Autogeneration of Fractal Photographic Mosaic Images

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Abstract—We present a novel method for the creation of photographic mosaic images using fractals generated via evolutionary techniques. A photomosaic is a rendering of an image performed by placing a grid of smaller images that permit the original image to be visible when viewed from a distance. The problem of selecting the smaller images is a computationally intensive one. In this study we use an evolutionary algorithm to create fractal images on demand to generate tiles of the photomosaic. A number of images and tile resolutions are tested yielding acceptable results.

Index Terms—Art, Evolutionary computation, photomosaic, fractal, on-demand image generation

I. INTRODUCTION

Artistic expression using fractals has seen an explosion in the age of computers. While the “monsters” had been generally known since the 1800s when Hausdorff quantified the fractal dimension [1], these sets were of general interest until the use of computer imagery allowed for detailed and artistic colour representations. The most notable and recognizable of these is the Mandelbrot set [2]. Interestingly, a number of artistic works prior to the discovery of fractals have been found to have fractal properties. For example, the paintings of Katsushika Hokusai [2] and Jackson Pollock [3] have been found to incorporate fractal elements.

The art form of mosaic images is broadly defined as a method of placing small pieces of differently coloured materials closely together to form a surface, usually with a pattern or pictorial representation created by this placement [4]. The mosaic has a history extending from the classical era and many works use geometrical representations. Some of the most fascinating mathematically are those from Islamic regions. Some believe these are due to a religious interdiction against the depiction of humans in art [5], while others believe that the artists found that geometry verified their Islamic beliefs, with their art allowing them to explore a natural law [6]. The best example is the Grand Mosque of Damascus, part of the UNESCO World Heritage Site of the Ancient City of Damascus [7].

The production of the modern photographic mosaic, also known under the trademark photomosaics, were first described by Silvers [8]. They are a matrix of individual images which when placed together create a combined image visually similar to a target image. This is a process called sub-picture resolution, which uses smaller images to fill in for both the content and colour values of the larger target image. The images used in the construction of the greater image are selected for their artistic effect/significance and are generally related either semantically or methodically to the larger produced image.

In previous work by Ashlock and Jamieson [9] methods for the exploration of the Mandelbrot set are explored in order to find aesthetic images. This method is expanded upon in order to create fractal photographic mosaics. The goal of the fractals generated by the evolutionary method is to create visually similar blocks to a target image. When a number of these masked areas of fractals are presented together they give a photographic mosaic comprised of fractal images created by an evolutionary process. The evolution takes into account the structure as well as the colour values of the image as the criteria for selection.

One of the benefits of using fractals to stand in for pixel elements is that a fractal can be zoomed in upon without loss of image quality. In fact more patterns of the fractal will become visible as long as the image is regenerated via calculation of the missing point values. However, this will not lead to a loss in the target image which will have the same degradation in quality as zooming upon the original image.
The primary focus of this initial study is to look at portraits. The majority of current photographic mosaic work has been in the creation of portraits of humans or cartoon characters with human features [10], [11], [12]. Portraits allow the sub-picture resolution to be enhanced by the psychological phenomenon of Pareidolia, where a random or vague stimulus is perceived to be significant. The subconscious compulsion to see faces in random images or objects, such as the man in the moon or the face on Mars, are part of this phenomenon. The perception of faces has been examined with magnetoencephalography by presenting subjects with images which were or were not “face like”; the activation in the ventral fusiform cortex was only seen in those images containing faces. This makes images presented in portraits come forward even in the presence of “noise”.

The remainder of this manuscript is organized as follows: Section II looks at the evolutionary photographic mosaic algorithm used in the generation of the fractals which created the final image. Section III examines the portraits used and the settings examined in this study. Section IV gives the results of this experimentation with the fractal generation of portraits. The conclusions reached as well as the next steps for the generative process are outlined in Section V.

II. METHODS

A. The Photographic Mosaic Algorithm

The algorithm scans the input image in small squares, calling an evolutionary algorithm to generate a fractal to match each square. The size of the squares is an algorithm parameter. Within each square each pixel of the input image is compared to each pixel of a square subset of size \( k \times k \) within the fractal. The mean squared error (MSE) of the red, blue, and green colour values for each pixel comparison is used to evaluate the merit of a given fractal. We call this the total mean squared error fitness of the fractal.

