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Abstract

This paper uses a Bayesian time-varying parameter vector autoregressive (TVP-VAR) model to assess the impact of alternative drivers of bitcoin returns. We consider an extended set of alternative drivers and select the most important variables using a Bayesian variable selection method. To examine the evolution of the Granger-causality relationship between the selected variables and bitcoin returns over time, we employ a new approach based on the estimates of the TVP-VAR model and heteroscedastic-consistent Granger-causality hypothesis testing. In addition, we perform impulse response function and forecast error variance decomposition analysis. The results indicate that investor sentiment and ethereum returns affect bitcoin returns over the entire sample. Trading volume emerges as an important determinant of bitcoin returns when bitcoin prices remain relatively steady.

Keywords: Bayesian VAR, time-varying Granger-causality, bitcoin, cryptocurrency, uncertainty, Google trends.
JEL Codes: C11, C12, C15, C52, D80, E58 G15.

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1 Introduction

More than ten years after its creation, bitcoin continues to attract the attention of both researchers and investors. Bitcoin has distinct differences from traditional currency as it is used neither as a medium of exchange (despite its low transaction costs and peer-to-peer network) nor as a store of value due to its price fluctuations, (Blau, 2017). Furthermore, it can not function as money due to its almost fixed supply (Yermack, 2015). However, bitcoin has features that make it appealing as an asset. Ghabri et al. (2021) provide evidence that bitcoin can function as a risk diversifier and reduce liquidity risk since it is characterised by (i) high average returns and (ii) low correlation with traditional financial assets. As an asset, bitcoin is used mainly for speculative purposes (among others, see Baur et al., 2018; Lee et al., 2020). This exploitation as a speculative asset leads to bubbles (Cheah and Fry, 2015, 2016; Ben Osman et al., 2024) and extreme volatility (Catania and Grassi, 2022; Bakas et al., 2022; Panagiotidis et al., 2022). Furthermore, Fry (2018) demonstrates that liquidity risks can result in boom-bust episodes even in the absence of a bubble and Xiang et al. (2022) argue that the bitcoin market capitalisation series exhibit explosive behaviour.

A strand of the bitcoin related literature focuses on the dynamics that drive bitcoin returns and the causal relationship between bitcoin returns and other factors. The literature identifies numerous factors as bitcoin returns determinants. Macroeconomic and financial variables such as stock market indices (Jia et al., 2024), exchange rates (Dyhrberg, 2016), exchange traded funds (Ciner et al., 2022) and economic policy uncertainty receive the most attention by researchers. The attractiveness of bitcoin as an asset is also documented as a driving force of its returns. As an investment asset, bitcoin draws the attention of retail investors who rely mostly on the internet as a source of information. Google searches (Li and Wang, 2017; Fry and Serbera, 2020; Liang et al., 2022; Raza et al., 2022), Wikipedia (Kristoufek, 2013), Twitter (Suardi et al., 2022) and Reddit (Rothman, 2019) are shown to have a significant impact on bitcoin returns. Another set of variables that can affect bitcoin are the supply and demand forces (de la Horra et al., 2019). However, supply forces have a smaller impact due to the nearly fixed amount of bitcoins. The fixed amount of bitcoins stems from the difficulty in creating new bitcoins. New bitcoins are generated through a mining process (validating transactions on the network) which requires enormous amounts of electricity and computational power.

Another strand of the literature employs a Granger-causality analysis (hereafter, all instances of the word causality refer to Granger-causality). Lin (2021) employs a VAR model and Granger-causality analysis to investigate the relationship between cryptocurrency returns and Google searches. The causal relationship between bitcoin returns and Google searches is also examined by Dastgir et al. (2019) and Li et al. (2021). Dastgir et al. (2019) employ the Copula-based Granger-causality in the distribution test and find a bidirectional causal relationship between Google Trends search terms and bitcoin returns only in the tails of the distribution. Li et al. (2021) use a wavelet-based quantile Granger-causality test and find a symmetric causal relationship between Google searches and cryptocurrency returns in the short term. The authors do not consider only a single search term but proxy investor attention by constructing an index similar to Da et al. (2011). The relationship between bitcoin returns and Google searches is also analysed by Zhang et al. (2021) using a dynamic conditional correlation multivariate GARCH model. Shen et al. (2019) use linear and nonlinear tests Granger-causality to examine the link between bitcoin returns and the number of tweets from Twitter. Bouri et al. (2018) study daily dependence between global financial stress and bitcoin using standard and quantile-based copula models. Their results suggest that financial stress causes bitcoin returns at the left and right tail of the latter's conditional distribution. Financial stress, however, has limited directional predictability for Bitcoin. Finally, Ji et al. (2018) use the directed acyclic graph approach to study the interrelation between bitcoin and other financial asset classes.

Despite the growing literature on the driving factors of bitcoin returns, the results are often diverse and contradicting. Panagiotidis et al. (2024) show that choosing the appropriate model is crucial and that misspecification issues can lead to misleading results. This paper aims to examine the effect of alternative factors on bitcoin returns. Contrary to previous studies, we do not focus only on a specific group of variables (e.g. stock markets or investor sentiment) but consider a wide range of factors from different areas. Specifically, we employ bitcoin volatility, investor sentiment indices, proxies for bitcoin supply and demand, stock market returns and volatility indices, commodities returns, exchange rates and interest rates. Overall, we consider 29 potential determinants of bitcoin returns. As indicated by the extended number of related studies, understanding what drives bitcoin returns is a subject of major interest since the rampant fluctuations of bitcoin can severely affect the investing decisions of individual investors and the policymakers who seek to regulate the cryptocurrency market.

The analysis is conducted on a time-varying framework using a Bayesian time-varying parameter vector autoregressive (TVP-VAR) model. In this framework, we model the evolution of the variance-covariance matrix of the errors using stochastic volatility, thus accounting for the volatility dynamics of bitcoin returns. The latter is often overlooked in the relevant literature. We perform the analysis using a two-step approach. In the first step, we estimate the TVP-VAR using all 29 potential factors (plus the bitcoin returns) and detect the most important variables of the model based on the Bayesian variable selection method by Korobilis (2013). In the second step, we reestimate the TVP-VAR model using only the variables selected in the previous step and perform the analysis. We perform Granger-causality, impulse response function and forecast error variance decomposition analysis.

To take advantage of the time-varying framework in the Granger-causality analysis, we propose a time-varying Granger-causality approach that allows us to examine the evolution of the causal relationship between two variables over time. The proposed methodology combines the estimated parameters from the TVP-VAR model with the heteroscedastic-consistent Granger-causality hypothesis testing. Moreover, our approach does not rely on rolling windows or recursive algorithms (as in Shi et al., 2018) that require the estimation of the model on subsamples of the initial sample and as a result does not suffer from inference issues caused by small sample sizes (Zapata and Rambaldi, 1997) and temporal aggregation (Otero et al., 2021). In addition, the causality analysis can be performed over the entire sample and does not require an initial window to initiate the algorithm, thus using all available information in the dataset. The intuition behind the methodology is that when a rolling window is applied, the aim is to find a window size sufficiently small without causing inference problems. A time-varying approach provides estimates for each observation in the sample which is the equivalent of setting the window size to one in a rolling window algorithm.

The contribution of this study is twofold. From a methodological perspective, we propose a new time-varying Granger-causality approach that does not depend on rolling windows or recursive evolving algorithms. We assess the performance of this new approach using Monte Carlo simulations. From an empirical perspective, we study the impact of alternative predictors on bitcoin returns using an extended set of potential predictors.[1] Similar studies that consider an extended set of bitcoin determinants often ignore the time-varying dynamics of bitcoin returns (for example, see Panagiotidis et al., 2018). The results, based on the Bayesian variable selection algorithm, suggest that the most important factors that drive bitcoin returns are investor sentiment (proxied by Google searches and uncertainty cryptocurrency index), bitcoin trading volume, Nasdaq and ethereum returns. The recently proposed approach quantifies the causal relationship between the five variables and bitcoin returns. We find that Google searches and ethereum returns Granger-cause bitcoin returns over the whole sample while the rest of the variables only for specific periods. In addition, the causal relationship from trading volume to bitcoin returns exists only for periods when bitcoin prices remain relatively steady.

The rest of the paper proceeds as follows. Section 2 describes the methodology and the new time-varying Granger-causality approach. Section 3 discusses the Monte Carlo simulations. Section 4 presents the data used in the empirical analysis. Section 5 discusses the main findings. Section 6 compares the empirical results with the results of a recursive

[1] Zhang et al. (2021) employ the time-varying Granger-causality test of Lu et al. (2014) that uses all information available in the sample but focuses exclusively on the relationship between bitcoin and internet attention.

algorithm. Section 7 discusses the implications of our findings. Section 8 concludes.

2 Methodology

2.1 The TVP-VAR model

Let y_t be a vector of n time series, $t = 0, \dots, T-1$. The VAR(p) model is written as:

$$y_t = b_t + \sum_{i=1}^{p} B_{i,t} y_{t-i} + \varepsilon_t, \tag{1}$$

where b_t is a $n \times 1$ vector of time-varying coefficients, $B_{i,t}$, $i = 1, \dots, p$ are $n \times n$ matrices of time-varying coefficients and $\varepsilon_t \sim N(0, \Sigma_t)$. The TVP-VAR is a state space model (Koop and Korobilis, 2010). Specifically, we define $X_t = I_n \otimes [1, y'_{t-1}, \dots, y'_{t-p}]$ (where I_n is the $n \times n$ unit matrix) and $\beta_t = \text{vec}([b_t, B_{1,t}, \dots, B_{p,t}]')$ (where vec() is the columnwise vectorisation of a matrix) and rewrite equation (1), the measurement equation, in block form as follows:

$$y_t = X_t \beta_t + \varepsilon_t. \tag{2}$$

Since the variance-covariance matrix of the error terms is allowed to smoothly evolve over time, we model the evolution of Σ_t as in Cogley and Sargent (2005). Specifically, we consider the following decomposition of Σ_t:

$$\Sigma_t^{-1} = A' D_t^{-1} A,$$

where A_t is a $n \times n$ lower triangular matrix with diagonal elements equal to one and D_t is a $n \times n$ diagonal matrix with diagonal elements, $e^{h_{i,t}}$. Contrary to Primiceri (2005), we assume that A is a time invariant matrix. If we denote by α the vector of unrestricted elements of A, the elements below the main diagonal, and $h_t = (h_{1,t}, \dots, h_{n,t})'$, we complete the state space model by obtaining the following state equations:

$$\begin{aligned} \beta_t &= \beta_{t-1} + u_t, \quad u_t \sim N(0, Q) \\ h_{i,t} &= h_{i,t-1} + u^h_{i,t}, \quad u_t \sim N(0, \sigma^2_{h,i}), \end{aligned} \tag{3}$$

where the initial conditions β_0 and $h_{i,0}$ are treated as unknown parameters and Q is assumed to be diagonal ($Q = \text{diag}(q_i)$, where $i = 1, \ldots, n + n^2 p$).

