I consider a two-stage elimination contest with uninformed and informed players. Informed players can signal their type to future uninformed opponents through their efforts in the first stage. Relative to the benchmark case of complete information, I find that an informed player exerts a higher effort in stage 1, if the uninformed future opponent is weaker than him. Conversely, he exerts a lower effort, if the uninformed opponent is stronger than him. This result is consistent with a conjecture in Rosen (AER, 1986). Intuitively, informed players may want to scare future uninformed opponents by exerting higher efforts in earlier rounds. However, trying to scare a stronger player may not be a sensible strategy because he might compete very fiercely.

Key words: asymmetric information, elimination contests, mutual consistency of beliefs, signaling.
JEL Classification: D72, D44, D74.
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1. Introduction

There is now a wide literature on contests. Earlier research focused on static, single-stage contests. In recent years there has been a growing literature on dynamic contests, and in particular, on elimination contests (Amegashie, 1999; Amegashie et al. (2006), Gradstein and Konrad, 1999; Groh, et al., 2003; Klumpp and Harbaugh, 2005; Moldovanu and Sela, 2006; Stein and Rapoport, 2005). Notice that not every dynamic contest is an elimination contest. Elimination contests involve multiple stages where the set of contestants is narrowed in successive stages of the contest until a winner is finally chosen. Elimination contests abound in sports, promotion tournaments in internal labor markets, firms competing for contracts, prototype development contests, etc. Rosen (1986) gives examples of elimination contests.2

All the papers above, with the exception of Moldovanu and Sela (2006), assume complete information. Moldovanu and Sela (2006, p. 72) write: “[i]n the dynamic, two-stage model, the winners of the sub-contests organized in the first stage compete against each other in a final at the second stage. In order to avoid signaling effects due to possibly sophisticated information manipulation across stages, we assume here that the contestants at stage two do take into account the fact that their opponents won at a previous stage, but they do not observe their opponents’ past actions.” However, in their conclusion, Moldovanu and Sela (2006, p. 84) write: “[w]hile our model is such that the information released in the dynamic contest has a very simple structure, we believe that

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1 Even in boxing, there are explicit elimination bouts. For example, in 2001 Bernard Hopkins, Keith Holmes, William Joppy, and Felix Trinidad were involved in elimination bouts to unify the three middleweight titles which were split between Joppy and Hopkins at the time. Hopkins defeated Holmes in one semi-final and Trinidad won the other semi-final against Joppy. On September 30, 2001, Hopkins defeated Trinidad in the final to become the undisputed Middleweight champion.

2 Rosen (1986) was the first article on elimination contests, at least, in economics.
an interesting avenue is to focus on the role of information in contests with multiple rounds.” The purpose of this paper is an attempt to respond to the challenge in Moldovanu and Sela (2006) by examining an elimination contest where, in subsequent stages, uninformed players observe and take into account the past actions of their opponents.³

I focus on whether informed players will exert more effort in stage 1 of a two-stage elimination contest with incomplete information as a way of signaling to future uninformed opponents that they are high-ability players. Relative to the benchmark case of complete information, I find that an informed player exerts a higher effort in stage 1, if the uninformed future opponent is weaker than him. Conversely, he exerts a lower effort, if the uninformed opponent is stronger than him. This is consistent with Rosen’s (1986, p. 714) conjecture that “[i]t is in the interest of a strong player to make rivals think his strength is greater than it truly is, to induce a rival to put forth less effort … However, it is in the interest of a weak player in a strong field to give out signals that he is even weaker than true, to induce a strong rival to slack off.”

My result accords with intuition. For example, most unknown soccer teams⁴ may not like to signal to a stronger team like Brazil that they are stronger than they really are by exerting high effort levels in the initial stages of the world cup competition. It may instead be strategically optimal to signal that you are the underdog, hopefully make a

³ In a related paper, Lai and Matros (2006) extend the model in Moldovanu and Sela (2006) in a setting where there is incomplete information in the first round. However, full revelation of individual efforts in the first round makes the second-round contest a complete information contest. In contrast, I consider a model where the revelation of first-round efforts does not necessarily make the contest in the second round a complete information contest. Besides, my contest success function is different from theirs and Moldovanu and Sela (2006). Finally, all players in Lai and Matros (2006) are symmetrically uninformed. In my model, there is asymmetric information.

