Burning Out in Sequential Elimination Contests*

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Abstract

Is it rational for contestants in a sequential elimination contest to expend all their efforts in earlier stages, get burnt out, and have nothing to offer in subsequent stages? This paper identifies three properties of sequential elimination contests which result in burning out: (i) a constraint on aggregate effort across stages or rounds (ii) extreme high-powered incentives in earlier rounds, and (iii) the playing field is even; no contestant is outstanding. I present a model which captures these three features. Burning out is shown to be an equilibrium rational behavior although the ultimate prize is won only if a contestant is successful in all stages including the last stage. I find two burning-out equilibria: a full burning-out equilibrium in which all the contestants burn out and a partial burning-out equilibrium in which some contestants do not burn out. I discuss some applications such as boxing contests, salary caps in the NBA, and incentives in academia.

Keywords: all-pay auction, burning out, caps, contests.

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1. Introduction

Is it rational for contestants in a sequential elimination contest to expend all their efforts in earlier stages, get burnt out, and have nothing to offer in subsequent stages? This paper identifies three features of sequential contests which result in burning out. Burning out is shown to be an equilibrium rational behavior although the ultimate prize is won only if a contestant is successful in all stages including the last stage.

I use examine a two-stage sequential elimination contest where the contest in the first stage is an all-pay auction[1]. The all-pay auction has been used to examine many allocation processes and interesting economic phenomena. Some examples include strategic trade policy (Konrad, 2000), lobbying (Baye, Kovenock, and de Vries, 1993, 1996; Che and Gale 1998; Ellingsen, 1991; Hillman and Riley, 1989) and the provision of public goods (Baik, et al., 2001). In an all-pay auction, the contestant with the highest bid or effort wins but both the winner and loser(s) forfeit their bids. Contests such as elections campaigns, R & D races, competition for monopolies, labor tournaments, litigation, wars, and sports could be modeled as all-pay auctions. In all these cases, a contestant's effort is sunk whether he wins or loses.

Baye, Kovenock, and de Vries (1996) have characterized the equilibria for the all-pay auction with complete information when there is a single prize at stake. Clark and Riis (1998) and Moldovanu and Sela (2001) consider the case of multiple prizes. Che and Gale (1997, 1998) and Gravious, Moldovanu, and Sela (2002) examine the case of a

[1] It is a standard result that the all-pay auction with complete information and exogenous valuations has no equilibrium in pure strategies [Hillman and Riley (1989), Ellingsen (1991), and Baye, Kovenock, and de Vries (1996)]. However, it is known that there exists an equilibrium in mixed-strategies with or without a cap on bids in a single-stage all-pay auction, where contestants randomize their bids on some finite support. Amegashie (2001) presents all-pay auction with a pure-strategy equilibrium.

single prize but with a cap on the bids of the lobbyists or contestants. In all these cases and in the papers cited above, it is assumed that the contest is a single-stage contest. However, it is usually the case that such contests are designed as sequential elimination contests. A sequential elimination contest is a contest with two or more stages in which a subset of contestants are successively chosen in each stage to compete in subsequent stages. The winner is the contestant who proceeds successfully through all the stages.[2]

The paper explains "burning out" in sequential elimination contests. A contestant in a sequential elimination contest is "burnt out" in a given stage and subsequent stages if he only has enough energy to exert the minimum acceptable effort in that stage and subsequent stages. In the model with an all-pay auction, all the contestants expend all their effort in stage 1. Contestants may expend all their efforts in the first stage because they cannot move to the next stage, if they do not do well in the first stage. If they face a fixed resource constraint, then they might get burnt out by the time they get to the second stage. This result is interesting given that the entire prize is concentrated in the second stage. It is also interesting because it holds even if the success probability (in the second stage) is extremely sensitive to effort. The first stage of the contest is a very important stage because it precedes the second stage. One cannot get to stage 2 without first making it through stage 1. Herein lies the importance of stage 1. This simple point has not been made in the literature. The paper captures three key features of contests which produce "burning out": (i) a constraint on aggregate effort across stages or rounds (ii) high-powered incentives in earlier rounds, and (iii) the playing field is even; no contestant is

[2] There is a growing but small literature on sequential elimination contests. See Amegashie (1999), Fullerton and McAfee (1999), Gradstein and Konrad (1999), Groh et al. (2003), Harbaugh and Klumpp (2004), and Rapoport and Stein (2003).

outstanding. The third feature is incorporated into the model by placing a common cap on effort. However, in a contest with three stages, we find that a burning out equilibrium is possible if the contestants have different caps, although the difference in caps must be small.

In a second model, I modify the contest in stage 2. I construct an equilibrium in which some contestants burn out but others do not. Thus the paper presents two burning-out equilibria: (a) a full burning-out equilibrium in which everyone (who competes) burns out, and (b) a partial burning-out equilibrium in which some contestants do not burn out but others do.

The paper shows that one cannot get a burning-out equilibrium (full or partial), if there are differences in the effort caps of the contestants, given extremely high-powered incentives in stage 1 (i.e., an all-pay auction) and a certain property of the contest success function in stage 2. If a contestant has a chance of winning the prize even if he exerts zero effort or the minimum effort in stage 2 and the incentive structure in the stage 1 is not extremely high-powered (not an all-pay auction), then it possible to construct an equilibrium in which some contestants burn out but others do not, even if the contestants have different caps on effort. I find that in this equilibrium, it is the high-valuation contestants who expend all their effort in stage 1. An implication of this result is that we should expect the top teams in the NBA and NFL to exhaust their salary cap earlier in the season than the mediocre teams.

The paper is organized as follows: the next section examines a sequential elimination contest; a sub-section discusses some applications. Section 3 examines a modified version of the model presented in section 2. Section 4 concludes the paper.

2. A sequential elimination contest with full burning out

Consider $N \geq 3$ risk-neutral agents contesting for a prize with valuations commonly known to be $V_1 \geq V_2 \geq \ldots V_{N-1} \geq V_N > 0$, where V_i is the valuation of the i-th contestant, i = 1, 2, …, N-1, N. The contest is in two stages. In the first stage F contestants are chosen to compete in a second stage from which the winner is chosen; $2 \leq F < N$. As in Che and Gale (1997, 1998) and Gravious et al. (2002), suppose the i-th contestant faces a budget or effort constraint, $B_i > 0$. These papers give examples of caps in contests: caps on campaign contributions, salary caps in US professional sports[3], and caps on how fast Formula 1 racing cars can move. Also, a cap on effort arises because human beings naturally have a limit on much effort they can expend.

