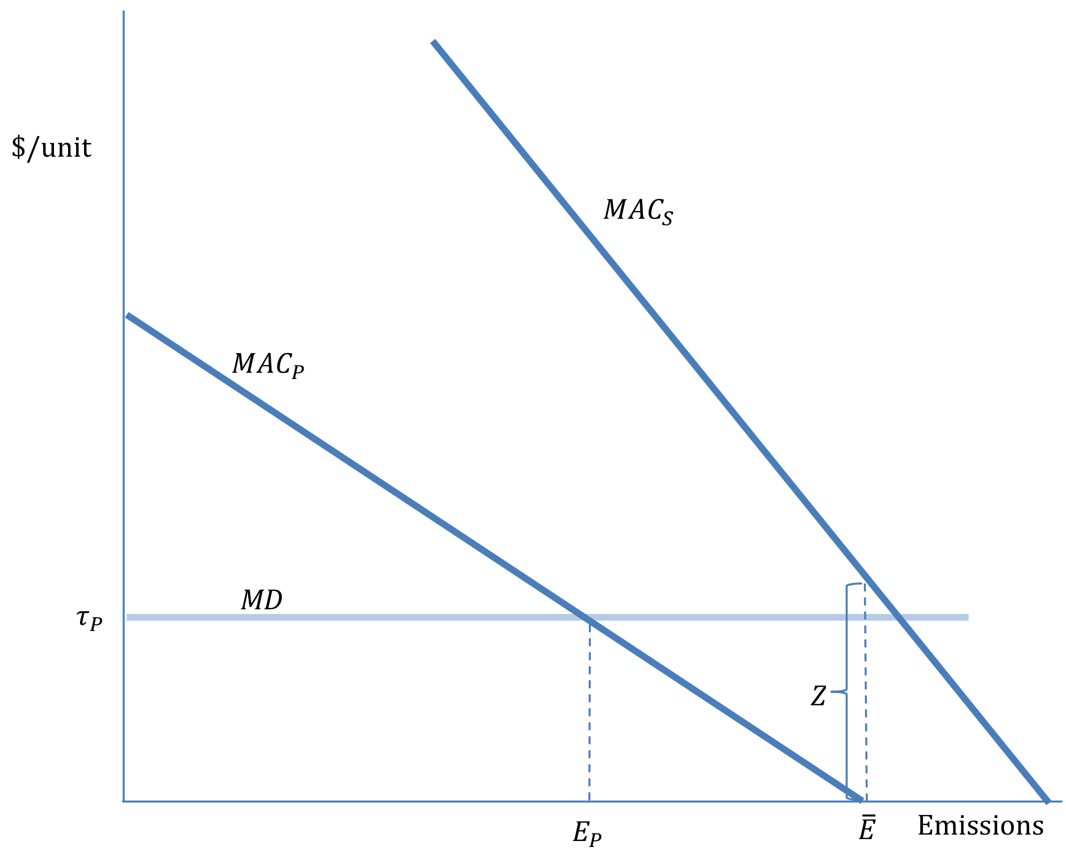

FIGURE 2: The classical Pigovian tax τ_P and the damage threshold Z.

8 APPENDIX

NBC+LME+GME implies GBC.

Equations (7), (5) and (1) and the numeraire condition imply

$$p_1X_1 + p_2X_2 + \omega H = (p_1F^1 - \omega L_1 - \tau_E E_1 + p_2F^2 - \omega L_2 - \tau_E E_2 + \omega L + \omega H) - \tau_y(\pi + \omega T).$$

Applying LME and GME yields

$$\cancel{p_1X_1} + \cancel{p_2X_2} + \cancel{\omega H} - \tau_Y \omega H = (\cancel{p_1F^1} - \cancel{\omega L_1} + \cancel{p_2X_2} + p_2G - \cancel{\omega L_2} - \tau_E E + \cancel{\omega L} + \cancel{\omega H}) - \tau_y(\pi + \omega T)$$

which in turn implies

$$-\tau_Y\omega H = p_2G - \tau_E E - \tau_Y\pi - \tau_Y\omega H - \tau_Y\omega L$$

which rearranges to $\tau_E E + \tau_Y(\pi + \omega L) = p_2G$. ■

Derivation of Equation (13)

Recall equation (12):

