

*Forthcoming in the European Journal of Political Economy*

## **Productive versus Destructive Efforts in Contests\*\***

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March 30, 2012

### **Abstract**

We consider a two-stage contest in which players choose destructive efforts (sabotage) in stage 1 and productive efforts in stage 2. When the value of the prize is sufficiently high, we find that the productive effort of the contestants is independent of the value but their destructive effort is increasing in the value of the prize. The players only engage in destructive activities after productive effort reaches a threshold and do not increase their productive effort beyond this threshold. This result is consistent with contests in which participants *increase* effort in sabotage and dirty tricks more than on productive effort when the stakes are high (i.e., when the prize is high). After some point, destructive effort is more responsive than productive effort to increases in the value of the prize. Hence the ratio of destructive effort to productive effort increases with the value of the prize after the value exceeds a threshold.

Keywords: contest, destructive effort, productive effort, sabotage, threshold.

JEL Classification: D72.

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\*\*I thank two anonymous referees, Gil Epstein, Joerg Franke, Qiang Fu, and Oliver Gurtler for helpful comments.

## 1. Introduction

Since the seminal works by Tullock (1980) and Lazear and Rosen (1981), there has been an explosion of literature on contests (see, for example, Konrad, 2009).

The efforts of participants in contest can be productive or unproductive (destructive). Such destructive effort or sabotage has been studied in the literature (e.g., Lazear, 1989; Konrad, 2000; Chen, 2003; Kräkel, 2005; Amegashie and Runkel, 2007; and Munster, 2007a).

By productive effort in a contest, I mean effort that directly affects the probability of success in the contest. Destructive effort or sabotage can be thought of as illegal effort that makes it difficult for one's opponent to exert effort. Examples are deliberately dirty and illegal tricks (fouls) in a game of soccer or basketball and time-wasting techniques to make it impossible for one's opponent to exert effort.<sup>1</sup> In internal labor tournaments, contestants may go out of their way to hide valuable effort-enhancing information from each other which is supposed to be shared (e.g., think about a sales contest where the firm cares not only about the relative individual volume of sales by the contestants but also cares about aggregate sales and so wants the workers to share information that will enhance each worker's ability to sell). For example, based on a survey of Australian manufacturing, Drago and Garvey (1998) found that when the incentives for promotion are high-powered, workers are less willing to help their fellow workers. Carpenter, Matthews and Schirm (2010) refer to this behavior as "office politics." Harbring, Irlenbusch, Krakel, and Selten (2007), Carpenter, Matthews, and Schirm (2010), and Harbring and Irlenbusch (2011) give examples of sabotage in corporate contests and confirm this behavior in laboratory experiments.

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<sup>1</sup> Of course, if detected by the referee, the player may be punished for these actions. In the formal model, this is captured as a cost to the player of engaging in destructive effort.

According to Harbring et al. (2007, endnote 1):

“When commenting on the possibility of sabotage at Merck, having introduced forced ranking in 1986, Murphy (1992, endnote 4) refers to John Dvorak, PC Magazine editor, who reports on a friend’s way how he attained promotion: This friend cracked the network messaging system which allowed him to read all memos. He sabotaged the workgroup software and manipulated the appointment calendars. According to Dvorak, stealing passwords and destroying important data is an easy task for many employees.”

These counterproductive efforts *indirectly* affect the probability of success. However, not all sabotage in contest is unlawful. For example, negative campaigning in electoral contests need not be illegal and may *directly* affect success in the contest since such negative messages about one’s opponents may sway voters (e.g., Skaperdas and Grofman, 1995). They may also be informative.