Suppose B_i can be allocated between the two stages. Let e_i and x_i be the effort levels of the i-th contestant in stages 1 and 2 respectively, where $e_i + x_i \leq B_i$. I assume that e_i and x_i are also the cost of expending effort (i.e., a linear cost function). In each stage, the contestants move simultaneously.

Let $P_{1i}(e_1, e_2, ..., e_N)$ and $P_{2i}(x_1, x_2, ..., x_F)$ be the success probability of the i-th contestant in stages 1 and 2 respectively. I shall sometimes refer to these functions as contest success functions. Denote the equilibrium success probabilities by $P^*_{1i}(e^*_i)$ and $P^*_{2i}(x^*_i)$, where I have suppressed the efforts levels of the other contestants in the probability function of the i-th contestant.

In stage 2, the equilibrium payoff of the i-th contestant, if he makes it to this stage, is

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3 As noted by Gravious et al. (2002), in the year 2000, NFL teams faced a salary cap of $62,172,000. This was a cap on the aggregate amount they could spend on their top 51 salaried players.

$$\Pi_{2i}^{*} = P_{2i}^{*}(x_i^{*})V_i - x_i^{*},$$

Focusing on a sub-game perfect Nash equilibrium (using backward induction), the equilibrium payoff of the i-th contestant in stage 1 is

$$\Pi_{1i}^{*} = P_{1i}^{*}(e_i^{*})\Pi_{2i}^{*} - e_i^{*}$$

I assume that the probability of success function in stage 2 has the following property: the i-th contestant wins the contest in stage 2 with probability 1 if $x_i > 0$ and $x_j = 0$ for all $i \neq j$.[4] This implies that a contestant who expends zero effort has no chance of winning the prize, given that some other contestant exerts a positive effort (no matter how small). For want of a better expression and for the purpose of exposition, I shall call this property the zero effort extreme penalty (**ZEEP**). An example of a contest success function which satisfies this property is $P_{2i} = x_i / (x_i + \sum_{j \neq i} x_j)$. This function is widely used in the literature on contests and has been axiomatized by Skaperdas (1996). Fullerton and McAfee (1999) and Baye and Hoppe (2002) derive this function from a model in which effort affects output with noise in a contest (see equation (1) in Fullerton and McAfee (1999) and Theorem 1 in Baye and Hoppe (2002)).

I wish to construct an equilibrium in which all the contestants expend all their effort in stage 1. To do so, I prove two lemmas.

Lemma 1: *In a two-stage sequential-elimination contest with complete information and* ***ZEEP****, a necessary condition for burning out is that the equilibrium contest success function in stage 1 must not be continuous for all levels of effort.*[5]

[4] Note, however, that the i-th contestant need not to win with certainty in stage 2 if $x_i > x_j > 0$ for all $i \neq j$.
[5] Of course, if the minimum effort required in stage 1 is equal to the cap on effort, then "burning out" will occur regardless of the contest success function in stage 1. However, this is a trivial and uninteresting example.

Proof: The proof is by contradiction. Suppose $P^*_{1i}(e^*_i)$ is continuous in the i-th contestant's effort level in stage 1 at all effort levels. Also, assume that $P^*_{1i}(0) = \alpha$, where $0 \leq \alpha < 1$. For now, assume that $\alpha = 0$. Consider a burning-out equilibrium in which all contestants expend their effort in stage 1. Note that if there remain subsequent stages of the contest, then given complete information and **ZEEP**, it does not make sense for some contestants to expend all their efforts in a given stage while other contestants do not expend all their effort. Hence, if the contestants get burnt out in stage 1, it has to be the case that all of them get burnt out. In a burning-out equilibrium, the payoff of the i-th contestant, in stage 1, is $P^*_{1i}(B_i)(1/F)V_i – B_i \geq 0$. If he deviates by allocating an effort level of $\varepsilon > 0$ to stage 2, he gets $P^*_{1i}(B_i - \varepsilon)(V_i – \varepsilon) – (B_i – \varepsilon)$, given that he will win the prize with certainty if he gets to stage 2. A deviation, $\varepsilon > 0$, is profitable if $P^*_{1i}(B_i - \varepsilon)(V_i – \varepsilon) – (B_i – \varepsilon) > P^*_{1i}(B_i)(1/F)V_i – B_i$. This holds if $\Omega \equiv [P^*_{1i}(B_i - \varepsilon) - P^*_{1i}(B_i)/F]V_i + \varepsilon[1 – P^*_{1i}(B_i - \varepsilon)] > 0$. Given $P^*_{1i}(B_i)(1/F)V_i – B_i \geq 0$, $\Omega \leq 0$ if $\varepsilon = B_i$. Also, $\Omega > 0$ if $\varepsilon = 0$. Now Ω is continuous in ε, given that $P^*_{1i}(e^*_i)$ is continuous. It follows that there exists a sufficiently small but positive ε such that $\Omega > 0$. Notice that the proof above holds even if $P^*_{1i}(0) = \alpha > 0$, given that $\Omega > 0$ still holds for $\varepsilon = 0$. Hence, burning out cannot be an equilibrium if the equilibrium contest success function is continuous at all effort levels for the i-th contestant. **Q.E.D**

The intuition for Lemma 1 is as follows. Starting from a burning-out equilibrium, a contestant who reduces his effort in stage 1 by a small amount and allocates it to stage 2 reduces his probability of success in stage 1 marginally but increases his success probability in stage 2 discontinuously from F/N to 1. Lemma 1 is consistent with a result in Stein and Rapoport (2003). In a two-stage sequential elimination contest with a budget constraint (which can be allocated between the two stages), using the probability functions $P_{1i} = e_i/(e_i + \sum_{j \neq i} e_j)$ and $P_{2i} = x_i/(x_i + \sum_{j \neq i} x_j)$, Stein and Rapoport (2003) find that the effort in stage 2 is always positive. That is, burning out never occurs. Note that, in a pure-strategy burning–out equilibrium, $e_i^* > 0$ for at least two contestants. So, in equilibrium, $P_{1i}^* = e_i^*/(e_i^* + \sum_{j \neq i} e_j^*)$ is continuous in e_i^*.