⁴ Indeed, since the composition of most teams change over time, the ability of most teams is not necessarily known with certainty.
strong team like Brazil complacent, and then surprise them. On the other hand, it may be optimal to scare a weaker future opponent by signaling that you are stronger.

The issue of signaling in dynamic contests has recently been studied in Horner and Sahuguet (2006), Munster (2004), and Netzer and Wiermann (2005). However, there is a key difference between my paper and theirs. These papers are repeated contests between the same pair of players over two rounds. So a player’s opponent in stage 1 will also be his opponent in stage 2. Therefore, these papers do not study elimination contests.

An interesting and novel result that emerges from Horner and Sahuguet (2006) is that both sandbagging (holding back effort to minimize competition) and bluffing (increasing effort to deter competition) are possible equilibria. In section 3, I discuss a key quantitative difference between this result and the result in this paper.

The next section presents an elimination contest with incomplete information and the possibility of signaling. Section 3 discusses the results and section 4 concludes the paper.

2. An elimination contest with incomplete information

Consider an elimination contest with four risk-neutral players, 1, 2, 3, and 4. There are two-stages of the game: stage 1 (the semifinal) and stage 2 (the final). Players 1 and 2 are in one semifinal and players 3 and 4 are in the other semifinal. The winners in each contest will meet in the final (i.e., stage 2). The contest prize is given to the player who wins the contest in stage 2. Players 1 and 2 have a common valuation of V for the
prize, while players 3 and 4 have a common valuation, \( \omega \). It is important to note that a player with a higher valuation in a contest can be thought of as a player with a higher ability (e.g., see Clark and Riis, 1998). Suppose players in a group know each other’s valuation. Also, suppose there is asymmetric information such that players 1 and 2 do not know the valuations of players 3 and 4 but players 3 and 4 know the valuations of players 1 and 2. Further, suppose that players 1 and 2 know that players 3 and 4 have the same valuation. In each semi-final and in the final, the players move simultaneously. Finally, assume that the semi-final between players 3 and 4 takes place before the semifinal between players 1 and 2. This is a reasonable sequence of actions since the uninformed players must get the signal (i.e., effort) from the informed players before choosing their (i.e., the uninformed players’) efforts in stage 1. This is similar to the timing in Horner and Sahuguet (2006), where the players move sequentially in stage 1 and the second-mover infers the first-mover’s type based on his bid in stage 1.

Let \( e_i \) be the effort of the \( i \)-th player in stage 1 and let \( x_i \) be the effort in stage 2. Let \( e_i \) and \( x_i \) also be the corresponding cost of effort. Let \( p_i^1 \) be the success probability of the \( i \)-th player in stage 1, and \( p_i^2 \) be similarly defined for stage 2. For simplicity, I use the ratio-form contest success function. I assume that \( p_i^1 = \frac{e_i}{e_i + e_j} \) and \( p_i^2 = \frac{x_i}{x_i + x_j} \), if player \( i \) meets player \( j \). Each player in a given contest has a success probability of 0.5, if they both players exert zero efforts. Single-stage incomplete information contests using the ratio-form contest success function have been studied in Hurley and Shogren (1998).
and Warynerd (2003). The ratio-form contest success function has also been axiomatized in Skaperdas (1996) and given micro-foundations in Fullerton and McAfee (1999) and Baye and Hoppe (2003).

Before I proceed to the solution of this incomplete-information elimination contest, I present the results for the complete information case.

2.1 Benchmark: complete information

Using results in Nti’s (1999) one-stage game, it is easy to show that, in the final, the equilibrium expected payoff of player 1 or 2 is $\Omega^{**} = \frac{V^3}{(V + \omega)^2}$. For player 3 or 4, it is $\Omega^* = \frac{\omega^3}{(V + \omega)^2}$. These equilibrium expected payoffs in stage 2 are the players’ valuations in stage 1. Hence, in stage 1, the symmetric subgame perfect equilibrium effort level for players 1 and 2 is $e_c^{**} = \Omega^{**}/4$, and for players 3 and 4, it is $e_c^* = \Omega^*/4$. It can be shown that the best-response functions are non-monotonic. Indeed, they are strictly concave. Since they cross only once and there is no equilibrium in which a player exerts a zero effort, the pure-strategy equilibrium in which each player exerts a positive effort is unique.
2.2 Incomplete information

Now suppose the type of the informed players is drawn from some continuous distribution \( G(\omega) \) with support \((0, \infty)\), where \( \omega \) is a random variable representing the type (valuation) of the informed players. I assume that valuations of the informed players are perfectly and positively correlated so that they get the same draw from \( G(\omega) \).\(^8\) Their draw is private information, so the uninformed players only know the distribution \( G(\omega) \) and its support. As before the valuation, \( V \), of the uninformed players is common knowledge.

