

## On the relationship between average cost and efficiency

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This is a pedagogical piece that explains the circumstances under which it is correct to claim that a lower average cost (i.e., per-unit cost) implies a higher level of economic efficiency (hereafter, efficiency).

Consider two modes of production (i.e., two technologies). Call them 1 and 2. Technology 1 produces  $q_1$  units of output at a total cost of  $C_1$  with an average cost of  $AC_1 = C_1/q_1$ . Technology 2 produces  $q_2$  units of output at a total cost of  $C_2$  with an average cost of  $AC_2 = C_2/q_2$ . If  $AC_2 < AC_1$ , does that mean that technology 2 is more efficient than technology 1? In other words, does a lower average cost necessarily imply a more efficient mode of production? It turns out that the answer depends on the relative sizes of the output produced. If the technology with the higher average cost can produce more output than the technology with the lower average cost, then the former technology may be more efficient than the latter. Therefore, conclusions about efficiency based on a comparison of average costs of production crucially hinge on the assumption that any output that the higher average-cost technology can produce can also be produced by the lower average-cost technology. But this assumption need not hold and it is *common* for people to argue that a lower average cost necessarily implies a more efficient mode of production without checking the validity of this assumption. Definitions of efficiency in Economics textbooks that use average costs are based on this implicit assumption. The application of this textbook definition, without checking if this assumption is satisfied, could lead to erroneous conclusions.

Let's make the point with reference to a June 15, 2003 article in the Washington Post titled "Third world nations dying at hands of U.S. agricultural policies". The author, Clyde Prestowitz, argues that:

"In Mali, farmers hitch their one-bladed plows to oxen and take two weeks to till 10- to 20-acre plots from which the cotton is eventually picked by hand. In contrast, the Mississippi Delta growers tend giant spreads of 10,000 acres or more in air-conditioned tractors using global positioning satellite systems to determine the proper amount of fertilizer to apply to sprouting seedlings. No wonder U.S. cotton growers have an average net household worth of nearly \$1 million. Sad though it may be, it would seem that oxen and plows on tiny plots are just no match for tractors and satellite systems on huge spreads. In fact, however, the U.S. growers are the higher-cost producers. All that high-tech equipment is expensive. Delta land is irrigated, and the seed is priced at a premium because it is genetically modified to resist pests. Then there are expensive fertilizers and

defoliants. In all, it costs 82 cents to produce a pound of cotton in Mississippi versus only 23 cents a pound in Mali.”

By comparing the average costs of producing a pound of cotton in Mali and in the Mississippi Delta, Clyde Prestowitz suggests that the Mississippi Delta farmers are less efficient than their the Malian counterparts. But is this necessarily correct? To answer this question, let us define what efficiency means. Efficiency is based on the generation of economic surplus (i.e., benefits minus costs). The higher is the size of economic surplus from a given activity, the higher is efficiency of that activity. Hence, the most efficient technology is the one that maximizes economic surplus. Note that there is no discussion of equity here; that is a different matter.

Wikipedia.com gives three correct and equivalent textbook definitions of economic efficiency:

1. No one can be made better off without making someone worse off.
2. More output cannot be obtained without increasing the amount of inputs.
3. Production proceeds at the lowest possible per-unit cost.

Consider a given allocation of resources. If you can *reallocate* resources to make everyone better off (increase in benefits) or can make some better off (increase in benefits) without making others worse off (no increase in cost), then the current allocation is not efficient because there is room for improvement. If there is room for improvement, you have not done your best and so there is waste in the system. The first definition means that, in the current state or allocation, there is no waste or no room for improvement. Therefore, you have done your best or have maximized economic surplus (i.e., benefits minus costs). The second definition follows the same logic. If you can increase output without increasing inputs, then you can increase benefits (i.e., output) without increasing cost (i.e., inputs). If this is possible, then you have not maximized economic surplus. The second definition means that you are efficient if this possibility does not exist. Clearly, the first two definitions are equivalent and are based on the maximization of surplus (i.e., benefits minus costs). I shall now demonstrate the conditions under which the third definition is equivalent to the first two definitions. In the process, the flaw in the argument by the author of the above Washington Post article, Clyde Prestowitz, will be obvious.

