

CH model for a 2-step thinker and higher-step thinkers using the normalized Poisson beliefs in Camerer, Ho, Chong (QJE, 2004)

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To follow the analysis in this note, see section 5 of our paper “Competitive Burnout: Theory and Experimental Evidence” available at http://www.uoguelph.ca/~jamegash/burn_out_GEB.pdf

We shall now solve a step 2-step thinker’s problem assuming that all players hold the normalized Poisson beliefs in Camerer et al. (2004). We consider the case of $N = 8$.

Consider a 2-step thinker. Note that a 2-step thinker knows that a 1-step thinker bids \hat{e} in stage 1 and $B - \hat{e}$ in stage 2. Let N_k be a 2-step thinker’s belief of the number of k -step thinkers, where $N_0 + N_1 = N - 1$. Suppose a 2-step thinker believes that $N_1 \geq 2$. That is, there are, at least, two 1-step thinkers. Then he will not advance to stage 2 if he bids less than \hat{e} , given $F = 2$. Therefore, his optimal bid is $e \geq \hat{e}$. Note that, in stage 2, a 1-step thinker bids $\hat{x} = B - \hat{e}$. Suppose a 2-step thinker bids more than \hat{e} in stage 1. Then he will lose with certainty, if he meets a 1-step thinker in stage 2. So if he bids more than \hat{e} , he has a chance of winning the prize in stage 2, if and only if he meets a 0-step thinker. Hence the optimal bid of a 2-step thinker is \hat{e} , if he believes that there is a sufficiently high number or proportion of 1-step thinkers. Using the Poisson distribution of thinking steps as in Camerer et al. (2004), we restrict the analysis to a mean number of thinking steps, τ , consistent with $N_1 \geq 2$. Using the *normalized* Poisson distribution in Camerer et al. (2004), a 2-step thinker believes that a proportion, $g_1 = \tau / (1 + \tau)$, of his opponents in

stage 1 are 1-step thinkers and $g_0 = 1 - g_1$ are 0-step thinkers. Given $N = 8$, $N_1 \geq 2$ requires $7g_1 \geq 2$. This gives $\tau \geq 0.4$.

As argued above, given $F = 2$ and a 2-step thinker's belief that the number of 1-step thinkers, $N_1 \geq 2$, it is not optimal for a 2-step thinker to bid less than \hat{e} . Hence he either has to match \hat{e} or bid more than it. We consider these two cases:

Case (i): 2-step thinker bids \hat{e} in stage 1

Consider stage 2. If a 2-step thinker bids \hat{e} in stage 1 and meets a 1-step thinker in stage 2, his expected payoff will be $0.5V - (B - \hat{e}) > 0$, if he bids $B - \hat{e}$, since a 1-step thinker will also bid $(B - \hat{e})$ in stage. If he bids below $(B - \hat{e})$, his expected payoff will be lower than $0.5V - (B - \hat{e}) > 0$. Indeed, it will be non-positive. Therefore, a 2-step thinker should bid $B - \hat{e}$, if he meets a 1-step thinker.

Recall that a 2-step thinker believes that the number of 0-step thinkers is N_0 . If he meets a 0-step thinker in stage 2, this player must have bid, at least, \hat{e} in stage 1 to have advanced to stage 2, given that there are, at least, two 1-step thinkers who bid \hat{e} in stage 1. So if he meets a 0-step thinker in stage 2, his expected payoff can be shown to be

$$\pi_2 = x \left(\frac{VN_0}{b^{N_0}} \int_{\hat{e}(N)}^{\tilde{e}^{49.99} N_0 - 1} \frac{d\tilde{e}}{B - \tilde{e}} - 1 \right),$$

