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A contest success function with a tractable noise parameter

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Abstract. I propose a simple contest success function which is a variant of the Tullock probability function under certain conditions. It relaxes two features of the Tullock probability function. I show that this contest success function could be used to obtain interesting results and is more tractable than Tullock's function in certain cases. In particular, researchers who are interested in examining the degree to which luck as opposed to effort affects behavior in different contest settings might find it easier to use this contest success function than the Tullock success function. Unlike the Tullock function, there always exists a pure-strategy equilibrium for all values of the parameter which captures the degree of noise. The proposed function has been used in Kolmar and Wagener (2004) with interesting results.

1. Introduction

Contests are an important fact and pervasive aspect of economic life. A contest is a game in which players compete over a prize by making irreversible outlays. Election campaigns, rent-seeking games, R&D races, competition for monopolies, litigation, wars, and sports are all examples of contests.

One of the key ingredients for modeling contests is the contest success function (CSF). This function specifies how effort in the contest maps into the probability of success. Let e_i be the effort level of the i-th player in a contest with $N \geq 2$ players, $p_i(e)$ be the i-th player's probability of success where $e = (e_1, e_2, \ldots, e_{N-1}, e_N)$ is a vector of the outlays of all contestants. One of the most widely used contest success function, if not the most-widely used function, is the probability function in Tullock (1975, 1980). This function gives the i-th player's probability of success as $p_i^{\mathrm{T}}(e) = \frac{e_i^r}{e_i^r + \sum_{j \neq i} e_j^r}$, where $r \geq 0$.[1]

This function has been axiomatized by Skaperdas (1996). Under this success function, the contestant with the highest outlay does not necessarily win the prize. Another widely used function guarantees that the contestant with the highest outlay wins with certainty (i.e., all-pay auction).[2] A feature of both success functions is that a contestant who expends zero effort has a zero probability of winning, if at least one other contestant expends some positive effort no matter how small this effort is. Thus in any Nash equilibrium, at least one contestant expends positive effort. Hirshleifer (1989) studied a contest with two players in which the success function depends on the difference between the contestants' efforts. He obtained results which were dramatically different from those of Tullock (1980). For instance, in any pure-strategy equilibrium, one or both contestants expend zero effort. However, Hirshleifer's success function has not been used by researchers working in the field of contests. The Tullock success function is very popular partly due to its tractability. Besides, applications of Hirshleifer's contest success function have only focused on the case of two contestants (e.g., Baik, 1998; Che & Gale, 2000).

In this paper, I propose a simple contest success function which has the Tullock success function as a special case, under certain conditions, but which does not have some arguably undesirable features of Tullock's success function. I show that this function is more tractable than the Tullock success function under certain conditions. In particular, researchers who are interested in examining the degree to which luck as opposed to effort affects behavior in different contest settings might find it easier to use this contest success function than the Tullock success function. In the next section, I present and discuss the properties of the contest success function. Section 3 discusses some applications of the proposed contest success function, for example, Kolmar and Wagener (2004). Section 4 concludes the paper.

2. A Proposed Contest Success Function

The success function I propose is the following: $p_i(e) = \frac{e_i+\alpha}{e_i+\sum_{j\neq i} e_j+N\alpha}$, where $\alpha > 0$. If $\alpha = 0$, then this success function gives the Tullock success function with $r = 1$.[3]

Before I proceed, let me indicate that after writing this paper, my attention was drawn to the fact that this contest success function has been studied in Dasgupta and Nti (1998). However, unlike the subsequent analysis in this paper, the analysis in Dasgupta and Nti (1998) was restricted to identical contestants and the parameter, α, was not defined as capturing the degree of noise. Indeed, no economic interpretation is given to α in Dasgupta and Nti (1998). In contrast, the *main* focus of this paper is the parameter, α and an economic interpretation is provided for this parameter. While the analysis in Dasgupta and Nti (1998) is interesting and useful, it will be obvious to the reader that the focus of my paper is entirely different from their paper. Finally, let me indicate that while I repeatedly use the phrases "I propose" and "proposed contest success function" throughout the paper, my aim is not to lay any claim to originality but instead to show that this contest success function is very tractable and can be used to yield interesting results. In short, my goal is to encourage the use of this contest success function *originally* discovered by Dasgupta and Nti (1998).

2.1. *Properties of the proposed contest success function*

Given $\alpha > 0$, the proposed contest success function has the following properties: first, it is increasing in e_i and decreasing in $e_j, \forall\, i \neq j$, and $\sum_{i=1}^{N} p_i = 1$.