For the sake of exposition, I make statements that compare the relative efficiencies of Malian farmers and Mississippi Delta farmers. However, what is actually being compared here is not the innate abilities of the farmers but instead the relative efficiencies of the technologies used by these farmers (i.e., the high-tech equipment of the Mississippi Delta farmers versus the plough/Oxen of the Malian farmers).

Let the Malian farmers use technology 2 and the Mississippi Delta farmers use technology 1. Without loss of generality, let's keep the price constant and normalize it to 1. Then the surplus generated by the Malian farmers is  $S_2 = q_2 - C_2$  and the Mississippi Delta farmers generate a surplus of  $S_1 = q_1 - C_1$ .

Then the Malian farmers are more efficient than the Mississippi Delta farmers if they generate a bigger surplus (i.e.,  $S_2 > S_1$ ). That is, if

$$q_2 - C_2 > q_1 - C_1, \quad (1)$$

where  $q_k - C_k > 0$ ,  $k = 1, 2$ .

Notice that the Mississippi Delta farmers work over an area which is, at least, 500 (i.e., 10000 acres/20 acres) times the area that the Malian farmers work on. It is therefore reasonable to say that the Mississippi Delta farmers produce more output than the Malian farmers. That is,  $q_1 > q_2$ . In particular, let  $q_1 = \alpha q_2$ , where  $\alpha > 1$  since  $q_1 > q_2$ .

Divide both sides of equation (1) by  $q_1$  to get

$$\frac{q_2 - C_2}{q_1} > \frac{q_1 - C_1}{q_1} \quad (2)$$

Substitute  $q_1 = \alpha q_2$  into the denominator of the left-hand-side of (2) and simplify to get

$$\frac{1 - AC_2}{1 - AC_1} > \alpha, \quad (3)$$

where  $q_k - C_k > 0$  implies that  $(1 - AC_k) > 0$ ,  $k = 1, 2$ .

Suppose, for a moment, that  $0 < \alpha \leq 1$ , so that  $q_2 \geq q_1$ . That is, the Malian farmers' output is, at least, equal to the output of the Mississippi Delta farmers. Given  $AC_2 < AC_1$  (i.e., the Malian farmers have a lower average cost), it follows that  $(1 - AC_2)/(1 - AC_1) > 1$ . Therefore, the inequality in (3) will necessarily hold if  $AC_2 < AC_1$  and  $\alpha \leq 1$ . So, in this case, a lower average cost necessarily implies a higher surplus. Under these conditions, using average cost alone to determine efficiency is correct.

However, if the Mississippi Delta farmers produced a bigger output than the Malian farmers, as was indeed the case, then  $\alpha > 1$  (i.e.,  $q_1 > q_2$ ). In this case, the inequality in (3) will not hold even if  $AC_2 < AC_1$  so long as  $\alpha$  is sufficiently high (i.e., so long as  $q_1$  is very big compared to  $q_2$ ). That is, a lower average cost does not necessarily imply that the Malian farmers are more efficient.

The intuition behind this result is the following: while the Malian farmers can produce a smaller output at a smaller average cost than the Mississippi Delta farmers, their inability to operate at the higher scale (i.e., higher output) of the Mississippi Delta farmers limits their ability to generate a greater surplus associated with a bigger output. Indeed, this capacity constraint (i.e., an inferior technology has tighter limits on the maximum output

that can be produced) faced by the Malian farmers means that they have a lower average cost up to some maximum capacity but beyond this capacity, the average cost of production is infinitely large (i.e., it is excruciatingly painful to produce beyond this point). So beyond this maximum capacity, the Mississippi Delta farmers actually have a much lower average cost than the Malian Farmers. The fact that I can grow tomatoes in my small backyard at almost zero average cost does not make me more efficient than someone who, using advanced technology, can grow tomatoes on a very large scale at a positive average cost. The author of the Washington Post article missed this point.

Given that the Mississippi Delta farmers work over an area which is, at least, 500 times (i.e., 10000 acres/20 acres) the area that the Malian farmers work on, it is reasonable to argue that  $\alpha \geq 500$ . It is unlikely that the ratio of average costs will be such that the inequality in (3) holds. Therefore, notwithstanding subsidies, there may well be a reason why U.S. cotton growers have an average net household worth of nearly \$1 million. They produce a much bigger surplus compared to Malian farmers. The author of the Washington Post article should not find this surprising.