for a bid of x , where we have made use of the probability density function of the largest order statistic among the N_0 random variables (i.e., the random bids of the N_0 0-step thinkers). Suppose that $\tau = 4/3$. Then a 2-step thinker believes that $N_0 = 7[1/(1+\tau)] = 3$ and $N_1 = 4$. It is easy to show that given $N_0 = 3$, $b = 49.99$, and $V \geq 100$, $\partial\pi_2 / \partial x > 0$, which implies that the optimal bid in stage 2 is $x = B - \hat{e}$. This gives

$$\hat{\pi}_2 = (B - \hat{e}) \left(\frac{VN_0}{b^{N_0}} \int_{\hat{e}(N)}^{\infty} \frac{\tilde{e}^{N_0-1}}{B - \tilde{e}} d\tilde{e} - 1 \right).$$

Hence a 2-step thinker who bids \hat{e} in stage 1 should bid $(B - \hat{e})$ in stage 2, regardless of the type of thinker he meets.

If a 2-step thinker bids \hat{e} in stage 1, his probability of advancing to stage 2 is

$$\alpha_1 = \frac{2}{(1 + N_1)} \left(\frac{\hat{e}}{b} \right)^{N_0} + \frac{1}{(1 + N_1)} N_0 \frac{b - \hat{e}}{b} \left(\frac{\hat{e}}{b} \right)^{N_0 - 1}.$$

Given that a 2-step thinker has advanced to stage 2, he knows that probability that a 1-step thinker will advance to stage 2 is $\hat{\rho}_1 = (\hat{e}/b)^{N_0}$ and hence the probability of a 0-step thinker advancing is $1 - \hat{\rho}_1$.

Hence his expected payoff in stage 1, is

$$\Omega_1 = \alpha_1 [\hat{\rho}_1 [0.5V - (B - \hat{e})] + (1 - \hat{\rho}_1) \hat{\pi}_2] - \hat{e},$$

Case (ii): 2-step thinker bids more than \hat{e} in stage 1

Suppose a 2-step thinker bids $\hat{e} + \Delta$ in stage 1, where $\Delta > 0$. Consider stage 2.

Suppose his opponent is a 0-step thinker. Then for any $x < B - \hat{e}$ in stage 2, his expected

$$\text{payoff is } \pi_2 = x \left(\frac{VN_0}{b^{N_0}} \int_{\hat{e}(N)}^{\infty} \frac{\tilde{e}^{N_0-1}}{B - \tilde{e}} d\tilde{e} - 1 \right).$$

If his opponent is a 1-step thinker his payoff is $-x$, since a 1-step thinker bids

$B - \hat{e}$ in stage 2. Hence his expected payoff in stage 2 with a bid of $x < (B - \hat{e})$ is

$$\Omega_2 = (1 - \hat{\rho}_1) \pi_2 - \hat{\rho}_1 x = x \left(\frac{VN_0(1 - \hat{\rho}_1)}{b^{N_0}} \int_{\hat{e}(N)}^{\infty} \frac{\tilde{e}^{N_0-1}}{B - \tilde{e}} d\tilde{e} - 1 \right)$$

It is easy to show that given $N_0 = 3$, $b = 49.99$, and $V \geq 100$, $\partial\Omega_2 / \partial x > 0$, which implies that the optimal bid in stage 2 is $x = B - (\hat{e} + \Delta)$, where $\Delta \in (0, B - \hat{e}]$. Therefore, his expected payoff is

$$\hat{\Omega}_2 = (B - (\hat{e} + \Delta)) \left(\frac{VN_0(1 - \hat{\rho}_1)}{b^{N_0}} \int_{\hat{e}(N)}^{\hat{e} + \Delta} \frac{\tilde{e}^{N_0 - 1}}{B - \tilde{e}} d\tilde{e} - 1 \right),$$

where $\Delta \in (0, B - \hat{e}]$.