The proof of lemma 1 hinges on **ZEEP**. This condition implies that luck or noise does not affect your success probability. If you expend the minimum effort (which is zero in this model), you have no chance of success. Unless luck, as opposed to effort, plays a major role in determining success in stage 2, it does not make sense for any contestant to allocate all his effort to stage 1, given that luck also matters in stage 1. We can obtain a burning-out equilibrium in stage 1 if luck plays a major role in stage 2. But that means that the contestants have allocated all their effort to stage 1 because they do not really have to work hard in stage 2. In section 3, I relax **ZEEP** to construct a burning-out equilibrium along these lines.

In view of lemma 1, I consider a discontinuous contest success function. As in Clark and Riis (1996, 1998) and Moldovanu and Sela (2001), I assume the following selection process. In the first stage, the contestants make a single bid (effort). Then the contestant with the highest bid is selected. This contestant's bid is then ignored and the contestant with the highest bid among the remaining contestants is selected. Then his bid is also ignored and the contestant with the highest bid among the remaining contestants is selected. This process continues till all F contestants are selected.[6] Ties are broken randomly; losers and winners forfeit their bids. Hence the contest in the first stage is an all-pay auction. The contest in the second stage need not be an all-pay auction. However, the success probability in this stage must be sensitive to effort.

Let me indicate that the "auction" success function is the most-widely used discontinuous function, if not the only discontinuous function, in the literature on contests. It is therefore the logical place to begin looking for a discontinuous contest success function for the purpose of constructing a burning-out equilibrium, given **ZEEP**. Formally, the contest success function in stage 1 is

$$P_{1i} = \begin{cases} 1 & \text{if } e_i \text{ is one of the top F effort levels} \\ (F-g)/(r+1) & \text{if g contestants bid higher than the i-th contestant and this contestant ties with r other contestants, where } g+r+1 > F \text{ and } 0 \le g \le F. \\ 0 & \text{otherwise} \end{cases}$$

Given that the contest in the first stage is an all-pay auction, the following lemma holds:

Lemma 2: *Given* **ZEEP***, a two-stage sequential-elimination contest, and an all-pay auction in stage 1, it is impossible to construct a burning-out equilibrium if the contestants have different caps on effort.*

Proof: Suppose the contestants have different caps (i.e., no two contestants have the same cap) and the contest in stage 1 is an all-pay auction. There will be no equilibrium in pure strategies in stage 1. If there is an equilibrium, it will be in mixed strategies (see, Che and Gale (1997)). In any mixed-strategy equilibrium, the contestant with the highest

[6] This is essentially choosing the contestants with the F highest bids.

cap will never bid more than the cap of the contestant with the second-highest cap, given that, at least, two contestants will be chosen. If this contestant bids the second-highest cap, he will surely be one of the contestants with the top two bids. Hence, he will be short-listed with certainty. It follows that for any set of active contestants in stage 1, the contestant with the highest cap will not burn out in any mixed-strategy equilibrium. Given that this contestant will not expend all his effort in stage 1, it does not make sense for any of the other contestants to expend all their effort in stage 1 since they will surely lose the contest in stage 2, if they did so. Hence, it is impossible to construct a burning-out equilibrium if the contestants have different caps. **Q.E.D**

Notice that the argument above will hold if all contestants only know the distribution of caps or only know the top two caps. With different caps, an all-pay auction in stage 1 and **ZEEP**, burning out can only occur if the contestants do not know the distribution of caps. Indeed, a contestant will only burn out if he under-estimates the abilities of his opponents. An example is the boxing contest between Thomas Hearns and Marvin Hagler in April 1985.[7] Hearns was burnt out in the second round of the twelve-round bout because he exerted too much effort in the first round. The reason why he did this was because he thought he could knock out Hagler by hitting him with his best shots very early in the fight.[8] He under-estimated Hagler's strength and got burnt out. He was knocked out in the third round. With different caps on effort and extremely high-powered incentives in preliminary stages, a contestant will get burnt out if he misjudged the abilities of other contestants (i.e., made a mistake).

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[7] I draw on examples from boxing. While boxing contests do not perfectly fit my model, they offer some insight into factors which account for burning out.

[8] Thomas Hearns was nick-named the "hit man" because of his devastating punches.

Given **ZEEP**, a two-stage sequential-elimination contest, and an all-pay auction in stage 1, I have shown that a sequential-elimination contest with different caps on effort cannot produce burning out. While incomplete information can produce burning out, the informational assumption required is very restrictive. In particular, a contestant will burn out if he misjudges the ability or cap of other contestants. Given **ZEEP**, burning out occurs when a contestant makes a mistake. In practice, this may be a reason why burning out occurs (e.g., in the Hagler-Hearns fight).

Based on lemmas 1 and 2, I consider a two-stage sequential elimination contest where each contestant has the same aggregate cap and the contest in the first stage is an all-pay auction. I state the following proposition:

Proposition 1: *Consider a two-stage contest where the contest in each stage is an all-pay auction and the contestants have valuations commonly known to be* $V_1 \geq V_2 \geq \ldots V_{N-1} \geq V_N$. *If* $F < N$ *contestants are chosen in the first stage to compete in the second stage and all the contestants face a common budget (effort) constraint,* B*, which can be allocated between the two stages, then there exist symmetric pure-strategy Nash equilibria in which each of the* N *players expend* e^* *in stage 1, where* $0 \leq e^* \leq B$, *and* $x^* = B - e^*$ *in stage 2, if* $(F/N)[(1/F)V_i] - B \geq 0$ *for all* $i = 1, 2, .. N\text{-}1, N$.

Proof: In any equilibrium the expected payoff for the i-th player is

$\Pi_i^* = (F/N)[(1/F)V_i - (B\text{-}e^*)] - e^* \geq 0, \ \forall e^* \in [0, B], i = 1, 2, \ldots N\text{-}1, N$. This condition holds if $(F/N)[(1/F)V_i] - B \geq 0$, $i = 1, 2, \ldots N\text{-}1, N$. If $F = 2$, a player who deviates by bidding marginally more than e^* in stage 1 guarantees entry to stage 2 but will be joined by a player who has a bigger cap in stage 2. There will be no equilibrium in pure-strategies but in any mixed-strategy equilibrium in stage 2, the player with the smaller cap will get a zero expected payoff.[9] Hence, it is not profitable for any player to deviate if F = 2. If F ≥ 3, a player who deviates from this equilibrium by bidding marginally more than e* in stage 1 guarantees entry to stage 2, but will then lose in stage 2 with certainty since he will be joined by, at least, two players who have bigger caps in stage 2. There is no equilibrium in mixed-strategies but a unique pure-strategy equilibrium in which the players with the bigger cap in stage 2 will bid their cap,[10] yielding an expected payoff lower than the equilibrium expected payoff for the player who deviated. A player who bids less than e* in stage one will lose with certainty in stage 1 yielding an expected payoff lower than the equilibrium expected payoff. Hence there is no profitable deviation from the equilibrium stated in the proposition. **Q.E.D.**[11]

All these equilibria can be Pareto ranked by noting that $\partial \Pi_i^* / \partial e^* = F/N - 1 < 0$. Hence e* = 0 gives the highest payoff and e* = B gives the lowest payoff. This game is a coordination game where the players are better off if they coordinate on a lower effort in stage 1.

