# (NON)INTERVENTION IN INTRA-STATE CONFLICTS**


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## APPENDIX A: PROOF OF LEMMA 2

Consider a contest with two risk-neutral players, 1 and 2. In each period, t = 0, 1, 2, 3, ..., an identical prize is awarded to one of the contestants. The value of this prize is V + k to player 1 and V- k to player 2, where $V > k \geq 0$. Note that given the specification of the valuations, the mean valuation remains unchanged.

Let $e_i^t$ be the effort of the i-th player in the t-th period and let the corresponding cost of effort be $e_i^t$. , i = 1, 2 and t = 1, 2, 3... Let $0 < \delta < 1$ be the common discount factor for both players.

Since the game in each period is identical and stationary, it is easy to show that, the sub-game perfect Nash equilibrium expected payoffs of players 1 and 2 in each period are $\pi_1^N = \frac{(V+k)^3}{4V^2}$ and $\pi_2^N = \frac{(V-k)^3}{4V^2}$ respectively (see Nti, 1999). The corresponding effort levels are $\hat{e}_1^t = \frac{(V+k)^2(V-k)}{4V^2}$ and $\hat{e}_2^t = \frac{(V+k)(V-k)^2}{4V^2}$ for t = 0, 1, 2, …, ∞

However, the players can do better in this game by colluding. Suppose they set

$e_i^t = 0$ in each period, i = 1, 2 and t = 0, 1, 2, … Then their expected payoff is now $\pi_1^C = \frac{(V+k)}{2}$ and $\pi_2^C = \frac{(V-k)}{2}$. Then $\pi_2^C \geq \pi_2^N$ if $V^2 + 2Vk - k^2 \geq 0$. This condition holds for all V and k, given V > k. For player 1, $\pi_1^C \geq \pi_1^N$ if $\mu \equiv V^2 - 2Vk - k^2 \geq 0$. This does not hold for all k and V. Note that $\mu > 0$ if k = 0 and $\mu = 0$ if $k = \bar{k} \equiv V\sqrt{2} - V > 0$. Since $\mu$ is monotonically decreasing in k, it follows that any $k \in [0, \bar{k})$ gives $\mu \equiv V^2 - 2Vk - k^2 > 0$. It is not surprising that player 1's per-period expected payoff under collusion is not necessarily higher than his per-period payoff in the Nash equilibrium. This is because when k is sufficiently high (i.e., $k > \bar{k}$), his success probability in the Nash equilibrium is sufficiently higher than his success probability under collusion to make the increase in effort worthwhile. In what follows, we restrict the analysis to $k \in [0, \bar{k})$, so that the collusive equilibrium of $e_i^t = 0$ in each period t, Pareto dominates the Nash equilibrium in each period t = 0, 1, 2, 3… and i = 1, 2.

We now want to show that the players can sustain a collusive outcome in this game. Without any loss of generality but for the sake of simplicity, we use the well-known folk theorem in Friedman (1971) where a player punishes a deviator by reverting to the Nash equilibrium forever.[1] Suppose a player decides to deviate from a collusive equilibrium, then a small but positive effort $\varepsilon > 0$ is optimal since that will guarantee success in that period. If player 1 is the deviator, his payoff is $\pi_1^D = V + k - \varepsilon$ and for

[1] Abreu (1986, 1988) deviced a punishment strategy which sustains cooperation in cases where a trigger strategy fails to do so. However, using a trigger strategy is sufficient to prove that collusion can be sustained in this environment and using better punishment strategies will not change the main result.

player 2, it is $\pi_2^D = V - k - \varepsilon$. Then assuming that players use a trigger strategy as in Friedman (1971), player 1 will cooperate in every period if[2]

$$\delta \geq \frac{\pi_1^D - \pi_1^C}{\pi_1^D - \pi_1^N} = \frac{2V^2(V + k - 2\varepsilon)}{3V^3 + kV^2 - 4\varepsilon V^2 - 3Vk^2 - k^3} \qquad \text{(A.1)}$$

and player 2 will cooperate in every period if

$$\delta \geq \frac{\pi_2^D - \pi_2^C}{\pi_2^D - \pi_2^N} = \frac{2V^2(V - k - 2\varepsilon)}{3V^3 - kV^2 - 4\varepsilon V^2 - 3Vk^2 + k^3} \qquad \text{(A2)}$$

Taking the limit of the expressions in (A.1) and (A.2) as $\varepsilon \to 0$ gives

$$\delta \geq \frac{2V^2}{3V^2 - 2Vk - k^2} \equiv \underline{\delta}_1 \qquad \text{(A.1a)}$$

and

$$\delta \geq \frac{2V^2}{3V^2 + 2Vk - k^2} \equiv \underline{\delta}_2 \qquad \text{(A.2a)}$$