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} - X_2\frac{dp_2}{d\tau_E} - H\frac{dw'}{d\tau_E} + \frac{dY'}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

Substitute in $\frac{dY'}{d\tau_e} = (1-\tau_Y)\frac{d\pi}{d\tau_E} - \pi\frac{d\tau_Y}{d\tau_E} + H\frac{dw'}{d\tau_E} + L\frac{dw'}{d\tau_E}$ to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} - X_2\frac{dp_2}{d\tau_E} - H\frac{dw'}{d\tau_E} + H\frac{dw'}{d\tau_E} + L\frac{dw'}{d\tau_E} + (1-\tau_Y)\frac{d\pi}{d\tau_E} - \pi\frac{d\tau_Y}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} - X_2\frac{dp_2}{d\tau_E} + L\left(\frac{dw}{d\tau_E} - \tau_Y\frac{dw}{d\tau_E} - w\frac{d\tau_Y}{d\tau_E}\right) + (1-\tau_Y)\frac{d\pi}{d\tau_E} - \pi\frac{d\tau_Y}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} - X_2\frac{dp_2}{d\tau_E} + L\frac{dw}{d\tau_E} - \tau_Y L\frac{dw}{d\tau_E} - \frac{d\tau_Y}{d\tau_E}(\pi + wL) + (1-\tau_Y)\frac{d\pi}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

Note that at the initial point $w = \omega$ so $\pi + wL = B$, and $\frac{dB}{d\tau_E} = \frac{d\pi}{d\tau_E} + \omega\frac{dL}{d\tau_E}$. Use these and equation

(9)

$$\frac{d\tau_Y}{d\tau_E} = \frac{1}{B}\left(G\frac{dp_2}{d\tau_E} - \tau_Y\frac{dB}{d\tau_E} - \tau_E\frac{dE}{d\tau_E} - E\right)$$

to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} - X_2\frac{dp_2}{d\tau_E} + L\frac{dw}{d\tau_E} - \tau_Y L\frac{dw}{d\tau_E} - B \times \frac{1}{B}\left(G\frac{dp_2}{d\tau_E} - \tau_Y\left(\frac{d\pi}{d\tau_E} + \omega\frac{dL}{d\tau_E}\right) - \tau_E\frac{dE}{d\tau_E} - E\right) +$$

$$(1-\tau_Y)\frac{d\pi}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} - X_2\frac{dp_2}{d\tau_E} + L\frac{dw}{d\tau_E} - \tau_Y L\frac{dw}{d\tau_E} - G\frac{dp_2}{d\tau_E} + \tau_Y\left(\frac{d\pi}{d\tau_E} + \omega\frac{dL}{d\tau_E}\right) + \tau_E\frac{dE}{d\tau_E} + E + \frac{d\pi}{d\tau_E} - \tau_Y\frac{d\pi}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

Use $\frac{d\pi}{d\tau_E} = -E$ and collect terms to reduce this to:

$$= -X_1\frac{dp_1}{d\tau_E} - (X_2 + G)\frac{dp_2}{d\tau_E} + L\frac{dw}{d\tau_E} + \tau_Y\left(\omega\frac{dL}{d\tau_E} - L\frac{dw}{d\tau_E}\right) + \tau_E\frac{dE}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

Use GME to reduce this to $\frac{dW}{d\tau_E}\frac{1}{v_y} = -F^1\frac{dp_1}{d\tau_E} - F^2\frac{dp_2}{d\tau_E} + L\frac{dw}{d\tau_E} + \tau_Y\left(\omega\frac{dL}{d\tau_E} - L\frac{dw}{d\tau_E}\right) + \tau_E\frac{dE}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}$.

The rest follows immediately. ■

Derivation of Equation (15)

Rearrange $\frac{dW}{d\tau_E}\frac{1}{v_y} = -F^1\frac{dp_1}{d\tau_E} - F^2\frac{dp_2}{d\tau_E} + L\frac{dw}{d\tau_E} + \tau_Y(\omega\frac{dL}{d\tau_E} - L\frac{dw}{d\tau_E})\tau_E\frac{dE}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E} = 0$ to get

$$\tau_E = \frac{\delta N}{v_Y} + \frac{Q}{dE/d\tau_E} - \frac{\tau_Y R}{dE/d\tau_E}.$$