The fractals used in each tile of the input image are selected by running a steady state evolutionary algorithm [13] for 10,000 mating events. The representation used by the evolutionary algorithm consists of nine real parameters that specify the fractal; three for the location on the Fractal where the square is extracted from and six for the colouring operations. The algorithm uses a single tournament selection with a tournament size of seven [14] on a population of 1000 fractal specifications.

Two variation operators are used. The first is probabilistic crossover with a probability \( p = 0.1 \) of exchanging the real parameters at a given location. The second is a point mutation applied to one of the nine real parameters making up the fractal description. This operator is designed to make domain-compliant changes in the parameters and so treats different parameters differently. We now explain how the nine parameters specify a fractal and how mutation modifies each parameter.

B. The Evolvable Fractal Representation

The fractals used in this work are views into the portions of the complex plane near the quadratic Mandelbrot set[2]. A view specifies a position \( x + iy \) in the complex plane that is the upper left corner of a square subset of the plane and a side length \( L \) for the square. The three parameters \( x, y, \) and \( s \) are the first three parameters of the nine used by the representation to describe fractal.

**Definition 1.** The Mandelbrot Set consists of those point \( z \in \mathbb{C} \) for which the Mandelbrot sequence

\[
\begin{align*}
    z_0 & = z, \\
    z_{i+1} & = z^2_i + z,
\end{align*}
\]

fails to diverge away from the origin of the complex plane.

Any point which reaches a distance \( |z| = 2 \) from the origin of the complex plane can be shown to diverge, motivating the following definition.

**Definition 2.** The divergence number of a point \( z \) in the complex plane is the index in the Mandelbrot sequence for which \( |z_i| \) first exceeds 2. If no such number exists, the divergence number is \( \infty \) and the point is in the Mandelbrot set.

The fractal images used in this study select a regular grid of points within a view corresponding to the pixels of the desired image. The divergence number for each pixel is computed, truncating all divergence numbers greater than 200 to a value of 200. The divergence numbers are then mapped onto pixel colours with a six parameter palette. The red, green, and blue values \( R(n), G(n), \) and \( B(n) \) for an divergence number \( n \) are given by the periodic functions:

\[
\begin{align*}
    R(n) & = [128 \cdot \cos(s_R \cdot n + p_R) + 127] \\
    G(n) & = [128 \cdot \cos(s_G \cdot n + p_G) + 127] \\
    B(n) & = [128 \cdot \cos(s_B \cdot n + p_B) + 127]
\end{align*}
\]

Where the brackets denote the greatest whole integer function. Table I summarizes the nine parameters used in the representation.

<p>| (x, y) | The coordinates of the upper left corner are mutated by adding a Gaussian variable, the parameters with log-uniform initial distributions are multiplied by log-uniformly distributed multipliers, and the starting | (L, s_{any}) | The side length of a view and the speed controls are mutated by multiplying them by a value ( e^{\Delta L} ) where ( \Delta L ) is uniformly distributed in the range ([-0.1, 0.1]). | (p_{any}) | The initial colour specification is mutated by adding a number uniformly distributed in the range ([-0.04, 0.04]). |
|---|---|---|---|---|
| (x, y) | The coordinates of the upper left corner are mutated by adding a Gaussian random variable with a standard deviation of 0.1. | (L, s_{any}) | The side length of a view and the speed controls are mutated by multiplying them by a value ( e^{\Delta L} ) where ( \Delta L ) is uniformly distributed in the range ([-0.1, 0.1]). | (p_{any}) | The initial colour specification is mutated by adding a number uniformly distributed in the range ([-0.04, 0.04]). |</p>
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Initialization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>Real part of upper left corner of view.</td>
<td>$-2 \leq x \leq 1$</td>
</tr>
<tr>
<td>$y$</td>
<td>Imaginary part of upper left corner of view.</td>
<td>$-1.5 \leq y \leq 1.5$</td>
</tr>
<tr>
<td>$L$</td>
<td>Side length of view.</td>
<td>$0.0001 &lt; L &lt; 0.1$*</td>
</tr>
<tr>
<td>$s_R$</td>
<td>Speed control for red component of palette.</td>
<td>$1 \leq s_R \leq e^{-4.4}$*</td>
</tr>
<tr>
<td>$s_G$</td>
<td>Speed control for green component of palette.</td>
<td>$1 \leq s_G \leq e^{-4.4}$*</td>
</tr>
<tr>
<td>$s_B$</td>
<td>Speed control for blue component of palette.</td>
<td>$1 \leq s_B \leq e^{-4.4}$*</td>
</tr>
<tr>
<td>$p_R$</td>
<td>Control for red shade at $n = 0$.</td>
<td>$0 \leq p_R \leq 2\pi$</td>
</tr>
<tr>
<td>$p_G$</td>
<td>Control for green shade at $n = 0$.</td>
<td>$0 \leq p_R \leq 2\pi$</td>
</tr>
<tr>
<td>$p_B$</td>
<td>Control for blue shade at $n = 0$.</td>
<td>$0 \leq p_R \leq 2\pi$</td>
</tr>
</tbody>
</table>