Notice that e* = B is the burning-out equilibrium. In this equilibrium, the contestants expend all their effort in stage 1 and nothing in stage 2.[12] Burning out can also occur in single-stage contests. This case is not particularly interesting. It is not surprising that the contestants will allocate all their efforts in a single-stage contest. There

[9] See appendix A for a proof of this result.
[10] See appendix B for a proof of this result.
[11] In single-stage contests, Che and Gale (1998) and Gravious et al. (2002) also find symmetric equilibria in which each contestant bids the cap, if the contestants' valuations are sufficiently high.
[12] This result should not be interpreted literally. Zero effort in stage 2 only means that the contestants will put in the minimum acceptable effort in stage 2. If a positive minimum effort is required in stage 2, the contestants will allocate the minimum effort to stage 2 and the rest to stage 1.

are no other stages for them to allocate their effort anyway. What makes burning out in sequential contests interesting is that the contestants may choose to allocate all their effort in earlier stages and ignore subsequent stages. Furthermore, in a single-stage contest, there can be no pure-strategy equilibrium with $e^* < B$.

While there are several equilibria to this game, the burning-out equilibrium is the unique pure-strategy equilibrium under certain conditions. If we use a refinement of Nash equilibrium (i.e., coalition-proof Nash equilibrium) where we allow for joint deviations, then the burning-out equilibrium could be the only pure-strategy equilibrium. To see this, consider an equilibrium in which all the contestants in stage 1 bid $e^* < B$. Suppose a group of m contestants deviate by bidding marginally more than e^* in stage 1. If $m = F$, then they all guarantee entry to stage 2. Their payoff will be $\Pi_i^d = (1/F)V_i - B > 0$. It is easy to show that $\Pi_i^d > \Pi_i^*$, if $(1/F)V_i - (B-e^*) > 0$, which is true for all active contestants. This deviation by these F contestants is immune to deviations by sub-coalitions of this group, since each coalition member's success probability in stage 1 is one. Hence we have shown that when $e^* < B$, there exists a profitable joint deviation. Note that a deviation below e^* is not profitable and a deviation above e^* by any number of contestants is not feasible at $e^* = B$. Thus, the burning-out equilibrium is the only pure-strategy coalition-proof Nash equilibrium (CPNE) (see Bernheim et. al (1987) for a discussion of CPNE).[13]

[13] Experimental results by Amegashie et. al (2004) show that the burning-out equilibrium is obtained in sufficiently large groups. Muller and Schotter (2003) test a single-stage all-pay auction model in an experiment. They find that high-ability contestants try too hard and low-ability contestants drop out.

The rest of the paper will now focus on the burning-out equilibrium. Note that we can also construct a burning-out equilibrium if the contestants have the same valuations but have different costs of exerting the maximum effort, B.[14]

It is important to note that given an all-pay auction in stage 1 and a common cap on effort, the burning-out equilibrium holds regardless of the contest success function in stage 2, so long as each contestant has a success probability, 1/F, if all the contestants tie. This equilibrium does not hinge on **ZEEP**.

An implication of Lemma 2 is that, given **ZEEP**, it is impossible to construct a burning-out equilibrium in a sequential-elimination contest with any number of stages such that the contestants have different caps on effort in the penultimate stage. Indeed, if the elimination contest has more than two stages, then one can construct a burning-out equilibrium even if the contestants have different caps on effort, but the difference in the caps must be very small. To see this, consider a three-stage sequential-elimination where the contest in each stage is an all-pay auction. Suppose K ($< N$) contestants will be chosen in the first stage, and F ($< K$) contestants will be chosen in the second stage; $K \geq 3$. The winner is chosen in stage 3. Without loss of generality, assume that player 1 has a cap of $B + \varepsilon$ and the rest of the players have a cap of B, where $\varepsilon > 0$. Then as $\varepsilon \rightarrow 0$, it is easy to construct an equilibrium such that in stage 1 each of the ($N - 1$) contestants other than 1 bid $e^* < B$ and player 1 bids $e^* + \varepsilon$; in stage 2, each of the K successful contestants bid $B - e^*$, and in the final stage each contestant bids zero, where $e^* > 0$. Hence all the contestants burn out in stage 2. In stage 1, the equilibrium expected payoff to player 1 is

[14] Baye et al. (1996) and Clark and Riis (1998) show that a high-valuation contestant could be thought of as a contestant with a low cost of making a bid.

Π_2^* – (e* + ε) > 0 and each of the other players gets [(K-1)/(N-1)] Π_2^* – e* > 0, where Π_2^* = (F/K)(1/F)V_i – (B - e*) is each player's equilibrium expected payoff in stage 2. Note that, in stage 1, player 1 is successful with certainty. A crucial requirement for constructing this equilibrium is that the difference in the caps (B – ε) – B = ε must be very small. In stage 1, it is not optimal for player 1 to bid less than e*, given that the other players are bidding e*. If he bids e*, it is optimal for him to deviate and instead bid marginally higher than e* (i.e., e* + ε) since his probability of success rises discontinuously from K/N to 1. But if his cap is greater than B + ε, then he will have more effort left than any of the other players to expend in stage 2 (i.e., he will have more than B – e*). But then that means that a burning-out equilibrium is impossible to construct according to Lemma 2. Shortly, I shall give an economic intuition for why ε must be very small (i.e., ε → 0).