Modeling sabotage can take various forms. It can be modeled as *directly* reducing the effectiveness of one’s opponent’s efforts or output which translates into a *direct* reduction in the opponent’s probability of success (e.g., Konrad, 2000; Chen, 2003; Munster, 2007a; Gurtler and Munster, 2010).<sup>2</sup> Sabotage can also be modeled as directly reducing one’s opponent’s valuation of the prize (e.g., Amegashie and Runkel, 2007). In this paper, sabotage is modeled as an increase in one’s opponent’s unit cost of productive effort which then leads to a decrease in (or destruction of) the opponent's productive effort (e.g., Carpenter et. al, 2010).<sup>3</sup> Making it difficult for an opponent to exert effort is clearly evident in the above quote by Harbring et al (2007) where an employee "... sabotaged the workgroup software and manipulated the appointment calendars" of his co-workers. "Stealing passwords and destroying important data" and examples mentioned above illustrate this point.

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<sup>2</sup> This paper endogenizes this effect.

<sup>3</sup> That a player's investment affects the unit cost of effort of his *opponent* is related to contests in which a player’s investment reduces his *own* unit cost of effort (e.g., Fu and Lu, 2009; Munster, 2007b).

We consider a two-stage contest in which players choose destructive efforts in stage 1 and productive efforts in stage 2. When the value of the prize is sufficiently high, we find that the productive effort of the contestants is independent of the value of the prize but their destructive effort is increasing in the value of the prize (i.e., higher stakes). This result is consistent with contests in which participants' effort in sabotage and dirty tricks tend to be more responsive than their productive effort as the *stakes get higher*. The paper finds that players engage in destructive activities only after productive effort reaches a threshold. This makes sense since the contest cannot be won with only destructive effort; productive effort is necessary.

There is some anecdotal evidence in support of the result in this paper. At World Cup competitions, the games in the group or earlier stages are cleaner with very few dirty tricks. However, in the final game, the players tend to engage in more dirty tricks if they can get away with it (e.g., Zinedine Zidane head-butting Marco Materazzi in the 2006 World Cup final between France and Italy after Materazzi had provoked him). In a related but different context, doping and other forms of cheating in sports may be seen as acts of *indirect* sabotage of one's opponent insofar as they give the cheater an illegal or undue advantage over his opponent (see Krakel, 2007). For example, some professional boxers have illegally padded their gloves (e.g., Antonio Margarito padded his gloves when he fought Miguel Cotto for a World Boxing welterweight title) and in athletics, sprinters like Ben Johnson have been caught using banned substances at Olympic games. Anecdotal evidence shows that these dirty or destructive tricks are less likely in amateur boxing and amateur athletics where the stakes are low compared to professional boxing and high-profile athletic events like the Olympics where the stakes are high.

The preceding argument is not to suggest that when the stakes are high, productive effort,  $e^*$ , is necessarily less than destructive effort,  $x^*$ . But rather that the ratio,  $x^*/e^*$ , of destructive

effort to productive effort may be higher when the stakes are high relative to the ratio when the stakes are low. After some point, destructive effort is *more responsive* than productive effort to increases in the value of the prize.

Kraker (2007, p. 989) observes that "... in any situation in which stress of performance is sufficiently high, individuals have to decide whether to keep the rules of the game or not. If not, illegal or manipulating behavior is used to gain an individual advantage ...." Extending this argument, it is reasonable to argue that the stress of performance is sufficiently high when the stakes are sufficiently high.

As Lazear (1989) originally pointed out, incentivizing contestants with a higher prize may be counterproductive. However, in Lazear's (1989) framework and other models of sabotage in contests, both productive and destructive effort are monotonically increasing in the prize (i.e., the players' valuation) when destructive effort is positive.

The paper is organized as follows: the next section presents a model of productive and destructive effort in a contest. It begins with a two-player version of the model and then considers the n-player case. A subsection presents the intuition for the main result. Section 3 concludes the paper.

