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Optimal Return and Rebate Mechanism in a Closed-loop Supply Chain Game

By:
Talat S. Genc

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Talat S. Genc

College of Business and Economics, University of Guelph, Guelph, Ontario, Canada

Pietro De Giovanni

Department of Operations Management, ESSEC Business School, Paris, France

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Abstract

Within a Closed-loop Supply Chain (CLSC) framework we study several consumer return behaviors for the used products which are based on the product prices and rebates. Consumers evaluate the rebate they receive as well as the price of the new product before deciding whether to dump a return. Therefore, the number of used products returned is examined under two types of rebates: a fixed rebate and a variable rebate. We search for the optimal rebate mechanism and find that the CLSC profits are higher under an variable rebate policy. This finding justifies the industry practices that employ a rebate mechanism based on both the value and the price of used item. We offer two types of solution concepts to the CLSC games: open-loop Stackelberg solution and Markov perfect Stackelberg solution, which are commonly employed in the dynamic games literature. While we mainly employ Markovian equilibrium, we also allow firms to utilize open-loop strategies so as to assess the impact of precommitment on the market outcomes. Therefore, we offer a comprehensive analysis of all possible market equilibrium solutions under different strategic considerations and the commitment deliberations. We show that under the fixed rebate regime open-loop solution coincides with Markov perfect solution. Furthermore, we show how consumer return behavior impacts the dynamic nature of the game. We find that the time frame is irrelevant if firms offer a fixed rebate. In contrast, the game will be fully dynamic when firms offer a variable rebate.

Keywords: Supply Chain Management, Rebate Policy, Return rate, Markov perfect Stackelberg equilibrium, open-loop Stackelberg solution.

1 Introduction

It is well documented that consumers adopt socially and environmentally responsible behavior by properly disposing off their end-of-use products rather than dispersing them into landfills (Souza, 2013). This result is due to the recent changes in business practices to manage the returns within Closed-loop supply chain (CLSC) frameworks. In the last two decades, firms designed ad-hoc policies to enhance the consumers' sensibility in environmental issues by sponsoring the "green consciousness" for the future generations (Bakker, 1999; Pattie, 1999) and their commitments to reduce the impact of their products and processes (Guide, 2009). Among the numerous environmental targets, firms posed a considerable attention on the backward flow management, which involves the implementation of atypical managerial practices, such as product acquisition, reverse logistics, points of use and disposal, testing, sorting, refurbishing, recovery, recycling, re-marketing, and re-selling (Guide and Van Wassenhove, 2009, Fleischmann et al., 2001). The literature called this strategy value stream approach or active return approach, to highlight the firms' commitments and efforts to perform the number of returns as well as economic convenience and environmental feasibility of these policies. For example, Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), and De Giovanni and Zaccour (2014) characterize an active return approach in which the returns increase in the collector's promotional efforts.

Although these frameworks demonstrate the economic advantages as well as the environmental and social benefits obtainable from an active return approach, the businesses realized that these policies do not provide competitive advantage any more (Simpson et al., 2004). Rather they are perceived as a default orientation to be established independently of other factors. In fact, firms in almost all sectors take care of their past-sold products, adopt an active return approach and continuously advertise their socially responsible corporate attitudes. In other words, all firms responsibly manage their returns; consumers are fully aware of the firms' green programs and know that their returns will be surely treated responsibly (Baker, 1999). Consequently, a marketing strategy aiming at increasing the number of returns as well as the environmental recognition is marginally effective because all firms within an industry do advertise their green initiatives. Indeed, when consumers must choose between goods produced by a grey and a green manufacturer, they will most likely choose the latter because of its environmental initiatives (Atkin et al., 2012). Nevertheless, when consumers must choose between goods produced by two green manufacturers, there is no competitive advantage linked to being green (De Giovanni, 2016). Rather, it becomes a compulsory feature. Thus, the firms' attention is moving from sponsoring their green orientations to putting in place some more efficient mechanisms to increase the returns. In particular, recent programs are based on providing some generous economic incentives in forms of rebates (e.g., trade-in programs) to engage consumers in returning their used products. For example, since H&M has launched its Garment Collecting Initiative in 2013, customers from every corner of the world have helped recycle 25,000 tonnes of their unwanted clothes (www.hm.com). H&M pays a fixed per-bin rebate to consumers, independently of the textile products put in the bin, to be used for future purchases. Similarly, Dell has initiated the DellReconnect project, which is a partnership between Dell and Goodwill, and encourages responsible electronics recycling. When consumers return their used electronics to a Goodwill (2,000 locations across the US), they receive a fixed per-ton tax discount rebate independently of the returns' types and conditions (www.dell.com). Lexmark started the "Prebate" program in 1998, where customers could get $30 rebate off a $230 toner cartridge if they return the used cartridges back (Majumder & Groenevelt, 2001). Apple Recycling Program allows PowerOn to administer and manage the return and recycle of Apple's products. For any return qualified for reuse, the consumers receive a gift card to be used in the Apple stores whose amount depends on the results of the product's evaluation (Apple.com). Similar mechanisms are used for automobiles (Autotrader.com), books (Amazon.com), video games (Gameshop.com), and consumer electronics (BestBuy.com).

According to these cases, the firms enhance the consumers' attention through environmental issues by providing two types of rebates: a fixed rebate that does not depend on the returns' type, original price or conditions (e.g., apparel of H&M, and electronics of DellReconnect) and a variable rebate that can depend on all of these features (e.g., electronics of Apple, cars of Peugeout, cartridges of Lexmark). In this paper, we will seek to capture this distinction by modeling an active return approach in which the rebate can be either fixed or variable and depending on market value/price of the product. We will formulate the variable rebate as a function of the purchase price. Clearly, there is a certain relationship between the rebate and the retail price. Firms offer a high rebate if consumers paid a high price to purchase a product. In fact, the retail price is a proxy of the product quality (e.g., technology updates). Furthermore, consumers are likely to properly treat and responsibly use the product when they spend a high amount. The comparison of the two types of rebates will inform on the best option that firms within CLSC should adopt to improve their economic and environmental performance. The return functions that we employ will not only depend on the rebate but also on the price that consumers pay for a new product, according to the intuitive principle for which consumers need to purchase a new product to continue to satisfy their needs after their return (e.g., De Giovanni et al., 2016). According to these business practices and evidences, we will model the return functions that will depend on both the rebate, which can be either fixed or variable and depending on the retail price, and the new product price.

This way of formulating the returns provides a novel approach in the CLSC framework. While the prior papers have developed several rebate programs, they have mainly focused on B2B relationships. Consequently, the incentive mechanism is designed for collectors rather than for consumers. For example, Ferguson and Tokay (2006) model a setting in which a rebate is offered to retailers when the false returns do not exceed a certain amount. Savaskan et al. (2004) and Savaskan and Van Wassenhove (2006) use an exogenous rebate (e.g., a fee) to retailers per each unit returned. Similarly, Atasu et al. (2013) characterize a fixed incentive for collectors and highlighting the conditions according to which the incentive pushes the collector to collect all past sold products. Nevertheless, no incentive is provided to consumers for impacting their return decisions. De Giovanni (2014) shows that CLSC can be effective when a fixed rebate embedded on a reverse-revenue-sharing contract is proposed to a retailer. Corbett and DeCroix (2001) examine exogenous shared-savings contracts to overcome incentive conflicts between a supplier and a buyer to reduce the use of indirect materials. Ferguson and Toktay (2006) model an incentive that assumes a form of target rebate for a retailer; the mechanism increases the retailer's wishes to invest more in green activity programs and perform the reverse flow management. Ray et al. (2005) use a price mechanism in the form of trade-in rebate to enhance customers' willingness to repurchase. Bakal and Akcali (2006) compares various forms of per-unit acquisition price showing that the exogenous rebate works well in terms of operational performance. De Giovanni (2015) shows that a per-unit rebate incentive given to a retailer is never preferable than a mechanism based on the overall CSLC performance and the retailers' commitments to environmental issues. Wu and Zhou (2017) model a CLSC in which the product collection can be done either by a manufacturer or by a retailer; in the latter case, the manufacturer supplies a fix incentive to the retailer to increase the return rate, but not to consumers. Similarly, Hong et al. (2017) compares a fixed fee to a royalty licensing mechanisms showing the conditions under which the former is preferable. Nevertheless, no incentive is formally granted to consumers when returning the products. Heese et al. (2005) explicitly model penalty associated with the collection of past-sold products but no incentive is supplied to consumers to impact the return behavior. Yan et al. (2015), Galbreth et al. (2012) and Orsdemir et al. (2013) model the quantity to be remanufactured as a decision variable without involving any type of incentives, neither for supply chain members nor for consumers. Differently, Wu (2012) assume that the return rate is a fixed percentage of the past-sold product, being therefore rebate-independent. This literature stream highlights that CLSC incentives have been mainly designed for collectors rather than for consumers.

Therefore, in our paper, we seek to contribute to this body of knowledge by modeling a return function in which the rebate is offered to the consumers rather than to collectors. Ostin et al. (2008) refer to this approach as "credit system" because the collector supplies an incentive directly to consumers to return their old cores according to some features (e.g., price and quality); these credits can be used for future purchases. We model a game in which the manufacturer manages the returns exclusively, and reinforces the B2C relationships by proposing either a fixed or a variable rebate. As reported in a recent review by Souza (2013), only two papers dealt with trade-in programs for consumers: Ray, Boyaci, and Aras (2005), and Li, Fong, and Xu (2011). Ray et al. (2005) assume that the returns may carry out some value and can be traded-in through discount policies that are dependent on the used product's age or independent of the product age, or there is no trade-in discount. Differently, Li et al. (2011) provide a forecasting method for trade-in programs based on customer segmentation. In addition, Govindan and Popiuc (2014) model a return rate that linearly depends on a discount offered to consumers. Kaya (2010) model a return quantity that linearly depends on the rebate (incentive) offered to consumers. She characterizes several scenarios considering deterministic and stochastic frameworks as well as centralized and decentralized solutions. In all cases, when the rebate plays an important role in the return function, the decision maker always supplies larger incentives and earns higher profits. He (2015) designs an incentive for consumers that can act either linearly or non-linearly in the return function showing that an optimal rebate always leads to a concave function and maximizes the decision maker payoffs. Our contribution takes position within this framework in which the incentive (a rebate) is offered to consumers. Differently from the literature, consumers' return behaviors are very sophisticated and depend on both the price to be paid for purchasing a new product and the rebate. So, when the price of purchasing a new product is large, consumers show a lower willingness to return products. When the rebate is large, consumers' returns increase. Contrary to the literature that mainly models a rebate as a decision variable, we model the rebate as a function of the price paid for purchasing the product in the past. Therefore, consumers will decide whether to return a product according to the evaluation of the good they purchased and the sacrifice linked to purchasing a new product. This return function in fact reflects the reality as such collectors evaluate a return according to its original market value, which is mainly exemplified by the original retail price (e.g., BestBuy, Apple).

The motivation for pursuing a framework in which the manufacturer handles the collection has both theoretical and practical dimensions. From a theoretical perspective, the research in the selection of a proper CLSC structure demonstrates that when gains from the collection process are high, the manufacturers prefer collecting themselves only (De Giovanni & Zaccour, 2013). For instance, Guide (2000) reports that 82% of firms collect directly from customers, while Xerox carries out the product collection process alone and performs 65% return rate. From a practical perspective, the cases in which the OEMs collect directly from the consumers either through ad-hoc programs or through their store-brands are very common, as mentioned for the cases of Apple, H&M, and Lexmark.

Besides providing some theoretical developments, we also propose some methodological advancements in the context of CLSCs. Specifically, we adopt two different solution concepts, namely, open-loop and close-loop (or Markov perfect) equilibria in Stackelberg setting. The validity and adoption of these concepts depends on the firms' availability to optimally decide their strategies according to the current state information (Markovian strategy), or by just committing to a set of actions to be adopted over the time (Open-loop strategy). The Open-loop equilibrium has been conveniently used in the dynamic games literature because of being tractable for solving large-scale dynamic games (Genc et al. 2007), providing a benchmark solution that can be compared to more complex strategies (Genc and Zaccour, 2013), and being (under certain conditions) a good approximation to closed-loop solutions (Van Long et al. 1999). Nevertheless, the open-loop approach may not be subgame perfect in general. More recently, Haurie et al. (2012) provide a good overview of the solution concepts and offer several reasons why one might be interested either in the open-loop or in the close-loop equlibria. There are a number of papers in the dynamic games literature focusing on the comparison of Markov perfect and open-loop strategies. To our knowledge, this is the first paper using both equilibrium concepts in the CLSC literature. Indeed, many papers have dealt with the comparison of open-loop and Markov strategies and equilibria in different areas. See, for example, Genc and Zaccour (2013), Dockner et al. (2000), and Figuières (2002), Genc (2017) for capital accumulation games, Kossioris et al. (2008) and Long et al. (1999) for examples in environmental and resource economics, and Piga (1998) and Breton et al. (2006) for examples of advertising investments. Open-loop and feedback strategies have been compared in several other contests such as in the supply chain (e.g., Gaimon, 1998; Kogan and Tapiero, 2007) and in the marketing channel (e.g., Jørgensen and Zaccour, 2004). In our CLSC framework, the comparison between open-loop and Markov perfect equilibria informs on the best approach that the firms should adopt to set their strategies in real businesses to better perform from an economic and the environmental perspectives.