2.1. Discussion

In this section, I revert to the two-stage game unless otherwise indicated. When contestants get burnt out in sequential contests, it is almost always the outcome of an intense, cut-throat competition in previous rounds which results in a binding constraint on effort.[15] My model captures this feature; the contest success function which guarantees that the contestant(s) with the highest effort(s) win with certainty results in a very intense competition in the first stage. But "burning out" does not occur in contests where there are substantial differences in the abilities of the contestants and extremely high-powered

[15] When past effort does not constrain current effort, then "burning out" is less likely to occur. Obviously, past effort will have no effect on current effort in contests if the contestants get rejuvenated in future rounds.

incentives in earlier stages. Outstanding contestants can advance to subsequent rounds without exerting too much effort. For "burning out" out to occur the playing field must be somewhat even. The common cap (or almost identical cap), B, on effort has the effect of making the playing field somewhat even. This point was first made by Che and Gale (1998) and it provides the intuition for their result that lobbying caps could increase aggregate lobbying expenditures because they make the playing field even. High valuations contestants will find it difficult to pre-empt low valuation contestants if there is a cap; it is easier for them do so in the absence of a cap. Although equally-matched contestants may not be captured by common caps on effort in the real world, the common cap or almost identical cap (i.e., ε is very small) in this paper could be seen as a simple and tractable analytical device of making the contestants equally matched.[16] Indeed, common salary caps in professional sports (e.g., NBA, NFL) are intended to make the playing field even (see, Fort and Quirk, 1995; Gavious et al., 2000). However, in the contest with three stages, we find that a burning out equilibrium is possible if the contestants have different caps, although the difference in caps must be small.

It is helpful to summarize the discussion in the preceding paragraph. There are three properties of sequential contests which produce full "burning out" (i.e., all active contestants burn out). This paper captures all three features: (i) a constraint on aggregate effort across stages or rounds[17] (ii) extreme high-powered incentives in earlier rounds, and (iii) the playing field is even; no contestant is outstanding.[18] As in Che and Gale

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[16] The burning out result will still hold if all the F successful contestants in stage 1 receive an interim prize. Suppose ν_i is the i-th contestant's valuation of this interim prize. Then burning out occurs if $[F/N][(1/F)V_i + \nu_i] - B \geq 0$.

[17] The cap must be sufficiently small (i.e., $B \leq (F/N)(1/F)V_i$ for all i).

[18] However, the multiplicity of equilibria implies that these conditions are not sufficient for full burning out. They are only necessary conditions.

(1998) and Gravious et al. (2002), the third feature is incorporated into the model by placing a common cap on effort.

In this model, all the contestants get burnt out at the same time (i.e., in stage 1). Given an all-pay auction in stage 1, **ZEEP**, and complete information, it will never make sense for the contestants to get burnt out in different stages of the contest. A contestant will not expend all his effort in a previous stage knowing that other contestants will have more than the minimum effort to exert in subsequent stages. Even if there is incomplete information, a contestant who expends all his effort in a previous stage must believe that all other contestants are doing likewise.

There are examples of sequential elimination contests in which contestants expend a lot of effort in earlier rounds but not as much in latter rounds. For example, in academia junior faculty have to work hard to get tenure but the few who get tenure sometimes do not work as hard afterwards. What is driving the result is the following: if the contestants face a fixed resource constraint which has to be allocated between the two stages; say, for example, junior faculty have a fixed number of very good papers that they can write over their academic life, then it is optimal for them to produce their best papers early in their academic life than later. It may not make sense to reserve your best for the second stage because you may not get to the second stage with a mediocre effort in the first stage (given that there are high-powered incentives in the first stage). Doing well in the first stage is a necessary condition for getting to the second stage. Junior faculty have to give off their best early in their career because getting tenure (being short-listed) is a necessary condition for other academic prizes. If they don't get tenure, they may not be able to prove later that they can write good papers. They might get "burnt out" if the academic contest has the three properties noted above.[19] I hasten to add that I am not justifying "burning out" in academia. My objective is to present a formal model which produces "burning out" in sequential elimination contests. However, from a welfare point of view, it may be optimal to design sequential contests such that contestants burn out. For example, given the positive externalities that stem from good papers and a sufficiently low discount factor, the contest-designer might find it optimal to induce contestants in academia to write their best papers earlier than later.

To give a somewhat different but related example, consider boxing contests. When the contestants are almost equally matched like Ali-Frazier III (i.e., the famous thriller in Manila), they give off their best in the earlier rounds such that they get "burnt out" in the latter rounds. There is no point in coasting in the earlier rounds when your opponent is winning these rounds, unless you are much better than him and can dominate the fight anytime you choose to. Indeed, you could get knocked out if you try to coast in the earlier rounds. Putting in your best effort in the earlier rounds may be a necessary condition for proceeding to future rounds (i.e., preventing a knock out in the earlier rounds) and for overall success in the contest.[20]

Rosen (1986) shows that to prevent contestants from coasting in earlier rounds of sequential contests, a disproportionate amount of the total prize should be put in the latter rounds. Indeed, in his model this is required to ensure that the effort in each stage

[19] See Lazear (2004) for an analysis of why some academics are burnt out after getting tenure.
[20] In boxing the contestants rarely get "burnt out" in the first round or first few rounds. This is because the time constraint per round (i.e., three minutes) and the one-minute rest between rounds makes it difficult to get "burnt out" so early in the fight. But as the fight progresses (especially midway), signs of "burning out" begin to show in fights involving *equally matched* boxers. Ali and Frazier had fought in two previous fights before this fight; Frazier won the first fight and Ali won the second. In Ali-Frazier III, both boxers were burnt out by the seventh round of a fifteen-round contest. The contest was over at the end of the fourteenth round. Joe Frazier did not answer the bell for the last round. Muhammed Ali won the fight but he had this to say: "It was like death. The closest thing to dying that I know of".

(including the final stage) is the same. In my model, a disproportionate amount (i.e., the entire prize) is concentrated in the final stage but the effort in this stage could still be zero even if the prize is increased. Having stated Rosen's (1986) result, it is important to note why the "burning out" result in this paper is interesting. The success probability in stage 1 is may be the same as the success probability in stage 2. That is, the contestant(s) with the highest bid(s) may win with certainty in either stage. Also, the entire prize is concentrated in stage 2. Yet the contestants expend all their efforts in the first stage. It may appear that stage 2 is more important than stage 1. However, the contestants may attach more importance to stage 1 because stage 1 precedes stage 2. One cannot get to stage 2 without first making it through stage 1. Herein lies the importance of stage 1. To be a lawyer or doctor, one has to first get into law school or medical school. Although the ultimate prize is obtained after completion of medical school or law school, contestants may work much harder to get into medical school or law school than they would after getting admission to these schools.

