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**Nuclear Energy, Economic Growth and the  
Environment: Optimal policies in a model with  
endogenous technical change and environmental  
constraints**

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# Nuclear Energy, Economic Growth and the Environment: Optimal policies in a model with endogenous technical change and environmental constraints

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## Abstract

We use a model of endogenous growth with vertical innovations in order to derive optimal energy policy under uncertainty. Innovation can be directed to dirty, green or nuclear technologies, which in turn can be used to produce different types of energy. We show that, nuclear energy usage is not only a necessary welfare maximizing condition but also a crucial determinant of economic growth in the long-run. In addition, we find no evidence supporting the Environmental Kuznets Curve hypothesis under optimal policy implementation. Lastly, empirical results based on a panel VAR specification suggest that increases in emissions are strongly persistent in the long-run. Thus, a worsening of environmental quality seems to create dynamics that lead to even higher levels of emissions in the future.

*Keywords:* Growth, Nuclear Energy, Innovation

JEL Classification: O31, O44, Q42, Q55

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## 1. Introduction

In modern economic growth theory, interest on the Schumpeterian approach, has substantially increased lately. The influential paper of [Aghion and Howitt\(1992\)](#) was crucial in both establishing and developing this approach and as a result, making it one of the dominant approaches in the field. In this paper, in the spirit of [Acemoglu et al.\(2012\)](#) we adopt this approach and develop an endogenous economic growth model with environmental constraints. However, there is a growing part of the literature suggesting that the importance of environmental quality is not limited to its productivity and welfare effects, but it also has a direct effect on economic growth, as it could affect (and could be affected by) the development stage of an economy. In that respect the literature on economic growth with environmental constraints, is closely related to the concept of the Environmental Kuznets Curve (hereafter EKC) and in that context in our endogenous growth model with environmental constraints we show that environmental quality affects -and is affected- in a differential manner under optimal policy implementation.

The EKC is the empirical hypothesis according to which, environmental degradation is increasing during the early stages of development of a country. However, this relationship is reversed, after a country reaches a certain threshold of wealth. According to the EKC hypothesis, countries will target -and eventually achieve- lower emission levels and thus better environmental quality as they develop, leading to the observation of a negative relationship between emissions and wealth for developed countries. Hence it is usually represented as an inverted U-shape graph with the stage of development in the horizontal axis and environmental degradation or various indicators of pollution on the vertical axis. The EKC has been used to describe cross-sectional differences, not only in the preferences for more environmentally friendly technologies, but also in the implementation of policies, resulting from these preferences. Ever since the EKC has been subject to a huge ongoing debate, on whether it should indeed be thought as a stylized fact of economic development, or as a set of spurious correlations. As a result the validity of the EKC hypothesis has been widely debated and there are plenty of important studies either supporting or rejecting it.

One important dimension in this debate is the nature of the energy sources that are used in production, namely the mix between traditional “brown” carbon based energy sources and the

cleaner forms of the “green” variety. However, the literature so far has ignored the use of nuclear energy as part of the mix of energy types that have low or zero emissions. Yet, the inclusion of nuclear energy usage, will add to the existing literature, as nuclear energy seems to be -in most occasions- a more efficient (and reliable) version of green energy, with the downside of course being the (low) probability of a serious nuclear accident. In our setting, both nuclear and green energy resemble abatement activities, which are costly in terms of risk in the case of nuclear energy usage and in terms of efficiency in the green energy case. Furthermore, renewable energy production, is not only less efficient compared to nuclear energy, but it also faces problems that arise from its nature, see [Pommeret and Schubert\(2019\)](#). Of course nuclear energy production and usage comes with the downside of a small -yet ever present- probability of a nuclear accident. Our approach to this issue, is to deal with this trade off problem as we would in any typical insurance problem.

In this paper we expand the analysis made by [Acemoglu et al.\(2012\)](#) by introducing uncertainty as well as the possibility of nuclear energy usage. There have been several empirical studies including [Wolde-Rufael and Menyah\(2010\)](#), [Apergis et al.\(2010\)](#), [Yoo and Jung\(2005\)](#) and [Iwata et al.\(2010\)](#) that try to establish a relationship between nuclear energy usage and economic growth. However all these attempts have been strictly empirical and lacked the theoretical background that could enhance their results’ validity. With this study, our primary purpose is to do exactly that. In addition, we try to derive welfare maximizing energy policies given the environmental constraints imposed by nature. Our work has direct and indirect implications on not only the endogenous growth theory with environmental constraints framework, but also on the EKC hypothesis literature, as we show that, environmental quality affects -and is affected- in a different manner under optimal policy implementation. Lastly, the use of a panel VAR specification, allows us to trace any possible short-run inter-dependencies between growth,emissions as well as nuclear and total energy usage.

The paper is structured as it follows. Section 2 provides a short literature review, while Section 3 gives an overview of our model. Section 4 presents our results as well as the comparative statics. Section 5 presents some limited empirical results based on a panel VAR model and finally Section 6 offers some concluding remarks.

## 2. Literature Review

The endogenous growth literature based on the Schumpeterian approach has risen in prominence in the literature due to the influential paper of [Aghion and Howitt\(1992\)](#). The main idea of their model is that economic growth is generated endogenously, by technological advance which in turn is driven by innovations. The innovation procedure takes place in an intermediate sector, which is governed by a monopolistic competition structure. This exact feature is the driving force for sustainable economic growth, since the possible future rents that a successfully innovative entrepreneur will attain, would lead to an increased probability of innovation. Innovation in turn leads to technical progress and thus economic growth. Although the economic growth rate is random, the average growth rate is constant and positively correlated with the number of scientists employed in the sector, as well as negatively correlated with the level of difficulty to innovate in any given intermediate sector.

This innovation procedure though, has its downside in welfare terms. When some entrepreneur succeeds in innovating they become the next period's monopolists. Thus not only an agent is attaining profits, but also another agent (the last period's entrepreneur who produced the intermediate product which is now obsolete) is losing profits. This process of creative destruction is the reason why this approach is also known as the Schumpeterian approach. For a more detailed overview of the Schumpeterian approach in Economic growth see [Aghion and Howitt\(2008\)](#). Ever since the development of the Schumpeterian approach in Economic Growth, there has been a growing part of the literature that tries to use its tools and methods to analyze topics that are closely related but not limited to growth. For instance using this exact framework, [Bucci\(2008\)](#) tries to shed light on the relationship between population growth and economic growth while [Acemoglu et al.\(2012\)](#) try to derive optimal policies that lead to sustainable economic growth, when we take into account the fact that, technologies might or might not be harmful for the environment. In that context, the literature on economic growth with environmental constraints, is closely related to the concept of the EKC. The connection between economic growth and the environment in the form of the ECK is based on the attempt to describe and explain the environmental degradation (in the form of pollutant emissions) and economic growth nexus, as the former could affect (and could be affected

by) the development stage of an economy. It is usually represented as an inverted U-shape graph with the stage of development in the horizontal axis and environmental degradation or various indicators of pollution on the vertical axis.

