Negative externalities and the private provision of public goods: a survey*

J. Atsu Amegashie**

Department of Economics
University of Guelph
Guelph, Ontario
Canada N1G 2W1

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Abstract

The under-provision of public goods under voluntary contributions is well understood as the result of bidirectional positive externalities associated with voluntary contributions. In this survey, I apply the reasoning in Morgan (2000) to argue that schemes which introduce negative externalities associated with voluntary contributions increase the level of public good provision because they fully or partially offset the standard positive externalities. I argue that other previous works, like Andreoni and Bergstrom (1996) and Falkinger (1996), have also used this idea without apparently recognizing it. I show how this paper differs from Brunner and Falkinger (1999).

Key words: Negative externality, positive externality, public goods, private provision.
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**E-mail: jamegash@uoguelph.ca; Phone: 519-824-4120, ext. 58945; Fax: 519-763-8497.
1. Introduction

There is a wide literature on the private provision of public goods.\(^1\) It is a standard result that private provision typically results in the under provision of the public good. The under-provision of public goods under voluntary contributions is well understood as the result of bidirectional positive externalities associated with voluntary contributions. Warr (1983) proved the remarkable result that lump-sum redistribution among contributing individuals does not change the equilibrium amount of the public good. Bernheim (1986) showed that this neutrality result holds even with distortionary taxation. In light of these results, research has focused on what policy instruments are available for influencing the private provision of a public good. Bergstrom et al. (1986) showed that the neutrality result does not hold if large redistributions that change the set of contributors are considered. Boadway et al. (1989) showed that suitably chosen transfer-tax policies could be used to achieve the efficient allocation of a public good, provided that the contributors do not see through the government’s budget constraint.\(^2\) Andreoni and Bergstrom (1996) and Falkinger (1986) show that tax-financed subsidies to private contributions can result in an increase in voluntary contributions even if contributors take the government budget constraint into account. Morgan (2000) shows that the use of raffle/lotto revenue to finance a public good can increase the provision of the public good but not to the first-best level. His intuition for this interesting result is that the purchase of raffle tickets involves a bidirectional negative externality--the purchase of a ticket reduces expected winnings for others -- which partially offsets the standard positive externalities. In this survey, I argue that this observation by Morgan can

\(^2\) See also Roberts (1987, 1992).
be extended to other models of the voluntary provision of public goods to understand why certain schemes result in an increase in the voluntary provision of public goods. Simply put, schemes which introduce negative externalities stemming from voluntary contributions will increase the provision of public goods. Indeed, the paper sheds light on previous works of authors who have applied this idea without apparently recognizing it.

I use very simple models to illustrate the idea of negative externalities in the voluntary contribution to public goods. It will become clear that the simplicity of the models does not affect my argument. Also, Brunner and Falkinger (1994) provide a more formal but non-intuitive analysis of the argument in this paper. It is, therefore, not necessary to do so in this survey. However, I shall argue that this paper offers some helpful insight into the effect of negative externalities on public goods provision. The paper is an intuitive survey of a limited part of the literature on public goods provision.

The survey does not discuss the more familiar and well-known Vickery-Clarke-Groves mechanisms. In these mechanisms, an agent pays for the negative externalities imposed on other agents. This is intended to force each agent to internalize the negative externality imposed on others. This is not the case in the schemes surveyed in this paper. Agents do not pay for the negative externalities present in these schemes. Thus, agents do not internalize the negative externalities. On the contrary, by ignoring these negative externalities, their actions may partially or fully offset the positive externalities that are ignored in public good provision. This narrows the gap between private and social marginal benefits. Hence the rationale behind the Vickrey-Clarke-Groves mechanisms is different from the schemes surveyed in this paper but they produce the same effect (i.e.,

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3 It will be clear to the reader that the negative externalities are introduced in different ways in these schemes.
they both narrow the gap between private and social marginal benefits). It also important to note that the Vickery-Clarke-Grove mechanisms do more than the mechanisms discussed below. The Vickrey-Clarke-Grove mechanisms try to induce agents to reveal their true preferences for the public good. As I explain below, the mechanisms discussed in this survey are not concerned with this kind of information. Finally, the schemes discussed here always have balanced budgets while the schemes in the Vickrey-Clarke-Groves mechanisms may not have balanced budgets.

