



DEPARTMENT OF MATHEMATICS AND STATISTICS

**Math 2160, F15**

## **Test 1**

1. This is a 60 minute test. Do NOT start until instructed.
2. Please fill out your personal details on the cover of the Examination Booklet(s). If you use more than one booklet, please indicate how many you use, e.g. "1 of 3", "2 of 3", "3 of 3" on each booklet.
3. You may quote results from lecture notes without proof, unless you are asked to prove a specific result. Notes or books are not permitted during the exam.
4. Give sufficient working as solutions with no justification will receive little or no credit.
5. You may use a calculator, but not a 'graphing calculator' that supports matrix algebra. No other electronic equipment is permitted. **Turn off or 'mute' your cell phone!**
6. If you copy the work of your neighbour this is considered **Academic Misconduct** and will be reported to the appropriate university authority.
7. There are a total of **32** points to be awarded on this test.



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1. (6 points)

(a) If

$$\begin{bmatrix} a - c & c - d \\ c + 2d & 2b + 1 \end{bmatrix} = \begin{bmatrix} 2c & c + d \\ d - c & a + b \end{bmatrix},$$

find  $a$ ,  $b$ ,  $c$ , and  $d$ .

(b) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 0 \\ 4 & -2 & 1 \\ 1 & 3 & 7 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -1 & 2 & 6 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}.$$

If possible, calculate  $3\mathbf{A} - 2\mathbf{B}$ .

(c) For the matrices given in (b), if possible, calculate  $\mathbf{AB}$ .

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2. (6 points)

(a) Let  $\mathbf{A}$  and  $\mathbf{B}$  be square matrices. Find an expression for the expansion of  $(\mathbf{A} + 2\mathbf{B})^2$ , clearly stating the algebraic properties of matrices you use.

(b) Determine a scalar  $\lambda$  such that  $\mathbf{Ax} = \lambda\mathbf{x}$ , where

$$\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(c) Let  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and  $\mathbf{D}$  be matrices. Using an appropriate algebraic property of matrices find an expression for  $(\mathbf{AD})^T$  where  $\mathbf{D} = \mathbf{BC}$ .

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Please Turn Over

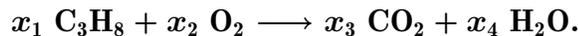
3. (10 points)

(a) Let

$$\mathbf{A}^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathbf{B}^{-1} = \begin{bmatrix} 1 & 5 \\ -2 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Solve the matrix equation  $\mathbf{ABx} = \mathbf{b}$  for the vector  $\mathbf{x}$ .

(b) The chemical equation below shows that  $x_1$  molecules of propane ( $\text{C}_3\text{H}_8$ ) burn with  $x_2$  molecules of Oxygen ( $\text{O}_2$ ) to produce  $x_3$  molecules of Carbon Dioxide ( $\text{CO}_2$ ) and  $x_4$  molecules of water ( $\text{H}_2\text{O}$ ):



The subscripts in each molecule indicate how many Carbon (**C**), Hydrogen (**H**) and Oxygen (**O**) atoms there are in that molecule.

(i) Write down the four linear equations for the unknowns  $x_1, x_2, x_3, x_4$  associated with the number of Carbon (**C**), Hydrogen (**H**) and Oxygen (**O**) atoms needed in the chemical reaction.

(ii) Write down the single matrix equation for the linear system given in (i).

(c) What do the following commands in MATLAB do?

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>> A = [1 2 3;4 5 6;7 8 9]
>> I = eye(3) % 3-by-3 identity matrix
>> (A^2)*I - A*A
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4. (10 marks) Use the Gauss-Jordan elimination method to find the unique solution of the following linear system of equations:

$$\begin{array}{rcccc} & - & y & + & z & = & 3 \\ x & - & y & - & z & = & 0 \\ -x & & & - & z & = & -3 \end{array}$$

**Clearly indicate the row operations you use** (answers with NO justification receive NO credit).

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**END OF TEST**