ASYMMETRY AND COLLUSION IN INFINITELY REPEATED CONTESTS

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Abstract

I show that in an infinitely repeated contest, collusion is easier to sustain when the players are more equal than when they are less equal. Therefore, aggregate rent-seeking expenditures may be lower when the contestants are more equally matched. This is in sharp contrast to the standard result in static rent-seeking contests where increasing asymmetries between players reduce aggregate expenditures. I also discuss applications of this result including its implications for signaling in contests with incomplete information.

Key words: aggregate expenditures, asymmetry, collusion, folk theorem.
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1. Introduction

There is now a wide literature on contests. Earlier research focused on static, single-stage contests. In recent years there has been a growing literature on dynamic contests (Amegashie, 1999; Annen, 2006; Gradstein and Konrad, 1999; Groh, et al., 2003; Halvor and Moene, 2006; Klumpp and Harbaugh, 2005; Moldovanu and Sela, 2004; Skaperdas and Syropoulous, 1996; Stein and Rapoport, 2005).[1]

In his note, I examine a complete-information, infinitely-repeated contest between two players using Tullock's framework. I shall show that in an infinitely repeated contest, collusion is easier to sustain when the players are more equal than when they are less equal. Therefore, aggregate rent-seeking expenditures may be lower when the contestants a more equally matched. This is in sharp contrast to the standard result in the literature on static rent-seeking contests where increasing asymmetries between players reduces aggregate expenditures (see Hillman and Riley, 1989; Baye et al, 1993; Che and Gale, 1998).

One of the earliest papers to study infinitely repeated contests using the Tullock framework was Leininger and Yang (1994). There are two key differences between my model and theirs: (1) they focus on homogenous players, while I focus on heterogeneous players, and (2) Their main concern is the comparison between expenditures in repeated contests and static contests, while I compare expenditures within the same infinitely repeated contest for different degrees of heterogeneity between the contestants.

[1] Rosen (1986) was the first article on elimination contests, at least, in economics.

I also discuss how the result that less asymmetry makes it easier to sustain collusion has implications for signaling in contests with incomplete information. I also discuss other applications.

2. An infinitely repeated contest with complete information

Consider a contest with two risk-neutral players, 1 and 2. In each period, t = 0, 1, 2, 3, ..., an identical prize is awarded to one of the contestants. The value of this prize is V + k to player 1 and V- k to player 2, where $V > k \geq 0$. Note that player 1 has a higher valuation than player 2, since $V + k \geq V - k$. Note also that given the specification of the valuations, the mean valuation remains unchanged. So a change in k may be considered as a mean-preserving spread.

Let x_i^t be the effort of the i-th player in the t-th period, i = 1, 2 and let the corresponding cost of effort be x_i^t. I use a logit contest success function. I assume that $P_i^t = \dfrac{x_i^t}{x_i^t + x_j^t}$ is the success probability of the i-th player in period t, $i \neq j$, t = 0, 1, 2, 3...

If both players exert zero effort, then each has a success probability of 0.5. Let $0 < \delta < 1$ be the common discount factor for both players.

Since the game in each period is identical and stationary, it is easy to show that, the sub-game perfect Nash equilibrium expected payoffs of players 1 and 2 in each period are $\pi_1^N = \frac{(V+k)^3}{4V^2}$ and $\pi_2^N = \frac{(V-k)^3}{4V^2}$ respectively (see Nti, 1999). The corresponding effort levels are $\hat{x}_1^t = \frac{(V+k)^2(V-k)}{4V^2}$ and $\hat{x}_2^t = \frac{(V+k)(V-k)^2}{4V^2}$ for t = 0, 1, 2, …[2]

However, the players can do better in this game by colluding. Suppose they set $x_i^t = 0$ in each period, i = 1, 2 and t = 0, 1, 2,… Then their expected payoff is now $\pi_1^C = \frac{(V+k)}{2}$ and $\pi_2^C = \frac{(V-k)}{2}$. Then $\pi_2^C \geq \pi_2^N$ if $V^2 + 2Vk - k^2 \geq 0$. This condition holds for all V and k, given V > k. For player 1, $\pi_1^C \geq \pi_1^N$ if $\theta \equiv V^2 - 2Vk - k^2 \geq 0$. This does not hold for all k and V. Note that $\theta > 0$ if $k = 0$ and $\theta = 0$ if $k = \bar{k} \equiv V\sqrt{2} - V > 0$. Since θ is monotonically decreasing in k, it follows that any $k \in [0, \bar{k})$ gives $\theta \equiv V^2 - 2Vk - k^2 > 0$. It is not surprising that player 1's per-period expected payoff under collusion is not necessarily higher than his per-period payoff in the Nash equilibrium. This is because when k is sufficiently high (i.e., $k > \bar{k}$), his success probability in the Nash equilibrium is sufficiently higher than his success probability under collusion to make the increase in effort worthwhile. In what follows, I restrict the analysis to $k \in [0, \bar{k})$, so that the collusive equilibrium of $x_i^t = 0$ in each period t, Pareto dominates the Nash equilibrium in each period t = 0, 1, 2, 3… and i = 1, 2.