In the next section, I review the model in Morgan (2000) and the extension of this model by Apinunmahakul and Barham (2003). In section 3, I present and extend the model in Andreoni and Bergstrom (1996). In a subsection, I examine the model in Falkinger (1996) and other models of public good provision which use the idea of negative externalities. In section 4, I discuss how this paper differs from Brunner and Falkinger (1999). Section 5 concludes the paper.

2. Morgan’s Model

In Morgan (2000), an economy consists of $n \geq 2$ consumers with quasi-linear utility function of the form $U_i = w_i + h_i(G)$, where $w_i$ is a private good which denotes the wealth of consumer $i$, $G$ denotes the level of a public good, $h'_i(\cdot) > 0$ and $h''_i(\cdot) < 0$ for all $i$. The marginal rate of transformation between the public good and the private good is 1. The efficient level of the public good, $G^*$, satisfies $\sum_{i=1}^{n} h'_i(G^*) = 1$. Let $\hat{G}$ be the level of the public good under voluntary provision. Then it is easy to show that $\sum_{i=1}^{n} h'_i(\hat{G}) > 1$, where
\( \hat{G} > 0 \) and \( h_i'(\hat{G}) \leq 1 \ \forall \ i. \) This gives the standard result that \( \hat{G} < G^* \). This result follows from the strict concavity of the \( h_i(G) \) function or more generally from the strict quasi-concavity of the utility function. For two or more players with positive contributions, the aggregate contribution level is unique but the individual contributions are not. The best response functions have slope -1 (i.e., \( g_i = h_i^{-1}(1) - \sum_{j \neq i} g_j \), where \( g_i \) is the i-th player’s contribution to the public good). This is because the utility function is quasi-linear.6

It is important to note that Morgan’s (2000) mechanism and all the mechanisms described in this survey do not require the government or mechanism designer to know any particular individual’s preference for the public good. In what follows, the government does not have to know the \( h_i(G) \) function for any individual after the game is played. All the government needs to know is that \( h_i(G) \) is increasing and strictly concave. The government does not have to know \( G^* \) but by implementing this scheme, it knows that the voluntary contributions are greater than \( \hat{G} \) but less than \( G^* \). Hence Morgan’s scheme, like the subsequent schemes discussed below, is not a preference revelation mechanism like the Vickrey-Clarke-Groves mechanisms.

I assume that each of the \( n \) consumers contributes a positive amount, so the effect of subsequent modifications of the game on equilibrium provision of the public good do not arise from changes in the set of contributors. Thus \( h_i'(\hat{G}) = 1 \ \forall \ i \) or \( \sum_{i=1}^{n} h_i'(\hat{G}) = n \).

As in Morgan (2000), suppose a government or charitable organization administers a raffle with a prize of some fixed amount \( R \). The i-th consumer chooses

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4 \( \hat{G} > 0 \) if \( h_i'(\hat{G}) = 1 \) for some \( i \).
5 Since \( h_i'(\cdot) \) is a monotonically decreasing function, its inverse function exists.
\( x_i \in [0, w_i] \) to spend on raffle tickets. His probability of winning the raffle prize is

\[
p_i = \frac{x_i}{x_i + \sum_{j \neq i} x_j}.
\]

Since the raffle must be self-financing, the public good provision is

\[
G = \sum_{i=1}^{n} x_i - R.
\]

The expected utility of the i-th consumer is

\[
EU_i = w_i - x_i + p_i R + h_i(\sum_{i=1}^{n} x_i - R) \tag{1}
\]