There are apparent cases of "burning out" in real-world sequential contests. For example, with reference to the British Navy during the age of fighting sail,[21] Dandeker (1978, p. 305) writes "[a]fter six years' service a young gentlemen, already a midshipman, could take his examination for lieutenant." He continues "[o]fficers were promoted by selection to post Captain, via the ranks of lieutenant and commander, and from then on by the *de facto* rule of seniority." That is, while promotion above the captain's rank was automatic (i.e., it was just a matter of time), promotion at the lower ranks was not. Hence officers exerted a lot of effort earlier in their career, but relaxed

[21] My thanks are due to Doug Allen for this example.

once they got to the rank of post captain. Here the contestants did not relax in the latter rounds because of a binding fixed resource constraint. In the context of my model, the automatic promotion above the captain's rank is the same as effort having no effect on a contestant's success probability in stage 2. My explanation does not rely on low-powered incentives in the second stage. It relies on the simple fact that stage 1 precedes stage 2, excellence in stage 1 is a necessary condition for advancement to stage 2, and there are high-powered incentives in earlier stages. Note that in the equilibrium in proposition 1, the incentive structure is such that any contestant in stage 2 would prefer to exert more effort, if he could. In the next section, I construct a burning out equilibrium by introducing sufficient noise in stage 2 such that contestant who exerts zero effort in this stage has a non-zero success probability even if some other contestant expends a positive effort. However, I find that it is possible to obtain an equilibrium in which some contestants burn out but others do not. So while some contestants burn out in stage 1 because of low-powered incentives in stage 2, others choose not to.

In sports like tennis and athletics the contestants exert greater effort in latter rounds than in earlier rounds. In the context of my model, one can think of different abilities as translating into different caps on output. High-ability contestants have higher caps on output than low ability contestants. Hence, high-ability contestants need not bid up to their own cap to make it to the next round. This implies that they may be able to exert more effort in latter rounds. As shown above, if the success probability is extremely sensitive to effort (i.e., an auction), then there will be no burning out if the contestants have different caps on output in a two-stage game or if the differences in the caps is big.

In sports like athletics, the contestants sometimes get rejuvenated between rounds because they are allowed to rest between rounds. So past effort may not constrain future effort. Burning out occurs when past effort constrains future effort.

3. A model with partial burning out

Suppose we relax **ZEEP**. In particular, the i-th contestant wins the contest in stage 2 with probability θ if $x_i > 0$ and $x_j = 0$ for all $i \neq j$, where $1/F < \theta < 1$. Note that $\theta > 1/F$ is a reasonable assumption because if all the contestants expend zero effort in stage 2, each of them has a success probability of 1/F. It makes sense to assume that if one of them exerts a positive effort but the rest do not, his success probability should exceed 1/F. In this section, I examine this issue in some detail by specifying a contest success function in stage which does not satisfy **ZEEP**. I shall show that it is possible to construct equilibria in which some contestants get burnt out but other contestants do not burn out, if **ZEEP** is relaxed.

Consider a two-stage sequential elimination contest with contestants who have valuations, V_i, and effort constraint, B_i. The contest success functions are

$P_{1i} = e_i /(e_i + \sum_{j\neq i} e_j)$ and $P_{2i} = (x_i + \beta)/(x_i + \sum_{j\neq i} x_j + F\beta)$, where β is a positive constant.

Notice that if $x_i > 0$ and $x_j = 0$ for all $i \neq j$, then $1 > P_{2i} \equiv \theta > 1/F$. Also, the i-th contestant has a non-zero probability of winning the prize in stage 2, regardless of his effort level and the effort levels of other contestants. Thus **ZEEP** is relaxed. As $\beta \rightarrow \infty$, $P_{2i} \rightarrow 1/F$. Indeed, β measures the degree of noise in this contest success function (see Amegashie (2003) for a detailed discussion of this contest success function).

Lemma 3: *For any finite value of* $\beta \geq 0$*, at least, one contestant will* not *burn out in any equilibrium, given that the contest success function in stage 1 is continuous and* $P_{2i} = (x_i + \beta)/(x_i + \sum_{j \neq i} x_j + F\beta)$. *Therefore a burning-out equilibrium, if it exists, must be a partial burning-out equilibrium.*

Proof: Consider an equilibrium in which all the contestants burn out, where the i-th contestant's payoff in stage 1 is $P^*_{1i}(B_i)(1/F)V_i - B_i \geq 0$. Note that $P_{2i} = (x_i + \beta)/(x_i + \sum_{j \neq i} x_j + F\beta)$ implies that if $x_i > 0$ and $x_j = 0$ for all $i \neq j$, then $P_{2i} \equiv \theta > 1/F$ if β is finite. Notice that, given the contest success function in stage 1, $P^*_{1i}(0) = 0$, if $e_j > 0$ for some $j \neq i$. This is true since it is not an equilibrium for all contestants to bid zero in stage 1. If the i-th contestant deviates from the burning-out equilibrium by allocating an effort level of $\varepsilon > 0$ to stage 2, he gets $P^*_{1i}(B_i - \varepsilon)(\theta V_i - \varepsilon) - (B_i - \varepsilon)$. A deviation, $\varepsilon > 0$, is profitable if $P^*_{1i}(B_i - \varepsilon)(\theta V_i - \varepsilon) - (B_i - \varepsilon) > P^*_{1i}(B_i)(1/F)V_i - B_i$. This holds if $\hat{\Omega} \equiv [\theta P^*_{1i}(B_i - \varepsilon) - P^*_{1i}(B_i)/F]V_i + \varepsilon[1 - P^*_{1i}(B_i - \varepsilon)] > 0$. Given $P^*_{1i}(B_i)(1/F)V_i - B_i \geq 0$ and $\theta > 1/F$, $\hat{\Omega} > 0$ if $\varepsilon = 0$ and $\hat{\Omega} \leq 0$, if $\varepsilon = B_i$. Now $\hat{\Omega}$ is continuous in ε, given that $P^*_{1i}(e^*_i)$ is continuous. It follows that there exists a sufficiently small but positive ε such that $\hat{\Omega} > 0$. **QED**.

