Adverse selection in competing all-pay auctions**

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Abstract

We study two all-pay auctions, where contestants cannot compete in both auctions. One contest has a higher prize than the other. Ideally, high-ability contestants should participate in the contest with a higher prize. We find that this is not necessarily the case. We show that the top contestant may choose to participate in the contest with a lower prize.

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1. Introduction

Contests are an important fact and pervasive aspect of economic life. A contest is a game in which players compete over a prize by making irreversible outlays. Election campaigns, rent-seeking games, R & D races, competition for monopolies, litigation, wars, and sports are all examples of contests.

There is now a wide literature on contests and tournaments. One area of study is the allocative efficiency of contests. Rosen and Lazear (1981) examine a model in which high-ability contestants have the incentive to participate in contests designed for low-ability contestants. Baye et al. (1993) find, in a single stage all-pay auction, that the contest-designer might find it optimal to exclude some high-ability agents from the contest. Arbatskaya (2003) finds that the incentive to exclude some contestants exists in contests with symmetric players. An inefficiency result in all-pay auctions is also found in Konrad (2004). In winner-pay auctions, it is known that there are inefficiencies in multi-unit auctions. For example, in a multi-unit auction in which two units of an object is auctioned off to two agents, Krishna (1993, p. 150) finds that "… even if one agent's marginal valuations are always greater than those of the other, he need not obtain all the units." There are also papers which elaborately study competition between sellers via auctions.[1] However, these auctions are not all-pay auctions.

In this paper, we consider two all-pay auctions each with a single prize. One contest has higher prizes than the other. *Ceteris paribus*, each player prefers to participate in the contest with higher prizes but strategic considerations may prevent her from doing so. A high-ability player might prefer to participate in the inferior contest in order to to avoid

[1] McAfee (1993), Peters and Severinov (1997), and Ellison, Fudenberg, and Moebius (2003).

competition with other high-ability contestants and thereby increase her chances of success. We shall show that this behavior may be an equilibrium strategy.

2. A model of competing all-pay auctions

Consider N risk-neutral contestants who may participate in *only one* of two contests: a high-quality contest and a low-quality contest. Each contest has a single prize and $N \geq 4$. One may think of the high-quality contest as competition into an elite school, journal, job, etc.

Suppose the i-th contestant has valuation $\overline{V}_i$ for the prize of the high-quality contest and $\hat{V}_i$ for prize of the low-quality contest, where $\overline{V}_i > \hat{V}_i \; \forall$ i. Let the valuations be ordered as follows: $\overline{V}_1 > \overline{V}_2 > ... > \overline{V}_{N-1} > \overline{V}_N$ and $\hat{V}_1 > \hat{V}_2 > ... > \hat{V}_{N-1} > \hat{V}_N > 0$. As shown by Baye et al. (1996) and Clark and Riis (1998), a contestant with a higher valuation is analytically equivalent to a contestant with a lower cost of expending effort. So the higher is the valuation of a contestant, the higher is his ability.

We assume that the valuation of a contestant is common knowledge among all contestants but the contest designers do not know a contestant's valuation. In each contest, the player with the highest effort wins the prize. Thus, for those in the same contest, the competition amongst them is an all-pay auction.

We restrict the analysis to the following question: do high-ability contestants necessarily participate in the high-quality contest? We do not focus on the welfare implications of the aggregate efforts expended by contestants in the contests.

As in Baye et. al (1996) and Clark and Riis (1998), a player who bids zero with probability 1 is considered to be in the set of contestants. In our model, this means that a contestant will participate in a contest, even if his payoff is zero or even if he intends to bid zero. Hence each contestant will participate in either the low-quality contest or the high-quality contest.

The timing of actions is as follows: In stage 1, contestants decide to participate in either the high-quality contest or the low-quality contest but not both. A contestant's choice of contest is common knowledge. In stage 2, each contestant chooses how much effort to expend.