The i-th contestant chooses \( x_i \), given the expenditures of the other consumer, to maximize (1). An interior solution gives \( \frac{\partial EU_i}{\partial x_i} = 0 \) \( \forall \ i \). Let the level of public good provision be \( \tilde{G} \). It can be shown that \( 1 < \frac{n}{\sum_{i=1}^{n} h_i(\tilde{G})} < n \). Hence \( \hat{G} < \tilde{G} < G^* \). By introducing a raffle in this public goods game, the level of public good provision increases.\(^7\) As Morgan (2000) observes the provision of the public good increases because the raffle introduces a negative externality which counteracts the positive externality ignored by the consumers in their contributions. This narrows the gap between private and social marginal benefits resulting in an increase in the level of provision. The negative externality is due to the fact each consumer reduces the probability of winning the prize by all other consumers when he increases his contribution. Morgan (2000) notes that the level of public good is below the efficient level because the negative externalities \textit{partially} offsets the positive externalities.

Apinunmahakul and Barham (2003) introduce a two-part tariff into Morgan’s model. A participation fee is paid for the right to participate in the lottery. The revenue from the fees is added to the prize pool, so the total prize is \( R + T \), where \( T \) is the revenue

\(^7\) Morgan (2000) also shows that individual contributions are unique, although the utility function is quasi-linear.
from participation fees. They show that the first-best level of public good can be provided by setting the participation fee appropriately. Again the intuition is straightforward. The participation increases the prize of the lottery from \( R \) to \( R + T \). This intensifies the negative externality. To see this, note that in Morgan (2000) the negative marginal externality that the \( i \)-th player imposes on the other players through is purchase of a ticket is \( \sum_{j \neq i} \frac{\partial p_j}{\partial x_i} R = -\frac{\partial p_i}{\partial x_i} R \). In Apinunmahakul and Barham (2003), this externality is \( \sum_{j \neq i} \frac{\partial p_j}{\partial x_i} (R + T) = -\frac{\partial p_i}{\partial x_i} (R + T) \). Hence Apinunmahakul and Barham (2003) intensify the negative externality. Indeed, Morgan (2000) finds that the provision of the public good is increasing in \( R \).

An alternative intuitive explanation is that the lottery reduces the marginal cost of a player’s contribution. I shall argue, when I discuss Falkinger’s (1996) model, that this interpretation may not always be helpful.

3. The Andreoni-Bergstrom model

Suppose \( n \) consumers have the quasi-linear utility function above. As in Andreoni and Bergstrom (1996), suppose that for any \( x_i \) contributed by consumer \( i \), the government refunds \( sx_i \) to this consumer, where \( 0 < s < 1 \). That is, the government subsidizes contributions to the public good at a common rate of \( s \). Suppose that the total cost of the subsidy is financed by imposing a tax rate of \( t_i \) on the \( i \)-th person such that he pays \( T_i = t_i \sum_{i=1}^{n} s x_i \) in taxes, where \( \sum_{i=1}^{n} t_i = 1 \). Note that \( \sum_{i=1}^{n} s x_i = \sum_{i=1}^{n} T_i \), given \( \sum_{i=1}^{n} t_i = 1 \). So the

\[ 8 \text{Andreoni and Bergstrom (1996) consider a more general utility function. See also the discussion of this model in Falkinger (1996).} \]
government, in aggregate, gives in subsidies what it collects as taxes. One can think of this scheme as a three-stage game. In stage 1, the government chooses the subsidy rate $s$, and tax shares, $t_1, \ldots, t_n$. In stage 2, the consumers simultaneously choose their contributions. In stage 3, the government observes the vector of contributions, collects taxes and then pays subsidies. Let $\bar{G}$ be the level of public-good provision under this tax-subsidy scheme. Andreoni and Bergstrom (1996) focus on the Nash equilibrium for the consumers in stage 2.

The $i$-th consumer chooses $x_i$ to maximize

$$U_i = w_i - x_i + sx_i - T_i + h_i(G)$$

where $G = \sum_{i=1}^{n} x_i$.

Taking the derivative of $U_i$ with respect to $x_i$, setting it to zero, and summing over all $n$ consumers gives $\sum_{i=1}^{n} h_i'(\bar{G}) = n - s(n-1) < n$, yielding the result that $\bar{G} > \hat{G}$.