## **2. The model**

Consider a two-stage contest with two risk-neutral players 1 and 2 each with valuation,  $V > 0$  for the prize. In stage 1, player  $k$  chooses his sabotage effort  $x_k \geq 0$ , which increases his opponent's unit cost of effort,  $k = 1, 2$ . In stage 2, the players choose their productive effort,  $e_k$ , in a Tullock contest,  $k = 1, 2$ . If player  $k$  invests  $x_k$  in sabotage, then player  $j$ 's unit cost of

productive effort is  $(1 + x_k)$ ,  $k \neq j$ ,  $j = 1, 2$ ;  $k = 1, 2$ . Hence, sabotage increases one's opponent's unit cost of productive effort. In each stage, the players move simultaneously and have complete information. This timing of moves is the same as in Fu and Lu (2009) which is a model with pre-contest but non-destructive investment.

Our solution concept is subgame perfection. We work backwards by starting from stage 2. In this stage 2,  $x_k$  is a parameter,  $k = 1, 2$ . The player's payoffs are:

$$\Pi_1 = \frac{e_1}{e_1 + e_2} V - (1 + x_2)e_1, \quad (1)$$

and

$$\Pi_2 = \frac{e_2}{e_1 + e_2} V - (1 + x_1)e_2. \quad (2)$$

This is a standard Tullock game whose solution is well known. The unique efforts levels in stage 2 are:

$$e_1^*(x_1, x_2) = \frac{\theta_1 V}{(\theta_1 + \theta_2)^2} \quad \text{and} \quad e_2^*(x_1, x_2) = \frac{\theta_2 V}{(\theta_1 + \theta_2)^2}, \quad (3)$$

where  $\theta_1 \equiv 1 + x_1$  and  $\theta_2 \equiv 1 + x_2$ .

Note that

$$\frac{\partial e_k^*}{\partial x_j} = -\frac{2(1 + x_k)V}{(2 + x_k + x_j)^3} < 0, \quad (4)$$

$k = 1, 2$ ;  $j = 1, 2$ ;  $j \neq k$ . As discussed in section 1, the derivative in (4) shows that player  $j$ 's sabotage effort,  $x_j$  indirectly affects player  $k$ 's probability of success through its effect on player  $k$ 's productive effort. In Konrad (2000), Chen (2003), Munster (2007a) and other papers on sabotage in contest, this effect is assumed. Hence, sabotage in these papers has a *direct* effect on

performance and therefore on the probability of success in the contest. The effect here is *indirect*: sabotage affects the cost of effort which then affects output.

In the unique equilibrium in stage 2, the payoffs are

$$\Pi_1^* = \left( \frac{\theta_1}{\theta_1 + \theta_2} \right)^2 V, \quad (5)$$

and

$$\Pi_2^* = \left( \frac{\theta_2}{\theta_1 + \theta_2} \right)^2 V. \quad (6)$$

Now consider stage 1. The players' payoffs are:

$$\Omega_1 = \Pi_1^* - \alpha x_1, \quad (7)$$

and

$$\Omega_2 = \Pi_2^* - \alpha x_2, \quad (8)^4$$

where  $\alpha > 0$  is the marginal cost of destructive effort.

The Kuhn-Tucker conditions for each player's problem are:

$$\frac{\partial \Omega_k}{\partial x_k} = \frac{2(1+x_k)V}{(2+x_k+x_j)^2} - \frac{2(1+x_k)^2V}{(2+x_k+x_j)^3} - \alpha \leq 0, x_k \geq 0, x_k \frac{\partial \Omega_k}{\partial x_k} = 0, \quad (9)$$

$j \neq k, k = 1, 2; j = 1, 2.$

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<sup>4</sup> Define  $P_j = (1+x_j)/(2+x_j+x_k)$ ,  $k, j = 1, 2; j \neq k$ . Then the payoffs in stage 1 are  $\Omega_j = (P_j)^2 V - \alpha x_j$ . Notice that  $P_j$  is an example of the contest success function (CSF) considered in Amegashie (2006) and Dasgupta and Nti (1998) and recently axiomatized by Rai and Sarin (2009). The difference, though, is that in (7) and (8), this CSF is squared in the players' payoff function which implies that  $P_1 + P_2 < 1$ ; it is *as if*, in stage 1, there is contest where there is always a positive probability that neither player will win the prize. Consistent with the analysis in Amegashie (2006), it is not surprising that, in subsequent analysis, we obtain corner solutions in stage 1.