Contestants can change their choice of contest in stage 1 but *not* in stage 2. Of course, rational players in stage 1 will make their choice of contest anticipating the outcome of the game in stage 2.

2.1 Equilibrium analysis

Our equilibrium concept is subgame perfection. A subgame perfect Nash equilibrium will have the following properties: (i) in stage 1, no contestant has the incentive to change her choice of contest, given the contest choices of all other contestants and anticipating the equilibrium of the game in stage 2; and (ii) in stage 2, no contestant has the incentive to change her effort allocation.

In stage 1, we first look for an equilibrium in pure strategies and then focus on mixed strategies. In stage 2, it is well known that the equilibrium is typically in mixed strategies.

The equilibrium of an all-pay auction with a single prize and complete information has been characterized by Baye et al. (1996).[2] We state, without proof, the following result in their paper:

Lemma 1 (Baye, Kovenock, and de Vries, 1996): *Consider a complete information all-pay auction where m contestants with valuations, $V_1 > V_2 > \ldots > V_{m-1} > V_m$, compete for a single prize. There exists a unique mixed-strategy Nash equilibrium in which the contestants with the two highest valuations participate actively in the contest with expected payoffs $V_i - V_2 \geq 0$ $(i = 1, 2)$. The other contestants, $i = 3, \ldots, m-1, m$ bid zero and get an expected payoff of zero.*

Without any loss of generality, we consider, for now, the case where there is a total of N = 4 contestants (1, 2, 3, and 4). We shall examine all the possible partitions of contestants between the high-quality contest and the low-quality contest. Let the set of players who participate in the high and low quality contests be S^H and S^L respectively.

It is important to note that there is no pure-strategy equilibrium in which 1 and 2 participate in the same contest.[3] Hence we restrict the analysis to the following eight partitions: (a) $S^L = \{1,4\}$; $S^H = \{2, 3\}$, (b) $S^L = \{1,3\}$, $S^H = \{2, 4\}$; (c) $S^L = \{2,3\}$, $S^H = \{1, 4\}$; (d) $S^L = \{2, 4\}$, $S^H = \{1, 3\}$; (e) $S^L = \{1\}$, $S^H = \{2, 3, 4\}$; (f) $S^L = \{2,3, 4\}$, $S^H = \{1\}$; (g) $S^L = \{2\}$, $S^H = \{1,3,4\}$; and (h) $S^L = \{1,3,4\}$, $S^H = \{2\}$. We assume that $\overline{V}_4 - \hat{V}_2 > 0$.

In all these partitions (i.e., (a) to (h)), players 3 and 4 will not change their choice of contest because their payoff will be zero if they did so; there is no gain from deviation. Since in all the partitions above, players 1 and 2 are in different contests, player 2 will

[2] Barut and Kovenock (1998), Glazer and Hassin (1988), and Moldovanu and Sela (2001) study all-pay auctions with multiple prizes.
[3] The proof is simple. If the top two players participate in the same contest, one of them will earn a zero payoff. If one of them were to deviate to the other contest, she will win earn a strictly positive payoff since no other contestant has a higher valuation than the top two players.

also not deviate because she will not like to be in the same contest with player 1. Hence we focus on only player 1 (i.e., the contestant with highest valuation).

Consider (a): Based on lemma 1, player 1 will not deviate if $\hat{V}_1 - \hat{V}_4 \geq \overline{V}_1 - \overline{V}_2$. So player 1 will not participate in the high-quality contest if

$$\overline{V}_1 - \hat{V}_1 \leq \overline{V}_2 - \hat{V}_4 \qquad (1)$$

Consider (b): Based on lemma 1, player 1 will *not* deviate if $\hat{V}_1 - \hat{V}_3 \geq \overline{V}_1 - \overline{V}_2$. So player 1 will *not* participate in the high-quality contest if

$$\overline{V}_1 - \hat{V}_1 \leq \overline{V}_2 - \hat{V}_3 \qquad (2)$$

If (2) holds, then (1) will also hold. Therefore, the partitions in (a) and (b) are equilibrium partitions if (2) holds.