The idea of an EKC inverted U-shaped curve, was initially introduced by the influential paper of [Grossman and Krueger\(1991\)](#). Their study deals with concerns that, trade liberalization resulted by the NAFTA agreement, will have a negative environmental impact for the participating countries. These concerns were based on the fact that trade liberalization can result to larger scale of production (and of course higher income) for the participants, so income increase may potentially be "fed" by an increase in pollution. However, [Shafik and Bandyopadhyay\(1992\)](#) argue that the dominant -until then- opinion, that economic growth causes environmental degradation, is a relatively "naive" approach because it does not take into account the fact that the elasticity of benefits with respect to income depends on several mechanisms, that the previous studies failed to include. They also argue that, the development level of a country affects its output composition, which in turn affects environmental degradation and they are able to establish empirical findings that support the presence of the EKC hypothesis. Two additional studies reached the same conclusion using slightly different variables on their specifications as well as different estimates for environmental quality. More precisely, [Selden and Song\(1994\)](#) also find evidence of the existence of the Environmental Kuznets Curve, however the predicted income level turning point, above which a country is expected to improve the quality of its environment as income increases is almost double than the one predicted by [Grossman and Krueger\(1991\)](#) and [Shafik and Bandyopadhyay\(1992\)](#). This implies that, although their results provide evidence supporting the hypothesis, a country needs to reach a higher income threshold first before being able to see its environmental quality rise as income further increases. [Kaufmann et al.\(1998\)](#) deviate slightly from this framework but they reach similar conclusions.

[Stern\(2004\)](#) can be considered as one of the most important papers that criticize the validity of the EKC hypothesis, which by that time was considered as the dominant view on the environment and economic growth nexus. First, he argues that most of the literature in favour of the hypothesis can be considered econometrically weak. That is because most of the studies not only fail to take into account problems resulting from the nature of the data like serial correlation and non-

stationarity, but also because they tend to systematically neglect estimation problems such as omitted variable bias. In that context the econometric evidence suggests that i) the relationship cannot be considered the same for both concentrations and emissions, ii) it is far more likely that the evidence found by other studies is mostly misinterpreted spurious correlations, iii) even if their estimation was valid econometrically they would massively underestimate the income threshold above which the relationship between income levels and environmental quality becomes positive. The last point suggests if the threshold income level is too high then the EKC could arise as the result of random factors rather than the actual causal relationship between income levels and environment.

After Stern's paper the vast majority of the literature followed his steps and not only criticized previous results but also found strong empirical evidence against them. That happened mostly by taking into account the econometric problems stated by [Stern\(2004\)](#) and making use of more sophisticated estimation techniques. [He and Richard\(2010\)](#) argue that most of the literature so far have estimated fully-parametric quadratic and cubic regression models. They instead employ non-linear parametric and semi-parametric estimation techniques and their results suggest that for Canada, emissions are strictly increasing in income while they initially seem to be a convex function of income, then concave followed by a convex pattern again, giving us an increasing(with trend) inverted U-shaped Kuznets Curve. Lastly, [Empora et al.\(2019\)](#), using nonparametric estimation as well as threshold regression, report a positive nonlinear relationship between emissions and output, rejecting the EKC hypothesis. The above paper demonstrates that the absence of abatement is one of the driving forces for an EKC to emerge. In our setting, both nuclear and green energy are abatement activities, which are costly in terms of risk in the case of nuclear energy usage and in terms of efficiency in the green energy case.

However, renewable energy production, is not only less efficient compared to nuclear energy, but it also faces problems that arise from its nature. [Pommeret and Schubert\(2019\)](#) build a model of optimal transition to renewable energy and they argue that problems arise, on not only the production but also the storage of such energy. Providing the example of solar panels, they argue that intermittency, makes electricity generation as well as storage procedure uncertain. In their model, production is divided into two types of energy, day and night electricity. Day electricity

is produced by solar panels (a clean but uncertain way of production), whereas night electricity is produced by burning fossil fuel. The issue of intermittency as well as variability of more clean methods of production (such as solar panels), end up affecting the optimal energy mix. That being said, we strongly believe that, the induction of nuclear energy, in models trying to analyze economies with environmental constraints is necessary. As we have already mentioned, in our view, nuclear energy is an environmental-friendly alternative of typical renewable alternatives, which is free of issues such as intermittency and storage. Of course nuclear energy production and usage comes with the downside of a small -yet ever present- probability of a nuclear accident. Our approach to this issue, is to deal with this trade off problem as we would in any typical insurance problem.

### 3. Model

We expand the model of [Acemoglu et al.\(2012\)](#) to account for uncertainty. In their framework (as well as in most endogenous growth models following the Schumpeterian approach), the economy consists of two sectors, namely final good and intermediate good sector. In our analysis the final good  $Y_t$  is produced using the following production function:

$$Y_t = \begin{cases} S_t^\lambda [a_1(\beta E_{Gt})^{\frac{\sigma-1}{\sigma}} + a_2 E_{Dt}^{\frac{\sigma-1}{\sigma}} + a_3 E_{Nt}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} & S_t \in [\underline{s}, \bar{s}] \\ \bar{s}^\lambda [a_1(\beta E_{Gt})^{\frac{\sigma-1}{\sigma}} + a_2 E_{Dt}^{\frac{\sigma-1}{\sigma}} + a_3 E_{Nt}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} & S_t \in [\bar{s}, \infty] \\ 0 & otherwise \end{cases} \quad (1)$$

The only consumption good in the economy is produced by three types of inputs  $E_{jt}$  namely green, dirty and nuclear energy. Furthermore,  $S_t$  is the environmental quality at date t,  $\bar{s}$  is the state of environment in the absence of human-made production activities on the planet and  $\underline{s}$  is the threshold of environmental degradation below which environmental disaster is irreversible. Also  $\sum a_i = 1$  for  $i = 1, 2, 3$  and  $0 < \beta < 1$  is the degree at which green energy is lost due to capacity capabilities(green energy inefficiency).

We choose this production function mainly because of its flexible nature. This type of CES production function allows as to see how our results change on the degree of substitutability between our three inputs. In our setting,  $\sigma$  is the elasticity of substitution between our inputs. As  $\sigma \rightarrow 1$  our production function resembles the Cobb-Douglas case with three inputs. Varying  $\sigma$ , will later



allow us to examine how the results regarding optimal policy, are changing on the assumptions we make on the production function.