An equilibrium of this game is such that (i) players’ strategies are sequentially rational given their beliefs and the strategies of their opponent, and (ii) the beliefs of uninformed players are based on the signals received from the semi-final efforts of the informed players.

Consider stage 2. If an uninformed player believes that his opponent is of type \( \omega^u \), then his effort in this stage, using the results in Nti (1999), is

\[ \hat{x}(\omega^u, \omega, V) = \frac{V^2 \omega^u}{(V + \omega^u)^2} \].

Knowing this an informed player of type \( \omega \) chooses his effort to maximize

\[ \Omega_j^2 = \frac{x_j}{x_j + \hat{x}_j} \omega - x_j \]

\( j = 3 \) or \( 4 \).

Denote the optimal choice by \( \tilde{x}(\omega^u, \omega, V) \) and the associated payoff by

\[ \tilde{\Omega}(\omega, \omega^u, V) \].\(^9\) Notice that mutual consistency of beliefs will be satisfied if \( \omega = \omega^u \) but not

\(^8\) Of course, given that \( G(\omega) \) is continuous, the probability that players 3 and 4 get the same draw is zero. However, for the sake of exposition, the reader should think of the process as working in the following way. One of them gets a draw and that draw becomes the other player’s type as well. Later, I shall consider the case of two different draws. The results do not change.

\(^9\) It is easy to show that the equilibrium in this stage is unique.
when $\omega \neq \omega^u$. That is, mutual consistency of beliefs is not satisfied if an uninformed player’s inference about his opponent type is not the same as his opponent’s true type. So when $\omega \neq \omega^u$, the uninformed player’s effort of $\hat{x} = V^2\omega^u / (V + \omega^u)^2$ is not a best response to the informed player’s strategy but he cannot do anything about it since, by assumption, it would be too late to react, even if he found out. But of course, given his beliefs he considers his effort as a best response. Notice also that an uninformed player believes that, in stage 2, the expected payoff of an informed player is

$$\hat{\Omega}(\omega^u, V) = (\omega^u)^3 / (\omega^u + V)^2 = \Omega(\omega, \omega^u, V), \text{ if } \omega \neq \omega^u.$$  

This is similar to the Cognitive Hierarchy solution in Camerer et al. (2004) where players best-respond but do not have mutual consistency of beliefs.\(^{10}\) Also in Eyster and Rabin’s (2005) “cursed equilibrium”, a player does not fully take into account that his opponent’s action may be influenced by his (i.e., the opponent) private information. I shall later explain why the informed players can fool the uninformed players in this game.

Now consider stage 1. I look for a symmetric equilibrium. The informed players of type $\omega$, who are in the same semi-final, individually choose their efforts to maximize

$$\Omega_j^1 = \frac{e_j}{e_j + e_k} \hat{\Omega}(\omega^u, V, \omega) - e_j$$  

(2)

$j = 3$ or $4$, $k = 3$ or $4$, $j \neq k$. In a symmetric equilibrium, the optimal effort is

$$\hat{e} = \hat{\Omega}(\omega^u, \omega, V) / 4.$$  

However, recall that an uninformed player believes that in stage 2,

---

\(^{10}\)Indeed, games and economic models with wrong beliefs, subjective beliefs, or without common priors have been examined, among others, in Kalai and Lehrer (1993), Brunnermeier and Parker (2005), and Ettinger and Jehiel (2005). Morris (1995) presents an interesting and thought-provoking discussion of why heterogeneous prior beliefs among agents might persist. Brandenburger et al. (1992) prove that differences in priors can be justified as the manifestation of information processing errors (i.e., bounded rationality) on the part of the players.
the expected equilibrium payoff of an informed player is \( \hat{\Omega}(\omega^u, V) = (\omega^u)^3 / (\omega^u + V)^2 \).

