Solutions to Test 2

Math 2160, F15

1. (a) $\begin{vmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \\ 0 & 1 & 5 \end{vmatrix} = -\begin{vmatrix} 3 & 1 & 2 \\ 2 & -1 & 0 \\ 0 & 1 & 5 \end{vmatrix}$ $r_1 \leftrightarrow r_2$

$= -\begin{vmatrix} 1 & 2 & 2 \\ 2 & -1 & 0 \\ 0 & 1 & 5 \end{vmatrix}$ $r_1 - r_2 \to r_1$ $= -\begin{vmatrix} 1 & 2 & 2 \\ 0 & -5 & -4 \\ 0 & 1 & 5 \end{vmatrix}$ $r_2 - 2r_1 \to r_2$

$= +\begin{vmatrix} 1 & 2 & 2 \\ 0 & 1 & 5 \\ 0 & -5 & -4 \end{vmatrix}$ $r_2 \leftrightarrow r_3$ $= +\begin{vmatrix} 1 & 2 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 21 \end{vmatrix}$ $r_3 + 5r_2 \to r_3$

$= (1)(1)(21) = \boxed{21}$

(Alternate correct row ops acceptable)

- method ——— 4
- answer ——— 1

(−1/2 each arithmetic error).

(b) $|A^{-1}BC| = |A^{-1}D|$

$= |A^{-1}|\cdot|D|$ ('$|AB| = |A|\cdot|B|$') 1

$= |A^{-1}|\cdot|BC|$

$= |A^{-1}|\cdot|B|\cdot|C|$ ('$|AB| = |A|\cdot|B|$' again) 1

$= \dfrac{|B|\cdot|C|}{|A|}$ ('$|A^{-1}| = \dfrac{1}{|A|}$') 1

$= \dfrac{(3)(4)}{(2)} = \boxed{6}$ 1

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2. (a)

$$\begin{vmatrix} 3 & 0 & -1 \\ 2 & x & 1 \\ 4 & 5 & (x-4) \end{vmatrix} = +3\begin{vmatrix} x & 1 \\ 5 & x-4 \end{vmatrix} + (-1)\cdot\begin{vmatrix} 2 & x \\ 4 & 5 \end{vmatrix} = -30$$ 1

i.e. $3[x(x-4)-5] - (10-4x) = -30$ 1

or $3x^2 - 8x + 5 = 0 \Rightarrow (3x-5)(x-2) = 0$ 1

$\Rightarrow \boxed{x = 5/3 \text{ or } x = 1}$ 1

($-1/2$ each arithmetic error)

(b)

"The adjoint of square matrix is the transpose of the matrix of cofactors" — (1)

OR

given $A = (a_{ij})$

then $\text{adj}\, A = (A_{ij})^T$

where A_{ij} is the cofactor associated with entry a_{ij}.

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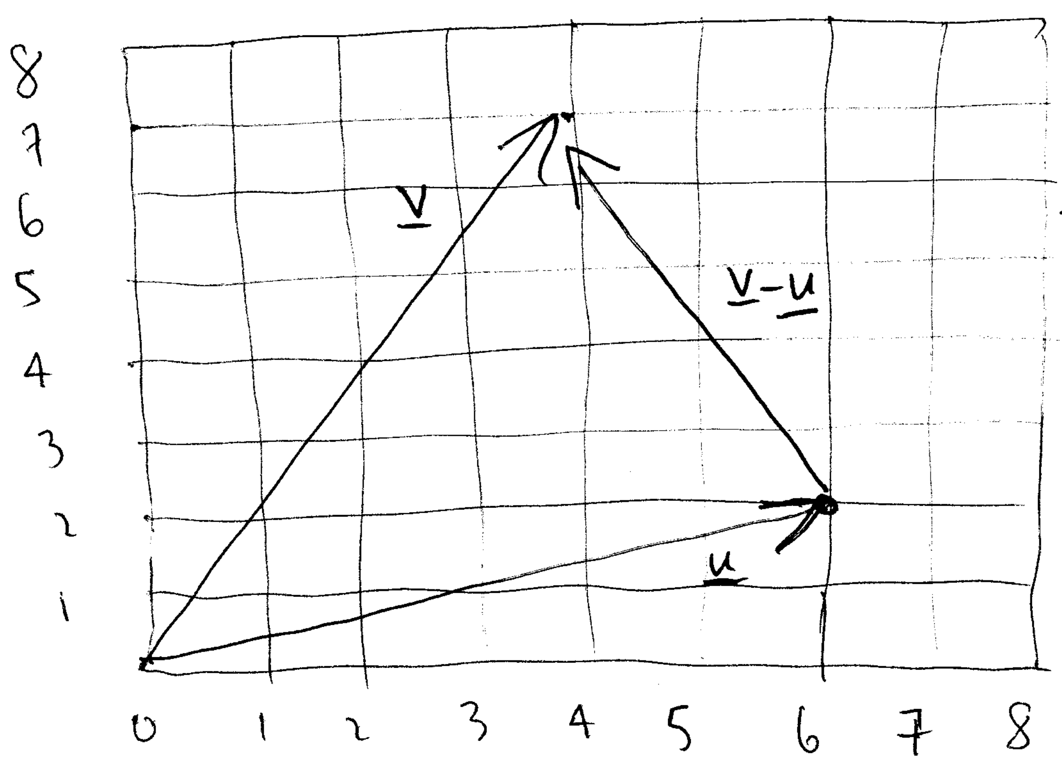
3. (a)