* - these distributions are log-uniform.

### Table I

The nine parameters used in the evolvable fractal representation together with their initializers.

#### III. Experimental Design

The purpose of this paper was to estimate suitable parameters to be used in the generation of photographic mosaics of known images using fractals. In this process our system has relied upon the researchers, and the readers of this paper, to give an opinion as to the aesthetics of the resulting images. As a proof of concept, this is appropriate. Clearly, however, such measurements are often subjective and thus for future work an important next step would be to incorporate automated fitness evaluation.

An example of such an evaluation may be found in [15], where a mathematical model of aesthetics is used. The model used in that paper results from an analysis of fine art showing that “many works consistently exhibit functions over colour gradients that conform to a normal or bell curve distribution”. An interesting point of the model is that photographs and paintings do not behave in the same way.

We selected four portraits with the goal of showing the effect of a wide variety of dimensions, as well as demonstrating the differences between photographs and paintings. The first selection was a test image of an author (Figure 1(a)) used in the debugging of the program and first set of tests on the process. The second is the standard test image of Lenna (Figure 1(b)), known for its use in image compression research. The third is the Mona Lisa (Figure 1(c)), selected as it is also a test image used in graphics, including evolutionary creations of filtering processes. The fourth is Van Gogh’s self portrait with bandaged ear (Figure 1(d)), selected to test smaller images. Both the Mona Lisa and Van Gogh paintings are also used to test if the method applies to painted works and not just photographs.

<table>
<thead>
<tr>
<th>Image</th>
<th>Type</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dan</td>
<td>photograph</td>
<td>640</td>
<td>480</td>
</tr>
<tr>
<td>Lenna</td>
<td>photograph</td>
<td>512</td>
<td>512</td>
</tr>
<tr>
<td>Mona Lisa</td>
<td>painting</td>
<td>280</td>
<td>394</td>
</tr>
<tr>
<td>Van Gogh</td>
<td>painting</td>
<td>175</td>
<td>214</td>
</tr>
</tbody>
</table>

The parameters which we will be using as free variables in this study are: pixel window size for matching (PX), sampling grid (SG), and rendering density (RD). The pixel window size determines the square target block size for which a fractal will be created. The rendering density is the size of the created fractal, the final image block size. The sampling grid is the number of locations over the block size which gauges the fitness function: the lower the number, the more samples taken in the space.

Note that the image size of the target to the fractal expression are linked in a ratio of RD:PX, hence these parameters were paired to maintain a ratio of 3:2 in order to visibly show more elements of the fractal in the final production image. PX and RD were set to 4/8, 8/12, and 16/24. The sampling grid was set to 2, 3, and 4.

#### IV. Results

We found that even with the largest block sizes, the basic outline of the figure is visible. However, smaller blocks are required in order to give details such as the eyes, nose, and mouth. All of these features are necessary to have the impression of a face. The large blocks however show the most features of the fractal. They are rather expressive of the underlying patterns of both image structure and colour.