Using equation (8) we have $\tau_Y R = \left(\frac{p_2 G}{B} - \frac{\tau_E E}{B}\right)R$. Making the substitution and rearranging yields

$$\tau_E = \frac{\delta N}{v_Y} + \frac{1}{\frac{dE}{d\tau_E}}\,Q - \frac{R}{\frac{dE}{d\tau_E}}\left(\frac{p_2 G}{B} - \tau_E\frac{E}{B}\right)$$

$$\tau_E\left(1 - \frac{R}{\frac{dE}{d\tau_E}}\frac{E}{B}\right) = \frac{\delta N}{v_Y} + \frac{1}{\frac{dE}{d\tau_E}}\,Q - \frac{R}{\frac{dE}{d\tau_E}}\left(\frac{p_2 G}{B}\right)$$

$$\tau_E = \left(1 - \frac{R}{\frac{dE}{d\tau_E}}\frac{E}{B}\right)^{-1} \frac{\delta N}{v_Y} + \frac{1}{\frac{dE}{d\tau_E}\left(1 - \frac{R}{\frac{dE}{d\tau_E}}\frac{E}{B}\right)} Q - \frac{R}{\frac{dE}{d\tau_E}\left(1 - \frac{R}{\frac{dE}{d\tau_E}}\frac{E}{B}\right)} \left(\frac{p_2 G}{B}\right)$$

$$\tau_E = \frac{\frac{dE}{d\tau_E}}{\frac{dE}{d\tau_E} - \frac{RE}{B}} \frac{\delta N}{v_Y} + \frac{1}{\frac{dE}{d\tau_E} - \frac{RE}{B}} Q - \frac{R}{\frac{dE}{d\tau_E} - \frac{RE}{B}} \left(\frac{p_2 G}{B}\right)$$

which corresponds to equation (15). ■

Derivation of Equation (17)

The derivative of W with respect to the emissions constraint yields

$$\frac{dW}{d\hat{E}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{E}} - X_2\frac{dp_2}{d\hat{E}} - H\frac{dw'}{d\hat{E}} + \frac{dY'}{d\hat{E}} - \frac{\delta N}{v_y}.$$

Since the policy does not raise revenue the GBC is $p_2 G = \tau_Y B$ which implies $-B\frac{d\tau_Y}{d\hat{E}} = (-G\frac{dp_2}{d\hat{E}} + \tau_Y\frac{d\pi}{d\hat{E}} + \tau_Y\omega\frac{dL}{d\hat{E}})$. Also note that $\frac{dY'}{d\hat{E}} = \frac{d\pi}{d\hat{E}}(1-\tau_Y) - \pi\frac{d\tau_Y}{d\hat{E}} + T\frac{dw'}{d\hat{E}}$. Combining these yields:

$$\frac{dW}{d\hat{E}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{E}} - X_2\frac{dp_2}{d\hat{E}} - H\frac{dw'}{d\hat{E}} + H\frac{dw'}{d\hat{E}} + \frac{d\pi}{d\hat{E}}(1-\tau_Y) - \pi\frac{d\tau_Y}{d\hat{E}} + L\frac{dw'}{d\hat{E}} - \frac{\delta N}{v_y}$$

$$\Rightarrow \frac{dW}{d\hat{E}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{E}} - X_2\frac{dp_2}{d\hat{E}} - \tau_Y\frac{d\pi}{d\hat{E}} - \pi\frac{d\tau_Y}{d\hat{E}} + L\frac{dw}{d\hat{E}} - L\tau_Y\frac{dw}{d\hat{E}} - Lw\frac{d\tau_Y}{d\hat{E}} + \left(\frac{d\pi}{d\hat{E}} - \frac{\delta N}{v_y}\right)$$

$$= -X_1\frac{dp_1}{d\hat{E}} - X_2\frac{dp_2}{d\hat{E}} + \left(\frac{d\pi}{d\hat{E}} - \frac{\delta N}{v_y}\right) - B\frac{d\tau_Y}{d\hat{E}} + L\frac{dw}{d\hat{E}}(1-\tau_Y) - \tau_Y\frac{d\pi}{d\hat{E}}$$

$$= -X_1\frac{dp_1}{d\hat{E}} - X_2\frac{dp_2}{d\hat{E}} + \left(\frac{d\pi}{d\hat{E}} - \frac{\delta N}{v_y}\right) - G\frac{dp_2}{d\hat{E}} + \tau_Y\omega\frac{dL}{d\hat{E}} + L\frac{dw}{d\hat{E}}(1-\tau_Y)$$

$$= -F^1\frac{dp_1}{d\hat{E}} - F^2\frac{dp_2}{d\hat{E}} + L\frac{dw}{d\hat{E}} + \tau_Y\left(\omega\frac{dL}{d\hat{E}} - L\frac{dw}{d\hat{E}}\right) + \left(\frac{d\pi}{d\hat{E}} - \frac{\delta N}{v_y}\right)$$

which yields Equation (17) when set equal to zero. ■

Derivation of Equation (21)