The evolution of  $S_t$  is given by:

$$S_{t+1} = \begin{cases} (1 + \theta)S_t - \xi_D E_{Dt} - \Delta & \text{nuclear accident occurs} \\ (1 + \theta)S_t - \xi_D E_{Dt} & \text{otherwise} \end{cases} \quad (2)$$

Here  $\theta$  is the degree of environmental regeneration and  $\xi_j$  is the environmental degradation caused by the use of energy of type  $j$ . We assume that  $\xi_D > \xi_G = \xi_N = 0$ . That is, green and nuclear energies are not as harmful for the environment as dirty energy is. As a matter of fact, green energies are not harmful at all, whereas nuclear energy is only potentially harmful if and only if a nuclear accident occurs.

Even though we take the potential loss  $\Delta \gg 0$  to be an exogenously given large constant, we assume that the small probability  $\delta(A_{jt}) \rightarrow 0$  of a nuclear accident occurring is a function decreasing in the nuclear technology knowledge stock ( $A_{jt}$ ). Hence, the technology stock is an index that summarizes not only the efficiency of a given technology, but also its safety.

The intermediate good is energy of type  $j$  is produced using labor as well as a variety of machines  $x_{jit}$  where  $i \in [0, 1]$ :

$$E_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^\alpha di \quad (3)$$

From now on we normalize labour  $L_{jt} = 1$ . So the profit maximization problem for the producer of energy  $j$  becomes:

$$\max_{x_{jt}} \left\{ P_{jt} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^\alpha di - w_t - \int_0^1 R_{jit} x_{jit} di \right\}$$

Where  $P_{jt}$  and  $R_{jit}$  are the per unit prices of type  $j$  energy and machine  $i$  used in the production of energy  $j$  respectively. The first order condition yields:

$$\alpha P_{jt} A_{jit}^{1-\alpha} x_{jit}^{\alpha-1} = R_{jit}$$

Thus the demand for machine  $i$  in sector  $j$  is given by:

$$x_{jit} = \left( \frac{\alpha P_{jt}}{R_{jit}} \right)^{\frac{1}{1-\alpha}} A_{jit} \quad (4)$$

The producers of every  $i$  machine used in sector  $j$  are successful (on innovating) entrepreneurs and so they have been awarded monopoly right on the production of the given intermediate good for that period. Given that their demand is given by (5) we can calculate their marginal revenue as  $MR_{jit} = \alpha^2 P_{jt} A_{jit}^{1-\alpha} x_{jit}^{\alpha-1}$ . Assuming that every monopolist have a constant marginal cost of  $c_j$  (same within sector), profit maximization implies that the supply for machine  $i$  in sector  $j$  is given by:

$$x_{jit} = \left( \frac{c_j}{\alpha^2} \right)^{\frac{1}{\alpha-1}} P_{jt} A_{jit} \quad (5)$$

And the supply of type  $j$  energy is:

$$E_{jt} = \left( \frac{c_j}{\alpha^2} \right)^{\frac{\alpha}{\alpha-1}} P_{jt}^{\frac{\alpha}{1-\alpha}} A_{jit} \quad (6)$$

A scientist can focus their research to one of the three sectors. If a scientist succeeds in innovating, they become the monopolist of producing energy  $j$  with technology  $A_{jt} = \mu_{jt}(1 + \gamma)A_{jt-1}$ , where  $A_{jt} = \int_0^1 A_{jit} di$  and  $\mu_{jt}$  is the probability of successful innovation for a researcher working on sector  $j$  at time  $t$ . If the attempt is unsuccessful monopoly rights are awarded randomly to one entrepreneur drawn by the pool of entrepreneurs. Thus a scientist's focusing their research on sector  $j$  expected profits are:

$$E\Pi_{jt} = \frac{1-\alpha}{\alpha^{\frac{\alpha+1}{\alpha-1}}} c_j^{\frac{\alpha}{\alpha-1}} P_{jt}^{\frac{1}{1-\alpha}} \mu_{jt}(1 + \gamma)A_{jt-1} \quad (7)$$

We assume that  $\mu_{D0} > \mu_{G0} > \mu_{N0}$ . That is, it is naturally more difficult to start innovating in the nuclear sector. Also it is more difficult to start innovating in the dirty sector. More importantly we assume that  $\mu_{jt}$  is a concave function of  $A_{jt-1}$ . This assumption captures not only the fact that as research stock increases, it becomes increasingly more difficult to keep increasing it but also complies with the current observation of difficulty differences between sectors (it can be accounted to

the current technology stocks).

The preferences of the consumers can be represented by the following social welfare function:

$$SW = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} U(c_t) = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} U\left(Y_t - \left(\int_0^1 \sum_{j=D,G,N} x_{jit} di\right)\right)$$

Obviously a policy would be welfare maximizing by simultaneously producing the maximum product in the most efficient way. Given our assumptions above, so far in modern economies dirty technologies are cheaper because  $\mu_{D0} > \mu_{G0} > \mu_{N0}$ . Keeping everything else constant this implies  $E\Pi_{D0} > E\Pi_{G0} > E\Pi_{N0}$ , which in turn means that economies in a development phase would use more dirty inputs. At this early stage policy makers wouldn't have an incentive to intervene because the marginal benefit of using the dirty technologies exceeds the marginal cost.

In general, since environmental quality affects productivity, the results of [Acemoglu et al.\(2012\)](#) suggest that policymakers need to intervene in order to fix not only the negative externality of the usage of dirty energy but also the other market imperfections(monopoly and the loss of profits due to creative destruction). These taxes-subsidies can be gradually levied as the green sector increases its knowledge stock.

However, our setting differs from theirs as we introduce uncertainty because of the possibility of a nuclear accident. We further assume that firms are risk neutral whereas consumers are risk averse. If at any point  $t^*$  the economy reaches a certain level of environmental degradation both the firms and the consumers receive a payoff of zero forever. That is if  $s_t < \underline{s}$  then  $SW_t = 0$  and  $\Pi_{jt} = 0$  for every  $t > t^*$ . Our goal now is to find the paths of technological stock that maximize social welfare. Later we can figure out how to redirect technological change so that we can induce these optimal energy usage paths. Thus, our target is to solve the following problem:

$$\begin{aligned} \max_{E_{jt}} \left\{ \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left( \delta U \left[ (S_{PI} - \Delta)^\lambda \left( \sum_{j=D,G,N} E_{jt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \int_0^1 \left( \sum_{j=D,G,N} x_{jit} \right) di \right] + \right. \right. \\ \left. \left. + (1-\delta) U \left[ (S_{PI})^\lambda \left( \sum_{j=D,G,N} E_{jt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \int_0^1 \left( \sum_{j=D,G,N} x_{jit} \right) di \right] \right) \right\} \end{aligned} \quad (8)$$

Above  $S_{PI} = (1+\delta)s_t - \xi E_{Dt}$  is the state of environment on the perfect information case.