Second, α is a measure of the degree of noise. The degree of noise captures the extent to which luck as opposed to effort determines success in the contest.

That is, α captures how sensitive the probability of success is to a player's effort. To see this, note that $p_i < 1/N$ if $e_i < \bar{e} \equiv \sum_{i=1}^{N} e_i/N$ and $p_i \geq 1/N$ if $e_i < \bar{e}$. Also, $\partial p_i/\partial\alpha > 0$, if $e_i < \bar{e}$ and $\partial p_i/\partial\alpha \leq 0$, if $e_i \geq \bar{e}$. Notice also that by L'Hopital's rule, $p_i \to 1/N$, as $\alpha \to \infty$. Hence, an increase in α favors contestants with lower efforts (i.e., those whose effort is below the mean effort). A smaller α means that the sensitivity of the success function to effort is higher. As $\alpha \to \infty$, success in the contest is entirely determined by luck, since $p_i \to 1/N$.

Third, it is not homogenous of any degree. In particular, it is not homogenous of degree zero as Tullock's function. This property of Tullock's function may not be desirable in certain situations. Consider two contestants, 1 and 2. Suppose $e_1 > e_2$. This gives $p_1^{\mathrm{T}}(e_1, e_2)$. Now suppose, both contestants increase their effort levels by the same proportion, where player 1's effort is now λe_1 and player 2's effort is λe_2 and $\lambda > 1$. Then $p_1^{\mathrm{T}}(e_1, e_2) = p_1^{\mathrm{T}}(\lambda e_1, \lambda e_2)$ under the Tullock function. But although the players increased their efforts by the same proportion, the absolute increase in player 1's effort is greater than the absolute increase in player 2's effort since $e_1 > e_2$; that is, $(\lambda - 1)e_1 > (\lambda - 1)e_2$. There are situations where it is conceivable that player 1's success probability should increase and player 2's probability should fall under this condition. The proposed contest success function satisfies this property. To see this, note that $p_i(\lambda e_1, \lambda e_2) - p_1(e_1, e_2) = \frac{\alpha(\lambda-1)(e_1-e_2)}{(\lambda e_1+\lambda e_2+2\alpha)(e_1+e_2+2\alpha)} > 0$, given $\lambda > 1, \alpha > 0$, and $e_1 > e_2$.

Fourth, the i-th player's probability of success is positive, even if he expends zero effort and some player expends positive effort. That is, $0 < p_i < 1, \forall e_i$ and e_j, where $i \neq j$. As noted by Grossman and Kim (1995), this may be the case if offensive weapons (in a contest) are not very effective. Hence this contest success function relaxes the feature of Tullock's success function where a contestant who expends zero effort has a zero probability of winning, if at least one other contestant expends some positive effort *no matter how small* this effort is. This feature of the Tullock contest success function implies that there is *no* noise for a given combination of effort levels (i.e., $e_i > e_j = 0, \forall i \neq j$) but displays some noise at other combinations of effort levels, even if r is finite. The proposed contest success function displays noise at *all* effort levels given $\alpha > 0$. Indeed, Hirschleifer (1989) argues that this property of the Tullock contest success function may be undesirable in modeling contests in certain situations.

Fifth, the contest success function satisfies the property of "independence of irrelevant alternatives" in Skaperdas (1996). Clark and Riis (1998) state this property as follows: the probability that player i wins if player k does not participate ($e_k = 0$) is equal to the probability that i wins when k participates ($e_k > 0$) given that k does not win. As in Clark and Riis (1998) this property can be written as

$$p_i(e_1, e_2, \ldots, e_{k-1}, 0, e_{k+1} \ldots, e_N) = \frac{p_i(e)}{1 - p_k(e)} = \frac{e_i + \alpha}{\sum_{i \neq k} e_i + (N-1)\alpha} \quad \forall\, k \neq i \text{ and } N \geq 3.$$

Finally, unlike the Tullock probability function, this success function is continuous at $e_i = e_j = 0\ \forall i \neq j$.

While I have discussed some properties of the proposed contest success function, I have not been able to axiomatize it as in Skaperdas (1996). However, the proposed contest success function satisfies axioms A1–A5 in Skaperdas (1996) and is therefore consistent with Theorem 1 therein.[4] There may well be other contest success functions which satisfy the properties above. However, I shall argue that the appeal of this contest success function lies in its tractability for examining contests where we care about the effect of the degree of noise on some variable(s) of interest. As noted by Skaperdas (1996, p. 290) "…axiomatizations by themselves are unlikely to settle the issue of appropriateness of CSF for any particular situation … [f]inding ways to discriminate among functional forms empirically would be a complementary and welcome endeavor." Ultimately, I hope that the reader would judge the proposed contest success function on these grounds. Indeed, this function has been used by Kolmar and Wagener (2004) with interesting results. I shall elaborate on this in Section 3.2.