We complete the model by specifying the prior distributions for β_0, q_i, α, $h_{i,0}$. We consider the following independent priors:

$$\begin{aligned} \beta_0 &\sim N(0, 1) \\ q_i &\sim IG(3, 0.001) \\ \alpha &\sim N(0, 0.01) \\ h^2_{h,i} &\sim IG(1, 1) \\ h_0 &\sim N(0.01, 0.05) \end{aligned}$$

where IG denotes the inverse gamma distribution. We consider an alternative set of prior values to assess the validity of our findings. In all cases, the results remain sshimilar to the ones presented here. The five model parameters and the two states are estimated using a Gibbs sampler described in Chan et al. (2019).

Once the model is estimated, we employ the Bayesian variable selection, proposed by Korobilis (2013), to select the most important variables in the model. We then re-estimate the TVP-VAR model using only the most important variables. In the new model, we employ the time-varying Granger-causality test as well as the impulse response function and forecast error variance decomposition analysis.

2.2 The Granger-causality hypothesis

To describe the methodology for the Granger-causality analysis, it is useful to define $Z_t = [1, y'_{t-1}, \ldots, y'_{t-p}]'$ and $R_{i,j}$ an appropriate $p \times n(np+1)$ coefficient restriction matrix for the null hypothesis that y_i does not Granger-cause y_j. Each row of $R_{i,j}$ picks one of the coefficients to set to zero under the non-causal null hypothesis. Next, we can calculate

$$\begin{aligned}
\Omega &= T^{-1} \sum_{t=1}^{T-p} (\varepsilon_t \otimes Z_t)(\varepsilon_t \otimes Z_t)', \\
F &= T^{-1} \sum_{t=1}^{T-p} Z_t Z_t', \\
G &= I_n \otimes F.
\end{aligned}$$

We estimate a series of heteroscedastic-consistent Wald statistics $W_{i,j,t}$, $t = p+1, \ldots, T$ as follows:

$$W_{i,j,t} = T(R_{i,j}\beta_t')'(R_{i,j}G^{-1}\Omega G^{-1}R_{i,j}')^{-1}(R_{i,j}\beta_t'). \tag{4}$$

For each t, $W_{i,j,t}$ follows a χ^2_q, where the degrees of freedom, q, is the number of restricted coefficients.

3 Monte Carlo simulations

3.1 Design of the Monte Carlo simulations

Next, we assess the performance of the proposed time varying Granger-causality test and perform several Monte Carlo experiments. We consider the following general data generating process (DGP):

$$\begin{aligned}
y_{1,t} &= b_{10} + b_{11} y_{1,t-1} + b_{12} y_{2,t-1} + \varepsilon_{1,t} \\
y_{2,t} &= b_{20} + b_{21} y_{1,t-1} + b_{22} y_{2,t-1} + \varepsilon_{2,t}
\end{aligned} \tag{5}$$

The DGP describes a bivariate VAR(1) model with a constant term. In all cases, $\varepsilon_{1,t}$

and $\varepsilon_{2,t}$ follow a normal distribution with zero mean and unit variance. Three alternative sample sizes are considered, $T \in \{100, 200, 400\}$. We examine the size and power properties of the test and the performance of the test in the presence of a structural break. To examine the size (incorrectly reject the null of no Granger-causality) properties of the test, we set $b_{21} = b_{12} = 0$, so that there is no Granger-causality between y_1 and y_2. The values of the rest of the parameters are all different from zero and are described in Table 1. To examine the power of the test, we use the same set of parameters as in the previous case but set $b_{12} = b_{21} = 0.4$. We consider a final case with a structural break in the parameters. Specifically, for the first half of the sample we set $b_{12} = 0$ and for the second half of the sample, we set $b_{12} = 0.4$. We do so to validate the ability of the test to detect Granger-causality in specific subsamples. The values of the coefficients in each case ensure that the series are stationary and that the VAR is stable. Figure 1 shows an example of two time series simulated using DGP3 and $T = 400$. Table 1 summarises the details for all DGPs.

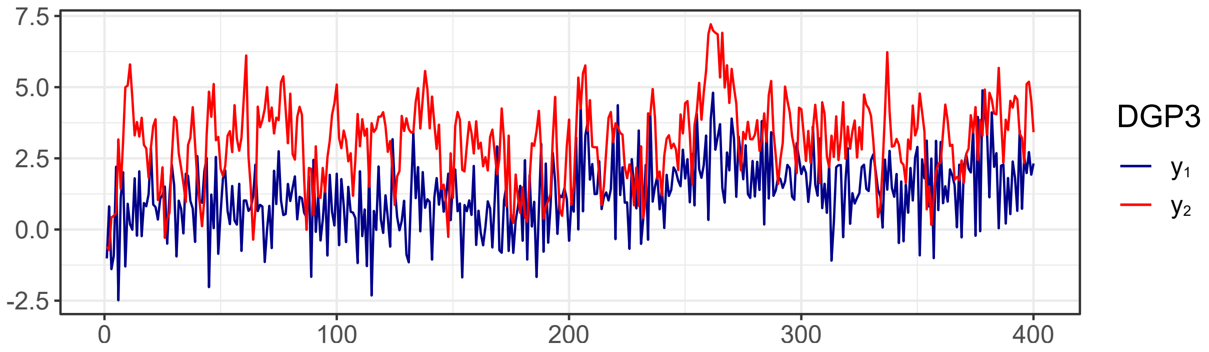
Table 1: Parameter values for the Monte Carlo simulations.

DGP/Parameter	b_{10}	b_{11}	b_{12}	b_{20}	b_{21}	b_{22}	
DGP1	1	-0.3	0.0	1	0.0	0.6	
DGP2	1	-0.3	0.4	1	0.4	0.6	
DGP3	1	-0.3	0.4	1	0.0	0.6	t = 1,..., T/2
	1	-0.3	0.4	1	0.4	0.6	t = T/2+1,..., T

Notes: i) DGP1 and DGP2 are used to examine the size and power properties of the test, respectively. In DGP3 Granger-causality exists only in the second half of the sample. ii) We consider three sample sizes, $T = 100, 200, 400$. iii) Both error terms are independently and identically distributed following the normal distribution with zero mean and unit variance.

The TVP-VAR model is fitted to the generated time series. To estimate the posterior distributions, we use 20.000 iterations of the Gibbs sampler algorithm. The first half of the iterations are used as burn-in sample and the remaining are used to build posterior sample. The results of the Monte Carlo simulations are based on 10.000 iterations. The reported simulations are programmed using the `bvartools` package of **R**. The code is available upon request.

Figure 1: Example of simulated time series using DGP3. For $t = 1, \ldots, 200$, there is a unidirectional Granger-causal relationship from y_2 to y_1. For $t = 201, \ldots, 400$ there is a bidirectional causal relationship.



3.2 Results

For all DGPs, we consider the unidirectional Wald statistics, both from y_1 to y_2 and vice versa. First, we evaluate the size properties of the test using DGP1. For the test to be successful, we require that it accepts the null hypothesis of no Granger-causality, at the 5% significance level, for every observation in the sample. The left part of Table 2 presents the failure frequency of the test. We observe that even for smaller samples (i.e. $T = 100$), the test handles size reasonably well.

We use DGP2 to assess the power of the proposed test. We assume that the test successfully detects Granger-causality if it rejects the null hypothesis for the entire sample. The middle part of Table 2 reports the success rate of the test. Regardless of the sample size, the test is able to detect the causal relationship more than 90% of the time.

The last Monte Carlo experiment, DGP3, is used to examine the behaviour of the test in the presence of a structural break. Specifically, we investigate whether the test detects Granger-causality from y_1 to y_2 during a specific subsample. The fifth column of Table 2 shows the number of times that the test rejects the null hypothesis only during the latter half of the sample. However, we consider a period of $T/10$ observations before and after the break point, where the test is allowed to produce a false result. The results suggest that the new test successfully identifies the change in causal relationship in more than 80% of the cases. The success rate increases as the number of observations increases. Furthermore, we notice that change in b_{21} does not affect the ability of the test to reject the null hypothesis of no Granger-causality from y_2 to y_1 for all observations in the sample. In particular, the last column of Table 2 indicates that the test indicates that y_2 Granger-causes y_1 for the entire sample, in more than 90% of the cases. This finding validates the results regarding the power of the test (obtained using DGP2).

Table 2: Power and size properties of the proposed time-varying Granger-Causality methodology.

	DGP1		DGP2		DGP3	
T	$W_{1,2}$	$W_{2,1}$	$W_{1,2}$	$W_{2,1}$	$W_{1,2}$	$W_{2,1}$
100	0.143	0.164	0.915	0.940	0.802	0.903
200	0.065	0.089	0.953	0.982	0.816	0.947
400	0.030	0.034	0.995	1.000	0.853	0.961

Notes: i) DGP1 and DGP2 are used to examine the size and power properties of the test, respectively. In DGP3 Granger-causality exists only in the second half of the sample.