Since all the players are *sequentially rational*, the uninformed players infer the type of the informed players in stage 1, given \( \tilde{e} \), by solving for \( \omega^u \) from the equation

\[
\tilde{e} = \hat{\Omega}(\omega^u, V) / 4 .
\]

The solution is \( \omega^u = \omega^u(V, \tilde{e}) \equiv \hat{\Omega}^{-1}(4\tilde{e}) \). Note that since \( \hat{\Omega}(\omega^u, V) = (\omega^u)^3 / (\omega^u + V)^2 \) is monotonic in \( \omega^u \), its inverse function, \( \hat{\Omega}^{-1} \), exists and since it is monotonically increasing in \( \omega^u \), \( \omega^u(V, \tilde{e}) \) is increasing in \( \tilde{e} \). Therefore, the uninformed players interpret a higher effort by the informed players in stage 1 as a signal of higher ability.\(^{11} \)

Note also that there exists a unique real number \( \omega^u(V, \tilde{e}) \) for a given real number \( \tilde{e} \geq 0 \), since \( \omega^u(V, \tilde{e}) \) is monotonic in \( \tilde{e} \).

At the risk of notational abuse, we may rewrite an informed player’s objective function in stage 1 as

\[
\Omega_j^1 = \frac{e_j}{e_j + e_k} \hat{\Omega}(\omega, V, \omega^u(V, e_j)) - e_j
\]

(3)

\( j = 3 \) or 4, \( k = 3 \) or 4, \( j \neq k \).

Then by finding the optimal effort choice of informed players using equation (3), we can determine whether or not \( \omega^u(V, e_j) \) is greater, equal, or less than \( \omega \) which corresponds to whether the informed players’ effort is greater, equal, or less than their effort, \( e^*_c = 0.25\omega^3 / (V + \omega)^2 \) in the complete-information case or if the type of players 3 and 4 was public information. I am unable to determine the exact value of informed

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\(^{11} \) This is the same as the requirement of a monotonic equilibrium or monotonic bidding functions in Lai and Matros (2006). Note that since our contest success function does not guarantee that the contestant with the higher or highest bid necessarily wins, we do not need to use order statistics in our analysis. Lai and Matros (2006) and Moldovanu and Sela (2006) use results from order statistics because the contest in the first round is an all-pay auction. See also Amegashie et al. (2006).
players’ optimal effort in stage 1. This is because $\omega^u(V, e_j)$ is a very complicated function of $e_j$, since it is the solution to a cubic equation. It turns out that $\tilde{\Omega}(\omega^u, \omega, V)$ is also a complicated function of $\omega^u$. So the function in equation (3) is very complicated since part of it is obtained by substituting $\omega^u(V, e_j)$ into $\tilde{\Omega}(\omega^u, V, \omega)$. However, I am still able to obtain analytically interesting qualitative results. To do so, I begin by taking the derivative of (3) with respect to $e_j$ to get

$$\frac{\partial \Omega^j}{\partial e_j} = \left( \frac{e_k}{(e_j + e_k)^2} \tilde{\Omega}(\omega, V, \omega^u(V, e_j)) - 1 \right) + \frac{e_j}{e_j + e_k} \frac{\partial \tilde{\Omega}}{\partial \omega^u} \frac{\partial \omega^u}{\partial e_j}$$

(4)

$j = 3$ or $4$, $k = 3$ or $4$, $j \neq k$. Given that $\omega^u = \omega$ and therefore $\tilde{\Omega}(\omega^u, V) = \tilde{\Omega}(\omega, \omega^u, V)$ when there is complete information (i.e., when $e_j = e_k = e^*_C$), it follows that the derivative in equation (4) evaluated at $e_j = e_k = e^*_C$ is

$$\left. \frac{\partial \Omega^j}{\partial e_j} \right|_{e_j=e_k=e^*_C} = \left( \frac{e_k}{(e_j + e_k)^2} \Omega(\omega, V) - 1 \right) + 0.5 \frac{\partial \tilde{\Omega}}{\partial \omega^u} \frac{\partial \omega^u}{\partial e_j}$$

(5)

