UNIVERSITY of GUELPH

DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 2160, F15

Test 2

1. This is a 60 minute test. Do NOT start until instructed.
2. Please fill out your personal details on the cover of the Examination Booklet(s). If you use more than one booklet, please indicate how many you use, e.g. "1 of 3", "2 of 3", "3 of 3" on each booklet.
3. You may quote results from lecture notes without proof, unless asked to do otherwise. But notes or books are not permitted during the exam.
4. Give sufficient working as solutions with no justification will receive little or no credit.
5. You may use a calculator, but not a 'graphing calculator' that supports matrix algebra. No other electronic equipment is permitted. **Turn off or 'mute' your cell phone!**
6. If you copy the work of your neighbour this is considered **Academic Misconduct** and will be reported to the appropriate university authority.
7. There are a total of **28** points to be awarded on this test.

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1. (9 points)

(a) Compute the following determinant by applying elementary row and/or column operations to reduce the determinant to triangular form:

$$\begin{vmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{3} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{5} \end{vmatrix},$$

Do NOT use a cofactor expansion. Clearly show all the steps in your calculation.

(b) For square matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ such that $|\mathbf{A}| = \mathbf{2}$, $|\mathbf{B}| = \mathbf{3}$ and $|\mathbf{C}| = \mathbf{4}$ find $|\mathbf{A}^{-1}\mathbf{B}\mathbf{C}|$. Justify each step of your calculation. (Hint: let $\mathbf{D} = \mathbf{B}\mathbf{C}$.)

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2. (5 points)

(a) Using a cofactor expansion *along the first row*, find the values of $\boldsymbol{x}$ so that

$$\begin{vmatrix} \mathbf{3} & \mathbf{0} & \mathbf{-1} \\ \mathbf{2} & \boldsymbol{x} & \mathbf{1} \\ \mathbf{4} & \mathbf{5} & (\boldsymbol{x} - \mathbf{4}) \end{vmatrix} = \mathbf{-30}$$

(b) State in words, or mathematics, the definition of the adjoint of a square matrix.

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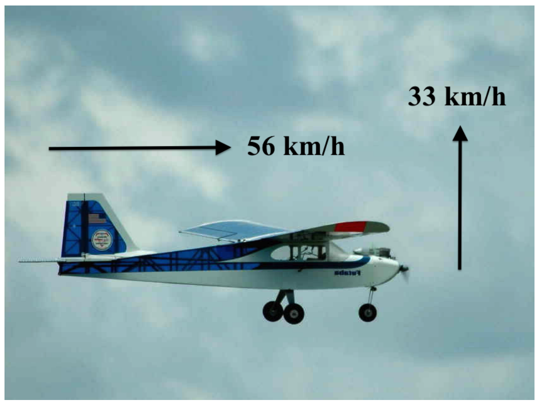
3. (8 points)

(a) Let

$$\mathbf{u} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}.$$

(i) Use directed line segments to represent the vectors $\mathbf{u}$ and $\mathbf{v}$. (ii) Calculate $\mathbf{v} - \mathbf{u}$ and indicate the resulting triangle on your diagram formed from $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{v} - \mathbf{u}$.

(b) An airplane flies horizontally with a constant velocity of 56 kilometres per hour (km/h). Suppose an updraft pushes the airplane vertically with a velocity of 33 km/h for three seconds (see the illustration below). How far does the airplane



travel **in metres** during the three seconds of the updraft? (Note: 1km $= 1000$m.)

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4. (6 marks)

(a) Consider the set of vectors in $\mathbb{R}^n$, with the usual rules for addition and scalar multiplication. For any $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, prove the 4th axiom of a vector space, namely, that

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

Clearly show *all* the steps in your argument.
(Hint: use the 'coordinate' form to represent vectors, e.g., $\mathbf{u} = (u_1, u_2, \ldots, u_n)$).

(b) Let V be a subset of $\mathbb{R}^3$ defined by

$$V = \left\{ \begin{pmatrix} a \\ b \\ c+1 \end{pmatrix} \in \mathbb{R}^3 \mid b = a - c \right\}$$

with the same rules of addition and scalar multiplication as for $\mathbb{R}^3$. Prove whether or not the set V is a subspace of $\mathbb{R}^3$.

END OF TEST