3. An Application to the Tullock (1980) Rent-Seeking Game

At this point, it is helpful to apply this contest success function to Tullock's (1980) rent-seeking game with non-identical contestants. That is, with contestants who have different valuations of the prize. An important question is the effect of the degree of noise on individual effort and aggregate efforts in the contest.[5] The parameter, r, captures the degree of noise in the Tullock probability function. The higher is r, the more sensitive is the success probability to effort. If $r = 0$, then each contestant has a success probability of $1/N$. This corresponds to the case of an infinite α.

Given $N > 2$ and $r \neq 1$, it appears that it is very difficult, if not impossible, to find a solution to Tullock's rent-seeking game with non-identical contestants. Hillman and Riley (1989) and Stein (2002) find the solution for $r = 1$. Baik (1994) and Nti (1999) find the solution for $N = 2$ and $r \neq 1$.

For $N > 2$, we do not know how individual and aggregate efforts change in response to changes in the degree of noise in the contest. In Tullock (1980) game with contestants with different valuations, Stein (2002, p. 326) writes "... in order to permit explicit solutions the constant returns to scale probability function (i.e., $r = 1$) will be used" (parenthesis mine). Most researchers in this field restrict their analysis to the case of $r = 1$. I shall show that the problem of examining a contest by allowing different values of a parameter which captures the degree of noise can be easily handled using the proposed contest success function.

In what follows, I assume risk-neutrality and complete information. Consider N contestants competing for a prize where the i-th contestant has valuation $V_i > 0,\ i = 1, 2, \ldots, N$.

The expected payoff of player i is

$$U_i = \frac{e_i + \alpha}{e_i + \sum_{j \neq i} e_j + N\alpha} V_i - e_i \tag{1}$$

It is easy to show that U_i is strictly concave in e_i.

Assume that this is a simultaneous-move game. I first look for an interior solution for all N players in a Nash equilibrium. This requires that

$$\frac{\partial U_i}{\partial e_i} = \left(\frac{1}{E + \alpha N} - \frac{e_i + \alpha}{(E + \alpha N)^2} \right) V_i - 1 = 0 \quad \forall i, \tag{2}$$

where $E \equiv \sum_{i=1}^{N} e_i$.

Summing over all N contestants and simplifying gives the equilibrium aggregate effort as

$$E^* = \frac{N-1}{\sum_{i=1}^{N} \frac{1}{V_i}} - \alpha N \qquad (3)^6$$

Putting (3) into (2) and solving for the equilibrium effort of player i gives

$$e_i^* = \bar{V} \left(1 - \frac{\bar{V}}{V_i} \right) - \alpha \tag{4}$$

where $\bar{V} \equiv \frac{N-1}{\sum_{i=1}^{N} \frac{1}{V_i}}$. For $N = 2$, we get $e_1^* = \frac{v_1^2 v_1}{(v_1+v_2)^2} - \alpha$ and $e_2^* = \frac{v_2^2 v_1}{(v_1+v_2)^2} - \alpha$.

Assuming that $e_i^* > 0\ \forall i$, it follows immediately that $\partial e_i^* / \partial \alpha < 0$ and $\partial E^* / \partial \alpha < 0$. Thus, an increase in the degree of noise reduces individual and aggregate effort. This result accords with intuition.[7] Note also that $e_i^* > e_j^*$ and $p_i^* > p_j^*$ if $V_i > V_j\ \forall i \neq j$.

Consider any mean-preserving spread in the valuations of the contestants. Then it is easy to show that $\bar{V}$ is maximized when all the valuations of the contestants are equalized. Hence this contest success function preserves the well-known result in contests that asymmetries reduce aggregate efforts (e.g., see Nti (1999) and Stein (2002)).

Note that when $\alpha = 1$, the solutions in (3) and (4) are identical to the solutions obtained in Hillman and Riley (1989) and Stein (2002) with $r = 1$.