To capture the dynamic aspects of the CLSC we model a two-period game (which can be easily extended to $T > 2$ periods for a fixed rebate case as discussed in the conclusions section), in which the manufacturer sets the wholesale price and the retailer sets the retail price in both periods. Consumers who purchase in the first period can decide to return their used goods in the second period according to a price-driven return function. We solve the games and compare the strategies using Markov perfect and open-loop equilibrium concepts under two different rebate policies.

Compared to the literature, we provide two main novelties:

1. We search for the best return policy by investigating two types of rebates, namely, fixed and variable rebates. Thus, consumers play a strategic role within our framework and their return behavior substantially influence the firms' strategies and payoff functions. Also, this is the first attempt to model some return functions that are consistent with real policies established by firms. For instance, the return strategies undertaken by H&M and Apple aim at increasing the returns but providing a fixed and a variable rebate, respectively. Therefore, we answer the question: Which of these mechanisms should firms implement to increase their payoff functions?

2. We adopt the two solution concepts, namely, open-loop and Markovian strategies. The literature focuses on the implementation of Markovian strategies only, while the dynamic game literature has also developed the concept of open-loop strategies. The two concepts are very relevant in supply chain contexts

because they are based on the principle of commitment: When firms write a contract based on open-loop strategies, they commit on some actions that cannot be modified over time; when the contracts are based on Markovian strategies, firms can adjust their actions according to the course of the game. We then answer the question: Which type of solution concept, between open and Markovian strategies, should firms use to improve their payoff functions within supply chains?

We offer significant methodological and conceptual contributions to the CLSC literature and find some new results. Our model specifically incorporates *i*) return functions involving current and future prices; *ii*) fixed and variable rebates in the return functions; *iii*) different information structures and equilibrium solutions; *iv*) dynamic interactions between the decision variables over time. To our knowledge, the previous papers have not considered comprehensive return behavior and rebate mechanisms to be implemented in the CLSC games involving B2C relations. Although some formulated CLSC games over finite time horizon they were static in the sense that the decisions were not interlinked over the periods of the game. Also, they have not provided equilibrium solutions under different information structures.

As we offer a richer modeling approach in CLSC context, we find some interesting results: *a*) when the CLSC adopts Markovian solution concept, larger fixed rebates have a positive impact on firms' profits and consumers' surplus. Nevertheless, this outcome is not environmentally sustainable as the returns decrease; *b*) when the CLSC adopts a Markovian solution concept, larger variable rebates show contrasting effect on firms' profits: the manufacturer prefers lower rebates while the retailer prefers larger rebates, which is mainly due to the independence between the retailer's profits and the remanufacturing outcomes; *c*) when the CLSC adopts an Open-loop solution concept, fixed rebates should always be preferred because they lead to higher profits and better social outcomes and environmental performance in most of the cases; *d*) when the consumer return behavior is reflected by a fixed rebate return function, firms will be indifferent between using either a Markovian or an Open-loop concept because of the independence between forward and backward flows; *e*) when the consumer return behavior is explained by a variable rebate return function, firms' preferences diverge: the manufacturer would always adopt a Markovian solution concept while the retailer would adopt an Open-loop concept. Further, a Markovian solution concept is socially optimal and environmentally unsustainable, while an Open-loop concept allows a CLSC to achieve good environmental performance it deteriorates the social welfare.

The paper is organized as follows. Section 2 introduces the CLSC model with endogenous return functions. Section 3 provides Markov perfect equilibrium outcomes with fixed and variable rebates. Section 4 reports the model solution using open-loop approach. Section 5 compares Markovian to open-loop solutions under the two different rebate types. Section 6 offers some practical managerial insights to business firms in CLSC, and section 7 concludes the paper with future research directions.

2 A two-period model of CLSC

We model a two-period game with a manufacturer, player M, and a retailer, player R. The two firms work in a closed-loop supply chain (CLSC) and strategically make pricing decisions over the two periods under investigation, $t = 1, 2$. In particular, M sets the wholesale price, ω_t, and R optimally sets the retail price, p_t. Consumers also take part in the CLSC by deciding whether to buy a product in $t = 1$ and return the used product and buy a new one in $t = 2$.

Assumption 1. Consumers set their purchasing decisions based on the retail price in each period according to the following demand function:

$$q_t(p_t) = \alpha_t - \beta p_t \tag{1}$$

where α_t and β denote the market potential and the consumer's sensitivity to price, respectively. Notice that demand function in Eq. (1) has been largely used in the CLSC literature (e.g., Savaskan et al. 2004) because it allows one to concentrate on the operational aspects of a problem while keeping the solution sufficiently tractable. Also, observe from the demand equation that demand intercept changes over time. This is because some customers who return their products can purchase in the second period. In addition, the market conditions as well as macroeconomic conditions may change in the second period. We reflect these changes in our demand function by the parameter α which has a time subscript and represent the market potential. Essentially demand in the second period can shift up or down that we do not restrict which way the market goes. On the other hand, we expect the same behavior by the consumers so that we keep the price sensitivity parameter β constant across the periods. This is because the consumers come from the same pool with certain price responsiveness. Of course, we will assume that the quantity demanded at a given price holds $q_t(p_t) > 0$ both in analytic and numerical solutions.

An important feature of a CLSC model is to uncover value-added in operations by saving costs through processing return (Souza, 2013). Accordingly, we assume the following return function.

Assumption 2. When consumers receive a variable rebate, the return function takes the form:

$$r(p_1, p_2) = \theta - \gamma(p_2 - \epsilon p_1) \tag{2}$$

The interpretation of Eq. (2) is as follows. When consumers seek to return a product, they indeed evaluate the price of new product available in the market to check whether they can afford it. Higher second-period retail price p_2 will discourage consumers to return their products. Similarly, when consumers wish to purchase a new product, they trade-in their old ones and get the rebate ϵp_1, which represents the rebate that consumers obtain when a product is returned, where $\epsilon\,(0,1)$ and represents the fraction of the original product price that is converted into a rebate. For example, in the automotive industry consumers get on average 79% of the original price (that is $\epsilon = 0.79$) that they paid if returning the car within 12 months from the purchase date (Gorzelany, 2016). Accordingly, we refer to Eq. (2) as *return function with variable rebate*, as the rebate depends on the first-period retail price.

Then, the term $\gamma(p_2 - \epsilon p_1)$ explains the consumers' willingness to return their old products according to the retail price differential over the two periods. Finally, there are maximum θ number of consumers who return their products independently of the price difference for whom $\gamma = 0$.

Assumption 3. Independent of the values of θ, γ and ϵ, $r(p_1, p_2) \leq \theta$.

Notice that this return function captures two fundamental paradigms that have been introduced in the CLSC literature, that is, a waste vs. a value stream collection approach. According to the waste stream approach, remanufacturers are barely interested in economic and operational perspectives; thus, firms do not invest in the implementation and management of recycling process, which implies that quality, quantity, and timing are uncertain while remanufacturing costs and opportunities are not at all aligned (Guide et al. 2003). Rather, they passively wait for the return of past-sold products. That is, remanufacturing is considered a cost-center practice that only creates marginal value and opportunities (Guide et al. 2006). In Eq. (2) a passive return approach is modeled when $\gamma = 0$ holds and $\theta > 0$, which represents the number of consumers who voluntarily return the product independently of the firm strategies. In contrast, the adoption of a value stream approach relates to all situations in which the remanufacturing process is largely convenient, thus the value that can be obtained from returns is considerably large. Accordingly, firms continuously invest in all backward activities to successfully perform the collection process (De Giovanni, 2015). In Eq. (2), $\gamma > 0$ captures the effectiveness of an active return approach, according to which consumers' willingness to return depends on the product prices over the two periods investigated. Therefore, firms adopt a passive return approach when $\gamma = 0$, and a value stream approach when $\gamma > 0$. Finally, θ is the maximum amount that the collector will receive back. Therefore, we will constrain the return rate to be at most equal to θ.

As mentioned earlier, some CLSCs (e.g., H&M's CLSC) base their rebate on a fixed amount (e.g., per-bin rebate) that does not depend at all on the original price that consumers paid for the product. In this case, Eq. (2) does not cover H&M's CLSC. Then, we assume an alternative return function, $v(.)$, that takes the form.

Assumption 4. When consumers receive a fixed rebate, the return function takes the form:

$$v\,(p_2) = \theta - \gamma\,(p_2 - k) \tag{3}$$

where k is a fixed rebate that consumers receive when returning their past-purchased products. Different than Eq. (2), consumers only evaluate the fixed rebate and the price of a new product in the second period before deciding whether to return the used product. The exogenous rebate represents a fixed amount that is independent of quality, condition of the good, and its original price. In the automotive sector, for example, this return function is generally applied to a consumer who returns a very old car, so he/she receives a lump-sum from either a dealer or the government who recycle the used cars responsibly. Similarly, H&M offers a fixed rebate for any bin that a consumer returns, independently of its content. This incentive payment is represented by k in the model.

Assumption 5. Independent of the values of θ, γ and k, $v\left(p_2\right) \leq \theta$.

Even when the CLSC offers a fixed rebate, the maximum amount of consumers who will return the product will be θ, thus we will consider the constraint $v\left(p_2\right) \leq \theta$. The market outcome comparison between these two return functions will lead the decision maker to determine the payment policy for the used products. Specifically, the differences between prices and profits will inform the firm preferences for the return policy.

Assumption 6. M's markup from selling the good is given by $\pi_{M_t} = \omega_t - c_t$.

ω_t represents its wholesale price, while $c_t > 0$ is the marginal production cost, which we assume to be constant over time. In solving firm's optimization problem we will constrain $\pi_{M_t} > 0$ to admit feasible equilibrium solutions.

Assumption 7. Under r return function, M's the value of a returned product is given by $\pi_{M_r} = \sigma - g - \epsilon p_1$.

σ is the return residual value that we assume to be constant, independent of time and condition of the good. g is the constant marginal collection cost and includes all costs associated with the backward logistics activities. ϵp_1 is the variable rebate that M pays to a consumer who returns a past-sold product according to (2). Finally, we constrain $\pi_{M_r} = \sigma - g - \epsilon p_1 > 0$ to highlight the economic convenience of remanufacturing and determine the M's willingness to shift from a waste to a value return approach or vice verse.

Assumption 8. R's markup is $\pi_{R_t} = p_t - \omega_t$.

Thus, R does not receive any benefit from the return process and consumers directly return products to M. In firm's maximization problem, we will constrain $\pi_{R_t} = p_t - \omega_t > 0$ to obtain feasible solution for the retailer. Although the R is not at all involved in the collection process, she has a substantial influence on the return quantity as the returns exclusively depend on the retail prices according to Eqs. (2) and (3).

Given the above assumptions 1-8, we write firms' profit functions under the variable rebate policy:

$$\max_{\omega_1,\omega_2} \Pi_M = \underbrace{q_1(.)(\omega_1 - c_1)}_{\text{Forward profits } t=1} + \delta[\underbrace{q_2(.)(\omega_2 - c_2)}_{\text{Forward profits } t=2} + \underbrace{r(.)(\sigma - g - \epsilon p_1)}_{\text{Backward profits}}] \tag{4}$$

$$\max_{p_1,p_2} \Pi_R = \underbrace{q_1(.)(p_1 - \omega_1)}_{\text{Forward profits } t=1} + \delta \underbrace{q_2(.)(p_2 - \omega_2)}_{\text{Forward profits } t=2} \tag{5}$$

where $q_1(.)$, $q_2(.)$, and $r(.)$ are functions of retail prices and δ is the discount factor, which is assumed to be common to both players. Also, the profit functions must satisfy $\Pi_M > 0$ and $\Pi_R > 0$.

Also, given the above assumptions, firms' profit functions under the fixed rebate policy can be written as follows:

$$\max_{\omega_1,\omega_2} \Pi_M = \underbrace{q_1(.)(\omega_1 - c_1)}_{\text{Forward profits } t=1} + \delta[\underbrace{q_2(.)(\omega_2 - c_2)}_{\text{Forward profits } t=2} + \underbrace{v(p_2)(\sigma - g - k)}_{\text{Backward profits}}] \tag{6}$$

$$\max_{p_1,p_2} \Pi_R = \underbrace{q_1(.)(p_1 - \omega_1)}_{\text{Forward profits } t=1} + \delta \underbrace{q_2(.)(p_2 - \omega_2)}_{\text{Forward profits } t=2} \tag{7}$$

Under the exogenous rebate regime, we employ the return function $v(p_2)$ rather than $r(p_t)$ and the rebate k instead of ϵp_1. Also profits should hold $\Pi_M > 0$ and $\Pi_R > 0$.

The game is played *á la Stackelberg* where M is the leader and maximizes his discounted sum of profits by optimally setting the wholesale prices over the two periods. Being the follower of the game, R maximizes its profit function by optimally choosing the retail prices in the two periods. In the first period, M chooses ω_1 to maximize his profit, while R takes ω_1 as given and chooses the retail price p_1 to maximize her profit. M also considers the second period collection decisions that affect his current and future profits. In fact, in the second period, some of the customers decide to dump their past-purchases. In period two, M introduces a new product. While the second period prices ω_2 and p_2 are obtained as described in $t = 1$, M also considers the return of past-sold products. The return process also affects the R's retail prices through the wholesale prices.

Table 1 displays the notations we employ.