We found that the photographs reveal more structure using the technique of fractal generation. This could be in part due to the harshness of lines in photographic works, compared to the relative smoothness of colour and line in the Mona Lisa and Van Gogh. This harshness might be more amenable to representation of lines produced by fractals. The algorithm shows the most difficulty in creating a suitable fractal images in areas of unvarying colour, for example the white border about the Mona Lisa image.

An early test with the Canadian flag also demonstrated this property. The flag, red and white with large fields of colour, was incomprehensible even to Canadian nationals who have the image pre-primed via culture. Large fields of the same colour have no structure upon which the algorithm can gain fitness. This leads to a fight between suboptimal
Fig. 1. Target Images

Fig. 2. Fractal Gallery of Dan
solutions which may have wildly different colour intensities or structural cues.

Figure 2 is the gallery for the Dan image. This image shows the process to be suitable for the creation of human figures. The midrange block size showing both the best features in the fractals as well as giving the impression of the image it has been targeted upon.

Figure 3 is the gallery for the Lenna image. This image is well known for having noise added to it, the fractal images produced for the block settings create a picture which is as visible as she has been in these experiments. The fractals are especially good in this image on picking out her eyes which give the best impression in the photograph. Figure 6 gives a closer look at the fractals which make up the final lenna image focusing on her eyes. The middle block size gives both a good visual of the original image while allowing the viewer to also see the details in the fractal subimages.

Figure 4 is the gallery of the Mona Lisa. The Mona Lisa in the small to medium block sizes gives a good feel for the image. The larger block size is mostly able to pick up colours and give an averaging of the colour over the blocks.

Figure 5 is the gallery for the Van Gogh self-portrait. This image is the smallest of those sampled and good resolution was found only for the smallest block size. The mid block
Fig. 4. Fractal Gallery of the Mona Lisa
size fares the worst of all tests in giving an impression of the image. The largest block size can pull out some of the colours, most notably in the hat, but structure is lost. Working from a small image presents a challenge as the data which gives the impression of the original is tightly compacted. Thus any loss of information due to noise leads to a loss of structure.

Changing the sample grid for matching pixels caused only trivial changes to the results. The major effect was simply to extend the runtime of the algorithm when the sample grid size was set too low. The size of the block created is the element which will most affect the final image. Not surprisingly, if set too small the process degenerates into just a reproduction of the original. Finding the happy medium between the image and seeing the fractal areas making up the image is therefore the objective of the fitness function selected.
V. CONCLUSIONS AND NEXT STEPS

In this study we have looked at the automatic generation of fractal images in order to create photographic mosaic images for portraits. We have shown how the process makes reasonable images given that the only creation media are fractals used to match both the structure and the colour values of square regions. In our opinion, they are as reasonable a match to the target image as the photographic mosaics in commercial production that use photographs.

The final images require some measure of aesthetic, i.e. a measure of whether the final image produced is appealing to the eye and if the image is recognizable as the original image without being primed for the viewer. A number of aesthetic measures exist for both fractal and evolved images[16], [17].

The algorithm for matching fractals to tiles of the input image, MSE of the RGB values averages over $k \times k$ squares of the fractal, is an ad hoc choice and should be re-examined. Simply scoring on direct match of pixels might work better, or permit the same quality of mosaic with larger tiles. There is also the possibility of saving libraries of pre-evolved fractals with their colour match information. Since fractals can be stored as nine real parameters, an enormous image library can be stored in a reasonable amount of space.

Some preprocessing of the images might lead to better fractals using larger squares. The use of image processing filters would allow for the discovery of edges in the graphic which a fractal discovery would be able to extract. The fractal would then follow these more significant features rather than taking the MSE over the entire space.

The techniques used to evolve views of the quadratic Mandelbrot set can also be used to evolve other fractals. In [18] Julia sets are evolved. Chaos Automata [19] are another form of evolvable fractal that yields a very different sort of appearance. There are many different colouring algorithms for the Mandelbrot set and these could replace the simple six-parameter periodic palette used in this study.

REFERENCES