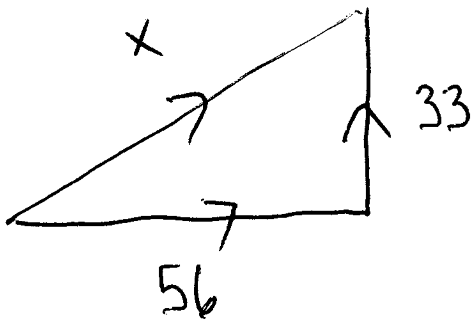
— (3)

(−1/2 each error).

$$\underline{v} - \underline{u} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}. \qquad \text{— } 1$$

(b) Let

x = speed in km/h the plane flies during the 3 second updraft



$$x = \sqrt{56^2 + 33^2}$$

$$= 65 \text{ km/h} \qquad \text{— } 1$$

(pythagoras)

The plane travels

65 km in 1 hour

i.e. 65 km in 3600 seconds

so $\frac{65}{3600}$ km in 1 second — 2

i.e. $\frac{65}{3600} \times 1000 = \frac{325}{18}$ m in 1 second,

thus $\frac{325}{18} \times 3 \approx \boxed{54.17 \text{ metres}}$ — 1

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4. (a)

$$\underline{u} + \underline{v} = (u_1, u_2, \dots, u_n) + (v_1, v_2, \dots, v_n) \quad \text{———} \quad 1/2$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \quad \text{———} \quad 1/2$$

$$= (v_1 + u_1, v_2 + u_2, \dots, v_n + u_n) \quad \text{———} \quad 1$$

$$= (v_1, v_2, \dots, v_n) + (u_1, u_2, \dots, u_n) \quad \text{———} \quad 1/2$$

$$= \underline{v} + \underline{u} \quad \text{———} \quad 1/2$$

(b) Let

$$u_1 = \begin{pmatrix} a_1 \\ a_1 - c_1 \\ c_1 + 1 \end{pmatrix} \in V, \quad u_2 = \begin{pmatrix} a_2 \\ a_2 - c_2 \\ c_2 + 1 \end{pmatrix} \in V \quad \text{———} \quad 1$$

$$u_1 + u_2 = \begin{pmatrix} a_1 + a_2 \\ a_1 + a_2 - (c_1 + c_2) \\ c_1 + c_2 + 2 \end{pmatrix} = \begin{pmatrix} a_3 \\ a_3 - c_3 \\ c_3 + 2 \end{pmatrix} \notin V \quad \text{———} \quad 1$$

$$a_3 := a_1 + a_2$$

$$c_3 : c_1 + c_2$$

Thus V <u>not</u> closed w.r.t. addition & so

V is not a subspace of $\mathbb{R}^3$. (1)

OR

Let

$$u = \begin{pmatrix} a \\ a-c \\ c+1 \end{pmatrix} \in V, \quad k \in \mathbb{R}$$

$$ku = \begin{pmatrix} ka \\ ka-kc \\ kc+k \end{pmatrix} = \begin{pmatrix} \hat{a} \\ \hat{a}-\hat{c} \\ \hat{c}+k \end{pmatrix} \notin V$$

unless $k=1$,

$$\hat{a} := ka$$
$$\hat{c} := kc.$$

OR

another method (e.g showing that V does not possess a zero vector).

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