First, note that the denominator of both expressions in (A.1a) and (A.1b) is positive since $k \in [0, \bar{k})$ implies that $\mu \equiv V^2 - 2Vk - k^2 > 0$. Therefore, $\underline{\delta}_1$ and $\underline{\delta}_2$ are both positive. Also, $\underline{\delta}_1$ and $\underline{\delta}_2$ are both less than 1, since $\mu \equiv V^2 - 2Vk - k^2 > 0$. Finally, note that $\underline{\delta}_1 \geq \underline{\delta}_2$. Then the result in Lemma 2 follows.[3]

## APPENDIX B: PROOF THAT IN THE CASE OF MILITARY INTERVENATION, THERE IS NO EQUILIBRIUM IN WHICH THE THIRD PARTY EXPENDS ZERO EFFORT, IF $\beta$ IS SUFFICIENTLY HIGH.

[2] We omit the proofs of these results, since this version of the folk theorem is well-known and standard in most elementary game theory textbooks (see for example, Gibbons, 1992).

[3] For details and further discussions of this result, see Amegashie (2005).

Faction i chooses $e_i$ to maximize

$\Pi_i = \rho_i V_i - e_i$,

i = 1,2.

The third-party chooses $e_3$ to maximize

$\Phi = \beta\theta + (1-2\beta)(\rho_1 V_1 + \rho_2 V_2) - \Sigma_{i=1}^{3} e_i$

We know that there is no equilibrium in which only one faction exerts a positive effort. So we focus on an equilibrium in which $e_1 > 0$, $e_2 > 0$, and $e_3 = 0$. In this equilibrium, we must have

$$\frac{\partial \Pi_1}{\partial e_1} = \frac{e_2}{(e_1+e_2)^2} V_1 - 1 = 0 \text{ and } \frac{\partial \Pi_2}{\partial e_2} = \frac{e_1}{(e_1+e_2)^2} V_2 - 1 = 0. \tag{B.1}$$

Let $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ be the equilibrium values. If $V_1 \geq V_2$, these equations imply that $\hat{e}_1 \geq \hat{e}_2$. These equilibrium values are independent of β, given $\hat{e}_3 = 0$.

Adding the equations in (B.1) and evaluating them at the equilibrium values gives

$$\frac{\hat{e}_2}{(\hat{e}_1+\hat{e}_2)^2} V_1 + \frac{\hat{e}_1}{(\hat{e}_1+\hat{e}_2)^2} V_2 = 2 \tag{B.2}$$

Then

$$\left.\frac{\partial \Phi}{\partial e_3}\right|_{e_1=\hat{e}_1, e_2=\hat{e}_2, e_3=0} = (2\beta-1)\left(\frac{\hat{e}_1}{(\hat{e}_1+\hat{e}_2)^2} V_1 + \frac{\hat{e}_2}{(\hat{e}_1+\hat{e}_2)^2} V_2\right) - 1 \tag{B.3}$$

Since $V_1 \geq V_2$ and $\hat{e}_1 \geq \hat{e}_2$, it follows using equation (B.2) that the second term in brackets in (B.3) is, least equal, to 2. Hence a sufficient but not necessary condition for a positive value for the derivative in (B.3) is $\beta > 0.75$. Clearly, this derivative is negative if $\beta \leq 0.5$. Since this derivative is increasing and continuous in β, it follows that there exists a cut-off value of $\beta$ such that the derivative in (B.3) is positive if $\beta \geq \tilde{\beta}$, where $\tilde{\beta} \in (0.5,1)$. Hence, if $\beta \geq \tilde{\beta}$, there is no equilibrium in which the third-party expends zero effort. To be precise, we can use the equations in (B.1) to obtain $\hat{e}_1 = \frac{V_1^2 V_2}{(V_1 + V_2)^2}$ and $\hat{e}_2 = \frac{V_2^2 V_1}{(V_1 + V_2)^2}$. Then $\tilde{\beta} = \frac{0.5(V_1^2 + V_2^2 + V_1 V_2)}{(V_1^2 + V_2^2)} > 0.5$.

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**Figure 1:** Third-party's intervention decision for different values of $\beta$, when $V_1 > 2V_2$, the conflict is one-shot, and with Tullock's (1980) conflict success function.

$0 \leftarrow$ non-military intervention $\rightarrow \hat{\beta} \leftarrow$ no intervention $\rightarrow \tilde{\beta} \leftarrow$military intervention $\rightarrow 1$

Notes: (i) $\hat{\beta} < 0.5$, (ii) $\tilde{\beta} > 0.5$, and (iii) The condition $V_1 > 2V_2$ is only required for non-military intervention.