4 Data

The empirical study utilises weekly data over the period 2014-2023. We download bitcoin closing prices from coinmarketcap.com and calculate the bitcoin returns as logarithmic differences of the closing prices. We examine the causal relationship between bitcoin returns and alternative drivers of bitcoin returns. The selection of potential drivers of bitcoin returns is based on findings from the related literature. To capture the volatility dynamics of bitcoin returns (see, for example, Catania and Grassi, 2022; Panagiotidis et al., 2022) we include the volatility of bitcoin returns as the square returns (Yousaf and Yarovaya, 2022). Moreover, we consider three indices regarding investor sentiment. The first is created using Search Volume Index (SVI) data, from Google Trends, for three search terms, 'bitcoin', 'bitcoin price' and 'BTC'.[2] Alternative queries such as 'bitcoin returns', 'bitcoin invest' or 'BTCUSD' are not used since the search volume for these queries is negligible compared to the search volume of the keywords used in the study. The three downloaded SVIs take

[2]Oil prices which are included in the dataset are also affected by Google searches Bampinas et al. (2023).

values from 0 to 100 and are scaled accordingly (meaning that at each date, the SVIs are comparable). Furthermore, the three indices maximise on the same date (on 30/08/2019). Using the maximum as values as weights (rescaled so their sum equals 1), we define the Google Search Volume Index (GSVI) as the weighted average of the three indices.[3] We use the GSVI as a measure of attention for retail investors. We assume that Google searches quantify information demand from individual investors given that it is the most popular search engine. The second index that reflects investor sentiment is the Uncertainty Cryptocurrency policy (UCRY policy) index and the third is the Uncertainty Cryptocurrency price (UCRY price) index. These two indices, proposed by Lucey et al. (2022), are used to evaluate the impact of policy and regulatory discussions on the returns and volatility of cryptocurrencies. The UCRY policy index gauges the uncertainty of informed investors who are more sensitive to policy changes while the UCRY price index measures the uncertainty of individual investors who pay more attention to price fluctuations and media coverage. Lucey et al. (2022) demonstrate that the two UCRY indices capture the impact of major events on bitcoin returns. The indices are available on weekly frequency from 1/3/2014 which determines the length and the frequency of the whole dataset.[4]

The next group of variables aims to capture supply and demand forces. We consider both supply and demand although Loi (2018) and de la Horra et al. (2019) argue that the effect of demand is stronger. Here, we measure bitcoin supply using the number of total bitcoins in circulation. We proxy bitcoin demand using the trading volume and a number of network related measures, the number of confirmed transactions, the number of unique addresses, hash rate and network difficulty. The latter two variables are proxies of the mining difficulty. Hash rate measures the computational power in the network. A higher hash rate indicates a more secure network and more users validating transactions. In addition, the higher the hash rate, the higher the chance that a user creates a new block

[3]Due to the small number of SVI series (three series) used for the construction of the GSVI, other methods such as Bayesian averaging, principal components analysis or factor models (these methods are suitable when a large number of series is available).

[4]The use of lower frequency data have only a small impact on the properties of VAR models (for example, see Otero et al., 2022).

in the blockchain and obtains bitcoin. Kubal and Kristoufek (2022) show that hash rate explains bitcoin dynamics. Network difficulty measures the computational power required to verify a transaction and is adjusted, based on hash rate and the total number of users, to maintain the time required to verify a transaction of approximately ten minutes.

We consider a set of traditional financial variables including stock market returns (Jia et al., 2024), commodity returns (Bouoiyour and Selmi, 2015; Ghabri et al., 2021), and exchange rates (Dyhrberg, 2016). Evidence in the literature indicates that the choice of sample period can substantially affect the results regarding the interaction between bitcoin and market returns. To analyse the relationship between stock markets and bitcoin returns, we employ the returns of five stock market indices, the S&P500, Dow Jones, Nasdaq, EURO STOXX 100, FTSE50, Nikkei 225 and the Shanghai composite index. We also include two stock market volatility indices, the CBOE market and the EURO STOXX 50 volatility indices. Regarding commodity markets, we consider two oil price indices, WTI and Brent and the price of gold. Finally, we examine the relationship between bitcoin returns and four exchange rates: euro, British sterling, Japanese yen and Chinese yuan. The exchange rates are expressed as the price of domestic currency to one US dollar, implying that an increase in the exchange rate denotes a depreciation of the domestic currency. We do not include non-fungible tokens since they exhibit a stronger relationship with ethereum than bitcoin (Panagiotidis and Papapanagiotou, 2024).

All variables are obtained in weekly frequency from their respective data sources (see Table A1 for the data sources). Table A2 reports the summary statistics and the results of the ADF test. Variables for which we do not reject the null hypothesis of the ADF test, at any level, are transformed to logarithmic differences.

5 Empirical results

In what follows, for each TVP-VAR model, we consider a burn-in sample of 50,000 iterations and build the posterior distributions from the next 50,000 draws by keeping only each 10^{th}

draw. This thinning process is used to mitigate auto-correlation among draws. The number of lags is based on the Schwarz information criterion and set equal to 1 in all cases.

5.1 Bayesian variable selection

We estimate a TVP-VAR model using all variables presented in Section 4 and use the Bayesian variable selection algorithm to detect the most important regressors in the equation where the bitcoin returns are the dependent variable. Table 3 presents the posterior inclusion probability for each variable in the bitcoin equation. We find that apart from the lag of bitcoin returns, the variables with the greatest posterior inclusion probability are the cryptocurrency policy uncertainty index and the GSVI, two variables that proxy investor sentiment. The finding is supported by evidence from previous studies that conclude that information demand is the most important determinant of bitcoin returns, (see Panagiotidis et al., 2018). The main bulk of (retail) investor sentiment is captured by GSVI which explains the smaller posterior inclusion probability of the cryptocurrency policy uncertainty index. We should note that the posterior inclusion probability of the cryptocurrency policy uncertainty index is larger than the threshold value 0.5.

Out of the ten stock and commodity markets, only one, the Nasdaq index, has a sufficient high posterior inclusion probability. This result reinforces the argument of Liu and Tsyvinski (2020) and Liu et al. (2022) who argue that cryptocurrency returns have low exposure to traditional asset markets.[5] Other variables with high posterior inclusion probability are the bitcoin trading volume and ethereum returns, indicating that the behaviour of bitcoin returns is better explained by cryptocurrency factors (Liu and Tsyvinski, 2020).

Using these five variables and bitcoin returns (i.e. cryptocurrency policy uncertainty index, the GSVI, the bitcoin trading volume, the returns of Nasdaq index, the ethereum and bitcoin returns) we estimate the main TVP-VAR(1). To assess the necessity for a time-varying framework, we compare the each of the two estimated TVP-VAR models with its

[5]The coefficients of oil prices indices remain small during the pandemic indicating that while stock markets are risk transmitters towards agricultural commodities and ESG markets (Wu et al., 2023; Wan et al., 2024), they have little to no effect towards bitcoin.

Table 3: Posterior inclusion probability of the coefficients for the bitcoin equation in the TVP-VAR model.

BTC : 0.952	**TV : 0.672**	EURO: 0.436	SP500 : 0.590	SSEC : 0.296
VOL : 0.046	ADD : 0.376	GBP : 0.440	DJ : 0.466	VIX : 0.000
GSVI : 0.786	CTR : 0.000	YUAN: 0.236	**NASDAQ: 0.748**	VSTOXX: 0.000
UCRY policy: 0.949	HASH: 0.118	YEN : 0.010	STOXX : 0.402	WTI : 0.070
UCRY price : 0.553	DIF : 0.254	FFER: 0.288	FTSE : 0.448	BRENT : 0.146
ETH : 0.834	NBC : 0.196	ECB : 0.144	NIKKEI: 0.462	GOLD : 0.458

Notes: i) The values denote the posterior inclusion probability and are obtained using the Bayesian variable selection method proposed by Korobilis (2013). ii) The results refer to the bitcoin equation. iii) The five variables with the highest probability, highlight in bold, are considered for the main model. iv) See Table A1 for the definitions of the mnemonics.

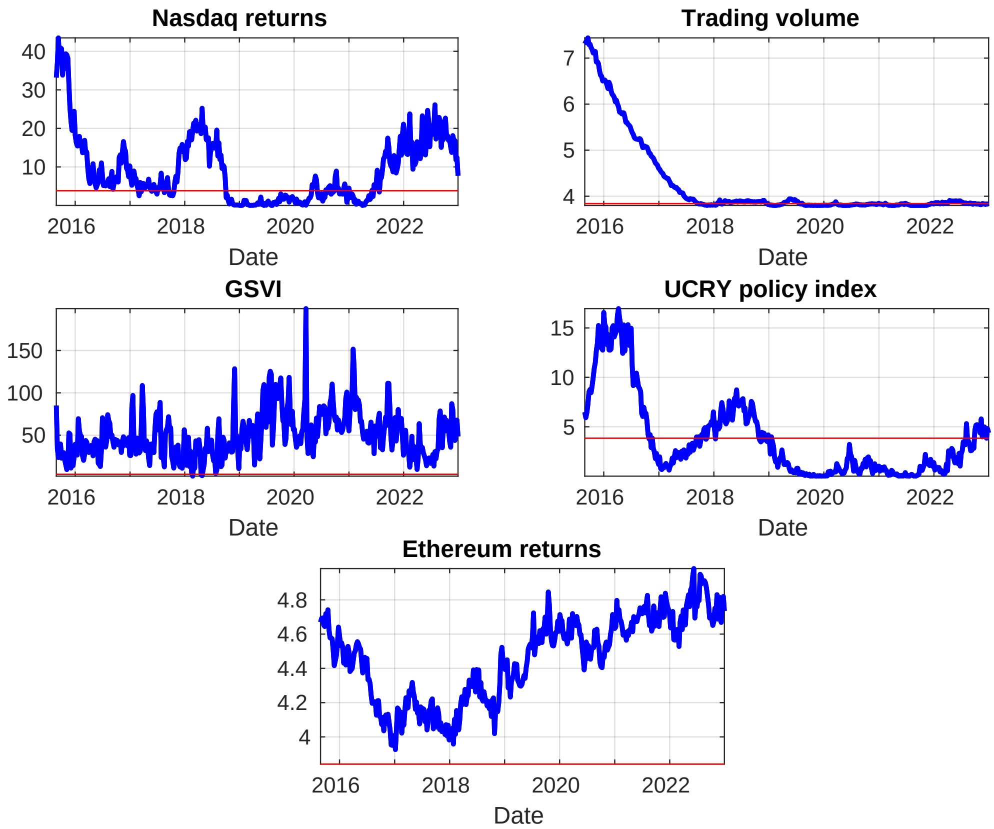
time invariant counterpart using the Bayes factor. Since the time invariant and the TVP-VAR models are nested models we are able to calculate the Bayes factor using the Savage-Dickey density ratio (see also Koop et al., 2010; Chan, 2023). Both for the model with all 30 variables and for the model with 6 variables, the results are in favour of the time-varying specification.