Note that

$$\left. \frac{\partial \Omega_k}{\partial x_k} \right|_{V=4\alpha} = -\frac{\alpha[(x_k + x_j)^3 + 2(3x_k^2 + 3x_j^2 + 2x_j x_k + 2x_j + 2x_k)]}{(2 + x_k + x_j)^3} \leq 0, \quad (10)$$

for all  $x_k \geq 0$  and  $x_j \geq 0$ ,  $k = 1, 2$ ;  $j = 1, 2$ ; and  $j \neq k$ . So when  $V = \hat{V} \equiv 4\alpha$ , the symmetric equilibrium is  $x_1^* = x_2^* = 0$  which gives  $e_1^*(0,0) = e_2^*(0,0) = V/4$ . Note that  $\partial \Omega_k / \partial x_k$  is strictly increasing in  $V$ , so  $\partial \Omega_k / \partial x_k \leq 0$  for all  $x_k$  if  $V \leq 4\alpha$ ,  $k = 1, 2$ . Accordingly, we state the following proposition:

**Proposition 1:** *If the players have a sufficiently low valuation (i.e.,  $0 < V \leq 4\alpha$ ), then there is a unique subgame perfect equilibrium with  $x_1^* = x_2^* = 0$  and  $e_1^*(0,0) = e_2^*(0,0) = V/4 > 0$ .*

*Therefore, when the stakes are low, (i) players do not invest in destructive efforts, and (ii) their productive effort is increasing in the prize,  $V$ .*

Given that the objective functions in (7) and (8) are very non-linear and the resultant non-linearity of the first-order conditions, it is difficult to solve for interior solutions for  $x_1$  and  $x_2$  without imposing further restrictions. So, like Fu and Lu (2009), we restrict ourselves to a symmetric equilibrium where  $x_1 = x_2 > 0$ .<sup>5</sup> At a symmetric interior equilibrium,  $\partial \Omega_k / \partial x_k = 0$ ,  $k = 1, 2$ . If  $V > 4\alpha$ , we get

$$x_1^* = x_2^* = (V/4\alpha) - 1 > 0 \quad (11)$$

At this symmetric solution, the second-order condition for a local maximum holds:<sup>6</sup>

$$\left. \frac{\partial^2 \Omega_k}{\partial x_k^2} \right|_{x_1^* = x_2^* = (0.25V/\alpha) - 1} = -\frac{2\alpha^2}{V} < 0. \quad (12)$$

<sup>5</sup> Fu and Lu (2009) solved a related problem by restricting their analysis in stage 1 to symmetric equilibria.

<sup>6</sup>Holding  $x_2$  at  $(V/4\alpha) - 1 > 0$ , several plots of  $\Omega_1$  against  $x_1$  show that the equilibrium is a global maximum.

Then the equilibrium effort in stage 1 is:

$$e_1^*(x_1^*, x_2^*) = e_2^*(x_1^*, x_2^*) = \alpha. \quad (13)$$

We may restate the symmetric subgame perfect equilibria of this two-stage game as follows:  $e_1^* = e_2^* = \min[\alpha, V/4]$  and  $x_1^* = x_2^* = \max[0, (V/4\alpha) - 1]$ . Figures 1 and 2 illustrate this point. Since  $e_1^* = e_2^* = \min[\alpha, V/4]$ , we know that productive effort does not exceed a threshold level,  $\hat{e} = \alpha$ . Intuitively, the threshold for productive effort is high when the cost of destructive effort is high. The threshold value of the prize is  $\hat{V} = 4\alpha > 0$ . So if  $V \leq \hat{V} = 4\alpha$ , the players do not invest in destructive effort. They invest in destructive effort only if  $V > \hat{V}$  when productive effort has reached the threshold,  $\hat{e} = \alpha$ . In this case, the equilibrium payoff is  $\Omega_1^* = \Omega_2^* = \Omega^* = \alpha$ . As the marginal cost,  $\alpha$ , of destructive effort approaches zero,<sup>7</sup>  $\hat{e}$ ,  $\hat{V}$ , and  $\Omega^*$  also approach zero.