In the case of $N = 2$, Nti (1999) using the Tullock probability function obtained the following expressions for the equilibrium effort levels:

$$e_1^{\mathrm{T}} = \frac{rV_1^{r+1}V_2^r}{\left(V_1^r + V_2^r\right)^2} \quad \text{and} \quad e_2^{\mathrm{T}} = \frac{rV_2^{r+1}V_1^r}{\left(V_1^r + V_2^r\right)^2} \tag{5}$$

Comparing Equations (4) and (5), it is obvious that the noise parameter, r, of the Tullock probability function appears in a much more complicated manner in the expressions for the equilibrium effort levels compared to the noise parameter, α, in our proposed contest success function. One can only imagine what the solution, *if possible*, will look like with $N > 2$ and $r \neq 1$. Contrary to my result, Nti (1998) finds that the individual efforts of two non-identical contestants are not monotonically decreasing in the degree of noise, r. I do not have an intuition for the difference in results.

I now look for an equilibrium in which the contestants expend zero effort. In models of rent seeking, Skaperdas (1992), Grossman and Kim (1995), and Neary (1997) find equilibria where the players expend zero effort. Skaperdas (1992, p. 721) shows that "... an important condition for such an outcome to be an equilibrium is that it is sufficiently difficult for an agent to increase her probability of winning in conflict." Grossman and Kim (1995, p. 1286) found that "... a non-aggressive equilibrium requires that either offensive weapons would not be too effective ... or that predation would be sufficiently destructive." The proposed contest success function has the feature that for sufficiently high values of α, the gains to rent-seeking are sufficiently small. It is important to note that the equilibria in the papers above involve only two players. The tractability of the proposed contest success function allows me to obtain a no rent-seeking equilibrium for $N \geq 2$ players.

An equilibrium in which $e_i = 0\ \forall i$ is easy to construct. This requires that $\partial U_i/\partial e_i \leq 0\ \forall i$. Putting $e_i = 0\ \forall i$ into $\partial U_i/\partial e_i \leq 0$ and simplifying gives $(N-1)V_i/N^2 \leq \alpha\ \forall i$. Given the strict concavity of U_i, $(N-1)V_i/N^2 \leq \alpha$ guarantees that this is indeed an equilibrium. This condition holds if the degree of noise, α, is sufficiently high.

Let $V_M > 0$ be the valuation of the contestant with the M-th highest valuation where $M \geq 1$. It is possible to construct an equilibrium in which all contestants with valuations, $V_i \geq V_M$, expend positive effort levels and those with valuations below V_M expend zero effort. Stein (2002) obtains a similar result but since he uses Tullock's contest success function, he requires $M \geq 2$ (see proposition 1 in Stein (2002)).

Hirshleifer (1989) suggests that ratio-form contest success functions like Tullock's cannot produce equilibria where all players exert zero effort or where some players exert zero effort and others exert positive effort. My analysis shows that a ratio-form contest success function can produce such equilibria.

It is important to note that there always exists a unique pure-strategy equilibrium for any value of the noise parameter, α. If we use the Tullock contest success function, there will no pure-strategy equilibrium for sufficiently high values of the noise parameter, r (see Baye et al., 1994).

3.1. *The Tullock rent-seeking game with identical contestants*

For the sake of exposition, let's turn to the case where $V_i = V$ for all i. Then, in a symmetric equilibrium, this gives $e^* = (N-1)V/N^2 - \alpha$ and aggregate effort is $E^* \equiv Ne^* = (N-1)V/N - N\alpha$. The equilibrium payoff is $U^* = (1/N)V - e^* = V/N^2 + \alpha$.

Notice that for $e^* > 0$, we require a sufficiently low α or a sufficiently high V. In Tullock (1980) and several variants of his model, an increase in the number of contestants results in an increase in aggregate efforts. This is not necessarily the case in this model, given that $\partial E^*/\partial N = V/N^2 - \alpha$. Hence, an increase in the number of contestants will result in an increase in aggregate efforts, if and only if the contestants' valuation, V, is sufficiently high. While I believe that V is very high in the real world, there is nothing in theory which precludes the possibility that an increase in the number of contestants could result in a fall in aggregate efforts. Essentially, this is an elasticity question. In Tullock's model with identical contestants, an increase in N results in a fall in e^* but the proportionate increase in N is greater than the proportionate fall in e^* resulting in an increase in E^*. *A prior*, there is no reason to think that the response of e^* to N is inelastic. It is interesting to note that by simply relaxing the feature of the Tullock function where a contestant's probability of success is zero if he expends zero effort and some contestants expend positive effort no matter how small this effort is, we obtain the possibility that an increase in the number of contestants could lead to a fall in aggregate efforts.