In what follows, I assume that N = 4 and F = 2. Assume that β = 20. Each contestant has an effort constraint, $B_i = 10$ for all i. Suppose that contestants 1 and 2 have the same valuation, $V_1 = V_2$. Let these contestants be in the same group in stage 1, where one of them will be chosen to compete in stage 2. Contestants 3 and 4 are in a different group from which one of them is chosen to compete in stage 2.[22] I assume that 3 and 4 have the same valuation, $V_3 = V_4$. In view of Lemma 3, I want to construct a sub-game perfect Nash equilibrium in which contestants 1 and 2 allocate all their effort in stage 1 but contestants 3 and 4 do not. I construct the equilibrium with the help of the Math software Maple V. I focus on an equilibrium in which the constraint binds for each contestant (i.e., $e_i + x_i = B$). Hence for each contestant, I shall write $x_i = B - e_i$, where necessary.

In what follows, I look for a symmetric equilibrium for contestants in the same group in stage 1. So I focus on an equilibrium in which the effort allocations of contestants 1 and 2 are identical. The same holds for 3 and 4. In each stage, the contestants move simultaneously.

Consider contestant 1. His payoff, in stage 1, is

$$\Pi_{11} = \frac{e_1}{e_1 + e_2}\left(\frac{(B - e_1) + \beta}{(B - e_1) + x_j + 2\beta} V_1 - (B - e_1)\right) - e_1,$$

j = 3 or 4. Contestant 2 has a similar payoff function.

Now consider contestant 3. His payoff, in stage 1 is

$$\Pi_{13} = \frac{e_3}{e_3 + e_4}\left(\frac{(B - e_3) + \beta}{(B - e_3) + x_k + 2\beta} V_3 - (B - e_3)\right) - e_3,$$

k = 1 or 2. Contestant 4 has a similar payoff function.

To construct a subgame perfect symmetric equilibrium in which contestants 1 and 2 are burnt out, I set

[22] This selection process is used in Amegashie (1999), Groh et al. (2003), Konrad and Gradstein (1999), Rosen (1986) and Stein and Rapoport (2002).

$\left.\frac{\partial \Pi_{11}}{\partial e_1}\right|_{e1=e2=B} = 0$. Using B = 10 and β = 20, I find that this derivative is equal to zero if

$x_3 = x_4 = 2\sqrt{5V_1}$ - 40. This gives $e_3 = e_4 = B - x_3 = 50 - 2\sqrt{5V_1}$, assuming that the effort constraint binds. Since I want $0 < e_3 < B$, I require that $V_1 \in (80, 125)$. A necessary condition for this interior solution, in stage 1, is

$\left.\frac{\partial \Pi_{13}}{\partial e_3}\right|_{x1=x2=0, e3=e4=50-2\sqrt{5V_1}} = 0$. This derivative is zero if

$V_3 = V_4 = \frac{60V_1^{3/2} - 2V_1^2\sqrt{5}}{V_1^{3/2} + 2V_1\sqrt{5} - 100\sqrt{V_1}}$. Denote this by $\hat{V}$. It is easy to show that $\hat{V} > 0$

given $V_1 \in (80, 125)$. Now $V_1 > \hat{V}$ if $V_1 + 4\sqrt{5V_1} - 160 > 0$. This holds given

$V_1 \in (80, 125)$. In this equilibrium, contestants 1 and 2 are burnt out but contestants 3 and 4 are not.

So given B = 10, N = 4, F = 2, $V_1 = V_2 \in (80, 125)$, $V_3 = V_4 = \hat{V}_3$, and β = 20, there exists a subgame perfect Nash equilibrium in which contestants 1 and 2 burn out but 3 and 4 do not burn out.[23] However, all the contestants exhaust their effort cap, B. Note that one could also construct this kind of equilibrium with different caps on effort. Obviously, the above analysis is in no way exhaustive. My objective was to construct a specific equilibrium (an example) in which some contestants burn out while others do not.[24]

[23] The equilibrium profits are non-negative and second-order conditions for a local maximum hold.

[24] A more general analysis gets very complicated, given non-identical contestants who compete in different groups in stage 1. For tractability, Groh et al. (2003) also consider an elimination contest with only four non-identical contestants with two contestants in two different groups in stage 1.

An alternative way of verifying the above equilibrium is as follows: consider contestant 1. His payoff in stage 2 is

$$\Pi_{21} = \frac{x_1 + \beta}{x_1 + x_j + 2\beta} V_1 - x_1,$$

j = 3 or 4.

If the effort allocations above are indeed equilibrium allocations, then in this equilibrium $\partial\Pi_{21}/\partial x_1 \geq 0$. That is, once stage 2 is reached it would not be optimal to decrease the equilibrium effort. If a contestant does not want to decrease his stage 2 equilibrium effort, then it trivially implies that he did not want to increase his stage 1 equilibrium effort when he played the game in stage 1. His stage 2 equilibrium effort cannot be increased since that would violate the constraint $e_1 + x_1 = B$.[25]

An interesting aspect of this equilibrium is that it is the contestants (1 and 2) with the higher valuation who allocate all their effort in stage 1. The intuition for this result is as follows: it is rational to allocate all your effort in stage 1 because a contestant still has non-zero probability of winning the prize in stage 2. But since one has to do well in stage 1 before proceeding to stage 2, it is possible that it is the contestants with very high valuations who will allocate all their effort to stage 1 because they really care about advancing to stage 2. An implication of this result is that, in the competition for players, we should expect the top teams in the NBA to exhaust their salary cap earlier than the mediocre teams. This is evidenced in top teams violating the cap in the NBA. Fort and Quirk (1995, p. 1281) write "[T]he profit incentives for strong-drawing teams to violate

[25] I have checked that, in the partial burning-out equilibrium above, $\partial\Pi_{2i}/\partial x_i \geq 0$ holds for all i

the cap certainly exist, and there have reportedly been a dozen or more cases that have gone to arbitration since the cap was instituted, most involving strong-drawing teams."