Accordingly, we obtain the following proposition:

**Proposition 2:** *If the players have sufficiently high valuations (i.e.,  $V > 4\alpha$ ), then (i) the players' productive efforts are independent of their valuations but their destructive efforts are increasing in their valuation, (ii) the ratio,  $x^*/e^*$ , of destructive efforts,  $x^*$ , to productive efforts,  $e^*$ , is increasing in  $V$ ,<sup>8</sup> and (iii) the players' productive effort does not exceed a threshold level,  $\alpha$ . The players do not engage in destructive effort until productive effort reaches this threshold.*

<sup>7</sup> Note that we cannot have  $\alpha = 0$  because that trivializes the economic problem facing the players.

<sup>8</sup> Note that the claim here is not that destructive effort is necessarily higher than productive effort as the stakes get higher. Destructive effort is higher than productive effort only if  $V > 4\alpha(1 + \alpha) > 4\alpha$ . Yet, the effect we have found kicks in when  $V > 4\alpha$ . When  $V > 4\alpha$ , the ratio,  $x^*/e^*$ , increases with  $V$  even if destructive effort is not higher than productive effort.

## 2.1 Discussion

It is interesting to note that although the cost of destructive effort (sabotage) and productive effort are both linear, they behave differently with respect to the prize. The intuition is as follows: at  $e_1 = e_2 = 0$ , the marginal impact of productive effort is very high. This is a well-known property of the Tullock contest success function (CSF) and it explains why productive effort is always positive in the model. It is important to bear in mind that while a player can win the contest with *only* productive effort, the contest cannot be won with only destructive effort. This is because productive effort *directly* enters the CSF while destructive effort does not. Furthermore, the marginal impact of player  $j$ 's destructive effort is zero if his opponent's (player  $k$ ) productive effort is zero because player  $j$  will have nothing to sabotage. Given that, in the absence of destructive effort, his opponent's productive effort is increasing in  $V$ ,<sup>9</sup> it follows that if  $V$  is sufficiently small, then his opponent's productive effort will also be very small. In this case, the marginal impact of player  $j$ 's destructive effort will also be very small (i.e., the absolute value of  $\partial e_k^* / \partial x_j$  in (4) is increasing in  $V$ ). This explains why there is no sabotage if  $V$  is sufficiently small. As the value of the prize increases, the marginal impact of a player's destructive effort increases while the marginal cost,  $\alpha$ , stays constant. Therefore, when  $V$  is sufficiently high, the players invest in destructive effort and this effort rises with  $V$ . Given the strict concavity of the Tullock CSF, the marginal impact of player  $j$ 's productive effort is decreasing as his productive effort increases while the marginal cost,  $1 + x_k$ , of his productive effort rises because player  $k$ 's destructive effort,  $x_k$ , is rising as  $V$  rises. This means that, after

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<sup>9</sup> In general, productive effort is necessarily increasing in  $V$  if destructive effort is fixed.

some point ( $V \geq 4\alpha$ ), it is not profitable to increase productive effort even if  $V$  increases.<sup>10</sup>

Being productive invites too much destructive effort.<sup>11</sup> After this point, the equilibrium payoff is independent of  $V$  since, as noted in the preceding section,  $\Omega_1^* = \Omega_2^* = \alpha$  if  $V > 4\alpha$ .

To summarize, four factors drive the results: (a) productive effort directly enters the contest success function (CSF) while destructive effort does not, (b) because the CSF is the Tullock ratio-form function, productive effort has a very high marginal impact at (0,0), (c) the marginal cost of a player's productive effort is increasing in his opponent's destructive effort, and (d) if a player's opponent's destructive effort is fixed, then the player's productive effort is increasing in the player's valuation.