The term in brackets in equation (5) is the derivative of the informed players’ payoff function in stage 1 with respect to their effort level under complete information. Therefore at $e^*_C$, this term must be zero. Hence equation (5) becomes

$$\left. \frac{\partial \Omega^j}{\partial e_j} \right|_{e_j=e_k=e^*_C} = 0.5 \frac{\partial \tilde{\Omega}}{\partial \omega^u} \frac{\partial \omega^u}{\partial e_j}$$

(6)

Since $\omega^u(V, e_j)$ is increasing in $e_j$, the sign of the derivative on the LHS of equation (6) depends on the sign of $\partial \tilde{\Omega} / \partial \omega^u$. Therefore, the derivative in equation (6) is positive,
zero, or negative if $\frac{\partial \Omega}{\partial \omega_u}$ is positive, zero, or negative. This leads to the following intuitive lemma:

**Lemma 1**: Consider a two-stage elimination contest with two identical and informed players in one semi-final and two identical and uninformed players in the other semi-final, where uninformed players interpret greater semi-final effort by informed players as a signal of higher ability. Then relative to the benchmark case of complete information, the informed players exert a greater or smaller semi-final effort, if an increase in their perceived ability increases or decreases their payoff in the final.

It can be shown that in stage 2, an informed player of type $\omega$, facing an uninformed player who thinks that the informed player has valuation $\omega_u$, exerts an effort

$$
\tilde{x}(\omega_u, \omega, V) = \frac{\sqrt{\omega V^2 \omega_u^2 + 2\omega V(\omega_u)^2 + \omega(\omega_u)^3} - V^2 \omega_u}{(V + \omega_u)^2}, \\
(7)
$$

where $V$ is the valuation of the uninformed player. Note if the players in the final have identical valuations and there is complete information, then each player’s equilibrium effort is $V/4$. Indeed, putting $V = \omega = \omega_u$ into equation (7) gives $\tilde{x}(\omega_u, \omega, V) = V/4$.

Using equation (7) and $\tilde{x}(\omega_u, \omega, V) = V^2 \omega_u / (V + \omega_u)^2$, it is straightforward to obtain an explicit function for the equilibrium expected payoff, $\tilde{\Omega}(\omega, \omega_u, V)$, in stage 2 for an informed player. We may write this as

$$
\tilde{\Omega}(\omega, \omega_u, V) = \frac{\tilde{x}(\omega, \omega_u, V)}{\tilde{x}(\omega, \omega_u, V) + \tilde{x}(\omega, \omega_u, V)} \omega - \tilde{x}(\omega, \omega_u, V) \\
(8)
$$
Taking the derivative of equation (8) with respect to $\omega^u$ and setting $\omega^u = \omega$, we can show after some algebra\(^{12}\) that

$$\text{sign} \left( \frac{\partial \tilde{\Omega}}{\partial \omega^u} \right)_{\epsilon_j = \epsilon_k = \epsilon_c}^* = \text{sign}(\omega - V) \quad (9)$$

Then lemma 1 and equation (9) imply the following proposition:

**Proposition 1:** Consider a two-stage elimination contest with two identical and informed players in one semi-final and two identical and uninformed players in the other semi-final, where uninformed players interpret greater semi-final effort by informed players as a signal of higher ability and the informed players have a valuation $V > 0$ for the prize. Then relative to the benchmark case of complete information, an informed player of type $\omega$ exerts a greater effort, a smaller effort, or the same effort in the semi-final, if $\omega$ is greater than, smaller than, or the same as $V$.

For completeness, let us consider the effort choice of uninformed players 1 and 2 in stage 1. To find the equilibrium of the game in stage 1, players 1 and 2 must know their expected equilibrium payoff in stage 2. This is

$$\tilde{\Omega}(\tilde{e}) = \frac{\hat{x}}{\hat{x} + \tilde{x}} V - \hat{x} \quad (10)$$

where $\hat{x} = V^2 \omega^u / (V + \omega^u)^2$, $\tilde{x} = \tilde{x}(\omega^u, \omega, V)$ and $\omega^u$ is a function of $\tilde{e}$. I have written equation (10) to capture the dependence of the uninformed player’s stage-2 payoff on the informed players’ stage-1 effort, $\tilde{e}$.