5.2 Time-varying Granger-causality

We now turn to the analysis of the main TVP-VAR model. We use the time-varying Granger-causality test, described in the section 2.2, to examine whether there is a unidirectional causal relationship between each variable and bitcoin returns. Figure 2 presents the sequences of Wald statistics for each variable, along with the critical value at the 5% significance level. The null hypothesis of no Granger-causality is rejected during the periods that the Wald statistic exceeds the critical value.

We observe that the GSVI and ethereum returns Granger-cause bitcoin returns for the entire sample. The strong effect of GSVI on bitcoin returns is explained by the fact that bitcoin is traded mostly by individual investors who rely on the information on the internet instead of specialised journals and media. GSVI captures the information demand by individuals and noise traders (see, Da et al., 2011) since bitcoin is often used as a speculative asset. Bitcoin returns are also affected by ethereum returns. Bitcoin and ethereum are the two biggest cryptocurrencies (in terms of market capitalisation and trading volume). Both are driven by technological factors and regulatory developments. For example, regulations which are favourable towards ethereum could lead to broader optimism and increase both ethereum and bitcoin returns.

Figure 2: Time varying Granger-causality results.



Notes:) i) The blue lines present the evolution of the Wald statistic during the sampling period. The red line presents the critical value at the 5% significance level. ii) The null hypothesis of Granger non-causality is rejected if the Wald statistic is greater than the critical value.

The results indicate that the other three variables (bitcoin trading volume, cryptocurrency policy uncertainty and Nasdaq returns) affect bitcoin returns only for specific periods. All three variables have a significant effect during the first part of the sampling period (approximately up to 2017). We observe that the policy uncertainty index became statistically significant during the third quarter of 2017, the period of the first major rise in the price of bitcoin. The index remains significant for one year, until 2018-10-26., long after bitcoin prices have returned to a steady level. After that period, the index becomes significant only during the last quarter of the sample. Regarding trading volume, we detect four periods where the null of Granger-causality is rejected. These are the first two years of the sample (up to 2017-09-15), during 2018, during the second quarter of 2019 and during the first three quarters of 2022. During these periods, bitcoin price was relatively constant, indicating that trading volume is better used to analyse the behaviour of bitcoin during periods when no bursts occur. Furthermore, if we consider a lighter confidence level (i.e. 10%), the null hypothesis is rejected for the majority of the sampling period, further supporting the results of Yarovaya and Zięba (2022) who show a causal relationship between trading volume and cryptocurrency returns. Finally, Wald statistics for Nasdaq returns exceed the critical value from the start of the sample up to the third quarter of 2018.[6] Nasdaq returns also Granger-cause bitcoin returns from June 2021 until the end of the sample. That is, there is no causal relationship between the Nasdaq and bitcoin returns during the second bubble burst at the end of 2020. However, there is a relationship during the most recent bubble burst, occurred in the second half of 2021.

We examine the existence of feedback loops by testing for inverse Granger causality between the variables. More formally, we examine whether bitcoin returns Granger-cause the rest of the variables and how these relationships change over time. Figure A2 presents the results. The results indicate that bitcoin returns with all variables except the Nasdaq returns. Specifically, the Wald statistics exceed the critical value during the entire sample. We observe no Granger-causality effect from the bitcoin returns to Nasdaq returns except during the latter part of 2022 when bitcoin prices hit a two-year low.

5.3 Structural analysis

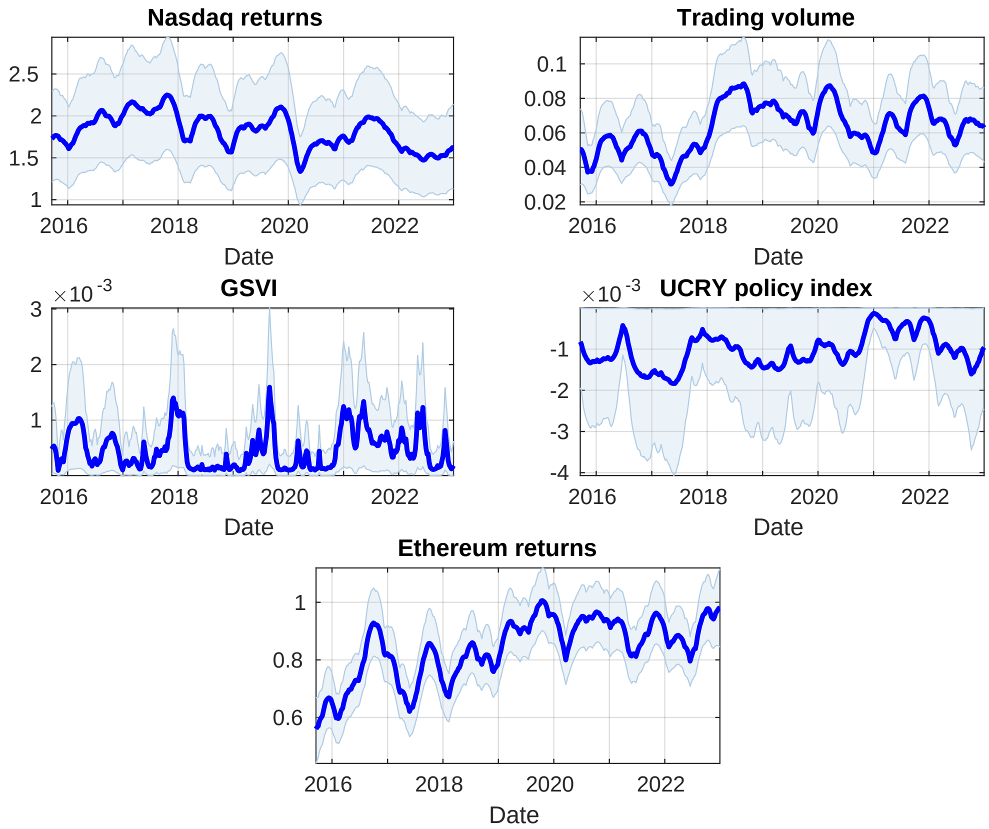
Although the main purpose of the analysis is to examine for changes in the Granger-causality relationship between bitcoin returns and potential determinants, we also perform impulse response function and forecast error variance decomposition analysis to get a thorough insight into the mechanisms that drive bitcoin returns. Both for the impulse responses and

[6]There are a few exceptions during this period but only last for up to two consecutive observations.

the forecast error variance decomposition, the identification of the model is based on the ordering of the variables. In particular, we set the vector of endogenous variables y_t = [Nasdaq returns, trading volume, GSVI, UCRY policy, ethereum returns, bitcoin returns]$'$. The ordering of the variables reflects our a priori assumption that a stock market the size of the Nasdaq can be considered largely exogenous in the context of a cryptocurrency system. We place bitcoin returns last in the VAR model on the grounds that the evolution of bitcoin returns is dictated by the rest of the variables (such as attention from institutional and individual investors). One could argue that spillovers from sudden shifts in bitcoin returns materialise immediately. To account for this possibility, we considered alternative orderings of the variables. We observe that in practice, placing bitcoin returns prior to cryptocurrency related variables makes no qualitative difference to our results.

Figure 3 presents the contemporaneous impulse responses of bitcoin returns to shocks to rest of the variables. Since we consider a TVP-VAR model, we are able to examine the effect of shocks occurring at different points in time. In particular, we consider a shock at each point in the sample. We focus only on the effect of the shocks on impact since the effect of each shock diminishes rather quickly and after approximately a month (4 periods) bitcoin returns reach pre-shock levels. All shocks significantly affect bitcoin returns over the entire sample. We observe that an unexpected increase in the UCRY policy index causes a feeble decrease in bitcoin returns, indicating that policy and regulatory discussions discourage informed investors who might find fewer opportunities for speculation. Positive shocks to the rest of the variables lead to a rise in bitcoin returns. According to the results, a shock to Nasdaq returns has the biggest impact on bitcoin returns during the first boom of bitcoin prices at the end of 2017. On the contrary, shocks to the trading volume of bitcoin cause the most severe responses during periods when bitcoin prices remain constant (during 2018 and 2020) which is in line with the results obtained from the Granger-causality analysis. Finally, we observe an upward trend in the impact responses of bitcoin to shocks to ethereum returns. This finding suggests that as the price of ethereum increases so does its ability to affect bitcoin. In general, the results obtained from the impulse response function analysis provide evidence of the time-varying dynamics between bitcoin returns and the rest of the variables.

Figure 3: Contemporaneous impulse responses of bitcoin returns to a shock in each variable.



Notes: i) The blue lines present the median impulse responses and the shaded area the 95% credible set. ii) Since stochastic volatility allows for the variance-covariance matrix to change over time, we estimate the impulse responses using the median matrix of the variance-covariance matrices.