Variables modified to take account of the change in their initial values due to the partial regulation are denoted with $\sim$. The derivative of W with respect to the emissions constraint yields

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} - H\frac{d\widetilde{w}'}{d\tau_E} + \frac{d\widetilde{Y}'}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}.$$

Substitute in $\frac{d\widetilde{Y}'}{d\tau_e} = (1-\tau_Y)\frac{d\widetilde{\pi}}{d\tau_E} - \widetilde{\pi}\frac{d\tau_Y}{d\tau_E} + H\frac{d\widetilde{w}'}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}'}{d\tau_E}$ to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} - H\frac{d\widetilde{w}'}{d\tau_E} + H\frac{d\widetilde{w}'}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}'}{d\tau_E} + (1-\tau_Y)\frac{d\widetilde{\pi}}{d\tau_E} - \widetilde{\pi}\frac{d\tau_Y}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} + \widetilde{L}\left(\frac{d\widetilde{w}}{d\tau_E} - \tau_Y\frac{d\widetilde{w}}{d\tau_E} - \widetilde{w}\frac{d\tau_Y}{d\tau_E}\right) + (1-\tau_Y)\frac{d\widetilde{\pi}}{d\tau_E} - \widetilde{\pi}\frac{d\tau_Y}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}}{d\tau_E} - \tau_Y\widetilde{L}\frac{d\widetilde{w}}{d\tau_E} - \frac{d\tau_Y}{d\tau_E}(\widetilde{\pi} + \widetilde{w}\widetilde{L}) + (1-\tau_Y)\frac{d\widetilde{\pi}}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}.$$

Using $r = \widetilde{B}_r/\widetilde{B}$ we have

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}}{d\tau_E} - \tau_Y\widetilde{L}\frac{d\widetilde{w}}{d\tau_E} - \frac{d\tau_Y}{d\tau_E}(r\widetilde{B}) + (1-\tau_Y)\frac{d\widetilde{\pi}}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}.$$

Following similar steps as before while noting that $\widetilde{p}_2$ does not change in response to the emission tax, we have $-\widetilde{B}\frac{d\tau_Y}{d\tau_E} = \tau_Y\frac{d\widetilde{B}}{d\tau_E} + \tau_E\frac{dE_1}{d\tau_E} + \widetilde{E}$ and $\frac{d\widetilde{B}}{d\tau_E} = \frac{d\widetilde{\pi}}{d\tau_E} + \omega\frac{d\widetilde{L}}{d\tau_E}$. Substitute these in to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}}{d\tau_E} - \tau_Y\widetilde{L}\frac{d\widetilde{w}}{d\tau_E} + r(\tau_E\frac{dE_1}{d\tau_E} + \widetilde{E} + \tau_Y\left(\omega\frac{d\widetilde{L}}{d\tau_E} + \frac{d\widetilde{\pi}}{d\tau_E}\right)) + (1-\tau_Y)\frac{d\widetilde{\pi}}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE_1}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}}{d\tau_E} - \tau_Y\widetilde{L}\frac{d\widetilde{w}}{d\tau_E} + r\tau_E\frac{dE_1}{d\tau_E} + r\widetilde{E} + r\tau_Y\omega\frac{d\widetilde{L}}{d\tau_E} + r\tau_Y\frac{d\widetilde{\pi}}{d\tau_E} + \frac{d\widetilde{\pi}}{d\tau_E} - \tau_Y\frac{d\widetilde{\pi}}{d\tau_E} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}$$