If the players were to simultaneously determine productive and sabotage efforts as in, for example, Konrad (2000), then there will never be sabotage in equilibrium since sabotage effort does not directly enter a player's benefit function and negatively enters his cost function. In Konrad (2000) which is a model in which the players simultaneously choose sabotage and productive effort and a player's own sabotage effort directly enters his benefit function through its effect on the contest success function, there may be positive sabotage in equilibrium. In fact, stage 1 of the model in this paper with the payoff functions in (7) and (8) is equivalent to the single-stage model of Konrad (2000). However, only sabotage effort is chosen in stage 1 and not productive effort. Unlike Konrad (2000), productive effort is not *independently* chosen in this stage because it is a function of sabotage effort. Note that Konrad (2000) does not obtain the

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<sup>10</sup> To see this, note that in a symmetric equilibrium, each player's destructive effort is  $V/4\alpha - 1$ , if  $V > 4\alpha$ . Put this into player  $j$ 's payoff in stage 2 to get  $\Pi_j = e_j V / (e_j + e_k) - V e_j / 4\alpha$ ,  $j, k = 1, 2; j \neq k$ . So as  $V$  rises, both the marginal benefit and marginal cost of productive effort rise. But they rise in such a way that, in equilibrium, each player chooses  $e_1^* = e_2^* = \alpha$  regardless of the value of  $V$ .

<sup>11</sup> In a different context, this is similar to arguments by Allen (2002) and Konrad (2002) that an asset owner may reduce investment in the asset in order to make the asset less attractive to encroachers or challengers.

result that the players do not engage in sabotage unless productive effort reaches a threshold and they do not exceed this threshold level of productive effort regardless of their valuation.

## 2.2 Extension to $n \geq 3$ players

In this section, I consider the case of  $n \geq 3$  players. When the other  $n - 1$  players sabotage player  $k$ , let his unit cost of productive effort be  $1 + \sum_{j \neq k} x_j$ , where  $x_j$  is the sabotage effort of player  $j$  and  $j = 1, 2, \dots, k-1, k+1, \dots, n-1, n$ . Notice that player  $k$ 's destructive effort is not player-specific. The same  $x_k$  affects the other  $n - 1$  players. This formulation is consistent with sabotage in team environments where one worker's destructive activity negatively affects the ability of all team members to generate productive effort.

In stage 2, player  $k$ 's payoff is:

$$\Pi_k^n = \frac{e_k}{e_k + \sum_{j \neq k} e_j} V - \left( 1 + \sum_{j \neq k} x_j \right) e_k, \quad (14)$$

$k = 1, 2, \dots, n$ . The first-order condition is:

$$\frac{\partial \Pi_k^n}{\partial e_k} = \left( \frac{1}{E} - \frac{e_k}{E^2} \right) V - \left( 1 + \sum_{j \neq k} x_j \right) = 0, \quad (15)$$

$\forall k$  where  $E \equiv \sum_{k=1}^n e_k$ . Noting that  $x_k$  enters a total of  $(n - 1)$  first-order conditions and summing

(15) over all  $n$  players gives

$$\sum_{k=1}^n \frac{\partial \Pi_k^n}{\partial e_k} = \left( \frac{n}{E} - \frac{1}{E} \right) V - n - (n-1) \sum_{k=1}^n x_k = 0. \quad (16)$$

Solving (16) for  $E$  gives the aggregate productive effort as:

$$E^* = \sum_{k=1}^n e_k^* = \frac{(n-1)V}{n + (n-1) \sum_{k=1}^n x_k}. \quad (17)$$

Put  $E^*$  into (15) and solve for  $e_k$  to get:

$$e_k^* = \frac{(n-1)V}{n + (n-1) \sum_{k=1}^n x_k} - \frac{(n-1)^2 (1 + \sum_{j \neq k} x_j) V}{\left( n + (n-1) \sum_{k=1}^n x_k \right)^2}, \quad (18)$$

$k = 1, 2, 3, \dots, n$ . Notice that (18) boils down to the equations in (3) if  $n = 2$ .