Notations	Description
M (R)	Player M (R)
$t = 1, 2$	Periods
α_t	Market potential in period t
β	Consumers' sensitivity to price in period t
c_t	Marginal production cost in period t
q_t	Demand in period t
δ	Discount factor
ω_t	Wholesale price in period t
p_t	Retail price in period t
σ	Return residual value
g	Per-return collection cost
k	Fixed rebate
θ	Passive return quantity
ϵ	Variable rebate rate
γ	Active return approach effectiveness

Table 1. Notations

3 Markovian equilibrium market outcomes

We start our analysis by using Markovian equilibrium analysis of CLSC games in the presence of the two return functions defined in Eqs. (2) and (3).

3.1 Markovian equilibrium with variable rebate (M-scenario)

In this section, we characterize outcomes of the CLSC game under Markovian equilibrium strategies with the variable rebate in $r(p_t)$, thus named $M - scenario$. In each period, the M is the leader and optimally chooses the wholesale price ω_t, and next R optimally chooses the retail price p_t. In addition, M collects some past-sold products to be remanufactured in $t = 2$. To figure out Stackelberg equilibrium outcomes we solve the game backwards. This is because the current decisions impact the future strategies and profits. The profit functions for both players are given in Eqs. (4)-(5) and the return function is as in Eq. (2). The detailed solution of the game is given in the Appendix and the equilibrium strategies are summarized in the following propositions.

Proposition 1 *With the variable rebate, the Markovian pricing strategies are given by:*

$$\omega_1^M = \frac{\Sigma}{32\beta^3\left(\epsilon^2\delta\gamma\left(4\beta-\gamma\right)+8\beta^2\right)} \tag{8}$$

$$p_1^M = \frac{\epsilon\delta\gamma\left(\left(g-\sigma\right)\gamma+\beta c_2\right)+\left(8\beta\alpha_1-\gamma\delta\epsilon\alpha_2\right)}{\beta\left(16\beta^2-\gamma^2\delta\epsilon^2\right)}+\frac{8\beta}{16\beta^2-\gamma^2\delta\epsilon^2}\omega_1^M \tag{9}$$

$$\omega_2^M = \frac{8\beta^2\left(\left(g-\sigma\right)\gamma+\alpha_2+\beta c_2\right)+\epsilon\gamma\left(4\beta\alpha_1-\gamma\delta\epsilon\alpha_2\right)}{\beta\left(16\beta^2-\gamma^2\delta\epsilon^2\right)}+\frac{4\beta\gamma\epsilon}{16\beta^2-\gamma^2\delta\epsilon^2}\omega_1^M \tag{10}$$

$$p_2^M = \frac{4\beta^2\left(\left(g-\sigma\right)\gamma+3\alpha_2+\beta c_2\right)+\epsilon\gamma\left(2\beta\alpha_1-\gamma\delta\epsilon\alpha_2\right)}{\beta\left(16\beta^2-\gamma^2\delta\epsilon^2\right)}+\frac{2\beta\gamma\epsilon}{16\beta^2-\gamma^2\delta\epsilon^2}\omega_1^M \tag{11}$$

where Σ is a constant term that consists of a complex network of relationships among all model parameters.

Proof. See the Appendix. □

Note that in addition to the Assumptions 1-8, the expressions in the strategies should be positive so that all prices are positive.

Observe that the first-period wholesale price heavily depends on all parameter values[1] and also influences the retailer's strategies. As there are 13 parameters, we face involved expressions. To make the model outcomes more tractable, we will fix some minor parameters in order to focus on the key features of the strategies. Thus, we fix the following parameters:

i. $\delta = 1$: firms give the same importance to present and future cash flows in two periods.

ii. $c_t = 0$: the marginal production cost is normalized to zero.

iii. $\beta = 1$: consumer price responsiveness is unity (any marginal price increase implies marginal sales decrease of the same amount).

iv. $\gamma = 1$: active return response to price changes is unity.

Note that if the return is assumed to be passive, then $\gamma = 0$ holds in any return function.

With this simplification the first-period wholesale price becomes:

$$\omega_1^M = \frac{7\epsilon\left(\epsilon^2+16\right)\left(\sigma-g+\alpha_2\right)+8\epsilon\theta\left(\epsilon-4\right)\left(\epsilon+4\right)+\alpha_1\left(\epsilon^2\left(\epsilon^2-136\right)+128\right)}{32\left(3\epsilon^2+8\right)} \tag{12}$$

thus entailing more analytically tractable function.

Proposition 2 *Higher consumers' willingness to voluntarily return past-sold products (θ) leads to lower pricing strategies.*

[1] Note that: $\Sigma = \begin{pmatrix} -8\beta^2\left(\gamma^2\delta\epsilon^2-16\beta^2\right)\left(\alpha_1+\beta c_1\right)+\alpha_2\epsilon\delta\gamma\beta\left(112\beta^2-\epsilon^2\delta\gamma\left(9\gamma-16\beta\right)\right) \\ +\epsilon\delta\gamma\beta\left(\gamma-8\beta\right)\left(16\beta^2+\gamma^2\delta\epsilon^2\right)\left(g-\sigma\right)-16\left(\gamma\delta\epsilon^2-\beta\right)c_2\epsilon\delta\gamma\beta^3 \\ -128\epsilon\delta\beta^3\left(\theta\beta+\gamma\epsilon\alpha_1\right)+\epsilon^3\delta^2\gamma^2\beta^2\left(8\theta+\gamma c_2\right)+\gamma^4\delta^2\epsilon^4\alpha_1 \end{pmatrix}$

Proof. Compute $\frac{\partial \omega_1^M}{\partial \theta}$ to show that $\frac{\partial \omega_1^M}{\partial \theta} = \frac{\epsilon\theta(\epsilon-4)(\epsilon+4)}{4(3\epsilon^2+8)} < 0$, as $\epsilon \in (0,1)$. Then, use the Envelop Theorem to check that:

1. $\frac{\partial p_1^M}{\partial \theta} = \frac{\partial p_1^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \theta} = \frac{\beta\epsilon\theta(\epsilon-4)(\epsilon+4)}{2(16\beta^2-\gamma^2\delta\epsilon^2)(3\epsilon^2+8)} < 0$;
2. $\frac{\partial p_2^M}{\partial \theta} = \frac{\partial p_2^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \theta} = \frac{\epsilon\theta\beta\gamma\epsilon(\epsilon-4)(\epsilon+4)}{(16\beta^2-\gamma^2\delta\epsilon^2)(3\epsilon^2+8)} < 0$; and
3. $\frac{\partial \omega_2^M}{\partial \theta} = \frac{\partial p_1^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \theta} = \frac{8\epsilon\theta\beta\gamma\epsilon(\epsilon-4)(\epsilon+4)}{2(16\beta^2-\gamma^2\delta\epsilon^2)(3\epsilon^2+8)} < 0$. $\square$

Interestingly, a large voluntary return rate θ leads firms to reduce their prices at all CLSC levels for all periods. The idea behind this result is intuitive: When more consumers are willing to voluntarily return the past purchases, M lowers p_1 to increase his profits. This is evident from the M's benefit from a return $\pi_{M_2} = \sigma - g - \epsilon p_1$. Consequently, the voluntary return rate has a positive influence on the firm sales (i.e., $\frac{\partial q_t^M}{\partial \theta} = \frac{\partial q_t^M}{\partial p_t^M}\frac{\partial p_t^M}{\partial \theta} > 0$). Also, because $\frac{\partial q_t^M}{\partial \theta} > 0$, $\frac{\partial \omega_t^M}{\partial \theta} < 0$ and $\frac{\partial p_t^M}{\partial \theta} < 0$, it implies that the industry profits also increase in θ: $\frac{\partial \Pi_M^M}{\partial \theta} > 0$ and $\frac{\partial \Pi_R^M}{\partial \theta} > 0$.

Proposition 3 *The first (second) period prices increase (decrease) in the residual value of return σ, and decrease (increase) in the marginal collection cost g.*

Proof. Use Eq.(12) to compute to check that $\frac{\partial \omega_1^M}{\partial \sigma} = \frac{7\epsilon(\epsilon^2+16)}{32(3\epsilon^2+8)} > 0$. Substitute the fixed parameters i-iv and Eq.(12) into Eqs. (9), (10) and (11) to check that: $\frac{\partial p_1^M}{\partial \sigma} = -\frac{(19\epsilon^2+144)\epsilon}{4(3\epsilon^2+8)(\epsilon-4)(\epsilon+4)} > 0$, $\frac{\partial \omega_2^M}{\partial \sigma} = -\frac{(7\epsilon^2+32)}{8(3\epsilon^2+8)} < 0$, and $\frac{\partial p_2^M}{\partial \sigma} = \frac{(304\epsilon^2+7\epsilon^4+512)}{16(3\epsilon^2+8)(\epsilon-4)(\epsilon+4)} < 0$. Opposite signs apply when computing these derivatives with respect to g. $\square$

This result is in line with the previous CLSC literature, e.g., Savaskan et al. (2004), according to which when the benefit of remanufacturing is large, i.e., $\sigma - g > 0$, the M conveniently remanufactures returns without worrying about the sale reduction that eventually occurs. Note that, in equilibrium, the first-period strategies change with opposite sign with respect to the second-period strategies. Higher remanufacturing efficiency leads firms to charge larger prices that, although it pushes down the demand, exert a positive influence to the profits, as it is displayed in Figures 1 and 2.

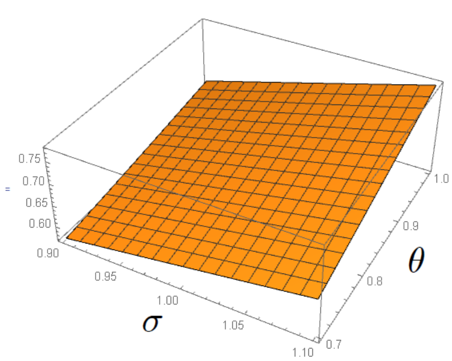


Figure 1 - Π_M^M in the $(\sigma,\theta) - space$

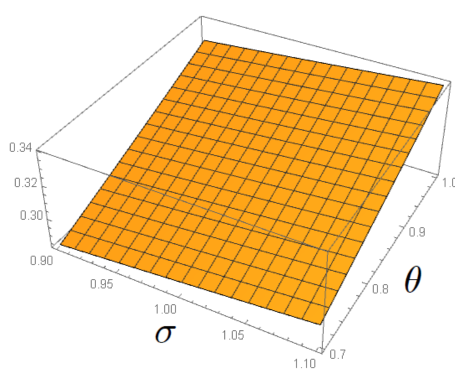


Figure 2 - Π_R^M in the $(\sigma,\theta) - space$

Proposition 4 *Any active return approach ($\gamma > 0$) results in higher prices.*

Proof. Substitute conditions $i - iii$ and $\gamma = 0$ in Eq. (8) to obtain $\omega^M_{1|\gamma=0} = \frac{\alpha_1 - \theta\epsilon}{2}$, which is the first-period wholesale price under a passive return policy. Use Eq. (12) to show that $\frac{\partial \omega_1^M}{\partial \gamma} > 0$ by computing $\omega^M_{1|\gamma=1} - \omega^M_{1|\gamma=0} = \frac{7\left(\epsilon^2+16\right)(\sigma-g+\alpha_2)+\epsilon\left(8(23\alpha_1-7\theta\epsilon)+\epsilon^2\alpha_1\right)}{32(3\epsilon^2+8)} > 0$. Then, use conditions $i - iii$ and the Envelop Theorem to show that:

1. $\frac{dp_1^M}{d\gamma} = \frac{\partial p_1^M}{\partial \gamma} + \frac{\partial p_1^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \gamma} = \frac{16\gamma\epsilon^2\omega_1 - 16(2\gamma(\sigma-g)+\alpha_2)+\epsilon\gamma(16\alpha_1-\gamma\epsilon\alpha_2)}{(\gamma\epsilon-4)^2(\gamma\epsilon+4)^2} + \frac{8}{16-\gamma^2\epsilon^2}\frac{\partial \omega_1^M}{\partial \gamma} > 0$
2. $\frac{d\omega_2^M}{d\gamma} = \frac{\partial \omega_2^M}{\partial \gamma} + \frac{\partial \omega_2^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \gamma} = \frac{4\left(2\left(16-\epsilon^2\gamma^2\right)(g-\sigma)+\epsilon\left((\alpha_1+\omega 1)\left(\gamma^2\epsilon^2+16\right)-4\gamma\epsilon\alpha_2\right)\right)}{(\gamma\epsilon-4)^2(\gamma\epsilon+4)^2} + \frac{4\gamma\epsilon}{16-\gamma^2\epsilon^2}\frac{\partial \omega_1^M}{\partial \gamma} > 0$
3. $\frac{dp_2^M}{d\gamma} = \frac{\partial p_2^M}{\partial \gamma} + \frac{\partial p_2^M}{\partial \omega_2^M}\frac{\partial \omega_2^M}{\partial \gamma} = \frac{2\left(2\left(16+\epsilon^2\gamma^2\right)(g+\sigma)+\epsilon\left((\alpha_1+\omega_1)\left(\gamma^2\epsilon^2+16\right)-4\gamma\epsilon\alpha_2\right)\right)}{(16-\gamma^2\epsilon^2)} + \frac{2\gamma\epsilon}{16-\gamma^2\epsilon^2}\frac{\partial \omega_1^M}{\partial \gamma} > 0$ □

When the consumer returns are based on the product prices ($\gamma > 0$), consumers tend to return a lower number of used products relative to the product returns under passive return approach ($\gamma = 0$). This causes less backwards profits. Also, as more consumers hold on to their used products, lower number of consumers is expected in the second period. These two reasons lead to firms to charge higher prices to offset the lost profits due to the lower sales under the active return approach. Alternatively, as the product is more "durable" (more consumers hold on to the used product), prices should be higher.