\(^{12}\) Part of the algebra was done with the help of the math software, Maple. The Maple output is available on request.
Therefore, in stage 1, player \( i \) chooses \( e_i \) to maximize

\[
\Omega_i^1 = \frac{e_i}{e_i + e_k} \hat{\Omega}(\tilde{e}) - e_i,
\]

\( i = 1 \) and \( 2 \), \( k = 1 \) and \( 2 \), and \( k \neq i \). Since the semi-final between players 3 and 4 takes place before the semifinal between players 1 and 2, it follows that players 1 and 2 know \( \tilde{e} \) before their own semifinal takes place and therefore can solve the optimization problem in (11). Given that \( \tilde{e} \) is not a choice variable for players 1 and 2, their symmetric equilibrium effort can easily be shown to be \( \hat{e} = \hat{\Omega}(\tilde{e})/4 \), \( i = 1 \) and \( 2 \).

2.3 A discussion of the belief process and the lack of mutual consistency of beliefs

Clearly, proposition 1 implies that informed players of type \( \omega \in (V, \infty) \) exert a higher semi-final effort than informed players of type \( \omega \in (0, V) \). However, in this equilibrium an uninformed player cannot correctly determine the type of the informed players, even though different types of the informed players exert different efforts in stage 1.

The lack of mutual consistency of beliefs and the inability of the uninformed players to correctly infer the type of the informed players is driven by the infinite type space of the informed players. To see this, suppose an informed player is of only two types: \( \omega_L \) and \( \omega_H \) and this type space is common knowledge. Then there is very little room for fooling the uninformed players. The informed players can only signal that they are either \( \omega_L \) and \( \omega_H \). Without loss of generality, suppose \( \omega_L < V < \omega_H \). Then, according to proposition 1, when the informed players’ type is \( \omega_H > V \), they will signal some

\[\text{13}I \text{ am unable to generally determine if higher-ability types exert higher semi-final efforts. A proof that } \tilde{e} \text{ is monotonic in } \omega \text{ will suffice. The difficulty is due to the complicated nature of the expression in equation (3) as argued above.}\]
\( \omega > \omega_H \). However, the uninformed players know that the informed players’ type is never greater than \( \omega_H \). Hence when they observe \( \omega^u > \omega_H \), they ignore it and instead correctly infer that \( \omega^u = \omega_H \). However, with our infinite type space the informed players can always fool the uninformed players because any \( \omega^u > 0 \) is in their type space.\(^{14}\)

What out-of-equilibrium beliefs support the equilibria in proposition 1? As noted above, there exists a unique real number \( \omega^u(V, e_j) \) for a given real number \( e_j \), since \( \omega^u(V, e_j) \) is monotonic in \( e_j \). Any \( e_j > 0 \) maps onto a unique \( \omega^u(V, e_j) > 0 \). Therefore, for \( e_j > 0 \), the uninformed players believe that \( \omega^u(V, e_j) > 0 \) is the informed players’ type, since the informed players’ types belongs to the set of all positive real numbers. Hence, we do not have to worry about out-of-equilibrium beliefs for \( e_j > 0 \). This is also the consequence of the infinite type space for the informed players and it is related to the argument in the preceding paragraph.

But suppose the uninformed players observe \( e_j = 0 \) for both informed players, then this will imply \( \omega^u(V, 0) = 0 \). But since \( \omega \in (0, \infty) \), an uninformed player puts a zero probability on the type of the informed players being zero. Suppose instead that an uninformed player believes that an informed player’s type is \( \omega > 0 \), when he observes \( e_j = 0 \) such that the corresponding conditional probability is \( \Pr(\omega|e_j = 0) = 1, j = 3, 4 \). Given this out-of-equilibrium belief, an informed player’s payoff in stage 1 when \( e_j = 0 \) for both