$$= -X_1\frac{dp_1}{d\tau_E} + \widetilde{L}\frac{d\widetilde{w}}{d\tau_E} + \left(r\tau_E - \frac{\delta N}{v_y}\right)\frac{dE_1}{d\tau_E} + \left(r\widetilde{E} + \frac{d\widetilde{\pi}}{d\tau_E}\right) + \tau_Y\frac{d\widetilde{\pi}}{d\tau_E}(r-1) + \tau_Y\left(r\omega\frac{d\widetilde{L}}{d\tau_E} - \widetilde{L}\frac{d\widetilde{w}}{d\tau_E}\right)$$

$$= -\widetilde{Q} + \left(r\tau_E - \frac{\delta N}{v_y}\right)\frac{dE_1}{d\tau_E} + \widetilde{E}(r-1) + \tau_Y\frac{d\widetilde{\pi}}{d\tau_E}(r-1) + \tau_Y\left(r\omega\frac{d\widetilde{L}}{d\tau_E} - \widetilde{L}\frac{d\widetilde{w}}{d\tau_E}\right)$$

$$= -\widetilde{Q} + \frac{dE_1}{d\tau_E}\left(r\tau_E - \frac{\delta N}{v_y}\right) + \widetilde{E}(r-1)(1-\tau_Y) + \tau_Y\left(r\omega\frac{d\widetilde{L}}{d\tau_E} - \widetilde{L}\frac{d\widetilde{w}}{d\tau_E}\right)$$

$$= -\tilde{Q} + \frac{dE_1}{d\tau_E}\left(r\tau_E - \frac{\delta N}{v_y}\right) + \tilde{E}(r-1) + \tau_Y\left(r\omega\frac{d\tilde{L}}{d\tau_E} - \tilde{L}\frac{d\tilde{w}}{d\tau_E} + \tilde{E}(1-r)\right).$$

Denote $\tilde{R} = \left(r\omega\frac{d\tilde{L}}{d\tau_E} - \tilde{L}\frac{d\tilde{w}}{d\tau_E} + \tilde{E}(1-r)\right)$. Use $\tau_Y = \frac{\tilde{p}_2 G}{\tilde{B}} - \frac{\tau_E\tilde{E}}{\tilde{B}}$ and set the above to zero to solve for the optimal value of τ_E:

$$\tau_E = \frac{1}{r}\frac{\delta N}{v_Y} + \frac{1}{r\frac{dE_1}{d\tau_E}}\tilde{Q} - \frac{1}{r\frac{dE_1}{d\tau_E}}\tilde{E}(r-1) - \left(\frac{\tilde{p}_2 G}{\tilde{B}} - \frac{\tau_E\tilde{E}}{\tilde{B}}\right)\tilde{R}\frac{1}{r\frac{dE_1}{d\tau_E}}$$

$$= \frac{1}{r}\frac{\delta N}{v_Y} + \frac{1}{r\frac{dE_1}{d\tau_E}}\tilde{Q} - \frac{1}{r\frac{dE_1}{d\tau_E}}\tilde{E}(r-1) - \frac{\tilde{p}_2 G}{\tilde{B}}\tilde{R}\frac{1}{r\frac{dE_1}{d\tau_E}} + \frac{\tau_E\tilde{E}\tilde{R}}{\tilde{B}}\frac{1}{r\frac{dE_1}{d\tau_E}}$$

$$\tau_E\left(1 - \frac{\tilde{E}\tilde{R}/\tilde{B}}{r\frac{dE_1}{d\tau_E}}\right) = \frac{1}{r}\frac{\delta N}{v_Y} + \frac{1}{r\frac{dE_1}{d\tau_E}}\tilde{Q} - \frac{1}{r\frac{dE_1}{d\tau_E}}\tilde{E}(r-1) - \frac{\tilde{p}_2 G}{\tilde{B}}\tilde{R}\frac{1}{r\frac{dE_1}{d\tau_E}}$$

$$\tau_E = \left(\frac{\frac{dE}{d\tau_E}}{r\frac{dE_1}{d\tau_E} - \frac{\tilde{E}\tilde{R}}{\tilde{B}}}\right)\frac{\delta N}{v_Y} - \left(\frac{1}{r\frac{dE_1}{d\tau_E} - \frac{\tilde{E}\tilde{R}}{\tilde{B}}}\right)\left(\frac{\tilde{p}_2 G}{\tilde{B}}\tilde{R} - \tilde{Q} - \tilde{E}(1-r)\right)$$

which corresponds to equation (21). ■