We also assume that  $\frac{\partial S_t}{\partial A_{Gt}} \geq 0$ ,  $\frac{\partial S_t}{\partial A_{Nt}} \geq 0$ ,  $\frac{\partial S_t}{\partial A_{Dt}} < 0$  as well as  $\frac{\partial E_{Dt}}{\partial A_{Gt}} \leq 0$ ,  $\frac{\partial E_{Dt}}{\partial A_{Nt}} \leq 0$  and  $\frac{\partial E_{Dt}}{\partial A_{Dt}} > 0$ .

In addition suppose that  $E_{jt}^{OPT}$  is the solution to the optimization problem above.

**Proposition 1.** *In a welfare maximizing balanced growth path  $E_{Dt}^{OPT} \neq 0$*

*Proof.* Assume otherwise. Then if we raise  $E_{Dt}$  by an amount (less than  $\theta S_t$ ) welfare will improve. That is because, since  $E_{Dt} = 0$ , the marginal benefit of increasing it is high (its marginal product is high), whereas the small increase is small enough to ensure that the state of the environment will not be worsened.  $\square$

As a matter of fact  $E_{Dt}^{OPT}$  could be even higher than  $\theta S_t$  (only for a short period of time and under the condition that  $E_{GT}$  is sufficiently small), however a level higher than that could not be indefinitely sustainable. In a later stage the optimal path of the production mix of  $E_{Gt}$  and  $E_{Nt}$  will pin down the optimal amount of overproduction of dirty energy (the deviation from its lower bound). It is now obvious that we can proceed on solving our optimization problem, without having to take into account corner solutions. To simplify the calculations and without loss of generality, we normalize  $\frac{c_j}{\alpha^2} = 1$ . So, using the fact that  $P_{jt} = \frac{\partial Y_t}{\partial E_{jt}}$  as well as assuming a isoelastic utility function  $U(c_t) = \ln c_t$ , the first order conditions with respect to  $E_{jt}$  yield:

$$(A - BC)(D + E) = F(G + H) \quad (9)$$

where

$$A = \frac{\beta_j^{\frac{\sigma-1}{\sigma}} a_j \sigma}{\sigma-1} \left( \sum a_j (\beta_j E_{jt})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}}, \quad B = \frac{\beta_j^{\frac{\sigma-1}{\sigma}} a_j}{1-\alpha} \left( \sum a_j (\beta_j E_{jt})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} E_{jt}^{\frac{1}{\sigma-1}} A_{jt},$$

$$C = \frac{\beta_j^{\frac{\sigma-1}{\sigma}} a_j}{\sigma} \left( \sum a_j (\beta_j E_{jt})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{2-\sigma}{\sigma-1}} E_{jt}^{\frac{2}{\sigma-1}} E_{jt}^{\frac{2}{\sigma-1}} + \frac{(\beta_j^{\frac{\sigma-1}{\sigma}} a_j)^2}{\sigma-1} \left( \sum a_j (\beta_j E_{jt})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} E_{jt}^{\frac{2-\sigma}{\sigma-1}},$$

$$D = \frac{(1-\delta)}{((1+\theta)S_t - \xi_j E_{jt})^\lambda \sum (\beta_j (a_j E_{jt})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma-1}{\sigma-1}} - \sum (\sum a_j (\beta_j E_{jt})^{\frac{1-\sigma}{\sigma}} E_{jt}^{\frac{1}{\sigma-1}})^{\frac{1}{1-\alpha}} A_{jt}},$$

$$E = \frac{1}{1+\rho} \left( \sum a_j (\beta_j E_{jt+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma-1}{\sigma-1}} \lambda \xi_j,$$

$$F = \frac{\delta[(1+\theta)S_t - \xi_j E_{jt}]}{((1+\theta)S_t - \xi_j E_{jt} - \Delta)^\lambda \left( \sum a_j (\beta_j E_{jt+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma-1}{\sigma-1}} - \sum (\sum a_j (\beta_j E_{jt+1})^{\frac{1-\sigma}{\sigma}} E_{jt+1})^{\frac{1}{1-\alpha}} A_{jt+1}} \quad \text{and}$$

$$G = \frac{(1-\delta)[(1+\theta)S_t - \xi_j E_{jt}]}{((1+\theta)S_t - \xi_j E_{jt})^\lambda \left( \sum a_j (\beta_j E_{jt+1})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \sum \left( \sum a_j (\beta_j E_{jt+1})^{\frac{1-\sigma}{\sigma}} E_{jt+1} \right)^{\frac{1}{1-\alpha}} A_{jt+1}}$$

Please note that the energy storage inefficiency  $\beta_N = \beta_D = 1$  (no inefficiency in storing nuclear and/or dirty energy),  $a_N = 1 - a_G - a_D$  and the environmental cost of green and nuclear energies are  $\xi_G = \xi_N = 0$ .

As we already mentioned, we are interested only in sustainable paths for  $E_{jt}$ . That is because, if degradation results to a state of environment which is lower than the threshold  $\underline{s}$ , then  $U = 0$ , which violates the transversality condition. That means that every optimal path for  $E_{jt}$  has to satisfy the following constraint:

$$\xi_D E_{Dt}^* \leq \Delta \Rightarrow E_{Dt}^* \leq \frac{\Delta}{\xi_D}$$

Also an additional constraint we impose is that at any time:

$$S_t \geq \Delta + u_t + \underline{s}$$

Hence an economy will never use nuclear energy if a possible accident would lead to irreversible environmental damage(i.e.  $y = 0$  and  $U = 0$  forever).