\(^{14}\) An infinite type space is sufficient but not necessary for the uninformed players to be fooled. They can be fooled so long as the type space is sufficiently large. For example, suppose the informed players’ types belong to the set \( \{\omega_L, \omega_M, \omega_H\} \), where \( \omega_L < \omega_M < \omega_H \). Then if the uninformed players draw \( \omega_M > V \), they could fool the uninformed by signaling that their true type is \( \omega_H > \omega_M \). Note, however, that if the gap between \( \omega_H \) and \( \omega_M \) is very large, then a very high effort level relative to the effort level consistent with their true type will be required to signal that their type is \( \omega_M \). In this case, they are better off signaling their true type, \( \omega_M \). This issue does not arise with our infinite type space.
informed players is \(0.5\tilde{\Omega}(\omega, V, \omega)\), if his type is \(\omega\). If the informed players play their equilibrium strategies, the expected payoff in stage 1 is \(0.25\tilde{\Omega}(\omega^u, \omega, V)\). Then this out-of-equilibrium belief will support the equilibria of the signaling game if \(0.25\tilde{\Omega}(\omega^u, \omega, V) \geq \tilde{\Omega}(\omega, V, \omega)\). Now since \(\tilde{\Omega}\) is increasing or decreasing in \(\omega^u\), if \(\omega\) is greater or smaller than \(V\) respectively, it follows that to satisfy \(0.25\tilde{\Omega}(\omega^u, \omega, V) \geq \tilde{\Omega}(\omega, V, \omega)\), we require a sufficiently low \(\omega > 0\), for \(\omega \in (V, \infty)\) (i.e., bluffing equilibrium) and a sufficiently high \(\omega\), for \(\omega \in (0, V)\) (i.e., sandbagging equilibrium). I assume that these conditions hold.

2.4 Non-identical informed players

In this section, I consider the case of non-identical informed players. So now suppose the informed players draw their type independently from \(G(\omega)\). Let \(\omega_j\) be the \(j\)-th player’s type, \(j = 3, 4\). Then an uninformed player believes that the \(j\)-th informed player’s payoff in stage 2 is \(\tilde{\Omega}(\omega_j, V) = (\omega_j)^3/(\omega_j + V)^2\), \(j = 3, 4\). Therefore, when the uninformed players’ observe the optimal efforts, \(\tilde{e}_3\) and \(\tilde{e}_4\) of the informed players in stage 1, they infer their types by solving the equations

\[
\tilde{e}_3 = -\frac{[\tilde{\Omega}(\omega_3, V)]^2 \tilde{\Omega}(\omega_4, V)}{[\tilde{\Omega}(\omega_3, V) + \tilde{\Omega}(\omega_4, V)]^2} \quad \text{and} \quad \tilde{e}_4 = -\frac{[\tilde{\Omega}(\omega_4, V)]^2 \tilde{\Omega}(\omega_3, V)}{[\tilde{\Omega}(\omega_3, V) + \tilde{\Omega}(\omega_4, V)]^2}.
\]

This gives \(\omega_3^u = \omega_3^u(V, \tilde{e}_3, \tilde{e}_4)\) and \(\omega_4^u = \omega_4^u(V, \tilde{e}_3, \tilde{e}_4)\). Since \(\tilde{\Omega}(\omega_j, V) > \tilde{\Omega}(\omega_k, V)\), if \(\omega_j > \omega_k\), it follows that \(\tilde{e}_j > \tilde{e}_k\), if \(\omega_j > \omega_k\), \(j = 3, 4\), \(k = 3, 4\), \(j \neq k\). Therefore, the uninformed players can uniquely determine each informed player’s type, so long as they know which player bid \(\tilde{e}_j\), \(j = 3, 4\). It also follows that \(\omega_j^u\) is increasing in \(\tilde{e}_j\), \(j = 3, 4\).
At their equilibrium effort levels in stage 1 when there is incomplete information, the derivative of an informed player’s payoff in stage 1 with respect his own effort would have a term which is zero as argued before. This gives an expression analogous to equation (6) as follows:

\[
\frac{\partial \Omega^1_j}{\partial e_j} \Big|_{e_j = e_j^*, e_k = e_k^*} = \frac{e_j^*}{e_j^* + e_k^*} \frac{\partial \tilde{\Omega}}{\partial \omega_j^u} \frac{\partial \omega_j^u}{\partial e_j},
\]

(12)

where \( e_j^* \) and \( e_k^* \) are the stage-1 equilibrium effort levels of the informed players under complete information, \( j = 3, 4 \). Then by using an analogous argument as before, it follows that

\[
\text{sign} \left( \frac{\partial \tilde{\Omega}}{\partial \omega_j^u} \right) \Big|_{e_j = e_j^*, e_k = e_k^*} = \text{sign} (\omega_j - V)
\]

(13)

\( j = 3, 4, k = 3,4, j \neq k \). Therefore, with non-identical informed players, the player who is stronger than the uninformed players overstates his true type and the player who is weaker than the uninformed players understates his true type. Hence, proposition 1 still holds.