I now want to show that the players can sustain a collusive outcome in this game. Without any loss of generality but for the sake of simplicity, I use the well-known folk theorem in Friedman (1971) where a player punishes a deviator by reverting to the Nash equilibrium forever. I recognize that the strategies in Abreu (1986, 1988) can sustain

[2] It can be shown that the best-response functions are non-monotonic. Indeed, they are strictly concave. Since they cross only once and there is no equilibrium in which a player exerts a zero effort, the pure-strategy equilibrium in which each player exerts a positive effort is unique.

cooperation for wider set of parameter values than the trigger strategy in Friedman (1971). Abreu (1986, 1988) devices a punishment strategy which sustains cooperation in cases where a trigger strategy fails to do so. However, using a trigger strategy is sufficient to prove my main result in proposition 1 and using better punishment strategies will not change this result.

Suppose a player decides to deviate from a collusive equilibrium, then a small but positive effort $\varepsilon > 0$ is optimal since that will guarantee success in that period. If player 1 is the deviator, his payoff is $\pi_1^D = V + k - \varepsilon$ and for player 2, it is $\pi_2^D = V - k - \varepsilon$. Then assuming that players use a trigger strategy as in Friedman (1971), player 1 will cooperate in every period if[3]

$$\delta \geq \frac{\pi_1^D - \pi_1^C}{\pi_1^D - \pi_1^N} = \frac{2V^2(V + k - 2\varepsilon)}{3V^3 + kV^2 - 4\varepsilon V^2 - 3Vk^2 - k^3} \tag{1}$$

and player 2 will cooperate in every period if

$$\delta \geq \frac{\pi_2^D - \pi_2^C}{\pi_2^D - \pi_2^N} = \frac{2V^2(V - k - 2\varepsilon)}{3V^3 - kV^2 - 4\varepsilon V^2 - 3Vk^2 + k^3} \tag{2}$$

Taking the limit of the expressions in (1) and (2) as $\varepsilon \to 0$ gives

$$\delta \geq \frac{2V^2}{3V^2 - 2Vk - k^2} \equiv \underline{\delta}_1 \tag{1a}$$

and

$$\delta \geq \frac{2V^2}{3V^2 + 2Vk - k^2} \equiv \underline{\delta}_2 \tag{2a}$$

[3] I omit the proofs of these results, since this version of the folk theorem is well-known and standard in most elementary game theory textbooks (see for example, Gibbons, 1992).

First, note that the denominator of both expressions in (1a) and (1b) is positive since $k \in [0, \bar{k})$ implies that $\theta \equiv V^2 - 2Vk - k^2 > 0$. Therefore, $\underline{\delta}_1$ and $\underline{\delta}_2$ are both positive. Also, $\underline{\delta}_1$ and $\underline{\delta}_2$ are both less than 1, since $\theta \equiv V^2 - 2Vk - k^2 > 0$. Finally, note that $\underline{\delta}_1 \geq \underline{\delta}_2$. Therefore, if the player with a higher valuation (i.e., player 1) is willing to collude, then player 2 will also collude. I summarize the analysis in the following lemma:

Lemma 1: *Consider an infinitely repeated contest between two players, 1 and 2, where there is a contestable prize in each period and the players have a common discount factor of* $0 < \delta < 1$. *Player 1 values the prize at* $V + k$ *and player 2 values the prize at* $V - k$. *Then there exists a collusive subgame perfect equilibrium in which each player exerts zero effort in each period, if* $k \in [0, \bar{k})$ *and* $\delta \geq \underline{\delta}_1$, *where* $\bar{k} \equiv V\sqrt{2} - V < V$ *and* $\underline{\delta}_1 \equiv 2V^2/(3V^2 - 2Vk - k^2) < 1$.

Notice that since $\underline{\delta}_1$ is increasing in k, collusion is easier to sustain when k is small than when k is large. Notice also that the higher is k, the more unequal are the players. Player 1 is much stronger than player 2. Indeed, the result that collusion can be sustained in this prisoner-dilemma-type environment is not surprising or novel. The novel result of this paper is that aggregate expenditures may be lower when the contestants are more equal since the collusive outcome gives the lowest aggregate expenditures (efforts) in the contest. Therefore, we state our main result in the following proposition:

Proposition 1: *In an infinitely repeated contest, aggregate expenditures may be lower when the players are equally matched than when they are less equal.*

3. Discussion and Conclusion

The result in proposition 1 is in sharp contrast to the standard result in the rent-seeking literature. In a static rent-seeking game, aggregate expenditures are lower, the more unequal the players are. This result was first shown by Hillman and Riley (1989) and it is the intuition or driving force behind the results in Baye et al (1993) Che and Gale (1998) and Epstein and Nitzan (2006). For example, in Epstein and Nitzan (2006), a reduction in the prize can result in higher aggregate expenditures, if it makes the players' valuations more equal (i.e., the valuation of the player with the higher valuations falls more than the valuation of the player with the lower valuation).