Proposition 5 *A large rebate rate given to consumers pushes firms to charge lower prices.*

Proof. Use Eq. (12) to check that $\frac{\partial \omega_1}{\partial \epsilon} = \frac{7(\sigma-g+\alpha_2)\left(3\epsilon^2\left(\epsilon^2-8\right)+128\right)+8\theta\left(72\epsilon^2+3\epsilon^4-128\right)+2\alpha_1\epsilon\left(16\left(\epsilon^2-92\right)+3\epsilon^4\right)}{32(3\epsilon^2+8)^2} < 0$.

1. $\frac{dp_1^M}{d\epsilon} = \frac{\partial p_1^M}{\partial \epsilon} + \frac{\partial p_1^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \epsilon} = \frac{\left(\epsilon^2+16\right)(g-\sigma-\alpha_2)+16\epsilon(\alpha_1+\omega_1)}{(\epsilon+4)^2(\epsilon-4)^2} + \frac{8}{16-\epsilon^2}\frac{\partial \omega_1^M}{\partial \epsilon} < 0$,
2. $\frac{d\omega_2^M}{d\epsilon} = \frac{\partial \omega_2^M}{\partial \epsilon} + \frac{\partial \omega_2^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \epsilon} = -\frac{4\left(4\epsilon(\sigma-g+\alpha_2)+(\omega_1-\alpha_1)\left(16+\epsilon^2\right)\right)}{(\epsilon+4)^2(\epsilon-4)^2} + \frac{4\epsilon}{16-\epsilon^2}\frac{\partial \omega_1^M}{\partial \epsilon} < 0$,
3. $\frac{dp_2^M}{d\epsilon} = \frac{\partial p_2^M}{\partial \epsilon} + \frac{\partial p_2^M}{\partial \omega_1^M}\frac{\partial \omega_1^M}{\partial \epsilon} = -\frac{2\left(4\epsilon(\sigma-g+\alpha_2)+(\omega_1-\alpha_1)\left(16+\epsilon^2\right)\right)}{(\epsilon+4)^2(\epsilon-4)^2} + \frac{2\epsilon}{16-\epsilon^2}\frac{\partial \omega_1^M}{\partial \epsilon} < 0$. □

The rate of rebate measured by ϵ has several effects for the manufacturer profits and prices. The first one is the "cost effect": the larger the rebate the larger the cost to the manufacturer. The second one is the "revenue effect": as the value of return is positive, i.e., $\pi_{M_2} = \sigma - g - \epsilon p_1 > 0$ it is economically convenient to remanufacture and give a rebate. Furthermore, the larger the rebate the higher the number of returns are (as r is increasing in ϵ). Therefore, M's backward profit increases in rebate rate. The third one is the "sales effect": the larger rebates can increase the sales of new product so that quantity demanded will increase. This will pressure down the final sale price for the retailer. M will react to lower retail price by decreasing its wholesale price charged to the retailer.

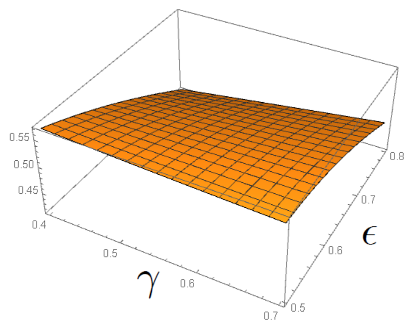


Figure 3 - Π_M^M in the $(\gamma, \epsilon) - space$

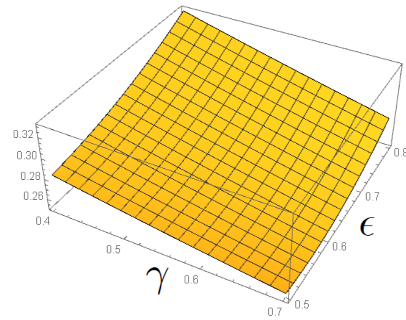


Figure 4 - Π_R^M in the $(\gamma, \epsilon) - space$

Figure 3 illustrates the relationship between M's profit and the two key parameters: active return parameter γ and the rate of rebate parameter ϵ. For "low" levels of γ, corresponding to higher number of returns, the M's profit decreases in the rebate rate. This is mainly due to the "revenue effect" stemming embedded in the backward profits. For "high" levels of γ, corresponding to a lower number of returns, M's profit still decreases in the rebate rate. This is because of the larger impact of the forward profits (than the backward profits) which decreases as a result of lower wholesale prices.

Figure 4 depicts the relation between R's profit and the parameters γ and ϵ. For all levels of γ, that is whether the number of returns are low or high, the R's profit increases in the rebate rate. This is mainly due to the increased number of sales. Because the wholesale and retail prices decrease in the rebate rate (Proposition 5), the quantities sold in both periods rise. As the change in price-cost margin (the retail and wholesale price differential) is lower than the rate of increase in the sales, the profits increase. Also, observe that the highest level of R's profit is attained when $\gamma = 0$, that is when the return quantity is the maximum.

3.2 Markovian equilibrium with fixed rebate ($\widetilde{M}$-scenario)

Similar to the M-scenario, we will characterize the closed-loop supply chain game assuming that the consumer rebate is fixed and the return function varies with the second-period price (Eq. (3)), namely $\widetilde{M} - scenario$. While this setting follows a similar structure with the previous one, the lower number of interactions among decision variables substantially simplifies the game solution. This is because the consumers know upfront the rebate they will obtain when returning the past-sold products, independently of their conditions. Some examples describing a fixed rebate is trade-in and save programs of BestBuy for used cell phones, of (the US and Canada) governments for used cars over 20 years old, and of H&M for used-clothes bins. The consumers look at the difference between the price for purchasing a new product and the fixed rebate, then decide whether to keep the product or return it. The market outcomes are obtained by solving the game backwards.

When the return function is $v(p_2) = \theta - \gamma(p_2 - k)$, the firms' profit functions are:

$$\max_{\omega_1^{\widetilde{M}},\omega_2^{\widetilde{M}}} \Pi_M^{\widetilde{M}} = \underbrace{q_1^{\widetilde{M}}(.)(\omega_1^{\widetilde{M}} - c_1)}_{\text{Forward profits } t=1} + \delta[\underbrace{q_2^{\widetilde{M}}(.)(\omega_2^{\widetilde{M}} - c_2)}_{\text{Forward profits } t=2} + \underbrace{v(p_2)(\sigma - g - k)}_{\text{Backward profits}}] \tag{13}$$

$$\max_{p_1^{\widetilde{M}},p_2^{\widetilde{M}}} \Pi_R^{\widetilde{M}} = \underbrace{q_1^{\widetilde{M}}(.)(p_1^{\widetilde{M}} - \omega_1^{\widetilde{M}})}_{\text{Forward profits } t=1} + \delta\underbrace{q_2^{\widetilde{M}}(.)(p_2^{\widetilde{M}} - \omega_2^{\widetilde{M}})}_{\text{Forward profits } t=2} \tag{14}$$

All stages of the games are solved in detail in the Appendix and summarized in the following propositions.

Proposition 6 *With the exogenous rebate, the Markovian pricing strategies are given by:*

$$\omega_1^{\widetilde{M}} = \frac{\alpha_1 + \beta c_1}{2\beta} \tag{15}$$

$$p_1^{\widetilde{M}} = \frac{3\alpha_1 + \beta c_1}{4\beta} \tag{16}$$

$$\omega_2^{\widetilde{M}} = \frac{(g - \sigma + k)\gamma + \alpha_2 + \beta c_2}{2\beta} \tag{17}$$

$$p_2^{\widetilde{M}} = \frac{(g - \sigma + k)\gamma + 3\alpha_2 + \beta c_2}{4\beta} \tag{18}$$

Proof. See the Appendix. □

Interestingly, we find that the first period decisions are independent of the customers' return decisions. This is because of the missing interface between the first and the second period strategies inside the return function that leads to a full independence of the first period prices ($\omega_1^{\widetilde{M}}$ and $p_1^{\widetilde{M}}$) with respect to the return and collection parameters (g, σ, k) namely, the collection cost, the return residual value, and the rebate. Consequently, the Stackelberg equilibrium strategies in this scenario will be much different than the ones in the previous return scenario. Indeed, to our knowledge, in all of the dynamic CLSC settings examined in the literature (e.g., Savaskan et al., 2004, vs. De Giovanni and Zaccour, 2014), the current decisions have not been interlinked with the future period decisions. Therefore, the qualitative behavior of the $\widetilde{M}$-strategies with respect to the M-strategies will be different.

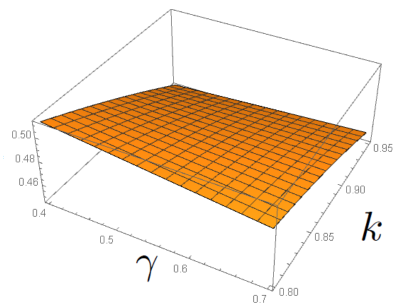


Figure 5 - $\Pi_M^{\widetilde{M}}$ in the $(\gamma, k) - space$

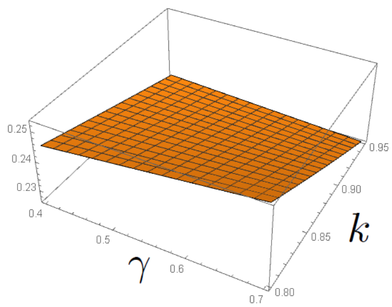


Figure 6 - $\Pi_R^{\widetilde{M}}$ in the $(\gamma, k) - space$

Figure 5 demonstrates that there is only one regime characterizing the equilibrium profit function for M such that its profit is decreasing in both γ and k. This result is the consequence of positive marginal benefit of remanufacturing and lower cost of payments to the customers. The maximum of M's profit is attained when the rebate is the lowest and the number of the customers who dump the used product is the highest.

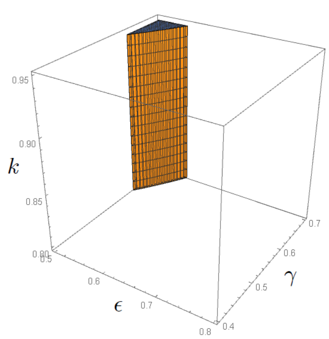
Figure 6 shows the relationship between R's optimal profit function and the two key model parameters. The retailer's profit function decreases in rebate k and increases in price sensitivity to returns γ. Note that the first period decisions will not change as a result of changes in γ and k. Only $\omega_2^{\widetilde{M}}$ and $p_2^{\widetilde{M}}$ will change, as it is clear from the above proposition. As k increases, the cost goes up for M. To offset the reduction in profit the M must increase its wholesale price $\omega_2^{\widetilde{M}}$, that is $d\omega_2^{\widetilde{M}}/dk > 0$, as obtained from the proposition. In response to increasing wholesale price (which is the marginal cost for R), the R must increase its retail price. This holds because $dp_2^{\widetilde{M}}/dk > 0$, by the proposition. Facing higher prices, consumers will buy less in the second period, which will result in profit decrease for the R. That is, $d\Pi_R^{\widetilde{M}}/dk < 0$ holds, which is also observed in Figure 6. Similarly, as γ increases, the number of returns goes down. This reduces the backward profit of M. To offset the reduction in the total profit M decreases its wholesale price $\omega_2^{\widetilde{M}}$, that is $d\omega_2^{\widetilde{M}}/d\gamma < 0$, which is also confirmed by the proposition. In response to decreasing wholesale price (which is the marginal cost for R), the R decreases its retail price. That is, $dp_2^{\widetilde{M}}/d\gamma < 0$ holds, which is also confirmed by the proposition. Facing lower prices, consumers will buy more in the second period, which will result in profit increase for R. That is, $d\Pi_R^{\widetilde{M}}/d\gamma > 0$ holds, as it is also observed in Figure 6.

3.3 Comparison of M and $\widetilde{M}$ scenarios

We investigate the differences in market outcomes for the proposed return functions (with price dependent variable rebate represented by r function, and with a fixed rebate represented by v function). We show the differences (in strategies and profits) algebraically as well as schematically. The wholesale price difference in period 1 under the two return functions is calculated below given the conditions $i - iv$.

$$\omega_1^M - \omega_1^{\widetilde{M}} = \frac{7\epsilon\left(\epsilon^2+16\right)(\sigma - g + \alpha_2) + 8\epsilon\theta\,(\epsilon-4)(\epsilon+4) + \alpha_1\left(\epsilon^2\left(\epsilon^2-184\right)\right)}{32\left(3\epsilon^2+8\right)} > 0 \tag{19}$$

Figure 7 exhibits a visual comparative statistics with respect to the key model parameters, where the solid area corresponds to the case in which $\omega_1^M - \omega_1^{\widetilde{M}} > 0$. It is clear from the Eq. (19), and also from the Figure 7 that the M charges a higher wholesale price to the R under the return function in which the rebate depends on the initial purchase price. Figure 7 provides further information and demonstrates that the farer the rebate to the initial buy price p_1, that is $\epsilon p_1 \to p_1$, which corresponds to a larger rebate, the wholesale price differential $\omega_1^M - \omega_1^{\widetilde{M}}$ decreases. Simply, the higher the rebate rate ϵ, the bigger is the wholesale price divergence. As k does not show up in the price differential (because it only impacts $\omega_2^{\widetilde{M}}$) the first period price differential $\omega_1^M - \omega_1^{\widetilde{M}}$ does not vary with k in the figure. However, the change in γ impacts the wholesale price differential. Specifically, decreasing γ, implying higher returns, reduces the wholesale price differential, but still $\omega_1^M - \omega_1^{\widetilde{M}} > 0$. This is because $\frac{\partial \omega_1^M}{\partial \gamma} > 0$ and $\frac{\partial \omega_1^{\widetilde{M}}}{\partial \gamma} = 0$.