Consider (c): player 1 will deviate and participate in the low-quality contest if

$\overline{V}_1 - \overline{V}_4 < \hat{V}_1 - \hat{V}_2$ or if

$$\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2 \qquad (3)$$

Consider (d): player 1 will deviate and participate in the low-quality contest if

$$\overline{V}_1 - \overline{V}_3 < \hat{V}_1 - \hat{V}_2 \text{ or if } \overline{V}_1 - \hat{V}_1 < \overline{V}_3 - \hat{V}_2 \qquad (4)$$

Equation (4) holds, if (3) holds.

Consider (e): player 1 will *not* deviate if $\hat{V}_1 \geq \overline{V}_1 - \overline{V}_2$. So player 1 will *not* participate in the high-quality contest if $\overline{V}_1 - \hat{V}_1 \leq \overline{V}_2$. This condition holds if (2) holds.

So a partition in which contestant 1 participates in the low-quality contest is an equilibrium partition if inequality (2) holds and a partition in which contestant 1 participates in the high-quality contest is *not* an equilibrium partition if inequality (3) holds. The inequality in (2) holds, if the inequality in (3) holds. Hence the inequalities above hold if $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$. It is straightforward to show that the partition in (g) is not an equilibrium if $\overline{V}_1 - \hat{V}_1 < \overline{V}_3 - \hat{V}_2$. This inequality holds given $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$. It is also straightforward to show that the partition in (h) is an equilibrium partition if $\overline{V}_1 - \hat{V}_1 < \overline{V}_2 - \hat{V}_3$. This also holds given $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$. So except, for the partition in (f), there is no equilibrium partition in which contestant 1 participates in the high-quality contest but there are equilibria in which this contestant participates in the low-quality contest, if $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$ (i.e., inequality in (3)).

Finally, consider (f): player 1 will *not* deviate and participate in the low-quality contest, regardless of the valuations of the other contestants. Hence, this is an equilibrium partition.

Hence there are five subgame perfect pure-strategy equilibrium partitions in stage 1: (a), (b), (e), (f), and (h). Now suppose $N \geq 4$. Notice that, except for the equilibrium in partition (f), each contest will have two active contestants as shown in Baye et al. (1996). It follows that for any $N \geq 4$, only the top four contestants will be active participants, since there cannot be an equilibrium in which a contestant with a lower valuation is an active participant but a contestant with a higher valuation is not. Hence the equilibrium partitions found will remain unchanged. This result is summarized as follows:

Proposition 1: If $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$, $N \geq 4$, then there are five subgame perfect pure-strategy Nash equilibrium partitions {(a), (b), (e), (f), and (h)} in stage 1. But in four of these five equilibria, the contestant with the highest ability participates in the low-quality contest (i.e., does not participate in the high-quality contest).

The condition $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$ holds, if $\overline{V}_4$ is sufficiently high and/or if $\hat{V}_2$ is sufficiently low. Given that contestant 1 is in the low-quality contest, he has no incentive to deviate if the weakest possible contestant she will compete with in the high-quality contest has a sufficiently high valuation (i.e., if $\overline{V}_4$ is sufficiently high). On the other hand, given that she is in the high-quality contest, she has the incentive to deviate if the strongest possible contestant he will compete with in the low-quality contest has a sufficiently low valuation (i.e., if $\hat{V}_2$ is sufficiently low). That is, contestant 1 will participate in the low-quality contest, if the competition in that contest is sufficiently less intense relative to the competition in the high-quality contest. Notice that the intensity of competition to any contest is determined by the marginal active contestant (i.e., the contestant who bids a positive bid with positive probability and gets a zero expected payoff).

We have not been able to use any of the known equilibrium refinements to choose among the pure-strategy equilibrium partitions in proposition 1.[4] To give us further insight into the conditions under which one of these equilibria may arise, we look for an equilibrium in mixed strategies in stage 1.