Andreoni and Bergstrom (1996) argue that this tax-subsidy scheme results in a higher level of public good provision because the government commits to making no direct contributions of public goods. My contention is not that this reasoning is wrong but instead to show that there is a different and more insightful explanation.

Note that tax paid by all consumers other than the $i$-th consumer is $\sum_{j \neq i} T_j = s(1-t_i) \sum_{i=1}^{n} x_i$. It is obvious that the $i$-th consumer increases the tax liabilities of all the other $(n-1)$ consumers when he increases his contribution. However, he ignores this

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9 Given a quasi-linear utility function, the aggregate value of private provision is unique but there is an infinity of individual equilibrium strategies supporting this unique aggregate contribution. With non-quasi linear preferences, individual contributions are unique if the subsidy rate, $s < 1$ and the private and public goods are normal (see Theorem 1 in Andreoni and Bergstrom, 1996).
negative externality in his choice of $x_i$. This negative externality, like in Morgan (2000), partially offsets the positive externality of public good contribution resulting in an increase in the provision of the public good.\textsuperscript{10} Andreoni and Bergstrom (1996) find that the level of public good provision is increasing in the subsidy rate, $s$

(i.e., $\frac{\partial \bar{G}}{\partial s} = -(n-1)/\sum_{i=1}^{n} h_i'(\bar{G}) > 0$). This is because the magnitude of the negative externality is increasing in the subsidy rate. Note that if instead of a negative externality we were to introduce a positive externality by specifying the tax paid by the $i$-th consumer as $t x_i$ and the subsidy received as $s_i \sum_{i=1}^{n} t x_i$, where $\sum_{i=1}^{n} s_i = 1$, the level of the public good will decrease.

Andreoni and Bergstrom (1996) argue that this tax-subsidy scheme may not have any effect on the level of public good provision if there is an “announcement stage” in which the $i$-th consumer announces a contribution function which is contingent on the contributions of all other consumers. To examine this, take the derivative of equation (2) with respect to $x_i$, taking note of the dependence of $x_i$ on $x_j$ ($\forall i \neq j$), and set it to zero. This gives

$$\frac{\partial U_i}{\partial x_i} = -1 + s - s t_i \left[ 1 + \frac{\partial (\sum_{j \neq i} x_j)}{\partial x_i} \right] + h_i'(\bar{G}) \left[ 1 + \frac{\partial (\sum_{j \neq i} x_j)}{\partial x_i} \right] = 0.$$

By assumption, $h_i'(\hat{G}) = 1 \ \forall i$ at $G = \hat{G}$. This gives $\frac{\partial (\sum_{j \neq i} x_j)}{\partial x_i} = s(t_i - 1)/(1 - s t_i) < 0 \ \forall i$. The negative sign of this derivative is consistent with the sign of the contribution function in Andreoni and Bergstrom (1996, p. 304). The negative sign implies that to neutralize the effect of the Andreoni-Bergstrom tax-subsidy scheme, each consumer must

\textsuperscript{10} This idea is also used in Varian (1994) where contributors announce subsidy rates by which they will subsidize other agents’ contributions to the public good.
announce that he will reduce his contribution, when all other consumers increase their
contributions. The intuition for this must now be obvious. By reducing his contribution in
response to an increase in the contributions of all other consumers, the i-th consumer
nullifies the effect of the negative externality\(^{11}\) which in turn reduces the equilibrium of
the game to the game with no taxes and subsidies.