Now consider stage 1. Without loss of generality, suppose that  $\alpha = 1$ . Player  $k$ 's payoff is:

$$\Omega_k^n = \frac{e_k^*}{e_k^* + \sum_{j \neq k} e_j^*} V - \left( 1 + \sum_{j \neq k} x_j \right) e_k^* - x_k, \quad (19)$$

$k = 1, 2, \dots, n$ . The first-order condition is:

$$\frac{\partial \Omega_k^n}{\partial x_k} \leq 0, \quad (20)$$

where the Kuhn-Tucker conditions imply that the derivative in (20) holds with strict equality if  $x_k > 0$ .

Consider a symmetric equilibrium:  $x_k = x_j = x^*(n)$  for  $k \neq j$ . Then

$$\frac{\partial \Omega_k}{\partial x_k} \Big|_{x_j = x_k = x^*} = \frac{(n-1)[2(n-1)V - n^3 x^*] - n^3}{n^3 [1 + (n-1)x^*]} = \frac{2(n-1)^2 V}{n^3 [1 + (n-1)x^*]} - 1, \quad (21)$$

for all  $k \neq j$ .<sup>12</sup>

Since the derivative in (21) is strictly decreasing in  $x^*(n)$ , it follows that there is no symmetric equilibrium with sabotage if

<sup>12</sup> This derivative was obtained with the assistance of the math software, Maple 14. Setting  $n = 2$  and  $V = 4$  in the derivative in (21) gives the same expression, after some straightforward algebra, as setting  $x_j = x_k$  and  $\alpha = 1$  in the derivative in (10).

$$\left. \frac{\partial \Omega_k}{\partial x_k} \right|_{x^*=0} = \frac{2(n-1)^2 V}{n^3} - 1 \leq 0. \quad (22)$$

The inequality in (22) holds if  $V$  is sufficiently low or  $n$  is sufficiently high. In particular, it is important to note that since  $n$  being sufficiently high implies zero sabotage, this is consistent with a result due to Konrad (2000, p. 164) that "... sabotage is a more important problem if few lobbying groups are in the contest, and under reasonable conditions sabotage disappears ... when the number of lobbying groups becomes large." Konrad (2000) correctly argues when player  $k$  sabotages player  $j$ , this acts as a positive externality to all other players. The larger is the number of players, the *smaller* is the benefit to the player who incurs the cost of sabotage since the total benefit of his destructive effort is shared by the other players except the player who he sabotaged (i.e., what Konrad (2000) calls the *dispersion* effect of sabotage). This discourages investment in sabotage.

The condition in (22) is a sufficient condition for the non-existence of a symmetric equilibrium with sabotage. It is not a necessary condition. To see this, note that the value of  $x$  which satisfies (20) with strict equality is  $x^*(n) > 0$ , given  $n$  players. We know that, given  $\alpha = 1$ ,  $x^*(2) = V/4 - 1 > 0$  for  $V > 4$ . It is instructive to choose numerical values for  $V > 4$  such (22) is violated, set  $x_j = x^*(n)$  for all  $j \neq k$  in  $\Omega_k^n$  as defined in (19) and plot  $\Omega_k^n$  against  $x_k$ . In several and all examples considered, we find that  $\partial \Omega_k^n \leq 0$  for  $n \geq 3$  with  $\partial \Omega_k^n = 0$  being a saddle point as illustrated in figure 3. This is the case when  $V$  is as high as 10000. Hence, there appears to be no *symmetric* equilibrium<sup>13</sup> with  $x^*(n) > 0$  when  $n \geq 3$ . Hence there may be no sabotage in equilibrium even if  $n$  is small (e.g.,  $n = 3$ ).

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<sup>13</sup> Although we cannot rule out asymmetric equilibria, we conjecture that they do not exist.

