UNIVERSITY of GUELPH

Fall 2017 MATH*1030 Midterm 1

Last name (print): ________________ **First** name (print): ________________

Student #: ____________________ Signature: ________________________

1. This is a 50 minute test. Do NOT start until instructed.
2. Please fill out your personal details above. Once the examination starts fill out your solutions in the spaces provided.
3. You may quote results used in your arguments from lecture notes without justification, unless asked to do otherwise. Notes or books are not permitted during the exam.
4. Give sufficient working as solutions with no justification will receive little or no credit.
5. You may use a scientific calculator and/or a 'graphing calculator'. Note however that graphical answers require justification. No other electronic equipment is permitted. **Turn off or 'mute' your cell phone! The use of cell phones during the exam is prohibited.**
6. If you copy the work of your neighbour this is considered **Academic Misconduct** and will be reported to the appropriate university authority.
7. There are a total of **36** points to be awarded on this test.

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1. (7 marks)

(a) Write as a *single* interval or set:

$$\{x \in \mathbb{R} \mid x > 3\} \cap \{x \in \mathbb{R} \mid x \leq 4\}.$$

$(3, 4]$ or $\{x \in \mathbb{R} \mid 3 < x \leq 4\}$ $\cdots\cdots$ (2)

($-\frac{1}{2}$ each error)

(b) A coffee and a cupcake together cost $\$1.30$. If the cupcake costs 30 cents more than the coffee, what are the prices of the coffee and the cupcake?

Let

$x =$ cost of a cupcake
$y =$ cost of a coffee

then from the given information we have

$$\begin{cases} x + y = 1.30 \\ x - y = 0.30 \end{cases}$$ $\cdots\cdots$ (1)

(method)

Subtracting the 2nd equation from the first yields $2y = 1.00$, thus
$y = 0.50$ or $50¢$ and $x = 0.80$ or $80¢$ $\cdots\cdots$ (2)

(full marks if answers correct)

(Question continued on next page)

(c) Simplify

$$\frac{\frac{1}{x} - 1}{x - 1} + 1, \quad x \neq 1.$$

$$\begin{aligned}\frac{\frac{1}{x} - 1}{x - 1} + 1 &= \frac{1 - x}{x} \cdot \frac{1}{x - 1} + 1 \\ &= -\frac{\cancel{(x-1)}}{x} \cdot \frac{1}{\cancel{(x-1)}} + 1 \\ &= -\frac{1}{x} + 1 \\ &= 1 - \frac{1}{x} \quad \text{or} \quad \frac{x-1}{x} \end{aligned} \qquad \cdots\cdots \quad (2)$$

($-\frac{1}{2}$ each error)

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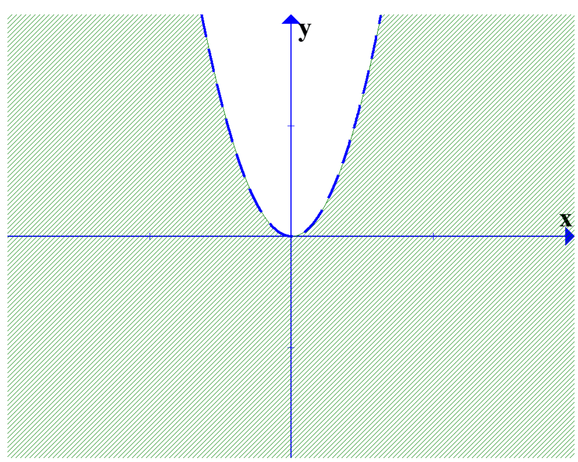
2. (10 points)

(a) Sketch below the region in the plane corresponding to the set:

$$\{(x, y) \quad | \quad y < x^2\}.$$

Use the convention that if the boundary of a region is included in the region it is indicated using a solid line, otherwise use a dashed/dotted line.



$\cdots\cdots$ (2)

($-\frac{1}{2}$ each error)

(Question continued on next page)

(b) Find the slope of the line joining the points $(-3, 6)$ and $(2, 0)$.

Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (-3, 6)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \cdots\cdots \ (1)$$

(formula)

$$= \frac{6 - 0}{-3 - 2} = -\frac{6}{5} \qquad \cdots\cdots \ (1)$$

(answer)

(c) Use the method of substitution to solve the system of equations below:

$$\begin{cases} x + y & = -3 \\ x^2 + y^2 & = 17 \end{cases}$$

From the 1st equation we have

$$y = -x - 3 \qquad (*)$$

Now substitute $(*)$ into the 2nd equation yielding:

$$\begin{aligned} x^2 + (-x - 3)^2 &= 17 \\ x^2 + (x^2 + 6x + 9) &= 17 \\ 2x^2 + 6x - 8 &= 0 \\ x^2 + 3x - 4 &= 0 \\ (x + 4)(x - 1) &= 0 \\ \implies x = -4 \text{ or } x &= 1 \end{aligned} \qquad \cdots\cdots \ (4)$$

(method)

$(-\frac{1}{2}$ each error)

Using $(*)$:
$x = -4 \implies y = 4 - 3 = 1$. $x = 1 \implies y = -1 - 3 = -4$.
Solutions are $(-4, 1)$ and $(1, -4)$. $\cdots\cdots$ (2)

(answers)

3. (10 points)

(a) Algebraically determine the range and domain of the functions:

(i) $f(x) = \sqrt{1+3x}$, (ii) $g(x) = 1 - \dfrac{2}{x-3}$.

Give full justification for your answers!

(i) **Domain**: need $1 + 3x \geq 0$ so $3x \geq -1$ thus $x \geq -1/3$ $\cdots\cdots$ (1) (explanation)

$D_f = [-1/3, +\infty)$. $\cdots\cdots$ (1) (answer)

Range: first note that $\sqrt{(\cdot)} \geq 0$. And by taking x as large as we like ($\geq -1/3$) we can make y as large as we like, thus $\cdots\cdots$ (1) (explanation)

$R_f = [0, +\infty)$. $\cdots\cdots$ (1) (answer)

(ii) **Domain**: $x - 3 \neq 0$ so $x \neq 3$, thus $\cdots\cdots$ (1) (explanation)

$D_g = \{x \in \mathbb{R} \mid x \neq 3\}$ or $(-\infty, 3) \cup (3, +\infty)$. $\cdots\cdots$ (1) (answer)

(Note: $x = 3$ is a vertical asymptote.)

Range: For large x (positive or negative), $\frac{2}{x-3} \approx 0$, so $g(x) \approx 1$, thus

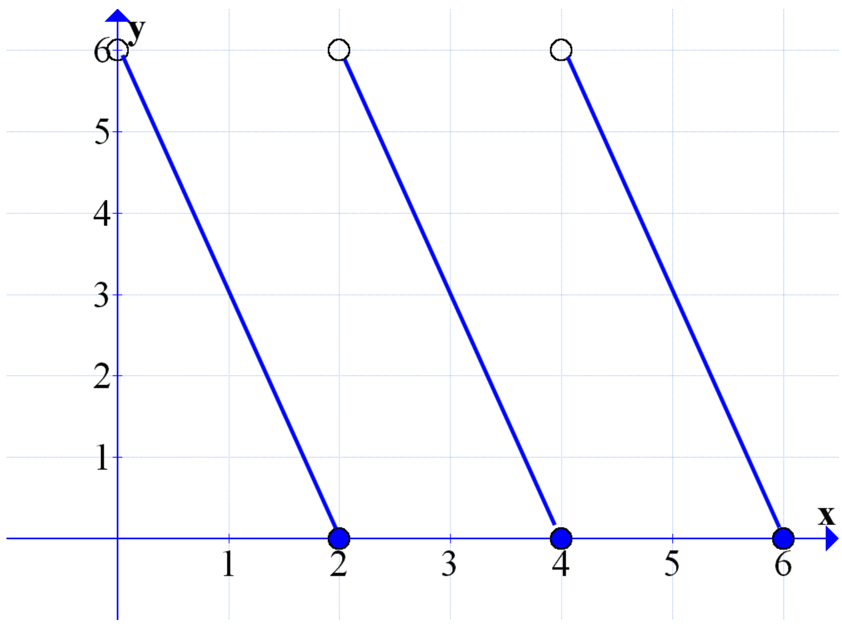
$\cdots\cdots$ (1) (explanation)

$R_g = \{y \in \mathbb{R} \mid y \neq 1\}$ or $(-\infty, 1) \cup (1, +\infty)$. $\cdots\cdots$ (1) (answer)

(Note: $y = 1$ is a horizontal asymptote.)