3.2. *Other applications*

Based on an earlier version of the present paper, Kolmar and Wagener (2004) use this contest success function in a model where a contest is used in the private provision of a public good (e.g., scientific knowledge). The main difference between their model and standard models of the private provision of public goods is that it is not efficient for all agents to contribute to the provision of the public good. In a model with heterogeneous agents who have different productivities in the provision of the public good, Kolmar and Wagener (2004) find that it is efficient for only high-productivity agents to contribute towards the public good (e.g., scientific knowledge) and for low-productivity agents to be non-contributors. They refer to this as the assignment problem. They find that a contributions game without a contest fails to solve the assignment problem. When a contest is introduced, the assignment problem cannot be solved under certain condition if the contest success function is the Tullock (1980) function. However, when the contest success function proposed in this paper is used, the assignment problem is solved in situations where the Tullock function is unable to do so. Indeed, they find that increasing the noise parameter, α, reduces the incentive for low-productivity types to enter the contest. Note that this contest success function will have no advantage over the Tullock function, if the agents are all identical since there is no assignment problem in that case.

For a given finite α, an interesting observation is that all the players expend zero effort if V_i is sufficiently low (i.e., $(N-1)V_i/N^2 \leq \alpha\ \forall i$) and at least two players exert positive effort if V_i is sufficiently high (i.e., $(N-1)V_i/N^2 > \alpha$ for, at least, two players). This could be used as simple but formal proof of Demsetz's (1967) hypothesis that private property rights over an asset will emerge if the asset becomes more valuable. That is, there will be rent-seeking efforts to move an asset from common property to private property if the asset becomes sufficiently valuable.

The proposed contest success function has also been used by Amegashie (2004) to examine burning out in elimination contests with non-identical contestants. In this contest, there are two stages. In stage 1, there is a preliminary contest where $2 \leq F < N$ contestants are chosen to compete in a contest in stage 2 from which a winner is chosen. The contestants have a finite cap on effort. The issue is to find an equilibrium in which some or all contestants expend their effort in stage 1 and have nothing to expend in stage 2 (i.e., they burn out). If one uses the Tullock success function in both stages, it is impossible to obtain an equilibrium in which some contestants burn out. By using the success function proposed in this paper, it is possible to construct such an equilibrium.

4. Conclusion

In this paper, I have proposed a contest success function originally discovered by Dasgupta and Nti (1998). This contest success function is useful and tractable in modeling contests with non-identical contestants. The success function has the Tullock (1980) contest success function as a special case under certain conditions (i.e., when $r = 1$ and $\alpha = 0$). It relaxes some features of Tullock's function. It is not homogenous of any degree and in particular it is not homogenous of degree zero in the efforts of the contestants. This feature is desirable if we are interested in contests where success is not entirely relative but absolute differences in effort matter. Also, it turns out to be more tractable than Hirshleifer's (1989) difference-form function since it is easy to apply it to contests with more than two players.

The proposed function relaxes a feature of the Tullock function and other variants of it. Unlike, the Tullock function, a player's probability of success is positive if he expends a zero effort and some contestants expend positive effort.

The proposed contest success function is not intended to be a substitute for Tullock's function or its other variants. However, it may be more appropriate for modeling contests in certain situations. An advantage of this success function is that it is more tractable than Tullock's success function for the case of $r \neq 1$. Researchers who are interested in examining the degree to which luck as opposed to effort affects behavior in different contest settings might find it easier to use this contest success function than the Tullock success function or *probably* any other function that one can think of. Unlike the Tullock function, there always exists a pure-strategy equilibrium in the case of identical contestants for all values of the parameter which captures the degree of noise.

Notes

1. See, for example, Amegashie (1999), Gradstein and Konrad (1999), Nitzan (1994), Szidarovsky and Okuguchi (1997) and Warynerd (2000).
2. See, for example, Baye, Kovenock, and de Vries (1996) and Hillman and Riley (1989).
3. Note that if we define $x_i \equiv e_i + \alpha$, then $p_i(e) = x_i/(x_i + \sum_{j \neq i} x_j)$.
4. The reader should refer to Skaperdas (1996) for these axioms.
5. The all-pay auction cannot be used to address this issue because there is no noise in the contest success function.
6. The first term of this equation can be expressed as a function of the harmonic mean of the valuations. For example, see Equation (4) in Stein (2002).
7. This is also the result obtained in the tournament model of Lazear and Rosen (1981) with two players.

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