### **3. Conclusion**

Destructive effort and productive effort as modeled here may be given a different interpretation. Consider a war. If defensive weapons are intended to make it difficult for an opponent to exert effort and offensive weapons directly affect the probability of winning the conflict, then the paper suggests as the stakes of the war become sufficiently high, the combatants may increase their investment in defensive weapons by a bigger amount than the increase in offensive weapons. But perhaps in wars, defensive weapons do not actually make it difficult for the enemy to exert effort but instead diminish the impact of the enemy's offensive weapons.

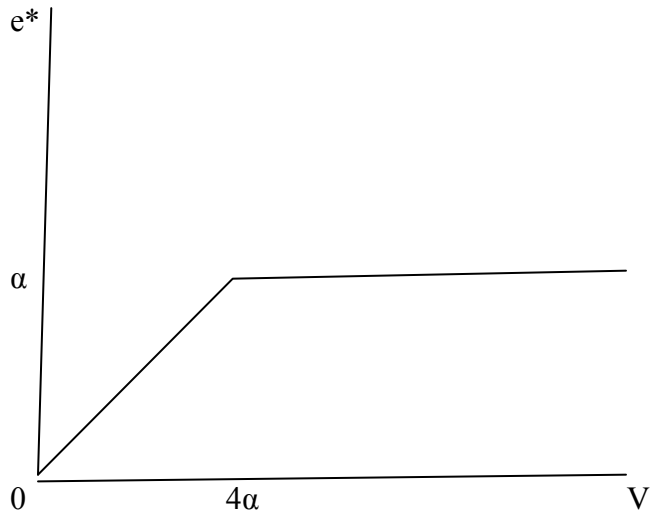
Notwithstanding the preceding interpretation, the result of this paper is consistent with contests in sports and in working environments where the players' destructive activities tend to be more responsive than their productive activities as the stakes get higher. Anecdotal and substantive evidence cited in section 1 confirm the presence of destructive activities in contests. We found that players only engage in destructive activities when productive effort reaches a threshold. This makes sense because the contest cannot be won with only destructive activities.

## References

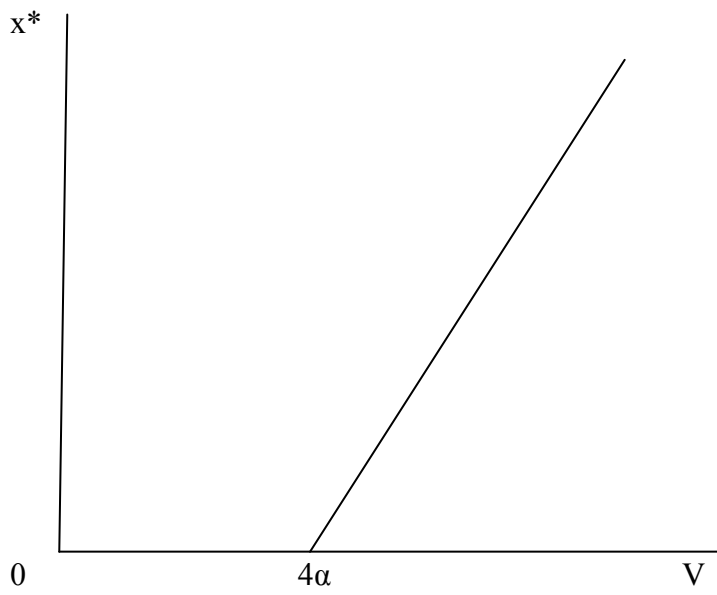
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**Figure 1:** Response of productive effort to changes in  $V$  in the two-player case



**Figure 2:** Response of destructive effort to changes in  $V$  in the two-player case



**Figure 3:** Plot of  $\Omega_k^n$  with respect to  $x_k$ , given  $V = 10$ ,  $\alpha = 1$ ,  $n = 3$ ,  $x_j = x^*(n)$  for  $j \neq k$ .