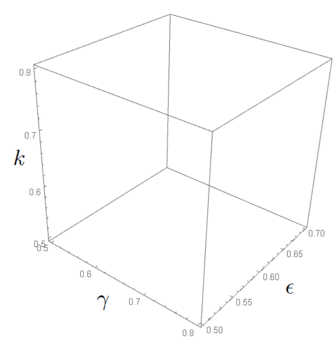


Figure 7 - $\omega_1^M - \omega_1^{\widetilde{M}}$ in the $(\gamma, k, \epsilon) - space$ Figure 8 - $p_1^M - p_1^{\widetilde{M}}$ in the $(\gamma, k, \epsilon) - space$

Next the comparison of the first-period retail prices is as follows:

$$p_1^M - p_1^{\widetilde{M}} = \frac{\left(-4\epsilon\delta\gamma\,(\alpha_2 - \beta c_2) + 16\alpha_1\beta\,(2-3\beta) + \epsilon^2\delta\gamma^2\,(3\alpha_1 + \beta c_1) - 16\beta^3 c_1 + 4\epsilon\delta\gamma^2\,(g-\sigma)\right)}{4\beta\left(16\beta^2 - \gamma^2\delta\epsilon^2\right)} + \frac{8\beta}{16\beta^2 - \gamma^2\delta\epsilon^2}\omega_1^M \tag{20}$$

which must be positive because the term in Eq. (19), the wholesale price differential, is positive. For a valid set of parameter regions, it is clear in Figure 8 that $p_1^M - p_1^{\widetilde{M}} > 0$ never holds. Thus, the adoption of a Markovian solution concept leads to larger first period wholesale price

Moreover we compare the second period prices under both return functions. We find that

$$\omega_2^M - \omega_2^{\widetilde{M}} = -\frac{\left(\epsilon\delta\gamma\left(\alpha_2 - \beta c_2\right) - 8\beta\alpha_1 - k\left(16\beta^2 - \gamma^2\delta\epsilon^2\right) + \epsilon\delta\gamma^2\left(\sigma - g\right)\right)\gamma\epsilon}{2\beta\left(16\beta^2 - \gamma^2\delta\epsilon^2\right)} + \frac{4\beta\gamma\epsilon}{16\beta^2 - \gamma^2\delta\epsilon^2}\omega_1^M > 0 \quad (21)$$

$$p_2^M - p_2^{\widetilde{M}} = -\frac{\left(\epsilon\delta\gamma\left(\alpha_2 - \beta c_2\right) - 8\beta\alpha_1 + k\left(16\beta^2 - \gamma^2\delta\epsilon^2\right) + \epsilon\delta\gamma^2\left(\sigma - g\right)\right)\gamma\epsilon}{4\beta\left(16\beta^2 - \gamma^2\delta\epsilon^2\right)} + \frac{2\beta\gamma\epsilon}{16\beta^2 - \gamma^2\delta\epsilon^2}\omega_1^M > 0 \quad (22)$$

whose signs can only be checked numerically (see Appendix 2.5). Considering all model parameters, the second period wholesale price under price-dependent rebate (or variable rebate) regime is lower than the price charged under the fixed rebate policy. As we observe from the expression (19) and Figure 8 the same result holds for the first period prices. This result is associated with the high rebate cost ϵp_1 which depends on the final product price. Similarly, the second period retail price under the variable rebate policy is lower than the price under the fixed rebate policy, because of the high cost (i.e., $\omega_2^M < \omega_2^{\widetilde{M}}$).

Figure 9 demonstrates that the manufacturer's profit under variable rebate regime/policy is higher than its profit under the fixed rebate regime. This holds for all parameter regions and is due to the high prices charged under the first regime. This figure also shows that the key parameter that impacts the difference in the profits is the rebate rate (ϵ) that is used under the first policy. The profit difference (in favor of the first policy) increases at a decreasing rate in the rebate rate. While the fixed rebate k has a little impact on the profit difference, the price sensitivity λ has more impact than ϵ to explain the profit difference under the two types of rebate policies. In Figure 10, we observe that the retailer is always better off under the variable rebate policy.

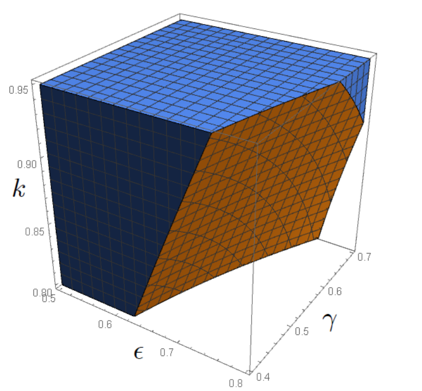


Figure 9 - $\Pi_M^M - \Pi_M^{\widetilde{M}}$ in the $(\gamma, k, \epsilon) - space$

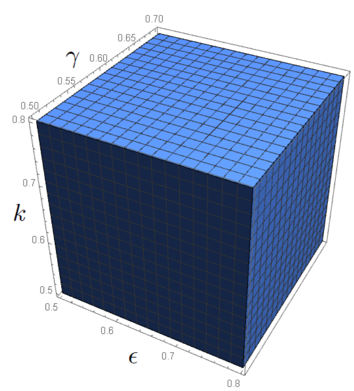


Figure 10 - $\Pi_R^M - \Pi_R^{\widetilde{M}}$ in the $(\gamma, k, \epsilon) - space$

3.4 Computational analysis of Markovian solutions

In this section, we fully compare the Markovian solutions under different rebate structures. This analysis is fully done numerically and considers all parameters. We start from a baseline set that consists of $\alpha_1 = \alpha_2 = 1$, $\beta = 0.5$, $c_1 = c_2 = 0.01$, $g = 0.001$, $\theta = 0.7$; $k = 0.8$, $\sigma = 0.9$, $\gamma = 0.4$, $\epsilon = 0.5$, and $\delta = 0.9$. This benchmark setting is based on recent studies on CLSC and takes into consideration of all assumption that we have introduced earlier. Appendix 2 reports the full numerical analysis when these parameters are varied in a range. When a parameter is varied the other remain at the benchmark value. The main row of each table in Appendix 2 contains the outcomes of the baseline parameter values while the main colon indicates the variations considered in each parameter. Appendix 2.1 and 2.2 display the numerical results on the $M-$ and the $\widetilde{M}$-scenario, respectively, while Appendix 2.3 reports their comparison.

According to Appendix 2.1, the following insights can be derived for the M-scenario:

- When the market potential in the first period, α_1, increases, M experiences increasing profits. This result derives from the variable rebate structure. Intuitively, increasing values of market potential lead to larger prices for both firms. Nevertheless, larger retail prices have an impact on both the returns and the marginal rewards linked to it. While the returns always increase in the market potential because the number of consumers returning the product increases, the margins linked to returns can decrease in the first period price, p_1, thus generating an overall issue of profitability of returns. The retailer is positively affected by larger market potential although the manufacturer directly manages the deals with consumers. Thus, she experiences larger profits due to the higher number of consumers in the first period. In sum, a variable rebate policy generates an important trade-off between sales and returns due to the higher number of consumers in the first period.

- When the market potential in the second period, α_2, increases, the manufacturer increases its profit. This finding comes from the impact of the second period prices on the returns. Larger prices have a negative influence on returns, which decrease in α_2. Increasing number of consumers in the second period entails an interesting trade-off between forward and backward rewards. Forward rewards are always increasing because demand in the second period increases accordingly. However, the returns decrease in p_2. Overall, the manufacturer is able to overcome this trade-off by favoring forward profits and denying the environmental performance.

- As expected, larger values of β lead to lower prices and profits. Interestingly, with the variable rebate structure the returns increase in β due to decreasing prices. Thus, higher consumers sensitivity to price entails a trade-off between forward and reverse flows generating a demand increase in all periods and a decrease in the forward margins, while increasing the returns and the backward margins.

- Increasing values of the marginal collecting profits, $\pi_{M_r} = \sigma - g - \epsilon p_1$, have a peculiar influence on the firms strategies and profits. While the prices in the first period increase in π_{M_r}, the second period prices decrease in its value. This disparity derives from the fact that all elements of π_{M_r} play a role on firm strategies. In the first period, the manufacturer increases the wholesale price in π_{M_r} to make the returns margin lucrative. In fact, increasing wholesale price leads to higher retail price in the first period, and thus larger returns margins. So the CLSC compensates the inefficiency due to returns by changing the pricing strategies accordingly. Nevertheless, increasing the prices also increases the number of returns. In the second period, increasing π_{M_r} intuitively leads to lower prices: firms seek to boost as much returns as possible by decreasing the prices while focusing on the forward flows. Interestingly, while the effects of σ and g on returns are clear, the influence of the rebate ϵp_1 on the returns substantially challenges the CLSC: contrary to g, increasing rebates leads to lower margins but increases the returns. Thus the CLSC should set the pricing strategies to manage the trade-off between returns and profitability.

- When consumers consider the price difference as an important element in their return decisions (e.g., through γ), the prices strategies increase in the first period and decrease in the second period at all levels of the CLSC. This has a dual effect within the supply chain: on one hand, increasing γ generates lower returns, thus the CLSC seeks to balance this loss by increasing the prices in the first period and reducing the prices in the second period. This strategy change has a negative impact on the forward flows due to low sales in the first period and scarce returns in the second period. Thus, high values of γ make the return policy challenging for the entire CLSC, which needs some other additional strategies (e.g., advertising or service) to counterbalance the effect of price difference in the consumers' returning decisions.

- While the discount factor δ does not influence the first period strategies at all, the second period prices increase in it. This deteriorates the prices from consumers' point of view, as such they will always pay more while diminishing the returns. Overall, increasing values of δ is economically sound while deteriorating the returns.

- The production costs c_1 and c_2 unsurprisingly decrease the profits and increase the prices. The most interesting result links to the impact of these parameters on the return function: the returns increase in c_1 and decrease in c_2. This peculiarity is linked to the rate of change of p_1 and p_2 with respect to c_1 and c_2. Increasing values of c_1 increase p_1 more than p_2 thus impacting the returns positively. In contrast, increasing values of c_2 increase p_2 more than p_1, hence influencing the return rate negatively.

- Increasing values of the passive return rate, θ, have positive effect on the businesses overall, exemplified by increasing profits, decreasing prices and increasing returns. Thus, CLSC should focus on regions in which consumers have a certain attitude of returning used goods, independent of the firms' pricing strategies and return behavior.

The Appendix 2.2 displays the sensitivity analysis of the Markovian solution under fixed rebate policy. From a qualitative point of view, firms strategies and profits change in the same direction as in the Markovian case with variable rebate policy. However, increasing the value of fixed rebate makes both players economically worse off. Thus, proposing a fixed rebate to consumers is convenient only when the rebate is sufficiently low. Indeed, as the fixed rebate policy implies an independence between the first period sales and returns, all changes in the first period parameters do not influence the second period strategies and returns. The results displayed in the Appendix 2.3 are relevant for the purpose of comparing variable and fixed rebate policies under the Markovian solution so as to identify the most suitable return strategy that CLSCs should prefer. Accordingly, the following findings can be derived:

- When the market expands in the first period (α_1), the manufacturer prefers adopting a fixed rebate. In fact, he knows that the retailer will post higher prices thus deteriorating the returns margins and quantity. Adopting a fixed rebate policy will make the manufacturer sufficiently safe from high prices charged by the retailer. On her side, the retailer prefers a variable rebate policy because it gives more power to her due to the influence of pricing on the returns function. From an environmental point of view, more people can return when the market expands, thus a variable rebate policy is more suitable to perform the environmental performance.

- When the market in the second period (α_2) expands, the firms' strategies change with respect to the same level as in the first period. When the second period market becomes important, the manufacturer prefers a variable rebate because he can better control the return flow by adjusting the wholesale price accordingly.

- When the consumers' sensitivity to price (β) enlarges, both firms prefer a variable rebate because they can adjust the rebate accordingly. In fact, a fixed rebate penalizes the pricing strategy to much and can lead to lower returns and sales over the entire planning horizon.

- The firms show contrasting preferences according to the remanufacturing parameters σ and g. When remanufacturing is convenient, the manufacturer prefers a variable rebate policy to positively influence the returns and get positive profits from remanufacturing. Instead, the retailer prefers a fixed rebate policy because her pricing strategies will be largely influenced by the wholesale price changes. Note that the retailer does not get any benefits from returns, which are fully retained by the manufacturer, hence the remanufacturing convenience is not balanced over the supply chain.

- Any increase in the marginal production costs leads both firms to prefer a variable rebate policy. This result is somehow expected due to the fact that a fixed rebate policy penalizes the prices and imposes firms to considerably adjust them to also consider the production costs. Under a variable rebate policy, this trade-off can be better managed.

- Increasing values of the passive return rate (θ) leads firms to prefer a variable rebate policy. This parameter plays the role of market expansion for returns, thus firms can better exploit its benefits by adjusting the pricing policy and return strategy accordingly.