3.1 The model of Falkinger (1996) and other models

Falkinger (1996) proposes the following incentive scheme for the private
provision of public goods: the net reward paid to the i-th individual who contributes \(x_i\) is
\[ \Omega_i = \beta(x_i - \bar{x}_{-i}), \]
where \(\beta \geq 0\) and \(\bar{x}_{-i} = G_{-i} / (n - 1)\) is the average contribution of the
(n-1) contributors. The i-th consumer pays a tax if his contribution falls below the
average contribution and receives a subsidy if he is above average. The parameter \(\beta\) is a
tax-subsidy rate. The government’s budget is balanced since \(\sum_{i=1}^{n} \Omega_i = 0\). Note that \(\frac{\partial \Omega_i}{\partial x_j} = -\beta / (n-1) < 0 \quad \forall \ i \neq j\). Hence Falkinger’s scheme introduces a negative externality
associated with contributions because the net reward received by an individual decreases
when other contributors increase their contribution. The magnitude of this negative
externality is increasing in \(\beta\). In view of our discussion, it is not surprising that Falkinger
(1996) finds that this scheme results in an increase in voluntary contributions and that
these contributions increase with \(\beta\). Falkinger (1996) finds that this scheme does not
provide the efficient level of the public good. To provide the efficient level of the public

\(^{11}\)Note that when the i-th consumer increases his contribution, the other n-1 consumers do not reduce their
aggregate contribution by the same amount. This because although the contribution of the i-th consumer
exerts a negative externality by increasing their tax liabilities, it also exerts a positive externality through its
effect on the level of the public good. That is, the absolute value of \(\frac{\partial (\sum_{j \neq i} x_j)}{\partial x_i} < 1 \quad \forall \ i.\)
good, he partitions the $n$ consumers into groups where the number of members in the $k$-th group is $n_k < n$ for all $k$. The same net reward function is used but the members in each group is now $n_k$. It easy to show that for the $k$-th group $\frac{\partial \Omega_{ik}}{\partial x_{jk}} = -\beta/(n_k-1) < 0$ $\forall$ $i \neq j$.

This design gives the efficient level of the public good. Falkinger (1996) does not provide an intuition for why this division of members into groups results in a higher level of public good provision.\textsuperscript{12} But the intuition is straightforward. By dividing the consumers into smaller groups, he intensifies the magnitude of the marginal negative externality. That is, $\beta/(n_k - 1) > \beta/(n - 1)$ for all $k$, given $n_k < n$. Notice that there is no change in the marginal cost of a player’s contribution when the consumers are divided into smaller groups, since $\beta$ remains unchanged. Hence an interpretation which relies on changes in the cost of contributions will not be helpful in this respect.

Andreoni and Bergstrom (1996) consider a second model due to Andreoni (1988). The only difference between this model and their model above is that the tax liability of consumer $i$ is a head tax, $\gamma_i$, and $G = \sum_{i=1}^{n} (\gamma_i - sx_i) + \sum_{i=1}^{n} x_i$. They find that for small changes in taxes and subsidies, there is no change in public good provision compared to the level of provision when taxes and subsidies are zero. Following my interpretation, I am inclined to argue that this tax-subsidy scheme has neutral effect because there is no negative externality to counteract the standard positive externalities. However, Andreoni and Bergstrom (1996) find that this neutrality result does not hold for large changes in taxes and subsidies.

Finally, consider the model in Bernheim (1986). I follow the version of Bernheim’s model described in Andreoni and Bergstrom (1996). In Bernheim (1986)\textsuperscript{12} He shows, however, that this division is necessary to guarantee that the equilibrium is unique.
distortionary transfers take the form of taxes on labor income. The economy has n
consumers and three commodities – an ordinary consumption good, leisure, and a public
good. Preferences of the i-th consumer are represented by a utility function of the form,
$U_i(G, c_i, l_i)$, where G is the total amount of the public good provided, $c_i$ is i’s
consumption of the private good, and $l_i$ is leisure. Let $m = (m_1, \ldots, m_n)$ be the vector of
labor supply choices by the n consumers. The sequence of actions is as follows: in stage
1, the government chooses a tax function, $t_i = t_i(m)$ for the i-th consumer. In stage 2,
consumers choose their labor supplies. In stage 3, consumers choose their voluntary
contributions to the public good. In stage 4, the government collects the contributions of
the consumers, taxes according to the functions, $t_i(m)$, and supplies an amount of the
public good equal to its total revenue. Bernheim (1986) finds that distortionary taxes and
transfers have no effect on the level of the public good provided. However, Bernheim
(1986, p. 791) notes that for this result to hold “…it is essen tial that contributions to the
public good be chosen subsequent to the choice of labor supply.” To see this, suppose
instead that the choice of labor supply is made after contributions to the public good.
Also, assume that $\sum_{i=1}^{n} t_i(m) = 0$. When labor supplies are chosen, aggregate contributions,
and therefore G are already known or fixed. Hence in this stage we may write the labor
supply choices as $m_i^*(G)$ for i, where $G = \sum_{i=1}^{n} x_i$. It follows that the tax function, $t_i(h)$
may be written as $t_i(m_1^*(G),\ldots,m_n^*(G))$. In the next stage, when the consumers choose
their contributions, they will impose externalities on each other through the derivative,
$\partial t_i/\partial x_j \forall i \neq j$. If $\partial t_i/\partial x_j \neq 0$, then an additional externality is generated by the choice of a