## 4. Results

### 4.1. Optimal energy policies

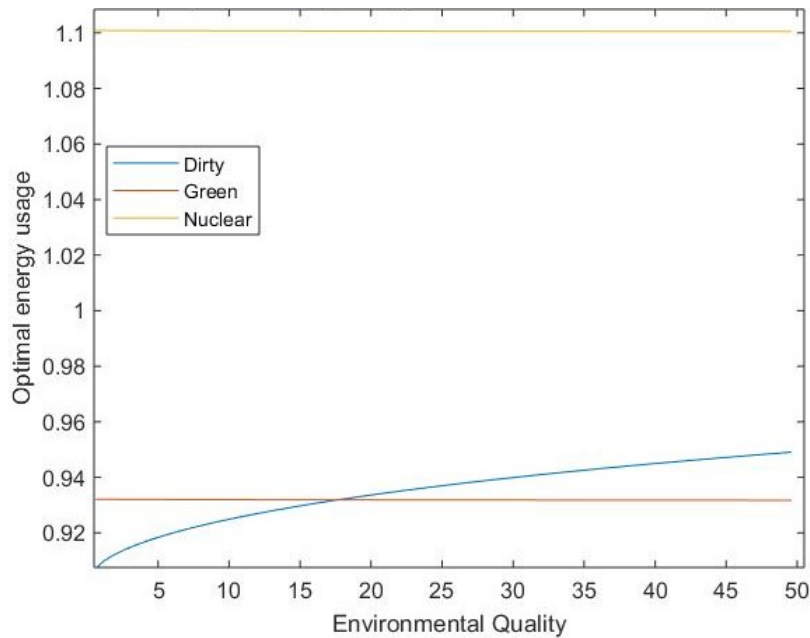
We now proceed on evaluating the future optimal energy usage given a baseline set of parameters. We assume that the labour income share,  $(1 - \alpha) = 1/3$ , the elasticity of substitution  $\sigma = 0.9$ (as we think that there is a certain level of complementarity between the three inputs), green energy storage capacity  $\beta_G = 0.80$ (only 80% of green energy will be able to be stored efficiently as well as utilized) ,  $a_D = 0.2$ ,  $a_G = 0.3$ ,  $a_N = 1 - a_D - a_G = 0.5$  and  $\lambda = 1/3$ . We furtherly assume that the patience parameter  $\rho = 0.5$  and  $A_{Dt} > A_{Gt} > A_{Nt}$ . For illustration purposes we arbitrarily set  $A_{Dt} = 5$ ,  $A_{Gt} = 3$  and  $A_{Nt} = 0.1$  as well as  $E_{Dt} = E_{Gt} = E_{Nt} = 1$ . Thus, in our numerical exercise, we are interested in finding the optimal energy usage levels, given that we depart from a scenario on which the economy is using all three types of energies equally. In other words we are interested in optimal changes in energy policy. Last but not least, the probability of

a nuclear accident  $\delta$  is assumed to be negatively correlated with the nuclear technology knowledge stock  $\delta = 0.0005 \frac{1}{A_{Nt}}$  (so in our numerical example, the probability of a nuclear accident occurring is  $\delta = 0.05\%$ ).

Thus, we solve the system of equations (9), for the optimal energy levels  $E_{Dt}, E_{Gt}, E_{Nt}$ , given the set of parameters above. By doing so -and alternating the values of one parameter of interest at a time- we are able to derive our comparative statics. Figures 1,2,3 and 4 summarize our results. In each one of them the vertical axis illustrates the optimal energy levels while the horizontal axis, the parameters of interest. The comparative statics that we will be presenting, are constructed with respect to the environmental quality  $S_t$ , the substitutability between our three intermediates  $\sigma$ , the patience parameter  $\rho$ , as well as both the nuclear technology knowledge stock  $A_{jt}$  and the probability of a nuclear accident  $\delta$ . Note that at this stage we treat  $A_{jt}$  as exogenous, although it's path is determined endogenously by the incentives described by equation (7). However, just for our numerical example and for illustrative purposes only we choose to treat it as exogenous as failing to do so, might unnecessarily increase computational burden.

Figure 1 illustrates the optimal next period's energy levels, for the three inputs, given the environmental quality today( $S_t$ ).

Figure 1: Optimal energy usage vs Environmental quality



First of all we can see that, it is optimal for an economy to choose a level of nuclear energy usage which is higher than both the green and the dirty energy one. That is because, in our numerical example:

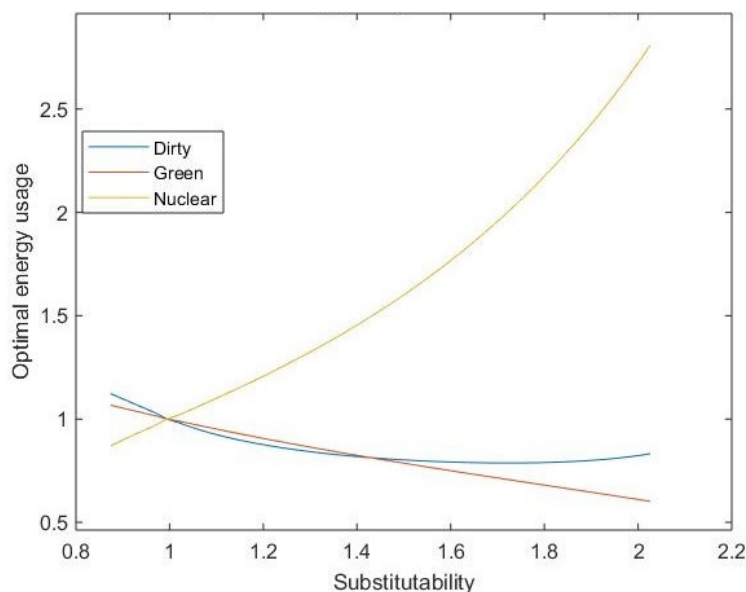
- A value of  $\rho$  significantly less than 1, results to optimal usage of dirty technology being lower than the nuclear one
- Green technologies are less efficient than the nuclear ones, as only 80% of the produced amount can be utilized. So even though they are more safe, the fact that the probability of a nuclear accident to occur is only 0.5% leads to the actual benefit of using nuclear energy over green exceeding the cost.

It is important to note that several studies such as [Barro and Jin\(2016\)](#), suggest that rare extreme events, might potentially lead to losses that are higher than the short-run direct effects of their occurrence. However in our model, a nuclear accident can only lead to a certain and exogenously given -strictly monetary- loss of  $\Delta$ . Thus, our model does not take into account any other potential long-run risks of a nuclear accident, as well as non monetary losses(for example losses of lives, ripple effects etc).

Secondly, our findings suggest that, the optimal dirty energy usage level is increasing in environmental quality. That is because, as the environmental quality rises, it is much harder for the "environmental constraint" to be binding. At the same time given that the production of the dirty intermediate does not feature any inefficiency -as the green one does-, it is optimal for the economy to increase dirty energy usage(as the environment is not in a fragile state).

As we already mentioned, our numerical exercise conditions on n-1 parameter values and aims to examine how sensitive are optimal policies to changes of a specific parameter of interest. These comparative statics will later help us better understand not only how these optimal policies could be applied in different settings but also how they could be induced. In addition, we have argued that the reasoning behind setting the substitutability parameter  $\sigma = 0.9$ , is that we believe that there is a weak but significant degree of complementarity between our three intermediates. However, it is crucial to examine how optimal policies change given different values of  $\sigma$ . Our findings are summarized on Figure 2.

Figure 2: Optimal energy usage vs Substitutability



Firstly we can see that as  $\sigma \rightarrow 1$  and thus our production function comes close to replicating the results we would have gotten if we were using the Cobb-Douglas production function, it would be optimal for society to use the same amounts of all of our three outputs. In addition, we can see that as the degree of substitutability increases, the economy should rely more heavily on nuclear technologies, whereas optimal green and dirty energy usage drops. However, we report that there are indications of a potential U-shaped optimal dirty energy curve.