4. Conclusion

The paper has examined sequential elimination contests. The paper explains "burning out" in sequential elimination contests. To the best of my knowledge, this is the first paper to obtain a "burning out" result in a sequential elimination contest. Contestants may expend all their efforts in the first stage because they cannot move to the next stage, if they do not do well in the first stage. If they face a fixed resource constraint, then they get burnt out by the time they get to the second stage. This result is interesting given that the entire prize is concentrated in the second stage. It is also interesting because it holds even if the success probability (in the second stage) is extremely sensitive to effort. The first stage of the contest is a very important stage because it precedes the second stage. One cannot get to stage 2 without first making it through stage 1. Herein lies the importance of stage 1. This simple point has not been made in the literature on sequential contests. The paper captures three properties of contests which produce full "burning out": (i) a constraint on aggregate effort across stages or rounds (ii) extreme high-powered incentives in earlier rounds (i.e., contestants with the highest bids win with certainty), and (iii) the playing field is even; no contestant is outstanding. The third feature is incorporated into the model by placing a common cap or almost identical cap on effort. The common cap in this paper could be seen as a simple and tractable analytical device of making the contestants equally matched; this is the rationale behind common salary caps in US professional sports. Indeed, Che and Gale (1998) have observed that a common cap makes the playing field even. If the contest in stage 1 is an all-pay auction, the contestants have different caps, and contest success function stage 2 satisfies a certain property (i.e., **ZEEP**), then burning out will occur only if the contestants misjudged the abilities (i.e., the caps) of other contestants. However, in a contest with three stages, we find that a burning out equilibrium is possible if the contestants have different caps, although the difference in caps must be small.

If a contestant has a chance of winning the prize in the second stage even if he exerts the minimum effort and the incentive structure in the first stage is not extremely high-powered, then it possible to construct an equilibrium in which some contestants burn out but others do not, even if the contestants have different caps on effort. The paper shows that the contestants who have the higher valuations for the prize may be those who will allocate all their effort in earlier stages.

If the efforts of the contestants have value, then a contest-designer who has a sufficiently low discount factor might design a sequential elimination contest such that some or all of contestants burn out.

Appendix A: Proof that in an all-pay auction with two players with different caps, the player with the smaller cap gets a zero expected surplus, if the cap is sufficiently small

Consider stage 2 of the two-stage game (an all-pay auction) where there are two players with different caps. For the sake of argument, suppose the players are 1 and 2, with valuations V_1 and V_2 and caps B_1 and B_2, where $B_2 < B_1 \leq B$ and $V_1 > V_2 > B_2$. Note that $V_2 > B_2$ since $(1/F)V_i – B > 0$, for all active players. We follow the proof in Che and Gale (1997), although in their model the players have different caps but the same valuations.

If $B_2 = 0$, then the only equilibrium is in pure strategies where player 2 bids zero and player 1 bids a small but positive amount.

Now suppose $B_2 > 0$. First, there is no equilibrium in pure strategies. The proof is straightforward, so it is omitted. There is an equilibrium in mixed strategies (Che and Gale, 1997). Second, no player has a mass point any bid $x \in (0, B_2)$ in stage 2. Without loss of generality, suppose the contrary that player 1 has a mass point at $x \in (0, B_2)$, say at x_1. Then the probability that player 2 wins rises discontinuously as a function of his bid at x_1. Hence there is some $\varepsilon > 0$ such that player 2 will bid on the interval $[x_1 - \varepsilon,\ x_1]$ with zero probability. But then player 1 is better of bidding $x_1 - \varepsilon$ instead of x_1 since his probability of winning is the same. This contradicts the hypothesis that putting a mass point at $x \in (0, B_2)$ is an equilibrium strategy. Third, only one player can receive a strictly positive expected surplus. Suppose the contrary that both players receive positive expected surpluses. Then both players must have the same infimum bid. If not, the player with the strictly lower infimum would lose with probability one when he bids below the other player's infimum bid, so his expected surplus cannot be strictly positive, since every bid in the support of his equilibrium mixed strategy must yield the same expected surplus. If both players have the same infimum bid, $\underline{x} > 0$, then in order for each of them to win with positive probability when bidding $\underline{x}$, they must both have mass points at $\underline{x}$. But this is not possible since no player puts a positive mass at $x \in (0, B_2)$ and B_2 cannot be either player's lowest bid since there is no pure-strategy equilibrium. Hence, only one player can have a strictly positive expected surplus. Finally, the player with the bigger cap (i.e., player 1) gets a positive expected surplus and therefore player 2's expected surplus is zero. To see this, note that player 1 can guarantee himself a positive expected surplus by submitting a bid above B_2. Since there exists a bid which guarantees player 1 a positive expected surplus, this player cannot make a zero expected surplus in a mixed-strategy equilibrium. Hence player 2 (i.e., the player with the smaller cap) gets a zero expected surplus. **QED.**

Appendix B: Proof that in an all-pay auction with $M \geq 2$ players and a common cap, the only equilibrium is in pure strategies if the cap is sufficiently small.

Che and Gale (1998) prove that in all-pay auction with two players who have a common cap, say b, the only equilibrium is in pure-strategies if $(1/2)V_i - b > 0$ for all i. Their proof also holds for any number of players $M \geq 2$ with a common cap, say b, and $(1/M)V_i - b > 0$ for all i. The proof requires the following: (i) no bidder has a mass point at any bid $x \in (0, b)$; this was shown in appendix A for two players but the proof still holds in case of more than two players (see also Che and Gale, 1997), (ii) all bidders cannot have a mass point at zero, since each player's probability of winning rises discontinuously at zero which is inconsistent with an equilibrium, and (iii) only zero or b can be infimum bids. To see this, note that if a player, say k, has an infimum bid $x \in (0, b)$, then no other player will bid below or equal to player k's infimum bid (given (i)), since such bids will lose with certainty. But then it is not optimal for player k to have an infimum bid $x \in (0, b)$. Hence, only zero or b can be infimum bids.

I shall show that if $(1/M)V_i - b > 0$, then all bidders have an infimum bid of b; the proof closely follows Che and Gale (1998). Suppose to the contrary that at least one player has an infimum bid of zero. Let this be player h. Bidding b guarantees at least a tie, so player h must get an expected payoff of at least $(1/M)V_h - b$ in a mixed-strategy equilibrium. Since player h's infimum is zero, a bid near zero must give the same expected payoff as a bid of b. Suppose one of the M-1 players does not have mass at zero, then bidder h receives less than $(1/M)V_h - b$ if he bids near zero. Hence all the M-1 bidders must have mass at zero. If the M-1 bidders have mass at zero, then their infimum bid is zero. But then a similar argument implies that player h also has mass at zero. Thus all M players have mass at zero. But that contradicts (ii). Hence the players cannot have infimum bids of zero, so the infimum bid must equal b according to (iii). But if the infimum bid is b, then we have a degenerate mixed-strategy equilibrium where the players bid b (i.e., a pure-strategy equilibrium where each player bids b). Note that this argument holds if there are M + 1 players, where all $M \geq 2$ players have a common cap but one player has a smaller cap. The optimal bid of the player with the smaller cap is zero. **QED**.

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