Suppose N = 4. Since players 3 and 4 earn a zero payoff in every equilibrium in proposition 1, we assume that they choose to participate in either contest with a probability of 0.5. Let p the probability that player 1 will participate in the high-quality

[4] One may argue that the equilibrium partition in which only contestant 1 participates in the high-quality contest (i.e., (f)) is the unique equilibrium of this game since it exists regardless of the contestants' relative valuations. However, note that, this equilibrium is weakly preferred by players 3 and 4. Hence even if player 1 commits to a strategy of participating in only the high-quality contest, players 3 and 4 may still participate in the high-quality contest. Then player 1 will have the incentive to change his contest choice if $\overline{v}_4$ is sufficiently high and/or $\hat{v}_2$ is sufficiently low.

contest and let q be similarly defined for player 2. We assume that in any mixed-strategy equilibrium, only the equilibria in proposition 1 can exist. For example, if players 1 and 2 happen to find themselves in the same contest, one of them will deviate to the other contest. One can think of a mixed-strategy equilibrium in this case as follows: players 1 and 2 decide to participate in the high-quality contest through some random process with probabilities p and q respectively. They apply to the low-quality contest with the probabilities (1-p) and (1-q) respectively. If one of the equilibria in proposition 1 is realized in this random process, then we have an equilibrium. Otherwise, the process is repeated till one of the equilibria in proposition 1 is obtained. Since it is not an equilibrium for players 1 and 2 to be in the same contest, it follows that in a mixed-strategy equilibrium, player 1 will apply to the low-quality contest with probability q(1-p).

Suppose player 1 participates in the high-quality contest. Then her expected payoff is $E_1(H) = 0.5(0.5)(1-q)\overline{V}_1$. This corresponds to the equilibrium in (f). If she participates in the low-quality contest, her expected payoff is

$$E_1(L) = 0.5(0.5)q(\hat{V}_1 - \hat{V}_4) + 0.5(0.5)q(\hat{V}_1 - \hat{V}_3) + 0.5(0.5)q\hat{V}_1 + 0.5(0.5)q(\hat{V}_1 - \hat{V}_3).$$

Each term corresponds to the payoffs in the equilibrium partitions (a), (b), (e), and (h).

Similarly, if player 2 participates in the low-quality contest, her expected payoff is $E_2(L) = 0.5(0.5)p(\hat{V}_2 - \hat{V}_3)$. If she participates in the high-quality contest, her expected payoff is

$$E_2(H) = 0.5(0.5)(1-p)(\overline{V}_2 - \overline{V}_3) + 0.5(0.5)(1-p)(\overline{V}_2 - \overline{V}_4) + 0.5(0.5)(1-p)\overline{V}_2 + 0.5(0.5)(1-p)(\overline{V}_2 - \overline{V}_3)$$

A mixed-strategy equilibrium requires that $E_1(H) = E_1(L)$ and $E_2(H) = E_2(L)$. This gives $p^* = \frac{\theta}{\theta + (\hat{V}_2 - \hat{V}_3)} < 1$ and $q^* = \frac{\overline{V}_1}{\overline{V}_1 + \phi} < 1$, where $\theta \equiv 4\overline{V}_2 - 2\overline{V}_3 - \overline{V}_4 > 0$ and $\phi \equiv 4\hat{V}_2 - 2\hat{V}_3 - \hat{V}_4 > 0$. Note that $p^* > 0.5$ since

$\theta \equiv 4\overline{V}_2 - 2\overline{V}_3 - \overline{V}_4 = \overline{V}_2 + 2(\overline{V}_2 - \overline{V}_3) + (\overline{V}_2 - \overline{V}_4) > \hat{V}_2 - \hat{V}_3.$

This leads to the following proposition:

Proposition 2: If N = 4, and $\overline{V}_1 - \hat{V}_1 < \overline{V}_4 - \hat{V}_2$ then there exists a mixed-strategy subgame perfect equilibrium in stage 1, where the most able contestant participates in the low-quality contest with probability, $(1-p^*)q^* \in (0, 0.5)$ and the second-best contestant participates in the high-quality contest with the *same* probability, where

$q^* = \overline{V}_1 / (\overline{V}_1 + \phi)$, $\phi \equiv 4\hat{V}_2 - 2\hat{V}_3 - \hat{V}_4 > 0$, $p^* = \theta / (\theta + \hat{V}_2 - \hat{V}_3)$ and

$\theta \equiv 4\overline{V}_2 - 2\overline{V}_3 - \overline{V}_4 > 0$.

Following a similar analysis, it straightforward to prove the following result:

Proposition 3: If $\overline{V}_1 - \hat{V}_1 \geq \overline{V}_2$, $N \geq 4$, then there are four subgame perfect pure-strategy Nash equilibrium partitions {(c), (d), (f), (g)} in stage 1. In each of these equilibria, the most able contestant participates in the high-quality contest.

3. Discussion and conclusion

It is important to point out a difference between our model and a model where effort affects success with noise as in Rosen and Lazear (1981). In their model, low-ability types strictly prefer to participate in contests which have several high-ability types because the prizes might be higher and their expected payoff need not be zero even if the number of high-ability contestants is equal to the number of prizes. In our model, this does not occur because in an all-pay auction with a single prize, any contestant whose valuation is below the highest valuation gets a zero expected payoff.

A limitation of our analysis is that we have not modeled a contest designer's choice of the parameters of the contest (i.e., mechanism design). We have restricted both contest designers to all-pay auctions. But as Baye et al. (1993), Clark and Riis (1998), Fullerton and McAfee (1999), Konrad (2004) and other authors have argued contest designers may find it optimal to run all-pay auctions.

Our analysis helps to reveal an important difference between competing contests *with* noise and those *without* noise. Our analysis shows that competing contests without noise could yield an inefficient result which is different from the kind of inefficiency found in Lazear and Rosen (1981). That is, high-ability contestants may strictly prefer to participate in contests intended for low-ability contestants.

References

Arbatskaya, M. (2003). The exclusion principle for symmetric multi-prize all-pay auctions with endogenous valuations. *Economics Letters* 80: 73-80.

Baye, M.R., Kovenock, D., and de Vries, C.G. (1993). Rigging the lobbying process: an application of the all-pay auction. *American Economic Review* 83: 289-294.

Baye, M.R., Kovenock, D., and de Vries, C.G. (1996). The all-pay auction with complete information. *Economic Theory* 8: 291-305.

Barut, Y., and Kovenock, D. (1998). The symmetric multiple prize all-pay auction with complete information. *European Journal of Political Economy* 14: 627-644.

Clark, D.J., and Riis, C. (1998). Competition over more than one prize. *American Economic Review* 88: 276-289.

Ellison, G., Fudenberg, D, and Moebius, M. (2003). Competing auctions. *Journal of the European Economic Association* 2: 30-66.

Fullerton, R.L., and McAfee, R.P. (1999). Auctioning entry into tournaments. *Journal of Political Economy* 107: 573-605.

Glazer, A., and Hassin, R. (1988). Optimal contests. *Economic Inquiry* 26: 133-143.

Konrad, K. (2004). Bidding in hierarchies. *European Economic Review* 48: 1301-1308.

Krishna, K. (1993). Auctions with endogenous valuations: the persistence of monopoly revisited. *American Economic Review* 83: 147-160.

Lazear, E., and Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy* 89: 841-864.

McAfee, R.P. (1993). Mechanism design by competing sellers. *Econometrica* 1281-1312.

Moldovanu, B., and Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review* 91: 542-558.

Peters, M., and Severinov, S. (1997). Competition among sellers who offer auctions instead of prices. *Journal of Economic Theory* 75: 149-171.