As  $\sigma \rightarrow \infty$  (meaning that the three intermediates are perfect substitutes), society should pick the most efficient per unit of cost one. So it is optimal to base their production on nuclear energies as the environmental shadow price of dirty energy as well as the green energy storage inefficiency increase their relative prices against the nuclear one. Hence, in the case of our intermediates being highly substitutable, production should be driven by nuclear technologies.

The other interesting comparative static results are the ones with respect to the patience parameter  $\rho$  as well as the nuclear technology knowledge stock  $A_{Nt}$  and the probability of a nuclear accident  $\delta$ . The latter ones are obviously closely related within our model as we assume that the probability of a nuclear accident is a decreasing function of the nuclear energy knowledge stock. Our results regarding the changes in both the patience parameter and the probability of nuclear accident/nuclear energy technology stock are summarized on Figure 3 and Figure 4.



Figure 3: Optimal energy usage vs patience

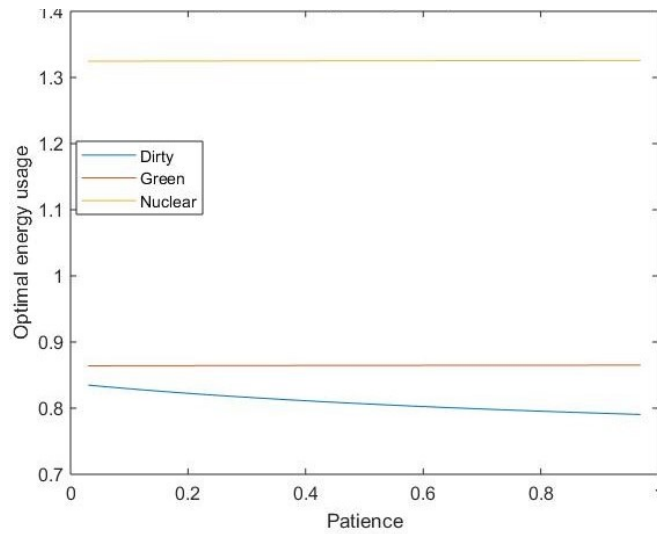
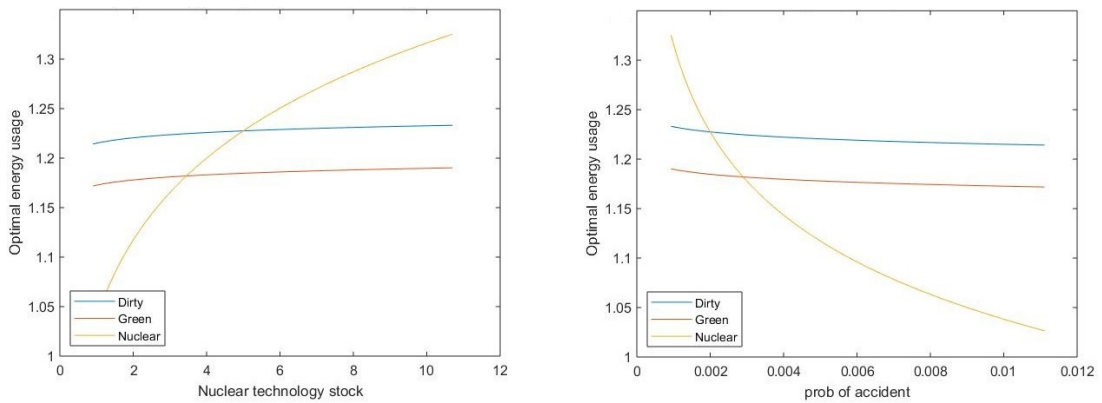


Figure 4: Optimal energy usage vs Nuclear energy technology stock and probability of nuclear accident



The patience parameter  $\rho$  is describing how much, people in our economy value future utility (and/or dis-utility). So it becomes apparent that, as society values the future more heavily, the optimal usage of dirty energy drops. The other two intermediates do not have any impact on future expected consumption. Of course, nuclear energy usage might potentially lead to a nuclear accident, but at the same time, in our model the probability of an accident occurring is only depending on the knowledge stock and not the usage level. Last but not least, the results illustrated in figure 4 suggest that optimal energy policies, for all three of our intermediates, are increasing

in the nuclear energy technology stock (and of course decreasing in the probability of a nuclear accident occurring). However, the absolute value of the slope of the optimal nuclear energy curve is in both cases significantly higher than the others.

As expected, optimal nuclear energy is decreasing in the probability of accident and increasing in its technology stock. That is because increasing technology stock not only increases the probability of an accident but also increases the productivity of the nuclear intermediate. Thus, as nuclear knowledge stock increases, the nuclear intermediate becomes not only more efficient but also safer. The reason why both the other intermediates feature the exact same patterns is because of the weak complementarity we assumed between all three types of energies.

#### 4.2. Optimal policy implementation

In section 4.1 we have found the optimal energy levels given our parameter values. It is apparent that the optimal energy policy mix should include all three types of energies/intermediates. In addition, we have to highlight the importance of nuclear energy in the production mix as it seems that high dependence on nuclear production is, against common belief, a necessary condition for welfare maximization.

Now a policy maker should simply try to induce this optimal energy levels. From equation (7), a scientist would direct their research efforts to some sector  $j$  if and only if:

$$E\Pi_{jt} > E\Pi_{-jt} \Rightarrow P_{jt}^{\frac{1}{1-\alpha}} \mu_{jt} A_{jt-1} > P_{-jt}^{\frac{1}{1-\alpha}} \mu_{-jt} A_{-jt-1} \Rightarrow \frac{P_{jt}}{P_{-jt}} > \left( \frac{\mu_{-jt} A_{-jt-1}}{\mu_{jt} A_{jt-1}} \right)^{1-\alpha}$$

So we can always redirect technical change towards a "preferred sector" by changing the relative price of its output(the intermediate) towards the others. That can be done by either imposing a tax on the other intermediate outputs and/or subsidizing the preferred one. Exactly as in [Acemoglu et al.\(2012\)](#) these taxes-subsidies can be later levied.