- When consumers evaluate the price difference before deciding to return (γ), firms have contrasting preferences relative to the return policy. The manufacturer prefers a variable rebate policy because he seeks to control the return function in both periods. The fixed rebate policy does not give any advantage to the first period strategies, thus he loses some control on the return function. When consumers disregard the price difference and the rebate, the manufacturer can opt for a fixed rebate because the return function is simply less important. On her side, the retailer always prefers a fixed rebate because the wholesale price in the first period is not influenced by the remanufacturing parameters, thus preserving the CLSC from the double marginalization problem.

- Increasing values of the variable rebate (ϵ) will lead to divergent preferences. The manufacturer will always prefer a fixed rebate. Indeed, the manufacturer seeks to give back to consumers a rebate that is as low as possible because the rebate directly influences the remanufacturing profitability. Nevertheless, increasing ϵ lead to higher returns. From her side, the retailer prefers always larger ϵ because she can charge lower prices in the first period and larger prices in the second period, thus increasing her profits.

- Increasing values of the fixed rebate (k) will make both firms economically worse off. This finding depends on the return structure and information availability. In this case, firms must always provide the same amount independent of the firms' strategies. CLSCs should prefer a variable rebate when the fixed rebate takes very large values.

To summarize, firms should prefer a variable rebate policy when facing highly price sensitive consumers (β) and high passive returns (θ). In contrast, they both prefer a fixed rebate policy for large marginal production cost values (c_t). In all other cases, firms show divergent preferences. Interestingly, in most of the cases in which the M's profits increase in the model parameters, the environmental performance are damaged, thus highlighting the serious difficulties that CLSCs encounter in selecting a rebate policy while balancing both the economic and the environmental outcomes. When the fixed rebate is high, both firms prefer the adoption of a variable rebate policy. When the variable rebate policy is high, the manufacturer would implement a fixed rebate policy while the retailer would always prefer a variable rebate policy.

4 Open-loop equilibrium market outcomes

We intend to examine the same CLSC game with endogeneous return functions in Eq. (2) and (3) within a closely related information structure which involves the characterization of Open-loop Stackelberg equilibrium (OLSE). Studying OLSE will disclose the strategic value of decisions and how market outcomes (profits, prices, outputs) could differ from the Markov perfect solution. As mentioned in the introduction, it is common to compare Markov perfect and open-loop strategies in dynamic games literature covering environmental and resource economics, capital accumulation games, advertising investments, and marketing channel. However, to our knowledge, this is the first paper comparing market outcomes under different equilibrium concepts within a CLSC framework.

Alternatively, the open-loop solution can be considered a benchmark case to differentiate the strategic value of production/sale that is observed under the Markov perfect behavior. Also, we note that open-loop equilibria can be used in a moving-horizon approach to approximate a Markov perfect (or closed-loop) equilibrium, see, e.g., van der Broek (2002) and Yang (2003). Further, some studies find that open-loop equilibria have some empirical support. For instance, Haurie and Zaccour (2004) and Pineau et al. (2011) compare the predicted open-loop equilibrium strategies to realizations in the European gas market and the Finnish Electricity industry, respectively, and find that the outcomes are close to each other. Similar to the open-loop concept, electricity traders in the wholesale markets regularly employ fixed-mix investment strategies for power portfolio optimization (see Sen et al. (2006)).

For a firm precommitting to a production profile (open-loop concept) could be an optimum strategy if its rival or a firm in the supply chain chooses its strategy at the outset of the game. In other words, in a CLSC game if M is playing an open-loop strategy (a vector of wholesale prices), R should also choose its open-loop strategy (a vector of retail prices). Characterization of open-loop strategies relies on optimally choosing all decisions at the beginning of the game, that is precommitting to the strategies, assuming that all players follow the suit. Note that in the open-loop solution, firms still respond to each other, (that is, R takes the wholesale price given and sells the same quantity it buys from M) and evaluate the impact of current decisions on the future profits given the available information at the beginning of the game.

4.1 Open-loop equilibrium with variable rebate (O-scenario)

Similar to the previous sections, we keep the leader-follower relationship between M and R in the CLSC. The game formulation is as in (1-5), which is solved in the proof of the following proposition in detail. There are two stages in the solution. In the first stage, R maximizes its profit function in (5) to choose both p_1 and p_2 functions (of wholesale prices) simultaneously. In the second stage, M optimally chooses both ω_1 and ω_2 by maximizing (4), given the R's strategies p_1 and p_2.

Under the open-loop approach, R ignores the indirect impact of p_2 and ω_2 on p_1. Rather, R chooses p_1 and p_2 simultaneously. Therefore, while the leader-follower game structure is preserved, firm(s) may lose the strategy update over the stages. However, as we show in the following subsection, it can be subgame perfect equilibrium to discard the strategy updates (i.e., R's ignorance of the impact of p_2 on p_1). Consequently, the firms' strategies are characterized in the following proposition.

Proposition 7 *With the variable rebate, the Open-loop Stackelberg equilibrium strategies are*

$$\omega_1^O = \frac{2\beta\left(4\beta^2 c_1 + \epsilon\delta\left(g\gamma^2 - 4\theta\beta\right)\right) + 2\epsilon\delta\gamma\beta\left(\beta\left(4\left(\sigma - g\right) + c_2\right) + 3\alpha_2 - \sigma\gamma\right) + \alpha_1\left(8\beta^2 + \epsilon^2\delta\gamma\left(\gamma - 8\beta\right)\right)}{\beta\left(16\beta^2 + \gamma\delta\epsilon^2\left(8\beta - \gamma\right)\right)} \quad (23)$$

$$\omega_2^O = \frac{2\beta\left(3\gamma\epsilon\alpha_1 + 4\beta\left(\alpha_2 + \beta c_2\right)\right) + 2\gamma\beta^2\left(4\left(g - \sigma\right) + \epsilon c_1\right) + \epsilon^2\delta\gamma\left(\alpha_2\left(4\beta + \gamma\right) + 2\beta\left(2\beta c_2 - \left(\theta + \gamma\left(\sigma - g\right)\right)\right)\right)}{\beta\left(16\beta^2 + \gamma\delta\epsilon^2\left(8\beta - \gamma\right)\right)} \quad (24)$$

$$p_1^O(\omega_1^O) = \frac{\alpha_1 + \beta\omega_1^O}{2\beta} \quad (25)$$

$$p_2^O(\omega_2^O) = \frac{\alpha_2 + \beta\omega_2^O}{2\beta} \quad (26)$$

Proof. *See the Appendix.* □

All prices in Eqs. (21-24) are decreasing in ϵ and increasing in γ, as in the Markovian solution. Also, the first and the second period decisions are interlinked. That is, initial period decisions impact the current profits as well as the future prices and the profits. This is contrary to the independence of the first period decisions from the second period ones observed under the fixed rebate policy (that is, v function). Furthermore, the major difference between the Markovian and open-loop strategies is that ω_1 only impacts p_1, and ω_2 only impacts p_2 in the open-loop solution. However, in the Markovian solution the first period wholesale price ω_1 impacts all of the prices. R cannot adjust the first period pricing strategies according to the M's second period strategies under an open loop solution, thus losing some decision power.

4.2 Open-loop equilibrium with exogenous rebate ($\widetilde{O}$-scenario)

When the consumer return behavior is characterized by $v\left(p_2\right) = \theta - \gamma\left(p_2 - k\right)$ as in Eq. (3), that is consumers are offered a fixed rebate for their used products and decide whether to return the product based on the difference between the new product price p_2 and the rebate k, the open-loop Stackelberg equilibrium strategies will coincide with the Markov perfect Stackelberg outcomes under exogenous rebate.

Proposition 8 *Under the exogenous rebate, firms' strategies do not vary according to the solution concepts (equilibrium types) adopted.*

Proof. See the Appendix. □

The intuition for this finding is that the first period decisions ω_1 and p_1 are totally independent from the second period prices ω_2 and p_2, when the rebate is fixed. The first period prices do not influence the second period decisions as well as the return decisions. Consequently, when the CLSC implements an exogenous rebate, the market outcomes are invariant to the solution concept adopted. The equilibrium strategies are as defined in Eqs. (13-16).

4.3 Comparison of O and $\widetilde{O}$ scenarios

The differences between the open-loop strategies under the two different $v(p_2)$ and $r(p_1, p_2)$ return scenarios will only spring from the nature of rebate type (fixed versus variable rebate). To explore the impact of return function on market outcomes in the open-loop framework, we will compare the equilibrium prices under these return functions. The initial period wholesale prices compare as follows:

$$\omega_1^O - \omega_1^{\widetilde{O}} = -\frac{\left(\gamma\beta\left(4\beta-\gamma\right)\left(4\left(\sigma-g\right)-\epsilon c_1\right)-12\gamma\beta\left(\alpha_2-2\epsilon\alpha_1\right)-3\gamma^2\epsilon\alpha_1+4\beta^2\left(4\theta-\gamma\left(c_2-\epsilon c_1\right)\right)\right)\delta\epsilon}{2\beta\left(16\beta^2+\gamma\delta\epsilon^2\left(8\beta-\gamma\right)\right)} \quad (27)$$

From Figure 11, we observe that the variable rebate approach leads to higher wholesale price to be charged to the retailer such that $\omega_1^O - \omega_1^{\widetilde{O}} < 0$ holds for all admissible parameter values. This finding is same as the one we obtained under the Markov perfect solution illustrated in Figure 7.

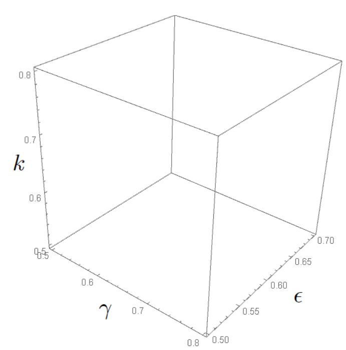


Figure 11 - $\omega_1^O - \omega_1^{\widetilde{O}}$ in the $(\gamma, k, \epsilon) - space$

Next we compare the retail prices under the two rebate policies. We find that

$$p_1^O - p_1^{\widetilde{O}} = -\frac{\left(\gamma^2\left(4\beta\left(\sigma-g\right)-\epsilon\left(3\alpha_1+\beta c_1\right)\right)-4\beta\left(3\gamma\left(\alpha_2-2\epsilon\alpha_1\right)-4\theta\beta\right)-4\gamma\beta^2\left(4\left(\sigma-g\right)+c_2-2\epsilon c_1\right)\right)}{4\beta\left(16\beta^2+\gamma\delta\epsilon^2\left(8\beta-\gamma\right)\right)} < 0 \quad (28)$$

The qualitative behavior of $p_1^O - p_1^{\widetilde{O}}$ will follow a similar shape as in Figure 11, because the retail price difference is linear in the wholesale price difference. This finding is also congruent to the one we obtained for the Markov perfect solution presented in Eq. (20).

Furthermore, we compare the second period open-loop equilibrium prices under both return functions and find that

$$\omega_2^{O} - \omega_2^{\widetilde{O}} = \frac{\left(4\beta\left(3\alpha_1 + \beta\left(c_1 - 4k\right)\right) + \epsilon\delta\left(4\beta\left(\left(\sigma - g\right)\gamma + \theta\right) + \gamma\left(3\alpha_2 + \beta\left(c_2 - 8k\epsilon\right)\right)\right) + \epsilon\delta\gamma^2\left(g - \sigma + k\epsilon\right)\right)\gamma\epsilon}{2\beta\left(16\beta^2 + \gamma\delta\epsilon^2\left(8\beta - \gamma\right)\right)} > 0 \tag{29}$$

$$p_2^{O} - p_2^{\widetilde{O}} = \frac{\left(4\beta\left(3\alpha_1 + \beta\left(c_1 - 4k\right)\right) + 4\epsilon\delta\beta\left(\left(\sigma - g\right)\gamma + \theta\right) + \epsilon\delta\gamma\left(3\alpha_2 + \beta\left(c_2 - 8k\epsilon\right)\right) + \epsilon\delta\gamma^2\left(g - \sigma + k\epsilon\right)\right)}{4\beta\left(16\beta^2 + \gamma\delta\epsilon^2\left(8\beta - \gamma\right)\right)} > 0 \tag{30}$$

Similar to the Markovian prices, open-loop prices are also higher under the variable rebate approach. That is, ω_2 and p_2 under O-scenario are always larger than those in the $\widetilde{O}-$scenario. These higher prices would reflect to higher profits under the variable rebate approach, that is $O-$scenario.

4.4 Computational analysis of the Open-loop solution

Appendix 2.4 displays the sensitivity analysis of the Open-loop solution with variable rebates. From a qualitative point of view, the results are aligned to the Markovian solution with variable rebate, whose sensitivity analysis is displayed in Appendix 2.1. Nevertheless, the comparison with the fixed rebate provides some different insights. Hereby, we comment on the results that differ from the ones in the Appendix 2.3, while the reader can refer to section 3.4 for the additional comments which also apply to the Open-loop solution:

- Increasing the values of γ changes the manufacturer's preferences with respect to the Open-loop solution. In fact, he will prefer the implementation of a fixed rebate when consumers' take into consideration the difference between the price of new releases and the rebate to return their used products. A variable rebate leaves the decision on the return basically to the retailer, whose strategies are only partially influenced by the wholesale price strategies due to the independence between strategies over time. Thus, a fixed rebate offers the possibility to lower the retailer's influence and adjust the strategies accordingly when the CLSC plays open-loop.

- The manufacturer will prefer a fixed rebate policy according to the marginal production costs (c_t) in both periods. The separation between strategies over time allows the double marginalization effect to decrease, thus the fixed rebate is much more manageable from an economic point of view.