The final part of our analysis regards the forecast error variance decomposition of the bitcoin returns. We consider two horizons, after 1 and 4 periods (weeks). Similarly to the impulse response function analysis, we are able to obtain different results for all points in the sample. However, in this case, we obtain little evidence of time variation, in the sense that the contribution of each variable to the forecast error variance of bitcoin returns does not fluctuate over the sampling period. Figures 4a and 4b present the forecast error variance decomposition of bitcoin returns at horizons 1 and 4, respectively. In both cases, shocks to bitcoin returns account for the majority of the forecast error variance of bitcoin returns, approximately 88%. The contribution of the remaining variables is negligible. On average, each variable accounts for less than 2% of the forecast error variance of bitcoin returns. After 4 periods, the contribution of shocks to bitcoin returns decreases by 5 percentage units. This difference is absorbed exclusively by Nasdaq returns, which explain approximately 12% of the forecast error variance of bitcoin returns. Finally, although not reported here, as the horizon increases, the percentage of the forecast error variance of bitcoin returns attributed to shocks to bitcoin returns reduces further to 60% on average and we observe more variation in the contribution of the shocks to each variable across the sample. To assess the validity of the results, we repeat the analysis and set the lag order equal to 2. The results remain qualitatively the same and are available upon request.

6 Discussion

In this section, we discuss the implications of our findings. To better demonstrate the advantage of detecting changes in the causal relationships of the analysed variables, we analyse the response of bitcoin returns during specific events. We consider three dates, January 2019, February 2020 and March 2022. The first date corresponds to a period of relative stability for the bitcoin prices (used as a benchmark). The second and third dates correspond to the early periods of the COVID-19 pandemic and the Russo-Ukrainian war. Figure A3 plots the median impulse response of bitcoin returns over a 16-weeks horizon for shocks occurring at the three selected dates. The results reveal only a small variation of the bitcoin's dynamic during these events. This is due to the use of aggregate weekly data (an analysis using daily data could reveal more differences in the responses of bitcoin returns during these events). A shock to Nasdaq returns has a greater impact on bitcoin returns during a period of tranquillity compared to periods of crisis. Similar results also hold for a shock in the GSVI. A shock occurring during January 2019 yields a greater increase in bitcoin returns than a shock occurring at the beginning of the pandemic. Furthermore, a similar shock during the early stages of Russo-Ukrainian war leads to a decrease in bitcoin returns after two weeks. A positive shock to cryptocurrency policy uncertainty index has a long-lasting effect on bitcoin returns which lasts up to 14 weeks.

The findings of this paper should be of interest both to policymakers and investors. Based on our findings we derive several implications. First, The interconnectedness between Nasdaq and bitcoin indicates the need for integrated regulatory oversight. Policymakers should consider coordinated regulations that address both sectors to manage systemic risks and prevent regulatory arbitrage. Second, the relationship between ethereum and bitcoin suggests a potential integration of the cryptocurrency market. As a result, policies should focus on the entire cryptocurrency ecosystem to minimise regulatory gaps. Third, institutional investors can exploit the impact of internet attention on bitcoin to drive prices up. On the contrary, retail investors ought to be cautious of unjustified increased media attention to bitcoin.

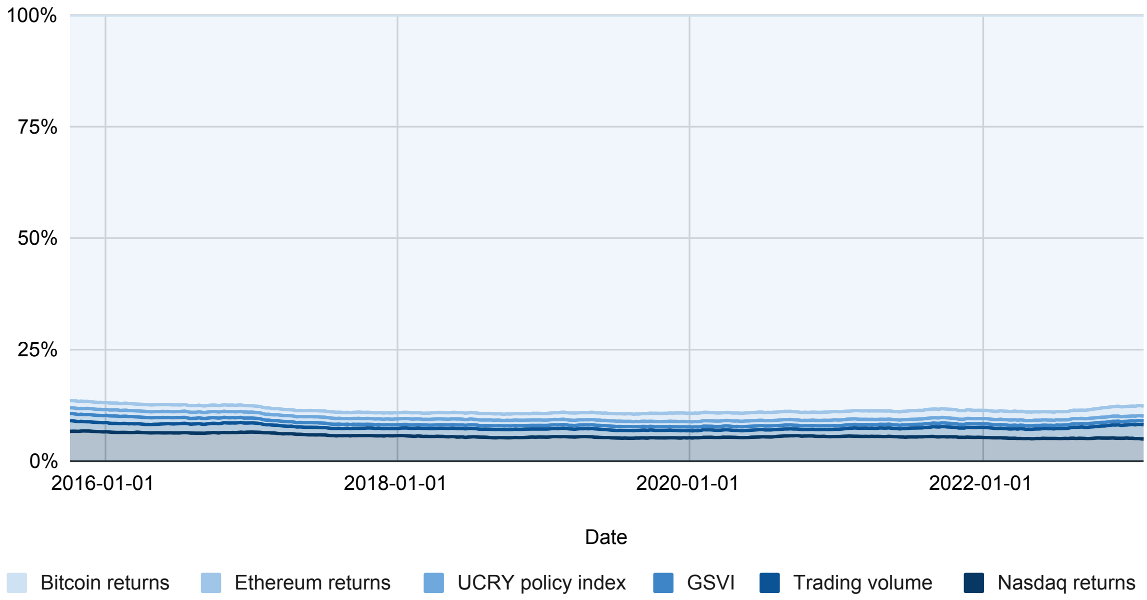
7 Conclusions

In this paper, we propose an approach to perform a time-varying Granger-causality test. The test combines the VAR estimates from a TVP-VAR model with heteroscedastic-consistent Granger-causality hypothesis testing. The proposed methodology does not rely on rolling window estimates. Consequently, our approach is not subjected to small sample size distortions and does not require a subsample to initiate the recursive algorithm, thus taking advantage of all available information in the dataset. We employ the new test to date-stamp the causal relationship between bitcoin returns and alternative variables. Specifically, in the empirical application, we consider a two-step approach. Initially, we estimate a Bayesian TVP-VAR with 30 variables (bitcoin returns and 29 potential determinants of bitcoin returns) and identify the most important variables of the model based on Bayesian variable selection algorithm. Next, we re-estimate the TVP-VAR model using only the variables selected in the previous step and perform the analysis. Specifically, we employ the time-varying Granger-causality test which we discussed earlier, as well as the impulse response function and forecast error variance decomposition analysis.

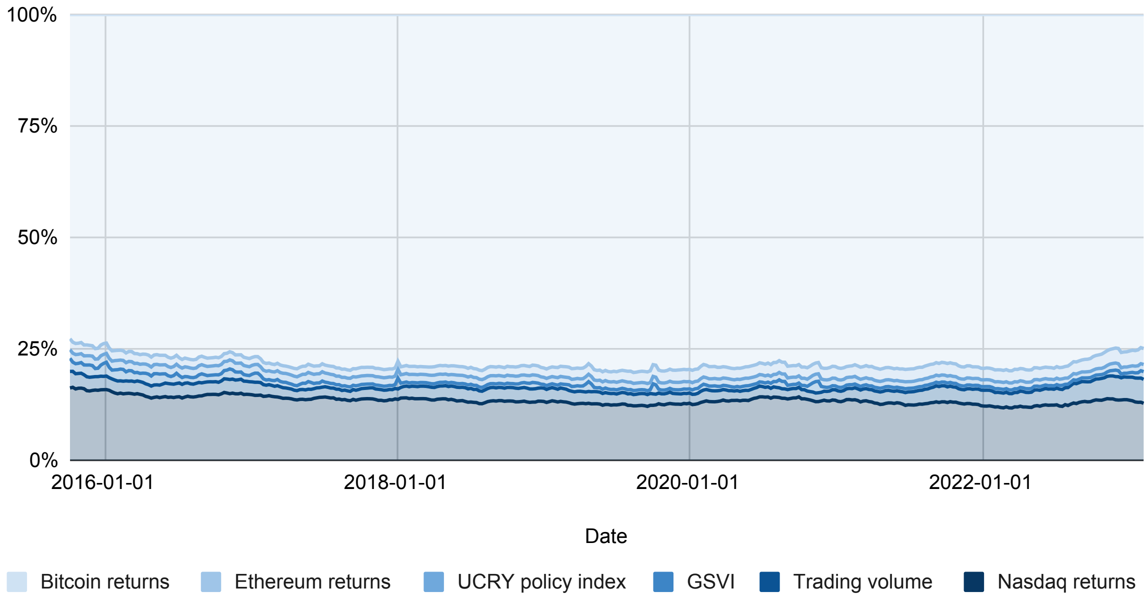
It is worth emphasising that the time-varying Granger-causality approach we employ in this study is very general and goes well beyond the current bitcoin application. It provides a rich and flexible environment for the detection of Granger-causality, that accounts for the disadvantages of similar methodologies. As a result, this methodology can be used in financial and economics applications where the dataset is characterised by structural breaks. The out-of-sample performance of the approach is also worth examining. This is left as future work.

Figure 4: Forecast error variance decomposition of bitcoin returns.

(a) Forecast error variance decomposition at horizon 1.



(b) Forecast error variance decomposition at horizon 4.



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Appendix A Supplementary tables and figures

Table A1: Data sources.