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13 The optimal labor supply will also be a function of parameters of the tax function.
consumer’s contribution. This will change Bernheim’s neutrality result.\textsuperscript{14} Note that if contributions to the public good are chosen \textit{subsequent} to the choice of labor supply, then a consumer’s choice of his contribution will not influence his labor supply and hence this will not influence the tax liabilities of other consumers.

4. \textbf{Relationship to the work of Brunner and Falkinger (1999)}

It is important to indicate how this paper differs from the related work of Brunner and Falkinger (1999). They formally show that taxation affects the equilibrium provision of the public good \textit{if and only if} at least one individual's voluntary contribution to the public good has an impact on the aggregate tax payments of others. Their paper formally strengthens the argument that equilibrium provision is affected if a negative externality associated with contributions is introduced. Drawing on the work of Morgan (2000), I have given an intuitive explanation of the condition found in Brunner and Falkinger (1999). Also, in addition to the introduction of a negative externality, I emphasize the role that the intensification of the externality plays. For example, this insight helps to \textit{intuitively} explain why the division of consumers into smaller groups in Falkinger (1996) results in a greater (the efficient) level of provision. Also, by emphasizing the general idea of negative externalities, this paper shows that proposed schemes need not be restricted to tax-subsidy schemes as examined by Brunner and Falkinger (1999). For example, Falkinger (1994) introduces a negative externality by assuming that deviations from the average contribution of others is an argument in a consumer’s utility function. This utility function may stem from a desire for status or “keeping with the joneses”. In this respect, the paper ties together seemingly different mechanisms like the tax-subsidy

\textsuperscript{14} See Brunner and Falkinger (1999) for a formal proof.
schemes of Falkinger (1996) and Andreoni and Bergstrom (1996) on one hand and the lottery mechanism of Morgan (2000) on the other hand. It does this by arguing that the common chord which runs through all these mechanisms is the introduction of a negative externality associated with voluntary contributions.

5. Conclusion

This survey has pointed out and emphasized an intuitive result in the voluntary provision of public goods: the voluntary provision of public goods will increase if negative externalities associated with contributions are introduced. As these externalities are intensified they might fully offset the standard positive externalities of public goods resulting in the efficient level of provision. This result is important because it can help us to understand why some schemes fail and others work. In this regard, the paper has shed light on the previous works of authors who have applied the idea of introducing negative externalities in public good games without apparently recognizing it.

The decentralized public goods provision problem has been largely solved (see, for example, Clarke (1971), Groves (1973), Groves and Ledyard (1977), Walker (1981), and the references cited above). However, some of these schemes are fairly complicated. The insight from this paper and Brunner and Falkinger (1999) might help us to device simpler schemes as in Falkinger (1996). As the work of Morgan (2000) shows the negative externality need not be introduced through explicit tax-subsidy policies. This is helpful especially in situations where the provider of the public good does not have the

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15 Sections 1 and 2 explained how the schemes surveyed in this paper differ from the Vickrey-Clarke-Groves mechanisms.
power to tax. In Morgan’s case, the losers of the raffle pay an implicit tax which is transferred to the winner.

**References**