- If the first and the second period profits are equally important, then the manufacturer prefers a fixed rebate policy. This depends on the moments in which firms optimize their profits. A fixed rebate policy avoids that the manufacturer mixes first and second period flows and strategies, thus he optimizes the second period while fully disregarding the first period and vice-versa. When this separation occurs in the market, the open-loop solution pushes for the adoption of a fixed rebate policy.

- Under the open-loop solution, the manufacturer experiences larger profits when passive return value

(θ) increases. With a fixed rebate, the manufacturer reduces the retailer's influence on the return rate considerably, while he optimizes his profits by fully taking into consideration of the number of passive returns.

Finally, when firms set their strategies by using an open-loop concept, they should most likely prefer the adoption of a fixed rebate to improve both economic and the environmental performance. Note that the retailer's profits tend to increase with any marginal increase in the parameters, thus both firms will prefer a fixed rebate policy when the business expands. Variations in the parameter values show improvements in both the profits and the returns for several parameters, specifically, α_1, α_2, g, γ, σ, ϵ, c_1, c_2, and θ. In all other cases, a variable rebate should be preferred although the trade-off between economic and environmental performance still exists.

5 Comparison of Markovian and Open-loop solutions

5.1 Variable rebate case

Although we have characterized open-loop and Markov perfect Stackelberg equilibrium productions, sales, and the profits in the previous sections, their analytical comparison is a daunting task. Therefore, we numerically compare the Markovian and the open-loop solutions when rebates are being offered. Appendix 2.6 reports the comparison between the optimal solutions for the $M-$Scenario and the $O-$Scenario. The comparison at the benchmark parameter values highlights an interesting finding: with the variable rebate policy M is better off under the Markovian solution, while R is better off under the Open-loop solution. Given that M is a leader and handles the collection, he will choose to play Markovian strategy. However, if he precommits to its wholesale price decisions at the outset of the game, R will choose to precommit to its retail decisions as well. This (open-loop strategy) will hurt M and provide benefit to R. But this will also hurt the consumers, as they will pay higher retailer prices under the precommitment strategy. Consequently, from a welfare point of view and from the perspective of the leader, Markovian solution is clearly preferred to the Open-loop solution under the variable rebate. In addition to this crucial finding, some other results can be obtained from the sensitivity analysis, specifically:

- When the market expands either in the first or in the second period (α_1 and α_2), M prefers to set Markovian strategies, while R opts for open-loop strategies. Under a Markovian concept, M can set a very large wholesale price compared to the open-loop case. In an open-loop framework, M faces the effect of the second period wholesale price on its first period wholesale price, causing an important decrease in its margins. While returns are larger within the open-loop framework, the double marginalization effect is prominent especially in the first period, thus suggesting the adoption of a Markovian concept from a social point of view, when the number of consumers enlarges.

- Increasing the values of the consumers' sensitivity to price (β) pushes M to espouse the Open-loop solution and R to implement the Markovian solution. High values of β can lead to price increases. In such a case, playing open-loop gives the possibility to keep the prices at sufficiently low levels. For R', increasing the values of β makes the Markovian pricing strategy more interesting because she can challenge M. In general, increasing values of β lead all prices as well as the returns in both solutions to converge.

- According to changes in all parameters of the marginal remanufacturing profit, $\pi_{M_r} = \sigma - g - \epsilon p_1$, M prefers a Markovian solution when the sign of their derivatives is positive. Thus, the higher the convenience to close the loop, the larger the M's willingness to play Markovian strategies. The intuition behind this result relies on the structure of the optimization problem as by setting the wholesale price in the second period he can influence the R's first period price decision. Otherwise, R will precommit the pricing strategies which will not be influenced by the convenience of remanufacturing reflected in the wholesale price in the second period. From her side, R does not get any benefit from remanufacturing, thus she prefers an open-loop solution concept to leave the full responsibility of closing the loop to M. Nevertheless, the Markovian solution is preferred by consumers who pay lower prices in both periods when remanufacturing is carried out although this leads to lower returns.

- The previous insights are corroborated by increasing values of returns parameters, namely, θ and γ. When the consumers show a certain willingness to return the old goods as well as a certain attitude in evaluating the difference between price of new products and rebates, M prefers the adoption of a Markovian concept to fully exploit the market potential linked to returns. Higher returns parameters also contributes positively to the environment under the Markovian concept. All these results also hold for increasing discount factor (δ) and marginal production costs (c_t).

To summarize, when consumers return behavior can be characterized by a variable rebate policy, a general trade-off exists in the selection of the solution concept. The adoption of a Markovian solution concept makes M economically better-off and leads to lower retail prices, thus being socially preferred. The implementation of Open-loop strategies makes R economically better-off and leads to larger returns, thus being environmentally preferred. Thus, when the rebates are variable and depending on the first period price, the selection of the solution concept is a challenging tasks. However, because M is the chain leader, he will opt for the adoption of a Markovian concept. This opens a warning on the environmental impact of this policy as well as the deterioration of some economic benefits for the retailer.

5.2 Exogenous rebate case

Instead of implementing a variable rebate policy, the manufacturer can choose a fixed rebate for the used products as defined in v function. In this case, given the model parameter regions studied, we find that market outcomes (prices, outputs, and profits) are identical under both equilibrium concepts. The main takeaway of this finding is that the Open-loop strategies are indeed sub-game perfect. Put differently, precommitting to the strategies (i.e., announcing all of the current and future prices at the beginning of the game) does not upset any firm. Alternatively, sequential pricing decision process, which is state-dependent, has no advantage over the precommitment process. Because, there is no transition or interlink between the periods, and the first period decisions are completely irrelevant for the second period decisions, when M applies the fixed rebate policy and consumers return as in v function.

6 Contributions and managerial insights

This research sheds light on an active return approach in dynamic CLSC games. It provides a new framework for consumer return behavior and offers comprehensive solutions for firms (such as Dell, Lexmark, H&M, BestBuy,etc.) under different information structures. Specifically, it includes the following contributions:

1. In modeling consumer return behavior for the used products, consumers respond to product prices and rebates (trade-in programs). Consumers evaluate the rebate they receive for the used product as well as the price of the new product, before they decide whether they should dump it. Therefore, the number of used products that are returned is determined. Surprisingly, this type of consumer behavior has been omitted in the CLSC literature.

2. We incorporate different rebate mechanisms into our CLSC games. We namely investigate two types of rebates, a fixed rebate and a variable rebate, which are commonly used by businesses. For example, Lexmark and BestBuy employ a variable rebate approach, while Dell and H&M implement a fixed rebate mechanism. In this case, it is imperative to know what would be the optimal rebate mechanism before they intend to offer a buy-back or a recycling program, because the payments to customers will directly impact their costs as well as the number of items to be re-manufactured. We explicitly entrench this rebate mechanism into the return function. We find that the variable rebate policy is optimal for the industry when Markov solution concepts are implemented: both retailer and manufacturer earn higher profits under the variable rebate policy than under the fixed rebate policy. This finding may justify the industry practices of Lexmark (and BestBuy) which employ a rebate mechanism based on quality and price of the used item. On the other hand, the practice of fixed rebate approach of Dell or H&M is also justifiable in our model because the quality and value of recycled computers or apparels are not important for Dell and H&M as the used items

are old and outdated. Ultimately, the goal of offering fixed rebates by these firms is to buyback the used products (computers and cloths) and sell new ones. A fixed rebate policy solves the problem between using a Markovian or a open-loop solution concept, as they coincide. A variable rebate makes the decision of the solution concept really challenging.

3. We offer two types of solutions to the CLSC games based on information structures: open-loop solution and Markov perfect solution, which are commonly employed in the dynamic games literature.[2] To our knowledge, open-loop solution has not been studied in the CLSC framework. While we keep the Markovian solution as our main solution concept, we allow firms to employ open-loop strategies so as to assess the impact of precommitment on market outcomes. Therefore, we offer a comprehensive market equilibrium solutions which would differentiate strategic considerations from the commitment deliberations. We show that under the fixed rebate policy open-loop solution coincides with Markov perfect solution. For instance, an implication of this result for H&M (buying back used apparels) is that H&M's fixed rebate policy will not impact its profits whether it announces its product prices sequentially over time or all at once.

4. We show how consumer return behavior (r and v functions) impact dynamic nature of firm interactions. We find that the time frame is irrelevant if the consumers base their return decisions according to the fixed rebate (as in v function). In this case, the first period decisions of firms do not impact their future decisions and profits. Therefore the game is reduced to a (repeated) static game. However, the market game will be fully dynamic, if the consumers base their return decisions with respect to variable rebate (as in r function). In that case, the first period strategies impact future decisions of all firms and their profits. An implication of this finding for Lexmark (buying back used cartridges) is that offering a variable rebate will complicate its pricing decisions as sophisticated consumers will impact the future product prices by their return decisions.

In light of these new findings, we offer some practical guidelines for firms operating in CLSCs:

i) Acknowledge the existence of sophisticated consumers who respond differently to different rebate mechanisms which will ultimately affect the industry profits and the number of returns.

ii) Offer a variable rebate program rather than applying a fixed rebate as it is more profitable when Markov strategies are implemented. If CLSCs seek to precommit their strategies, they should prefer an fixed rebate return policy;

iii) Take into account of the impact of information structure and the equilibrium solution concept on market outcomes (prices and profits). Precommitting to decisions at the outset may not cause a loss of profit for firms, but it is always preferable for them to consider the impact of current decisions on future outcomes as time evolves.

iv) Recognize the influence of the rebate type on the dynamic nature of market interactions. The game will

[2] In actual markets, it is an empirical question whether firms play open-loop or Markovian strategies.

be simplified and formulated as a time-independent repeated static game, if the rebate is constant. Otherwise, decision making process will be complex, as profits and prices will be time-dependent and interlinked.

v) Offering larger rebates to consumers can be beneficial for the CLSC when under a fixed rebate return function;

vi) When consumers' returning behavior can be explained by means of a variable return function, the choice between Markovian and open-loop solution concept is very challenging due to the trade-offs between economic, social and environmental performance.

7 Conclusions

This paper studies new CLSC games with various forms of return functions embedding the characteristics of price and rebate sensitive consumers. It addresses the best form of rebate type to be applied in the CLSC industries by utilizing several equilibrium solution concepts relevant to leader-follower type industry relations.

We offer significant methodological and conceptual contributions to the CLSC literature by exploring the impacts of return functions with variable or fixed rebates and the solution concepts (Markov or open-loop). Under the Markov solution concept, offering fixed rebates exert positive impacts on firms' profits and consumers' welfare, while entailing low environmental performance. In contrast, the adoption of a Markov solution concept with variable rebate show contrasting effect on firms' profits: the manufacturer prefers low variable rebates while the retailer prefers large variable rebates, highlighting that the implementation of a variable rebate policy can be challenging for the CLSC. When the CLSC adopts an Open-loop solution concept, fixed rebates should always be preferred: firms can expand their profits and consumers enjoy higher surplus with better environmental outcomes. In general, when the consumer return behavior can be explained by a fixed rebate return function, firms are indifferent between using either Markovian or Open-loop framework, thus writing contracts between the business parties becomes less complicated. Finally, when consumer return behavior can be explained by a variable rebate return function, firms' preferences diverge: the manufacturer would always adopt the Markovian solution concept while the retailer would adopt an Open-loop concept. However, the Open-loop concept allows a CLSC to achieve better environmental performance while deteriorating the social welfare. Offering a variable rebate to consumers complicates the CLSC decisions which will influence economic, social and the environmental performance.

Although we have examined the CLSC games over two periods, they could be extended to T finite periods. In fact, if the fixed rebate policy (as in v function) is implemented in the market, then it does not matter how many periods we would have in the game. Furthermore, the Markov solution will coincide with the Open-loop solution for all firms. This is because the decisions in a given period do not affect the future decisions, and therefore the game can be solved as static game, repeated T times. However, when the variable rebate policy is implemented (as in r function), all decisions in all periods will be interlinked (period t decisions will impact period $t+1$ decisions and outcomes). Therefore, the Markov perfect solution will diverge from the open-loop one. The Markovian strategies will facilitate higher profits for the manufacturer as the leader will take into account of impact of current decisions on future profits.

A future research direction could involve increasing the number of firms in both upstream and downstream layers of the CLSC. The manufacturer has an incentive to sell its products to many retailers to eliminate the double marginalization problem in the current setting. We believe our results will hold if the downstream industry would be competitive. However, competition in the upstream industry as well as product differentiation, and vertical controls would complicate the CLSC structure, but could lead to new managerial insights.

8 Appendix

Proof. of Proposition 1. The players optimize their objective functions over two periods, each of which is characterized by two stages. We seek to obtain a sub-game perfect Stackelberg equilibrium over the stages and the periods. When the rebate is variable, the players' optimization problems read as follows:

$$\begin{aligned}\Pi_M &= (\alpha_1 - \beta p_1)(\omega_1 - c_1) + \delta\left((\alpha_2 - \beta p_2)(\omega_2 - c_2) + (\theta - \gamma(p_2 - \epsilon p_1))(\sigma - g - \epsilon p_1)\right) \\ \Pi_R &= (\alpha_1 - \beta p_1)(p_1 - \omega_1) + \delta(\alpha_2 - \beta p_2)(p_2 - \omega_2)\end{aligned}$$

Because we have two stages per period, the course of the game is as follows:

Stage 4: To optimize her second period profit, R chooses the price p_2. Assuming an interior solution, the retailer's reaction function takes the form

$$p_2(\omega_2) = \frac{\alpha_2 + \beta\omega_2}{2\beta}$$

Stage 3: M optimizes its second period profits by choosing the wholesale price ω_2 and taking R's reaction function into account. That leads M's profits to become:

$$\Pi_M = (\alpha_1 - \beta p_1)(\omega_1 - c_1) + \delta\left(\left(\frac{\alpha_2 - \beta\omega_2}{2}\right)(\omega_2 - c_2) + \left(\theta - \gamma(\frac{\alpha_2 + \beta\omega_2}{2\beta} - \epsilon p_1)\right)(\sigma - g - \epsilon p_1)\right)$$

M's first order necessary condition yields in the second period to:

$$\omega_2(p_1) = \Omega_1 + \Omega_2 p_1$$

where $\Omega_1 = \dfrac{\alpha_2 + \beta c_2 - \gamma(\sigma - g)}{2\beta} > 0$ and $\Omega_2 = \dfrac{\epsilon\gamma}{2\beta} > 0$.