	Variable	Source
	Bitcoin returns (BTC)	Yahoo Finance
	Bitcoin returns' volatility (VOL)	authors' calculations
	Ethereum returns (ETH)	Yahoo Finance
Sentiment	SVI (GSVI)	Google Trends
	Uncertainty Cryptocurrency policy (UCRY policy)	Cryptocurrency indices
	Uncertainty Cryptocurrency returns (UCRY price)	Cryptocurrency indices
Supply	Total number of bitcoins in circulation (NBC)	blockchain.com
Demand	Trading volume (TV)	Yahoo Finance
	Confirmed transactions per day (CTR)	blockchain.com
	Unique addresses used (ADD)	blockchain.com
	Hash Rate (HASH)	blockchain.com
	Network Difficulty (DIF)	blockchain.com
Stock market	S&P500 (SP500)	Yahoo Finance
	Dow Jones NYSE index (DJI)	Yahoo Finance
	NASDAQ index (NASDAQ)	Yahoo Finance
	EURO STOXX 50 index (STOXX)	Yahoo Finance
	FTSE 100 index (FTSE)	Wall Street Journal
	Nikkei 225 index (NIKKEI)	Yahoo Finance
	Shanghai Composite Index (SSEC)	Yahoo Finance
	CBOE Market Volatility Index (VIX)	Yahoo Finance
	EURO STOXX 50 Volatility Index (VSTOXX)	Wall Street Journal
Commodities	Brent (BRENT)	FRED
	WTI (WTI)	FRED
	Gold (GOLD)	Yahoo Finance
Exchange rate	Euro (EURO)	FRED
	Sterling (GBP)	FRED
	Yuan (YUAN)	FRED
	Yen (YEN)	FRED
Financial variables	Fed Funds effective rate (FFER)	FRED
	ECB deposit facility rate (ECB)	FRED

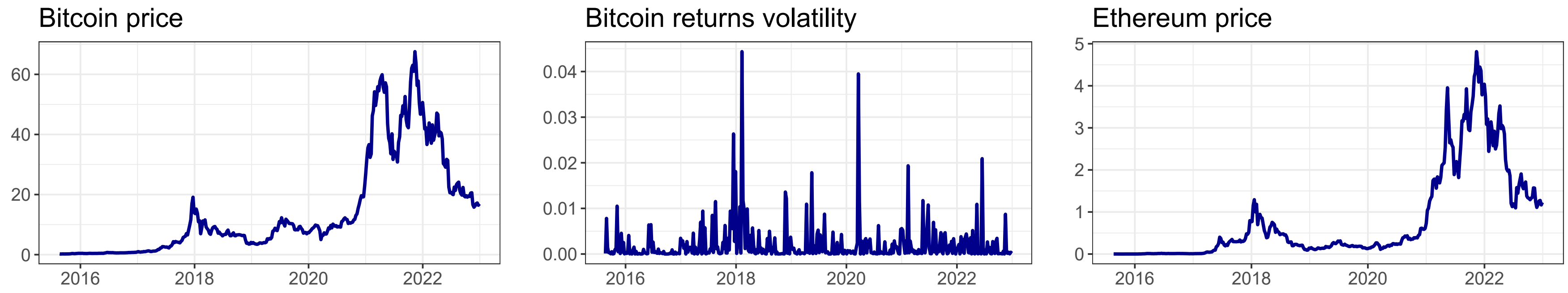
Notes: i) We calculate the volatility of bitcoin returns as the square of bitcoin returns. ii) The mnemonics in the parentheses are used to denote the variable in the Tables and Figures.

Table A2: Descriptive statistics.

Variable	Mean	Standard deviation	Kurtosis	Skewness	ADF
BTC					
VOL	0.002	0.004	37.472	5.213	-10.92***
ETH	0.019	0.163	1.837	0.148	-11.608***
GSVI	15.863	14.792	5.408	2.004	-4.933***
UCRY policy	101.455	2.959	2.78	1.841	-1.871
UCRY price	101.473	2.946	2.578	1.797	-2.7*
NBC	18888096481.897	19987985987.813	2.254	1.283	-1.893
TV	17451247.576	1324646.308	-0.953	-0.463	-2.705*
CTR	264088.701	53726.11	-0.039	-0.241	-3.597***
ADD	546421.255	127852.246	-0.273	-0.015	-2.895**
HASH	80523202.883	76669175.396	-0.751	0.65	0.516
DIF	11135864350553.9	10648347800965.4	-0.759	0.648	0.816
SP500	3055.668	802.386	-0.952	0.492	-1.103
DJI	25934.627	5768.66	-1.045	0.07	-1.252
NASDAQ	8833.534	3284.201	-0.943	0.556	-1.165
STOXX	3470.654	361.956	-0.24	0.291	-2.462
FTSE	6976.502	553.712	-0.348	-0.841	-2.800*
NIKKEI	22630.909	3973.685	-0.989	0.218	-1.494
SSEC	3154.762	274.114	-0.567	-0.084	-2.818*
VIX	18.826	7.849	7.308	1.986	-4.427***
VSTOXX	21.154	7.83	9.757	2.159	-5.205***
BRENT	62.763	20.568	0.64	0.723	-2.081
WTI	58.812	19.049	0.878	0.821	-1.543
GOLD	1484.881	272.208	-1.458	0.314	-1.226
EURO	1.129	0.055	-0.032	-0.299	-1.801
GBP	1.319	0.084	0.509	0.512	-2.971**
YUAN	0.889	0.064	2.125	-1.499	-1.139
YEN	113.092	9.219	3.561	1.924	-0.966
FFER	1.027	0.974	0.067	0.857	-0.406
ECB	-0.353	0.348	22.076	4.606	-3.662***

Notes: i) See Table A1 for the description of the variables. ii) ADF statistic is calculated with intercept and no trend. iii) *, **, *** denote statistical significance at the 10%, 5%, and 1% significance level, respectively.

Figure A1: Data in levels: Bitcoin and ethereum.



Notes: i) Bitcoin, ethereum and the stock market indices are in thousands of US dollars. Volume is in millions of US dollars. The number of unique addresses and daily confirmed transactions is in thousands. Hash rate is millions of TH/s. Difficulty is in trillions. The exchange rates are expressed as the price of domestic currency to one US dollar.

Figure A1: Data in levels (continued): Bitcoin related indices.

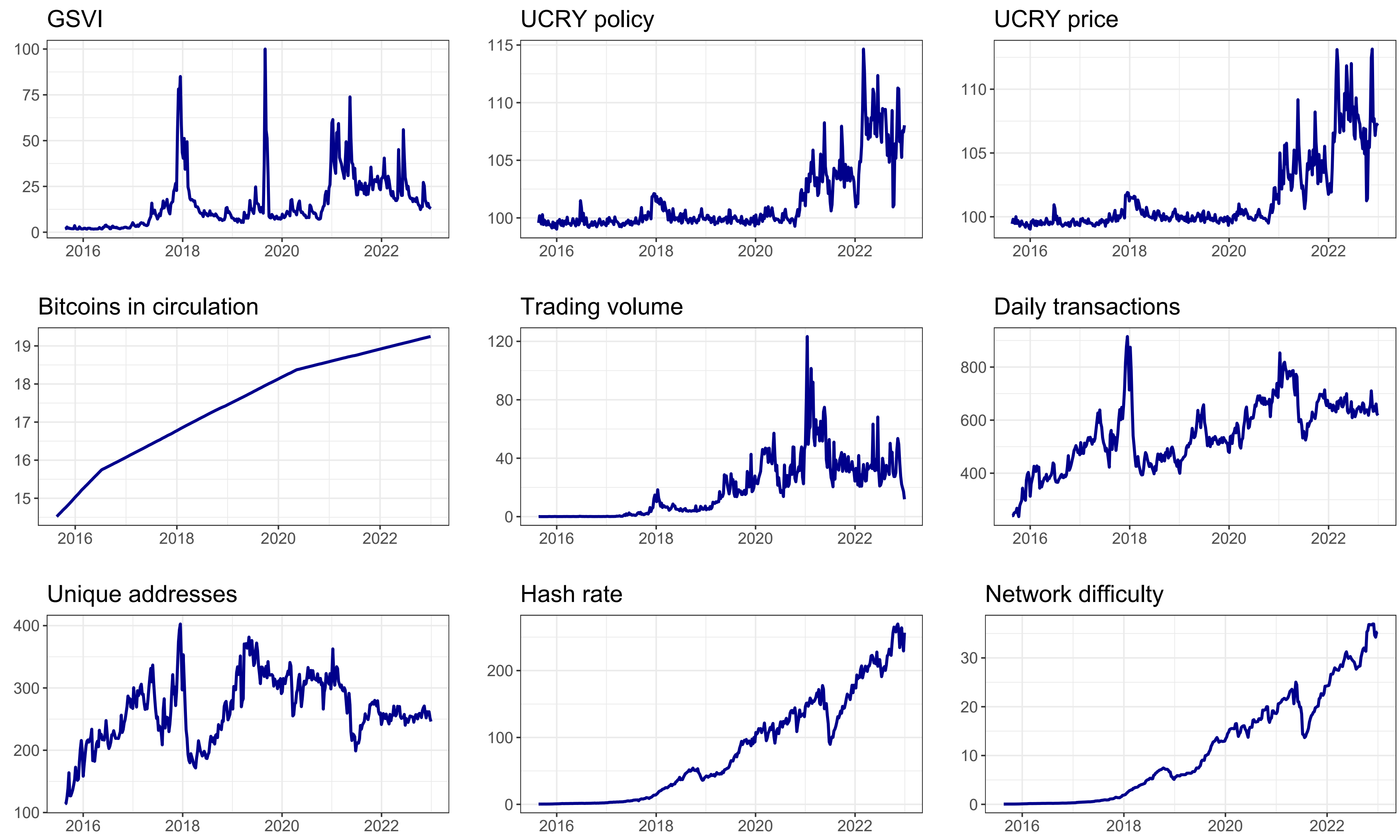


Figure A1: Data in levels (continued): Stock market prices and volatility.