Lemma 1 shows that in a repeated contest, collusion is easier if the players are more equal which could lead to a reduction in aggregate expenditures. The intuition is simple. If the contest is too intense because the players are equally-matched, then in repeated interactions there are more gains from "smoking the peace pipe" rather than engaging in a wasteful and protracted battle. This might explain why an arms race can eventually lead to a truce or disarmament. As the competing factions or countries become more equal through an arms race (e.g., the USA and the former Soviet Union during the cold war), they might find it worthwhile to collude in latter periods (i.e., stop accumulating arms, even if they do not disarm). This may also partly explain why some conflicts begin with fierce fighting in the initial stages where each party exaggerates its relative strength as a result of incomplete information but might be less intense in latter periods as the parties begin to find out each other's true abilities and discover that they are almost equally matched after all.

As shown in Amegashie and Kutsoati (2005), in a one-shot conflict or a conflict which is finitely repeated, a third-party will not intervene by helping the weaker faction, if his goal is to minimize the aggregate cost of conflict. Helping the weaker faction makes the combatants less heterogeneous which increases aggregate effort. However, in an infinitely repeated conflict or a conflict which is expected to be very protracted in the absence of third-party intervention, proposition 1 suggests that a third party might intervene by helping the weaker faction because this may induce cooperation.[4]

Proposition 1 may also explain why two very strong personalities can choose to co-exist peacefully or work together in a team, organization, or household while a weak and a strong personality may always clash with each other. It also explains why a fight between two very high-ability boxers may not live up to the hype surrounding it. This is because the boxers tend to have mutual respect for each other leading them to be overly cautious in the fight.[5] Being overly cautious may be observationally equivalent to collusion.

The result in this paper has implications for signaling in incomplete-information contests. A recent line of research focuses on signaling in two-stage contests (Horner and Sahuguet (2005), Munster (2004) and Netzer and Wiermann (2005)).[6] An interesting and novel result that emerges from these papers (e.g., Horner and Sahuguet (2005)) is that both *sandbagging* (holding back effort to minimize competition) and *bluffing* (increasing effort to deter competition) are possible equilibria. In these papers, an informed player signals his ability to an uninformed player through his effort in the first round of the

[4] For a more elaborate discussion of third-party intervention in conflicts, see Amegashie and Kutsoati (2005).
[5] For example, Oscar dela Hoya-Shane Mosley (I and II) and Bernard Hopkins-Jermaine Taylor II.
[6] There is also a literature on signaling in standard auctions (i.e., *only* winner pays his bid). This literature is reviewed and extended in Goeree (2003) and Horner and Sahuguet (2005).

contest. A high initial effort is intended to signal high ability and to deter the uninformed player from competing fiercely in subsequent rounds. However, this strategy will not be used if the uninformed player is sufficiently strong. That is, the informed player will not use a high effort to scare off a very strong player. For example, in Horner and Sahuguet (2005), if the informed player knows that a high initial bid will be matched by the uninformed player, then he will rather bid low to signal to the uniformed player to soften competition in subsequent rounds. That is, if he knows that his bluff will be called, then he will not bluff. For example, Horner and Sahuguet (2005, p. 17) note that player 2 (i.e. the second-mover) "… has therefore two reasons to give up after a high bid: covering is expensive, and the opponent is strong." While this intuition is correct in Horner and Sahuguet's (2005) two-round contest, proposition 1 suggests that, in an infinitely-repeated contest, an informed player might use a jump bid in the opening round, even if he knows that his opponent is very strong and will match his bid. This is because the first-mover in the opening round might use a high bid as a signal to the first player that they should collude in subsequent rounds. And therefore the second-mover will not give up if he observes a high bid by the first-mover nor will the first-mover give up if his high bid is matched.[7]

[7] In an infinitely repeated contest, Mehlum and Moene (2006) obtain a result similar to proposition 1 but for a different reason. In their model, the asymmetry between two contestants is captured through differences in their cost of exerting effort. An incumbent contestant has a lower cost of exerting effort. If the challenger wins today, he enjoys the lower cost advantage in the next period and the former incumbent will have a higher cost disadvantage. Therefore, the contestants are fighting over the cost advantage of incumbency. The bigger is the difference between the high marginal cost of effort and the low marginal cost of effort, the bigger is this cost advantage. Hence aggregate expenditures may be higher, the higher is the asymmetry in costs. A key difference between my model and theirs is that asymmetry in my model is permanent. It is player-specific and remains so in every period of the contest. In their model, these asymmetries are contestable. Hence my result applies to infinitely repeated contests where there are permanent player-specific cost advantages.

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