(Question continued on next page)

(b) Suppose a function has period $\mathbf{2}$ and on the interval $(\mathbf{0}, \mathbf{2}]$ has the form $\boldsymbol{y = -3x + 6}$, then sketch the graph of the periodic function on the $(\mathbf{0}, \mathbf{6}]$. Show the numbers on your scale.



$\cdots\cdots$ (2)

($-\frac{1}{2}$ each error)

4. (9 marks)

(a) Let $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = x - 3$. Find the composite function $(f \circ g)(x)$ and its domain.

$D_g = \mathbb{R}$ (g is a polynomial). $\cdots\cdots$ (1/2)

$f(g(x)) = f(x-3) = \frac{1}{\sqrt{x-3}}$. $\cdots\cdots$ (1)

Need $x - 3 > 0$ (not $= 0$ otherwise we divide by zero). I.e., $x > 3$, so $D_{f(g(x))} = (3, \infty)$. $\cdots\cdots$ (1/2)

Thus (see lecture notes)

$$D_{f\circ g} = D_g \cap D_{f(g(x))} = \mathbb{R} \cap (3, \infty) = (3, \infty). \qquad \cdots\cdots \quad (1)$$

(full marks if correct, and $-1/2$ if $[3, \infty)$)

(Question continued on next page)

(b) Given the function

$$f(x) = \frac{x+4}{3x-2}$$

find the inverse function f^{-1}. (Note: for this problem there is no need to discuss the range and domain.)

step 1: $y = \dfrac{x+4}{3x-2}$ $\cdots\cdots$ (1/2)

step 2: $x = \dfrac{y+4}{3y-2}$ (swap x and y) $\cdots\cdots$ (1/2)

step 3: Solve for y

$$\implies x(3y-2) = y+4$$
$$\implies 3xy - 2x = y+4$$
$$\implies 3xy - y = 2x+4$$
$$\implies y(3x-1) = 2x+4$$
$$\implies y = f^{-1}(x) = \frac{2x+4}{3x-1}$$ $\cdots\cdots$ (1)

(answer - full marks if correct)

$\cdots\cdots$ (2)

(method)

(-$1/2$ for each error if answer not correct)

(Question continued on next page)

(c) Let $f(x) = \dfrac{1}{\sqrt{x}}$. Describe how the rules of transformations are applied to the graph of f to obtain the graph of

$$g(x) = 1 + \frac{2}{\sqrt{x-1}}.$$

Does the graph of $y = g(x)$ have a y-intercept value? Explain.

step 1:
Shift the graph of f to the *right* by 1 unit, i.e.
$f(x) \longrightarrow f(x-1) = \dfrac{1}{\sqrt{x-1}}$ $\cdots\cdots$ (1/2)

step 2:
Stretch the graph from step 1 in the y direction by a factor of 2, i.e.
$f(x-1) \longrightarrow 2f(x-1) = \dfrac{2}{\sqrt{x-1}}$ $\cdots\cdots$ (1/2)

step 3:
Shift the graph of from step 2 *up* by 1 unit. i.e.
$2f(x-1) \longrightarrow 1 + 2f(x-1) = 1 + \dfrac{2}{\sqrt{x-1}}$ $\cdots\cdots$ (1/2)

The graph of $y = g(x)$ does NOT have a y-intercept value because when $x = 0$ we are taking the square root of a negative number. $\cdots\cdots$ (1/2)

END OF TEST

TA Initials (print) ________ Total: / **36**