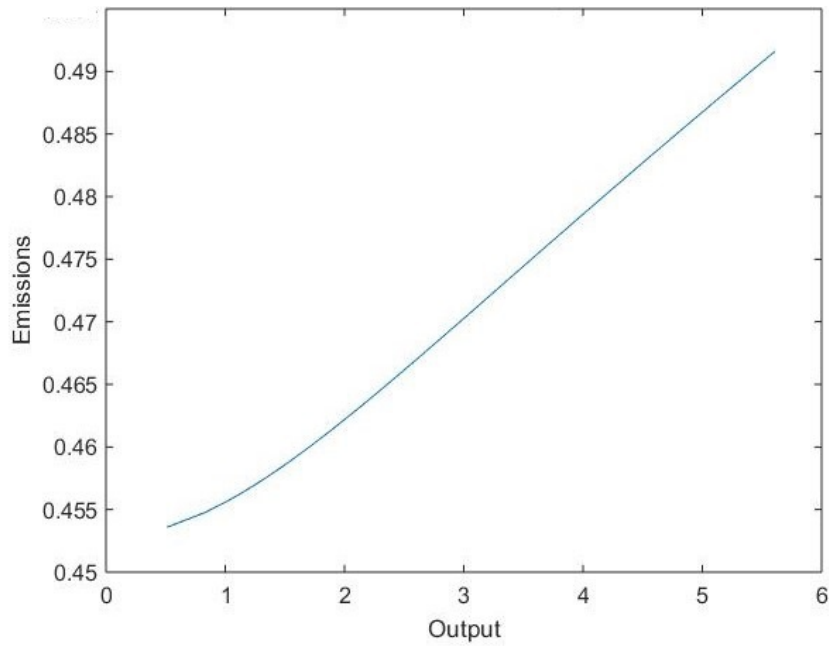
If  $E\Pi_{jt} > E\Pi_{-jt}$ , then all research effort will be focused on sector  $j$ . Thus  $A_{jt+1}$  will grow with a rate of  $\gamma$  with a probability  $\mu_{jt}$ . Hence, given that the inequality still holds(taxes/subsidies have not been levied yet), the expected growth rate of technology in sector  $j$  would be  $Eg_j = \mu_{jt}(1 + \gamma) > 0$ . Given that all technical progress is directed towards sector  $j$ , there will be some future point in time  $t'$ , that the  $E\Pi_{jt} > E\Pi_{-jt}$  will hold even in the absence of taxes/subsidies. Thus, technical

change will have been redirected, inducing the optimal policies, while the average growth rate in the economy will be  $\bar{g} = \bar{g}_j = \mu_{jt}(1 + \gamma)$

Last but not least, our results have indirect -yet clear- implications on the EKC literature. Both nuclear and green energies, resemble abatement activities, that differ in terms of risk as well as efficiency. At the same time, dirty energy usage, can be thought as a proxy for the change in environmental degradation, as higher usage of dirty energy usage today, will lead to a reduction of environmental quality. Suppose that we depart from a state with a relatively high level of environmental quality but low production. Our results suggest that, policy makers should increase the level of dirty energy, while keeping abatement actions at a constant level(i.e. green and nuclear energy usage levels). An increase of the dirty energy usage levels results to a worsening of the environmental state. Now suppose that environmental degradation was initially at a high level. That would translate to the policy makers, making moves towards an increase of abatement and dirty energy usage reduction. So, an economy that starts off with low GDP would most likely also have the characteristic of low environmental quality(dirty energy is cheap and efficient). As the economy grows(with a constant average growth rate), environmental degradation increases, and so optimal policies demand the reduction of dirty energy usage, accompanied by an increase in abatement(green and nuclear energy usage).

Keeping all that in mind, we could use our model, to see whether or not the EKC holds true, under optimal policy. After using the same parameter values that led to our other results, we can forecast the evolution of emissions as output grows. In order to achieve that, we use the optimally derived policy mix for every period while leaving output to grow in the context of the model. Following [Empora et al.\(2019\)](#), our proposed EKC under optimal policy, will account for a relationship between emissions and output, as opposed to the classical EKC hypothesis, which ties down emissions with output per capita.

Figure 5: Proposed Environmental Kuznets curve under optimal policies



Our results regarding the validity of the EKC hypothesis, are in accordance with [Empora et al.\(2019\)](#), in the sense that we report a positive relationship between output and emissions. However, our case is different from most studies on the subject because of three main reasons. First, our calibration is based on a theoretical model that takes into account uncertainty as well as, the ever-present possibility of nuclear energy usage. Secondly, our estimated EKC is derived under the assumption that optimal policies are implemented by the policy makers (optimal policies are being realized). Lastly, despite the fact that we report a positive relationship (correlation) between output and emissions, that relationship is notably weak. A 6-fold increase in output would lead to a 7.7% increase in emissions (or alternatively a 7.7% increase in emissions would result to a 6-fold increase in output). That is because, in the context of our model, there is an additional -and significantly productive- abatement alternative, nuclear energy. Another reason, for the weakness of the relationship, is the fact that our calibration has been done strictly under optimal policy implementation. Any policy, that might potentially result to the destruction of the environment (and thus any production), is eliminated as it is by definition not optimal.

### 4.3. Dynamic inter-dependencies

As we already mentioned, we are going to employ a panel VAR specification, which will allow us to trace any possible short-run inter-dependencies between growth, emissions as well as nuclear and total energy usage. That is rather crucial because it allows us to make sure that our long-run results would not be affected by partial short-term shocks in our variables of interest.

Our panel VAR specification is the following:

$$Y_{it} = A_0 + A_1 Y_{i,t-1} + V_{it}$$

$$\text{where } Y_{it} = \begin{bmatrix} y_{1it} \\ y_{2it} \\ y_{3it} \\ y_{4it} \end{bmatrix}; A_0 = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix}; A_1 = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \end{bmatrix} \text{ and } V_{it} = \begin{bmatrix} u_{1it} \\ u_{2it} \\ u_{3it} \\ u_{4it} \end{bmatrix}$$

Also, for every country  $i$  and year  $t$ ,  $y_{1it}$  denotes wealth,  $y_{2it}$  nuclear energy usage, while  $y_{3it}$  and  $y_{4it}$  represent emissions and total energy usage. The indicators that we are going to use are the GDP per capita (constant 2010 US\$), electricity produced by nuclear energy, Emissions (Millions Metric Tons of CO2) and Primary Energy consumption (Million kwh). Our data on all three variables except from the electricity produced by nuclear energy, are extracted by the dataset constructed by [Chen et al.\(2020\)](#), while the "Electricity production from nuclear sources (% of total)" variable is extracted by the World Bank Indicators database. Our combined dataset consists of a panel of 26 countries for the time period from 1990 to 2015. Our data can be summarized on table 1.