3. Discussion

For the sake of exposition, I focus on the case of identical informed players.

As indicated in the introduction, Horn er and Sahuguet (2006) find that both sandbagging (holding back effort to minimize competition) and bluffing (increasing effort to deter competition) are possible equilibria. This is consistent with the result in proposition 1. In my model, bluffing occurs when the informed player is stronger (i.e., \( \omega > V \)) and sandbagging occurs when the informed player is weaker (\( \omega < V \)). For example,
recall that an uninformed player’s effort in the final is \( \hat{x} = V^2 \omega^u / (V + \omega^u)^2 \). Then it follows that \( \hat{c} \hat{x} / \hat{c} \omega^u = V^2 (V - \omega^u) / (\omega^u + V)^3 \) is positive, zero, or negative if \( V > \omega^u \), \( V = \omega^u \) or \( V < \omega^u \), respectively. Notice that if \( \omega > V \), then any \( \omega^u > \omega \) implies \( \omega^u > V \). Hence if \( \omega > V \), a \( \omega^u \) higher than \( \omega \) stemming from a higher semi-final effort by the informed players will cause a future uninformed opponent to decrease his effort in the final.

The previous discussion suggests that the main result of Horner and Sahuguet (2006) does not only hold in repeated contests but also in an elimination contest with asymmetric information and a ratio-form contest success function. But there is a key quantitative difference here. In Horner and Sahuguet (2006), a player may find it optimal to scare an opponent who is stronger than him. While this result holds in their cases with continuous distributions, it is straightforward to see this by looking at the example with a discrete distribution in section 2.1 of Horner and Sahuguet (2006). A first-mover with an intermediate valuation of \( \frac{1}{2} \) in their model finds it optimal to intimidate a stronger uninformed second-mover, whose valuation is either 3/5 or 3/2. First-movers with a valuation 1 randomize between intimidation and accommodation. Therefore, there is a pooling equilibrium in which an intermediate-valuation type – who is weaker than the uninformed second-mover - pools with the high-valuation type. This result in Horner and Sahuguet (2006) is driven by the fact that the first-mover is uninformed about the type of the second-mover. Since a first-mover who has an intermediate valuation drops out of the contest, if his bluff is called, it follows that he would not have bluffed if he knew that his bluff would be called (i.e., if he knew the type of the second-mover). In my model,

\[15 \text{ In Horner and Sahuguet (2006), the contest is an all-pay auction with players who have symmetric information.}\]
informed players know the type, V, of the uninformed players. Thus the result in Horner and Sahuguet (2006) is not possible in my model given that informed players of type \( \omega \in (V, \infty) \) exert a semi-final effort that is different from the effort of informed players of type \( \omega \in (0,V) \).

In Horner and Sahuguet (2006), the bluff of the intermediate type backfires with a probability less than 1, since the second-mover might mistakenly infer that he is high type and drop out. In my model, trying to scare an opponent who is stronger than you will definitely backfire. It will cause him to exert a greater effort in the final which will make the player sending the signal worse off. In such cases, it is better to hold back effort. The example of a soccer powerhouse like Brazil, in section 1, captures this point. It is better for a player to signal to a stronger opponent that he is weaker than he actually is, get the opponent to be complacent, and then surprise him. Surprises are possible in my model because beliefs are not mutually consistent if \( \omega^u \neq \omega \). As indicated in section 1, my results are consistent with a conjecture in Rosen (1986).

4. Conclusion

This paper has examined signaling in a two-stage elimination contest. The main results have been discussed in the section 3. In particular, the paper demonstrates Rosen’s (1986) conjecture that a weak player in a contest may signal that he is weaker than he actually is and a strong player may signal that he is stronger than he is actually is.

The model has been kept very simple in order to obtain the main result. However, it can be extended in other ways. Examples are two-sided asymmetric information,
informed and uninformed players in the same semi-final, etc. We suggest these extensions as interesting topics for further work on signaling in elimination contests.

References