Given $\Delta > 0$, his bid of $\hat{e} + \Delta$ in stage 1 gives a success probability of

$$\alpha_1^d = \left(\frac{\hat{e} + \Delta}{b} \right)^{N_0} + N_0 \frac{b - (\hat{e} + \Delta)}{b} \left(\frac{\hat{e} + \Delta}{b} \right)^{N_0 - 1}.$$

Therefore, his expected payoff in stage 1 is

$$\Omega_1^d = \alpha_1^d \hat{\Omega}_2 - (\hat{e} + \Delta).$$

As in the main text, we set $\hat{e} = 40$ for $N = 8$. Given $b = 49.99$ and looking at the plots of $\Omega_1^d - \Omega_1$ on $[0, B - \hat{e}]$, we find that there exists a $\hat{\Delta} < B - \hat{e}$ such that $\Omega_1^d > \Omega_1$ for $\Delta \in (0, \hat{\Delta})$ and $\Omega_1^d \leq \Omega_1$ for $\Delta \in [\hat{\Delta}, B - \hat{e}]$. Indeed, the best response for the 2-step thinker is to bid marginally above \hat{e} . For discrete bids, this optimal bid is $\hat{e} + 1$ in stage 1 and $B - (\hat{e} + 1)$ in stage 2.

For any higher-step thinkers beyond a 2-step thinker, their belief of the proportion of 0-step thinkers is less than a 2-step thinker's belief, given the normalized Poisson distribution in Camerer et al. (2004). Notice that a 2-step thinker bids more than \hat{e} , if he believes that the proportion of 1-step thinkers is sufficiently small. This is because bidding more than \hat{e} in stage 1 yields a non-positive payoff in stage 2, if a 2-step thinker meets a 1-step thinker. Given $\tau = 4/3$, a 3-step thinker believes that the number of 0-step

thinkers is $7/(1 + \tau + 0.5\tau^2) = 2.172 \approx 2 < 3$. Therefore, a 3-step thinker believes that a higher proportion of his opponents are strategic (i.e., 1-step or 2-step thinkers) compared to the belief held by a 2-step thinker of the proportion of strategic opponents (i.e., 1-step thinkers). In this example, the problem for the 3-step thinker is analogous to the problem for the 2-step thinker, since he believes that there are, at least, two 2-step thinkers. The 3-step thinker must choose to either match $\hat{e} + 1$ or bid more than it. In doing so, he ignores the bids of all 1-step thinkers, since a bid of \hat{e} cannot outbid one which is, at least, equal to $\hat{e} + 1$. Hence a 3-step thinker considers the probability of meeting a 0-step thinker or a 2-step thinker but not a 1-step thinker. An analysis similar to that for the 2-step thinker is undertaken for the 3-step thinker by setting $N_0 = 2$ in the above analysis, noting that $N_1 = 2$ is now redefined (in the above equations) to be a 3-step thinker's belief of the number of 2-step thinkers, and \hat{e} is replaced with $\hat{e} + 1$. We find that a 3-step thinker also bids $\hat{e} + 1$. A similar argument applies to other higher-step thinkers. Thus all higher-step thinkers will also bid $\hat{e} + 1$. This is because higher step thinkers face the same problem as a 3-step thinker: match $\hat{e} + 1$ or bid more than $\hat{e} + 1$. However, they have a lower belief of meeting a 0-step thinker.

The main result we wish to emphasize is that when we use the normalized Poisson belief assumption in Camerer et al (2004), it is possible for a 2-step thinker and higher-step thinkers to bid more than a 1-step thinker. In contrast, a 2-step thinker and all higher-step thinkers will match the bid of a 1-step thinker, when we use the k-1 belief assumption in the main text. It must be clear to the reader that the solution for the normalized Poisson case is more difficult because it depends on the value of τ . Also, we

have to make approximations for the number of thinking types since these can only be positive integers.

Finally, we wish to note that the difference between the bids of the 2-step thinker based on the $k-1$ assumption and the normalized Poisson assumption is very small. Hence, using the normalized Poisson assumption does not yield substantially different predictions.