Table 1: Summary Statistics

Variable	Obs	Mean	Standard Dev.	Min	Max
GDP per capita	637	23183.69	16799.01	530.8947	55395.06
Production from nuclear sources	641	27.05817	21.52913	0	87.98622
Emissions	638	784.9398	1505.875	11.82992	9222.335
Primary Energy consumption	638	3775411	6615017	65373.7	3.51e+07

Figure 6: Impulse Response Functions

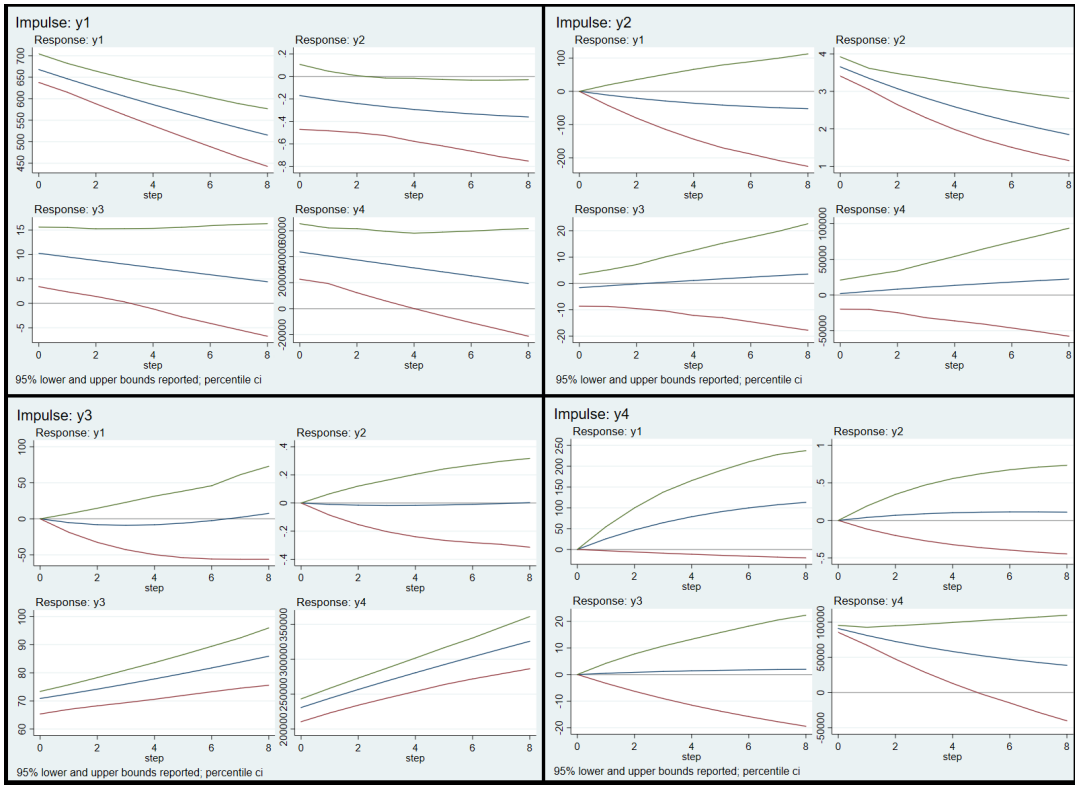


Figure 6 illustrates our Impulse Response Functions. First of all, we can see that a positive shock on  $y_1$  (GDP per capita) would lead to minor -but rather permanent- reduction of  $y_2, y_3$  and  $y_4$  (nuclear energy usage, emissions and total energy production). A possible interpretation of that could be that the economy is benefiting from the extra resources and so production becomes more efficient. Using the additional wealth, the economy may not only achieve lower emission levels by reducing total energy production, but also reduce any risk that might arise from excessive nuclear energy usage.

Another interesting finding is that an increase of emissions seems to have a positive effect on total energy production (both in the short-run and the long-run). Since the same shock does not seem to affect nuclear energy usage, we can be quite certain that this effect can be accounted for, solely to the increase in green energy usage. Thus, the economy responds to higher emission levels (and to a risk of possibly violating its environmental constraints), by increasing green energy usage. This result is mainly expected because increasing nuclear energy usage, is a highly unlikely response in the short-run (mainly because of the sunk costs associated with nuclear energy production).

However, the most important aspect of it, is that it seems to provide evidence that real life policies are indeed derived using welfare maximizing criteria (as assumed by our model).

Last but not least, it is worth noting that a positive shock in  $y_3$  (increase in emissions) will be persistent and will lead to a notable increase in emissions even in the long-run. That is in contrast with the results regarding all the other variables as, as in the case of  $y_1, y_2$  and  $y_4$ , a positive shock, will generally be generally cancelled off in the long-run. That is an indication of dynamics in the economy that can lead to environmental traps. More precisely, a worsening of environmental quality seems to be persistent as policies prove to be far from responsive. Thus, we find evidence that the worsening of environmental quality creates dynamics that lead the economy to an even worse environmental state.

## 5. Concluding Remarks

In this paper, we are expanding the model of [Acemoglu et al.\(2012\)](#), by including the possibility of nuclear energy. The inclusion of nuclear energy introduces uncertainty which in turn, is affecting society's welfare maximization problem and as a result, optimal policy decisions. Our strategy is to firstly, find the optimal energy policies, as well as how these can be differentiated and be applied in different cases. After doing so, we can induce the optimal policy levels by creating the incentives for entrepreneurs, to invest in each sector up, until their usage reaches its optimal level.

Our results suggest that, against popular belief, nuclear energy usage is a really important factor, as far as society's welfare maximization is concerned. Secondly, we show that, total elimination of more dirty inputs is not optimal. In addition, optimal dirty energy usage is increasing in environmental quality and decreasing in the extent people discount their future utility. Last but not least, we find that, the optimal energy policy mix is highly dependent on the assumptions regarding the production function. In other words, the degree of substitutability between the three inputs can dramatically affect the results and thus the policy implications.

This paper was an attempt to find optimal energy policies under uncertainty. As we have argued, policy makers can find -and later implement- the optimal energy policy mix. This implementation procedure though, involves the cost of maintaining distortionary policies over time(taxes and/or subsidies). Future research on the subject, could evaluate this cost and potentially re-evaluate the

optimal policy design results, as in some cases, it might be too costly to redirect technical change to a low technology stock sector.

In addition to the direct implications, our study has on the endogenous growth with environmental constraints literature, we believe that there are some indirect implications to the Environmental Kuznets Curve literature. As we already mentioned, the EKC hypothesis, suggests that there is a correlation between the GDP per capita level and the level of environmental degradation in an economy. In our setting, abatement activities take the form of either nuclear or green energy. In addition, usage of more "dirty" energy can be thought as a proxy for environmental degradation. Our results imply that, under optimal policy, there is a weak positive relationship between output and emissions.

Furthermore, our results suggest that, the average growth rate on the economy will decline over time. That is because, the average growth rate is a constant mark-up of the probability of innovation in the prevalent sector. As we assume that this probability is a concave function of the sector's knowledge stock, the growth rate of technology and thus the growth rate of output in the economy will be declining. Relaxing the assumption of concavity though, would result to a constant average growth rate. However, just because modern production is relying firstly on dirty inputs and secondly on green, an increase in nuclear energy usage is expected to have a positive effect not only on economic growth, but also on welfare.

Lastly, we are employing a panel VAR specification, which allows us to trace any possible short-run inter-dependencies between growth, emissions and nuclear and total energy usage. We report an (expected) increase of green energy usage after an exogenous rise in emissions. The most important finding of our VAR specification is that, higher emissions seem to create dynamics that lead to even higher emissions in the future, implying the possibility of the existence of environmental traps.



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