Substituting ω_2 in p_2, the second-period price becomes:

$$p_2\left(p_1\right) = \Omega_3 + \Omega_4 p_1$$

where $\Omega_3 = \dfrac{\alpha_2 + \beta\Omega_1}{2\beta} > 0$ and $\Omega_4 = \dfrac{\Omega_2}{2} > 0$, thus $\Omega_3 > \Omega_1$ and $\Omega_4 < \Omega_2$.

Stage 2: Moving to the first period, R optimally chooses its price p_1 to maximize its sum of discounted profits. After substituting for ω_2 and p_2, the R's profits becomes:

$$\Pi_R^M = (\alpha_1 - \beta p_1)(p_1 - \omega_1) + \delta[(\alpha_2 - \beta\left(\Omega_3 + \Omega_4 p_1\right))(\Omega_3 - \Omega_1 + \left(\Omega_4 - \Omega_2\right) p_1)]$$

whose optimization with respect to p_1 gives:

$$p_1\left(\omega_1\right) = \left(\Omega_5 + \Omega_6\omega_1\right)$$

where $\Omega_5 = \left(\frac{\alpha_1 - \delta\beta(\Omega_3 - \Omega_1)\Omega_4 + \delta(\alpha_2 - \beta\Omega_3)(\Omega_4 - \Omega_2)}{2\beta(1 + \delta\Omega_4(\Omega_4 - \Omega_2))}\right) > 0$ and $\Omega_6 = \left(\frac{\beta}{2\beta(1 + \delta\Omega_4(\Omega_4 - \Omega_2))}\right) > 0$. Substituting $p_1\left(\omega_1\right)$ into the second period strategies we obtain:

$$\begin{aligned} p_2 &= \Omega_3 + \Omega_4\Omega_5 + \Omega_4\Omega_6\omega_1 \\ \omega_2 &= \Omega_1 + \Omega_2\Omega_5 + \Omega_2\Omega_6\omega_1 = \left(\Omega_7 + \Omega_8\omega_1\right) \end{aligned}$$

where $\Omega_7 = \left(\Omega_1 + \Omega_2\Omega_5\right) > 0$ and $\Omega_8 = \Omega_2\Omega_6 > 0$.

Stage 1: Plugging $\omega_2\left(\omega_1\right)$ and $p_1\left(\omega_1\right)$ into the M's objective functional gives:

$$\Pi_M = \left(\alpha_1 - \beta\left(\Omega_5 + \Omega_6\omega_1\right)\right)\left(\omega_1 - c_1\right) + \delta\left(\begin{array}{c} \left(\dfrac{\alpha_2 - \beta\left(\Omega_7 + \Omega_8\omega_1\right)}{2}\right)\left(\Omega_7 + \Omega_8\omega_1 - c_2\right) + \left(\sigma - g - \epsilon\left(\Omega_5 + \Omega_6\omega_1\right)\right) \\ \left(\theta - \gamma(\dfrac{\alpha_2 + \beta\left(\Omega_7 + \Omega_8\omega_1\right)}{2\beta} - \epsilon\left(\Omega_5 + \Omega_6\omega_1\right))\right) \end{array}\right)$$

while the optimization with respect to ω_1 yields:

$$\omega_1 = \frac{\left(-\alpha_1 + \beta\Omega_5 - \beta c_1\Omega_6 - \frac{1}{2\beta}\left(\begin{array}{c} g\beta\gamma\delta\Omega_8 - 2\theta\beta\delta\epsilon\Omega_6 - \sigma\beta\gamma\delta\Omega_8 + \beta\delta\alpha_2\Omega_8 - 2g\beta\gamma\delta\epsilon\Omega_6 + 2\sigma\beta\gamma\delta\epsilon\Omega_6 \\ +\gamma\delta\epsilon\alpha_2\Omega_6 + \beta\gamma\delta\epsilon\Omega_5\Omega_8 + \beta\gamma\delta\epsilon\Omega_6\Omega_7 + \beta^2\delta c_2\Omega_8 - 2\beta^2\delta\Omega_7\Omega_8 - 4\beta\gamma\delta\epsilon^2\Omega_5\Omega_6 \end{array}\right)\right)}{-2\beta\Omega_6 + \frac{1}{2\beta}\left(2\beta\gamma\delta\epsilon\Omega_6\Omega_8 - 2\beta^2\delta\Omega_8^2 - 4\beta\gamma\delta\epsilon^2\Omega_6^2\right)}.$$

Substitute ω_1 into the other strategies to find the other prices. $\square$

Proof. Proposition 6. The players optimize their objective functions over two periods, each of which is characterized by two stages. We seek to obtain a sub-game perfect Stackelberg equilibrium over the stages and the periods. When the rebate is exogenous, the players' optimization problems read as follows:

$$\begin{aligned}
\Pi_M &= (\alpha_1 - \beta p_1)(\omega_1 - c_1) + \delta\left((\alpha_2 - \beta p_2)(\omega_2 - c_2) + (\theta - \gamma(p_2 - k))(\sigma - g - k)\right) \\
\Pi_R &= (\alpha_1 - \beta p_1)(p_1 - \omega_1) + \delta(\alpha_2 - \beta p_2)(p_2 - \omega_2)
\end{aligned}$$

Because we have two stages per period, the course of the game is as follows:

Stage 4: To optimize her second period profit, R chooses the price p_2. Assuming an interior solution, the retailer's reaction function takes the form

$$p_2(\omega_2) = \frac{\alpha_2 + \beta\omega_2}{2\beta}$$

Stage 3: M optimizes its second period profits by choosing the wholesale price ω_2 and taking R's reaction function into account. That leads M's profits to become:

$$\Pi_M = (\alpha_1 - \beta p_1)(\omega_1 - c_1) + \delta\left(\left(\frac{\alpha_2 - \beta\omega_2}{2}\right)(\omega_2 - c_2) + \left(\theta - \gamma(\frac{\alpha_2 + \beta\omega_2}{2\beta} - k)\right)(\sigma - g - k)\right)$$

M's first order necessary condition yields in the second period gives:

$$\omega_2 = \frac{(g - \sigma + k)\gamma + \alpha_2 + \beta c_2}{2\beta}$$

Substituting ω_2 in p_2, the second-period price becomes:

$$p_2 = \frac{(g - \sigma + k)\gamma + 3\alpha_2 + \beta c_2}{4\beta}$$

Stage 2: Moving to the first period, R optimally chooses its price p_1 to maximize its sum of discounted profits. The optimal p_1 can be obtained even without substituting the second period optimal strategies into the R's objective function as there is no interdependence between first and second period strategies. The optimization with respect to p_1 gives:

$$p_1 = \frac{\alpha_1 + \beta\omega_1}{2\beta}$$

Stage 1: By plugging $p_1(\omega_1)$ into the M's objective functional and taking the derivative with respect to

ω_1 gives: :

$$\omega_1 = \frac{\alpha_1 + \beta c_1}{2\beta}$$

so that the optimal retail price becomes:

$$p_1 = \frac{3\alpha_1 + \beta c_1}{4\beta}$$

□

Proof. of Proposition 7. The players maximize their objective functions to choose all period decisions at the outset of the game. We will obtain open-loop Stackelberg equilibrium over two stages. When the rebate is variable, the players' optimization problems read as follows:

$$\begin{aligned}
\Pi_M &= (\alpha_1 - \beta p_1)(\omega_1 - c_1) + \delta\left((\alpha_2 - \beta p_2)(\omega_2 - c_2) + (\theta - \gamma(p_2 - \epsilon p_1))(\sigma - g - \epsilon p_1)\right) \\
\Pi_R &= (\alpha_1 - \beta p_1)(p_1 - \omega_1) + \delta(\alpha_2 - \beta p_2)(p_2 - \omega_2)
\end{aligned}$$

Stage 2: As the retailer is follower we start with R's maximization problem. To optimize her profits, R simultaneously chooses the prices p_1 and p_2. Assuming an interior solution, the retailer's reaction function takes the form

$$\begin{aligned}
p_1(\omega_1) &= \frac{\alpha_1 + \beta\omega_1}{2\beta} \\
p_2(\omega_2) &= \frac{\alpha_2 + \beta\omega_2}{2\beta}
\end{aligned}$$

Stage 1: Next M optimizes its total profit function by simultaneously choosing the wholesale prices ω_1 and ω_2, taking R's reaction functions given. This leads to the following M's profit function:

$$\Pi_M = \left(\alpha_1 - \beta\frac{\alpha_1 + \beta\omega_1}{2\beta}\right)(\omega_1 - c_1) + \delta\left(\begin{array}{c} \left(\alpha_2 - \beta\frac{\alpha_2 + \beta\omega_2}{2\beta}\right)(\omega_2 - c_2) \\ + \left(\theta - \gamma(\frac{\alpha_2 + \beta\omega_2}{2\beta} - \epsilon\frac{\alpha_1 + \beta\omega_1}{2\beta})\right)(\sigma - g - \epsilon\frac{\alpha_1 + \beta\omega_1}{2\beta}) \end{array}\right)$$

Maximizing this function with respect to the wholesale price strategies we obtain:

$$\begin{aligned}
\omega_1 &= \frac{2\beta\left(4\beta^2 c_1 + \epsilon\delta\left(g\gamma^2 - 4\theta\beta\right)\right) + 2\epsilon\delta\gamma\beta\left(\beta\left(4\left(\sigma - g\right) + c_2\right) + 3\alpha_2 - \sigma\gamma\right) + \alpha_1\left(8\beta^2 + \epsilon^2\delta\gamma\left(\gamma - 8\beta\right)\right)}{\beta\left(16\beta^2 + \gamma\delta\epsilon^2\left(8\beta - \gamma\right)\right)} \\
\omega_2 &= \frac{2\beta\left(3\gamma\epsilon\alpha_1 + 4\beta\left(\alpha_2 + \beta c_2\right)\right) + 2\gamma\beta^2\left(4\left(g - \sigma\right) + \epsilon c_1\right) + \epsilon^2\delta\gamma\left(\alpha_2\left(4\beta + \gamma\right) + 2\beta\left(2\beta c_2 - \left(\theta + \gamma\left(\sigma - g\right)\right)\right)\right)}{\beta\left(16\beta^2 + \gamma\delta\epsilon^2\left(8\beta - \gamma\right)\right)}
\end{aligned}$$

□

Proof. Proposition 8. The players optimize their objective functions over two stages. We seek to obtain open-loop Stackelberg equilibrium over the stages. When the rebate is exogenous, the players' optimization problems read as follows:

$$\begin{aligned}\Pi_M &= (\alpha_1 - \beta p_1)(\omega_1 - c_1) + \delta\left((\alpha_2 - \beta p_2)(\omega_2 - c_2) + (\theta - \gamma(p_2 - k))(\sigma - g - k)\right) \\ \Pi_R &= (\alpha_1 - \beta p_1)(p_1 - \omega_1) + \delta(\alpha_2 - \beta p_2)(p_2 - \omega_2)\end{aligned}$$

Because we have two stages per period, the course of the game is as follows:

Stage 1: To optimize her second period profit, R chooses the prices p_1 and p_2. Assuming an interior solution, the retailer's reaction function takes the form

$$\begin{aligned}p_1(\omega_1) &= \frac{\alpha_1 + \beta\omega_1}{2\beta} \\ p_2(\omega_2) &= \frac{\alpha_2 + \beta\omega_2}{2\beta}\end{aligned}$$

Stage 2: M optimizes its second period profits by choosing the wholesale prices ω_1 and ω_2 and taking R's reaction functions into account. That leads M's profits to become:

$$\Pi_M = \left(\frac{\alpha_1 - \beta\omega_1}{2}\right)(\omega_1 - c_1) + \delta\left(\left(\frac{\alpha_2 - \beta\omega_2}{2}\right)(\omega_2 - c_2) + \left(\theta - \gamma(\frac{\alpha_2 + \beta\omega_2}{2\beta} - k)\right)(\sigma - g - k)\right)$$

M's first order necessary condition yields in the second period gives:

$$\begin{aligned}\omega_1 &= \frac{\alpha_1 + \beta c_1}{2\beta} \\ \omega_2 &= \frac{(g - \sigma + k)\gamma + \alpha_2 + \beta c_2}{2\beta}\end{aligned}$$

By pluging ω_1 and ω_2 into the prices, the R's strategies read:

$$\begin{aligned}p_1 &= \frac{3\alpha_1 + \beta c_1}{4\beta} \\ p_2 &= \frac{3\alpha_2 + \beta c_2 - \gamma(\sigma - g - k)}{4\beta}\end{aligned}$$

□

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