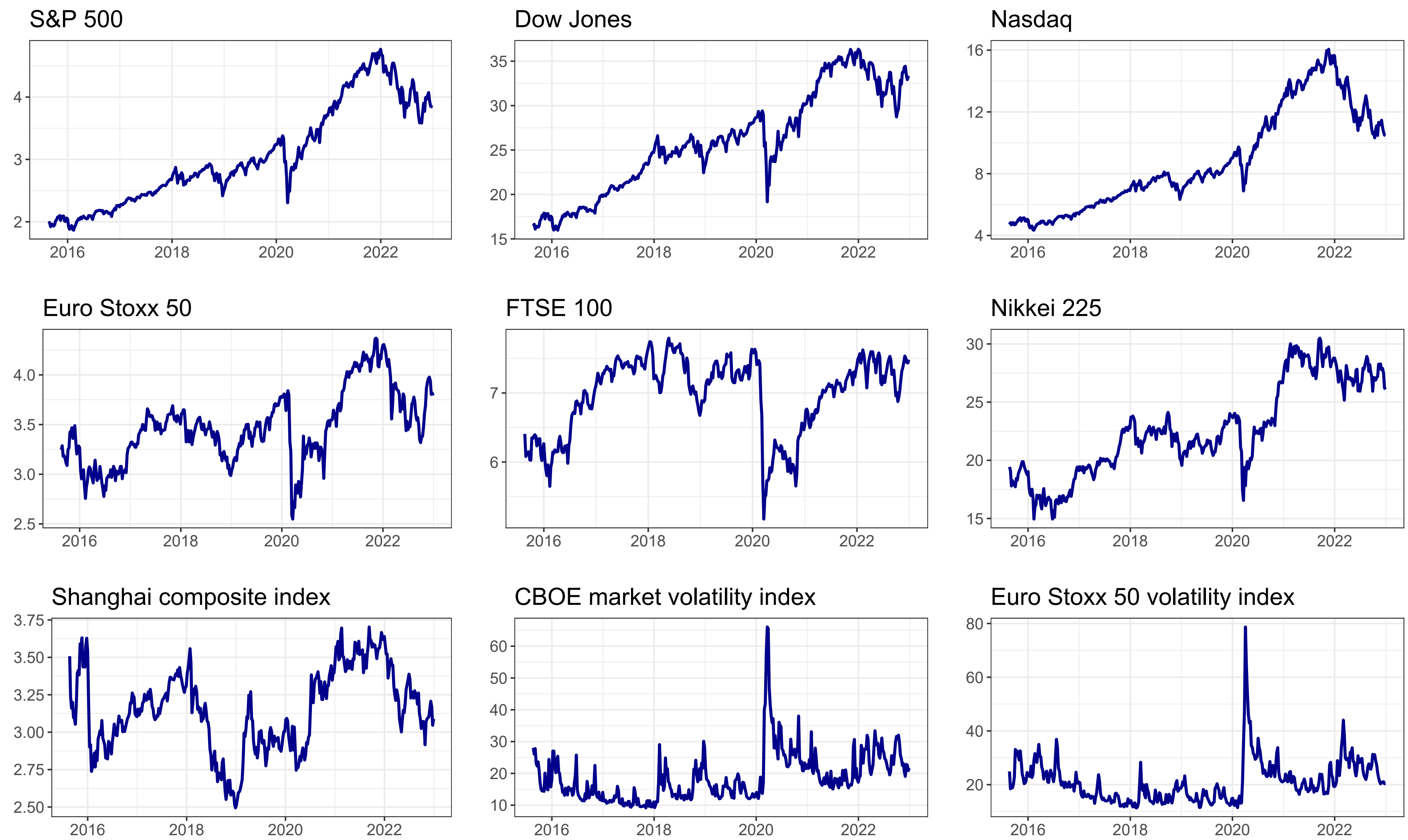


Figure A1: Data in levels (continued): Commodities, exchange rates and interest rates.

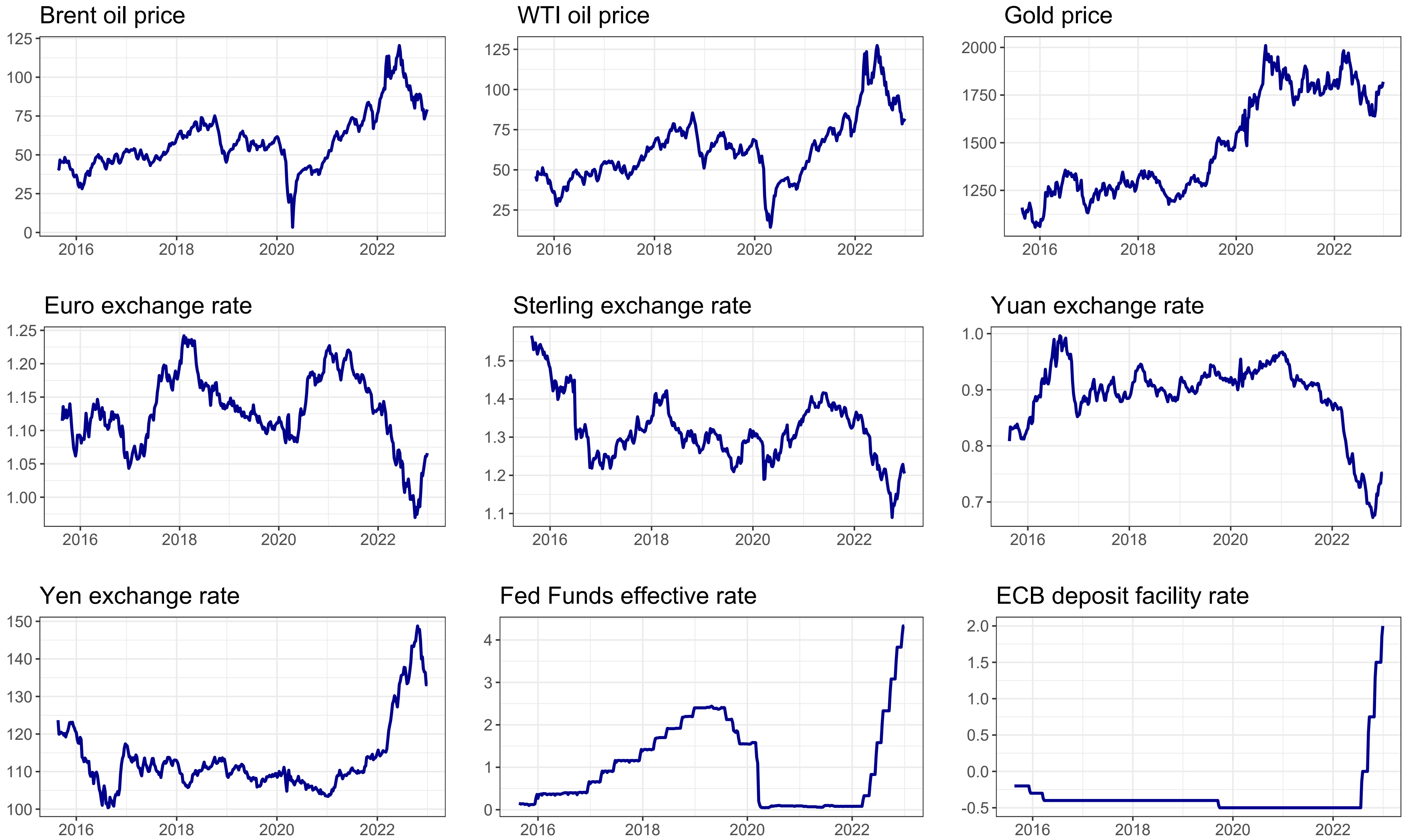
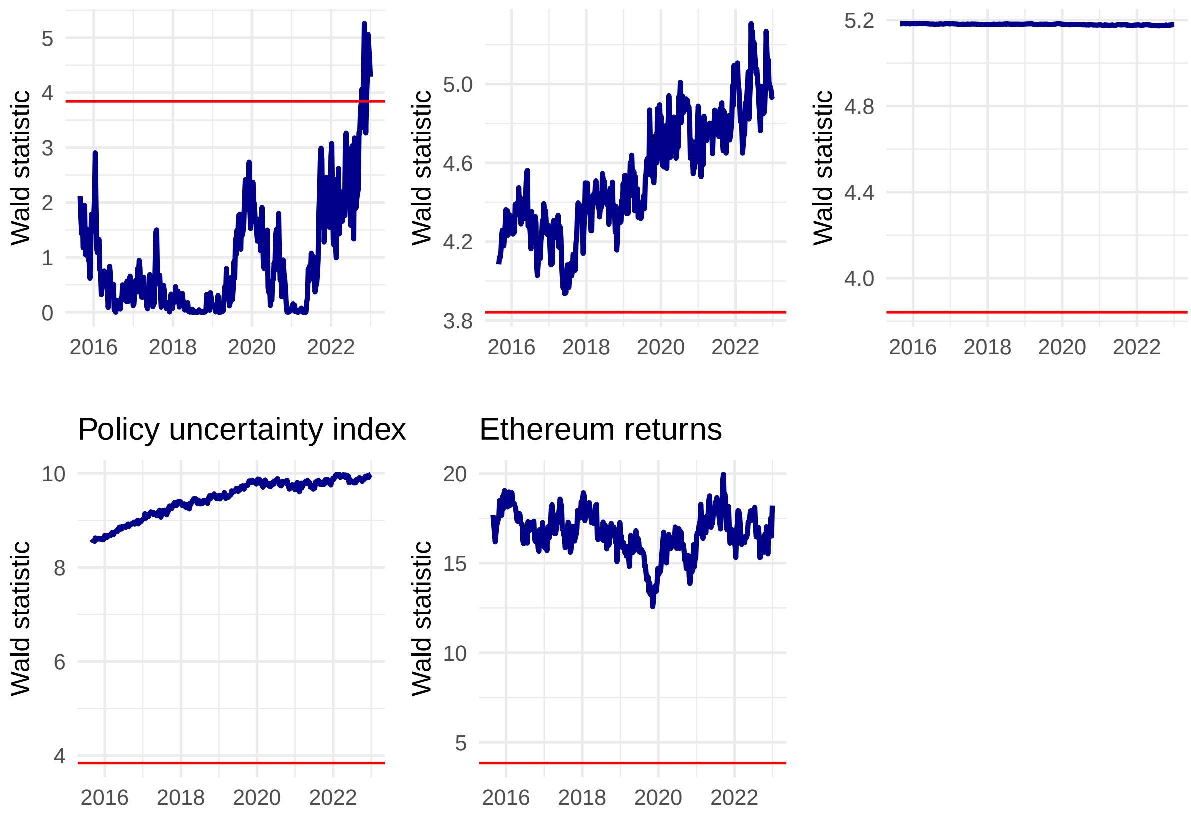


Figure A2: Reverse Granger-causality results.

