UNIVERSITY of GUELPH

DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 1160, F16

‘Mock’ Final: Solutions

Note:

If no working is given that is because the result follows immediately from lecture notes.

1. Answer = (D).

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2. After multiplying the 1st equation by 2 the result is obvious.

   Answer = (A).

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3. Just do the arithmetic.

   Answer = (D).

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4. Add a_{11} and a_{12} !

   Answer = (E).

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5. Answer = (E).

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6. We have $(kA)^T(kA) = k^2A^TA = 9k^2 = 1$ etc.

   Answer = (B).

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7. Review the algebraic properties of matrices. We have

$$\begin{aligned}(A + rB^TC)^T &= A^T + (rB^TC)^T \\ &= A^T + r(B^TC)^T \\ &= A^T + rC^T(B^T)^T \\ &= A^T + rC^TB.\end{aligned}$$

Answer = (A).

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8. $x = (A^T)^{-1}b = (A^{-1})^T b = (3, 6)^T$, using the given matrices.

   Answer = (C).

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9. $(A^2)^{-1} = (A \cdot A)^{-1} = (A^{-1})^2 =$ etc, using the given matrices.

   Answer = (E).

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10. Answer = (C).

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11. Answer = (A).

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12. Answer = (B).

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13. $x_1 = 2$ implies $10 - 2x_2 = 2$ implies $x_2 = 4$, which do not satisfy the 1st equation.

    Answer = (E).

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14. Answer = (C).

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15. $3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = -\begin{vmatrix} c & d \\ a & b \end{vmatrix} = -(1/2)\begin{vmatrix} 2c & 2d \\ a & b \end{vmatrix} = -(1/2)|B|$ which implies $|B| = -6$.

    Answer = (D).

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16. $|A^T B^{-1}| = |A^T||B^{-1}| = |A| \cdot \frac{1}{|B|} = 2/3$.

Answer = (E).

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17. A has a row of zeros, thus from lecture notes $|A| = 0 \implies$ the homogeneous system $Ax = 0$ has a non-trivial solution.

Answer = (D).

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18. Answer = (A).

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19. Answer = (D).

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20. The given matrix equations yields two linear equations:

$$3x + y = 1$$
$$2x + 4y = 2$$

Solving these equations yields $x = 1/5$ and $y = 2/5$ thus $x + y = 3/5$.

Answer = (B).

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21. Answer = (C).

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22. Apply the parallelogram law twice.

Answer = (C).

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23. By definition of scalar multiplication here, the right hand side of (C) should be $c \odot u \oplus d \odot u$.

Answer = (C).

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24. Answer = (D).

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25. Just because the set is a subset of $\mathbb{R}^2$ is not sufficient. We also need closure.

    Answer = (D).

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26. (B) is clearly nonsense.

    Answer = (B).

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27. Seek x, y, z s.t.

$$t^2 + 3t - 4 = x(t^2 + t - 1) + y(3t^2 + 1) + z(5t + 2).$$

Collecting like terms on the right hand side and equating coefficients on both sides of like powers of t yields three linear equations in x, y, z. After expressing these equations as a single matrix equation the associated augmented is easily found.

Answer = (A).

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28. Seek x, y, z s.t.

$$x(t^2 + t - 1) + y(3t^2 + 1) + z(5t + 2) = 0.$$

Now apply a similar strategy to Q 27.

Answer = (C).

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29. (E) is nonsense.

    Answer = (E).

30. Answer = (E).

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31. $\dim \mathbb{R}^2 = 2$ so we need 2 vectors for a basis. So we would need the matrix whose columns correspond to the given vectors to be square.

    Answer = (A).

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32. Elementary row ops yields that we have the single equation $2x - 5y = 0$, or $y = (2/5)x$.

    Answer = (D).

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33. First note that A is in RREF. Then the number of free variables is

$$p = n - r = 5 - 2 = 3,$$

    where n is the no. of variables in the homogeneous equation $Ax = 0$ and r is the number of pivot columns. The result then follows after recalling that the number of free variables is equal to the dimension of the null-space.

    Answer = (B).

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34. The matrix is in RREF, so just count the number of non-zero rows.

    Answer = (D).

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35. If the matrix has full rank then it is nonsingular which implies $|A| \neq 0$.

    Answer = (D).

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36. Ans $= \sqrt{(1+1)^2 + (2-0)^2 + (3-2)^2} = \sqrt{9} = 3$.

Answer = (A).

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37. Answer = (C).

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38. From lecture notes

$$\|f\|^2 = \int_0^1 [f(x)]^2\,dx = \int_0^1 (2x)^2\,dx = 4\int_0^1 x^2\,dx = \frac{4}{3}[x^3]_0^1 = 4/3.$$

Thus $\|f\| = 2/\sqrt{3}$.

Answer = (D).

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39. $|A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0$ so $\lambda^2 = 1 \implies \lambda = \pm 1$.

Answer = (D).

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40. Note: statement (C) implies the matrix is diagonalizable, but the converse statement isn't necessarily true.

Answer = (E).