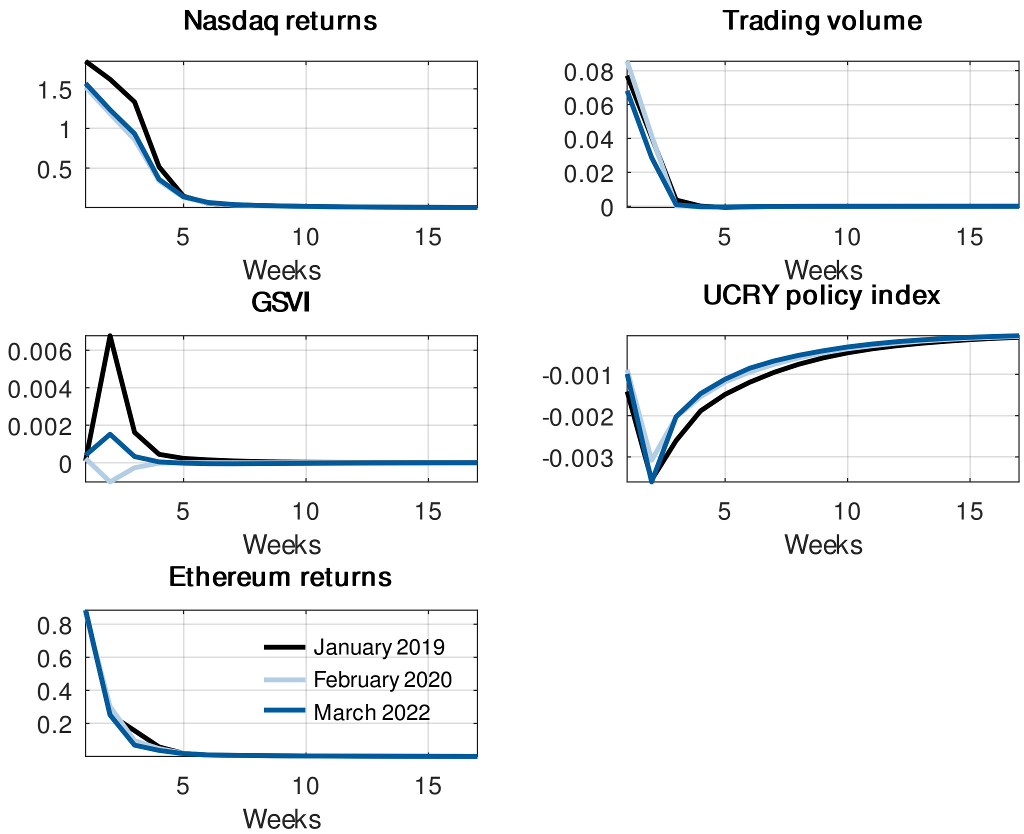


Notes: i) The blue lines present the evolution of the Wald statistic during the sampling period. The red line presents the critical value at the 5% significance level. ii) The null hypothesis of Granger non-causality is rejected if the Wald statistic is greater than the critical value.

Appendix B Comparison with alternative algorithms

The methodology proposed in this paper relies on Bayesian methods. Recent advances in Bayesian econometrics make Bayesian models particularly suitable for high-dimensional models since they can avoid over-parameterisation and provide accurate estimates. In addition, the use of shrinkage priors allows the model to eliminate insignificant variables faster compared to other estimators and improves the model's ability to exclude non-significant variables. Finally, the proposed approached can be easily modified for alternative Bayesian time-varying models (for example, quantile models). In what follows, we compare our methodology with alternative existing approaches.

Figure A3: Impulses responses of bitcoin returns to shocks in the rest of the variables during specific periods.



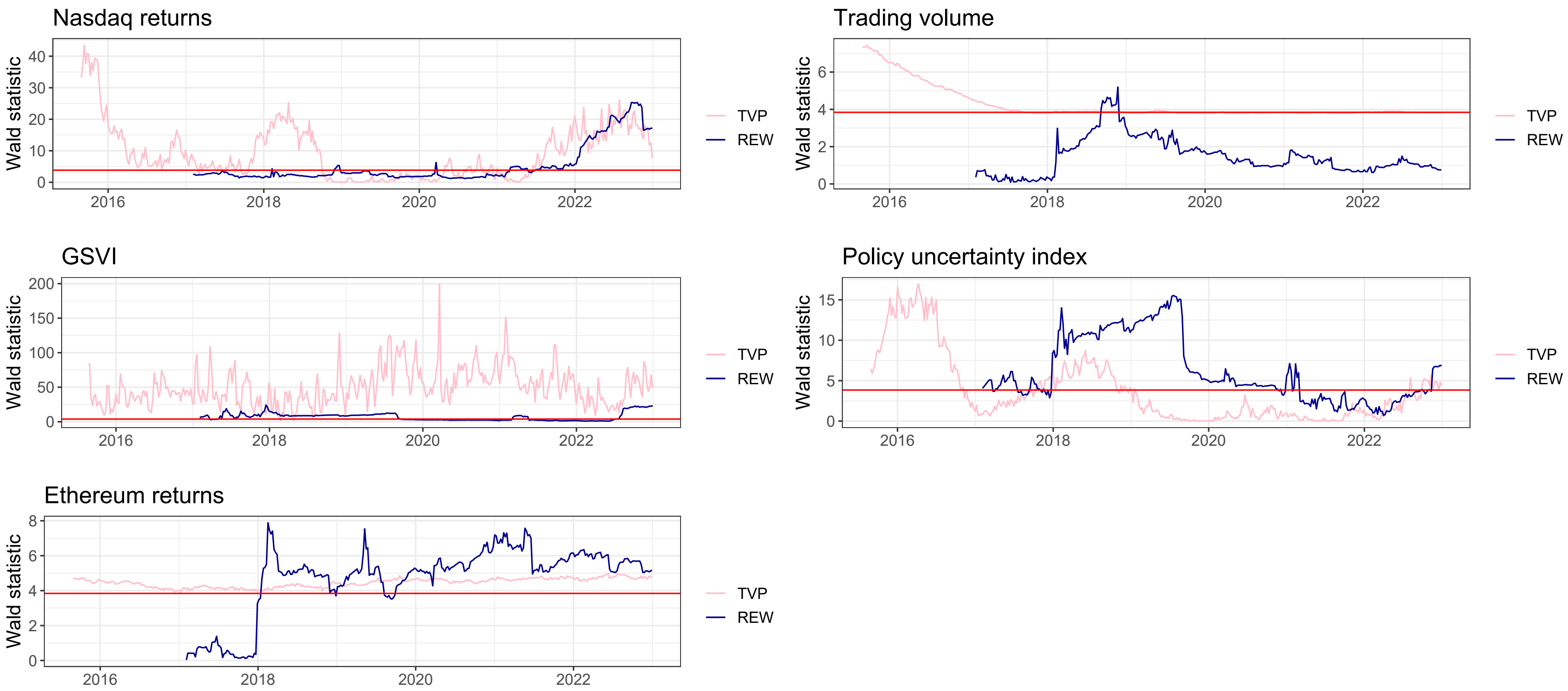
Notes: i) The lines denote the median response of bitcoin returns. ii) The events occurring the three selected dates are stable bitcoin prices, the COVID-19 pandemic and the Russo-Ukrainian war.

Lu et al. (2014) propose a time-varying Granger-causality test that uses all information of the sample. The main difference of this approach is that, in the first step, ARMA-GARCH models are estimated, while in this paper we estimate a Bayesian TVP-VAR. The estimates from the TVP-VAR model can be used for further inference apart from the Granger-causality test (for example, structural analysis). Moreover, recent advances in Bayesian econometrics make Bayesian models particularly suitable for high-dimensional models, since they can avoid over-parametersation and provide accurate estimates. In addition, employing shrinkage priors allows the model (i) to eliminate insignificant variables faster than other estimators and (ii) improves the model's ability to exclude non-significant variables. Finally, the proposed approach can be easily modified for alternative Bayesian time-varying models (for example, quantile models).

Next, we compare the proposed time-varying methodology with the recursive evolving window algorithm proposed by Shi et al. (2018) (for an additional empirical application see, Baum et al., 2021). For a given window of size r_0, the algorithm runs the Granger-causality test for all possible subsamples of length greater or equal to r_0. As a result, every point in the sample, except the ones in the initial window, is associated with a set of statistics. From each such set, we retrieve the supremum norm statistic and base statistical inference on the supremum norms. The recursive evolving algorithm is a generalisation of both the rolling window and the forward expanding window algorithms.

Here, we consider a window size equal to 20% of the sample size (i.e. 77 observations). For all subsamples with length greater or equal than the selected window size, we estimate a time invariant VAR model with six endogenous variables (i.e. bitcoin returns, ethereum returns, Nasdaq returns, bitcoin trading volume, GSVI and the cryptocurrency policy uncertainty index) and then, the heteroskedastic-consistent Wald statistic for the null hypothesis that a variable Granger-causes bitcoin returns. Figure B1 presents the sequences of Wald statistics obtained from the recursive evolving algorithm. Note that there are no Wald statistics during the initial window size. The Figure also plots the series of Wald statistics obtained using the Bayesian time-varying methodology for ease of comparison between the two approaches. In the case of Nasdaq returns and the GSVI, we observe that the recursive evolving algorithm fails to reject the null hypothesis of no Granger-causality during specific periods. For example, according to the recursive algorithm, there is no Granger-causal effect from the GSVI to bitcoin returns during 2020 which contradicts the existing literature (for example, see Raza et al., 2022). In the case of the policy uncertainty index and ethereum returns, the recursive algorithm indicates that there is a unidirectional Granger-causal relationship from the variables to bitcoin returns. Furthermore, the periods that the causal relationship is present, coincide with the periods indicated by the Bayesian time-varying approach. However, the critical value presented in Figure B1 is not size-adjusted. Shi et al. (2019) propose a bootstrapping algorithm that controls for size distortions (caused by the small sample size). If we consider a size-adjusted critical value, then the recursive algorithm detects a causal relationship for substantially shorter periods compared to the time-varying approach (note that the approach we propose in this paper does not require size-adjusted critical values). These results are available upon request.

Figure B1: Granger-causality results from the recursive evolving window algorithm



Notes: i) The red blue lines denote the sequence of Wald statistics obtained using the algorithm of Shi et al. (2019). The pink lines denote the sequence of Wald statistics obtained using the Bayesian TVP methodology. The red line represents the critical value at the 5% significance level. ii) The null hypothesis of Granger non-causality is rejected if the Wald statistic is greater than the critical value.