UNIVERSITY of GUELPH

Winter 2018 MATH*1030 Midterm 1

Last name (print): ________________ **First** name (print): ________________

Student #: ____________________ Signature: ________________________

1. This is a 50 minute test. Do NOT start until instructed.
2. Please fill out your personal details above. Once the examination starts fill out your solutions in the spaces provided.
3. You may quote results used in your arguments from lecture notes without justification, unless asked to do otherwise. Notes or books are not permitted during the exam.
4. Give sufficient working as solutions with no justification will receive little or no credit.
5. You may use a scientific calculator and/or a 'graphing calculator'. Note however that graphical answers require justification. No other electronic equipment is permitted. **Turn off or 'mute' your cell phone! The use of cell phones during the exam is prohibited.**
6. If you copy the work of your neighbour this is considered **Academic Misconduct** and will be reported to the appropriate university authority.
7. There are a total of **31** points to be awarded on this test.

THIS PAGE INTENTIONALLY LEFT BLANK

1. (5 marks)

(a) Write as a *single* interval or set:

$$\{x \in \mathbb{R} \mid x > 3\} \cap \{x \in \mathbb{R} \mid x \leq 1\}.$$

The null ('empty') set $\emptyset$ $\cdots\cdots$ (1)

(answer)

(b) The argument below is clearly incorrect ($1 \neq 0$!!). Explain what is wrong with the reasoning:

Step 1: $x = 1$ (initial assumption)
Step 2: $x^2 = x$ (multiply both sides by x)
Step 3: $x^2 - 1 = x - 1$ (subtract 1 from both sides)
Step 4: $x^2 - 1^2 = x - 1$ ($1^2 = 1$)
Step 5: $(x + 1)(x - 1) = x - 1$ (Using difference of 2 squares)
Step 6: $x + 1 = 1$ (From comparing both sides in Step 5)
Step 7: $x = 0$ (Follows from Step 6)

To go from Step 5 to Step 6 we divide both sides by $x - 1$, so need $x - 1 \neq 0$, or $x \neq 1$. But $x = 1$ (Step 1) so this is invalid. (Note: it is interesting that the equation in Step 5 is satisfied by 2 values of x, namely $x = 1$ and $x = 0$, but of course the $x = 0$ solution is invalidated by Step 1.)

$\cdots\cdots$ (2)

(Explanation)

(Question continued on next page)

(c) Simplify

$$\frac{1}{x} + \frac{\frac{1}{x}}{1+\frac{1}{x}}, \quad x \neq 0.$$

$$\begin{aligned}
\frac{1}{x} + \frac{\frac{1}{x}}{1+\frac{1}{x}} &= \frac{1}{x} + \frac{\frac{1}{x}}{\frac{x+1}{x}} \\
&= \frac{1}{x} + \frac{1}{\cancel{x}} \cdot \frac{\cancel{x}}{x+1} \\
&= \frac{1}{x} + \frac{1}{x+1} \\
&= \frac{(x+1)+x}{x(x+1)} = \frac{2x+1}{x(x+1)}
\end{aligned}$$

...... (2)

($-\frac{1}{2}$ each error)

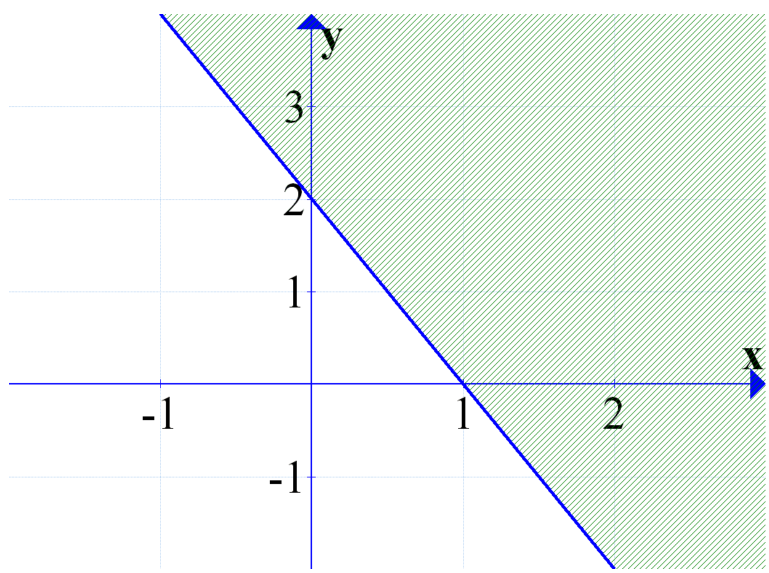
---

2. (10 points)

(a) Sketch below the region in the plane corresponding to the set:

$$\{(x,y) \quad | \quad y \geq -2x+2\}.$$

Use the convention that if the boundary of a region is included in the region it is indicated using a solid line, otherwise use a dashed/dotted line. Clearly show the x and y intercepts of your graph.



...... (2)

($-\frac{1}{2}$ each error)

(Question continued on next page)

(b) Calculate the midpoint of the line segment joining the points $(2, 4)$ and $(-6, 18)$.

Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (-6, 18)$. Then

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \cdots\cdots \quad (1)$$

(formula)

$$= \left(\frac{2-6}{2}, \frac{4+18}{2}\right)$$
$$= (-4/2, 22/2)$$
$$= (-2, 11) \qquad \cdots\cdots \quad (1)$$

(answer)

(c) Use the method of substitution to solve the system of equations below:

$$\begin{cases} xy &= 1 \\ x + y &= 2 \end{cases}$$

From the 2nd equation we have

$$y = 2 - x \qquad (*)$$

Now substitute $(*)$ into the 1st equation yielding:

$$x(2 - x) = 1$$
$$\implies 2x - x^2 = 1$$
$$\implies x^2 - 2x + 1 = 0$$
$$\implies (x - 1)(x - 1) = 0$$
$$\implies x = 1 \quad \text{(twice)} \qquad \cdots\cdots \quad (4)$$

(method)

$(-\frac{1}{2}$ each error)

Using this x value in $(*)$ yields:

$y = 2 - 1 = 1$. Thus the solution is the single point $(1, 1)$. $\cdots\cdots$ (2)

(answers)

3. (9 points)

(a) Algebraically determine the range and domain of the functions:
(i) $f(x) = \sin(x)$, (ii) $g(x) = \dfrac{1}{2x - 1}$.
Give full justification for your answers in (ii)!

(i) $D_f = \mathbb{R}$, $R_f = [-1, 1]$ ...... (2)
(see your notes)

(ii) **Domain**: $2x - 1 \neq 0$ so $x \neq 1/2$, thus ...... (1)
(explanation)

$D_g = \{x \in \mathbb{R} \mid x \neq 1/2\}$ or $(-\infty, 1/2) \cup (1/2, +\infty)$. ...... (1)
(answer)

(Note: $x = 1/2$ is a vertical asymptote.)

Range: For large x (positive or negative), $\frac{1}{2x-1} \approx 0$, thus

...... (1)
(explanation)

$R_g = \{y \in \mathbb{R} \mid y \neq 0\}$ or $(-\infty, 0) \cup (0, +\infty)$. ...... (1)
(answer)

(Note: $y = 0$ is a horizontal asymptote.)

(Question continued on next page)

(b) Is the function

$$f(x) = x + x^3 - 2x^5,$$

odd, even, or neither? Justify your answer.

Consider

$$f(-x) = -x + (-x)^3 - 2(-x)^5 \qquad \cdots\cdots \quad (1)$$

(method)

$$= -x - x^3 + 2x^5$$

$$= -(x + x^3 - 2x^5) = -f(x), \qquad \cdots\cdots \quad (1)$$

(working)

thus f is an odd function ······ (1)

(answer)

---

4. (7 marks)

(a) Let $f(x) = \sqrt{x-9}$ and $g(x) = \dfrac{1}{x^2}$. Find the composite function $(g \circ f)(x)$ and its domain.

Need $x - 9 \geq 0$ so $x \geq 9$ thus $D_f = [9, +\infty)$ ······ (1/2)

$g(f(x)) = g(\sqrt{x-9}) = \frac{1}{(\sqrt{x-9})^2} = \frac{1}{x-9}$, ······ (1)

with $D_{g(f(x))} = \{x \in \mathbb{R} \mid x \neq 9\}$. ······ (1/2)

Thus (see lecture notes)

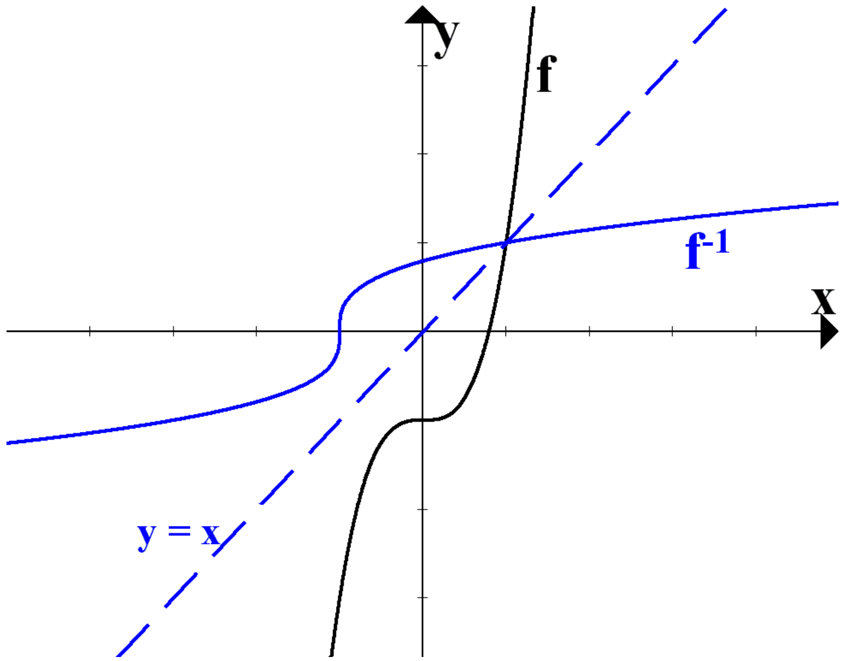
$$D_{g \circ f} = D_f \cap D_{g(f(x))} = [9, +\infty) \cap \{x \in \mathbb{R} \mid x \neq 9\} = (9, \infty).$$

······ (1)

(full marks if correct, and $-1/2$ if $[9, \infty)$)

(Question continued on next page)

(b) The graph of the function f is shown below. On the same diagram sketch the graph of the inverse function f^{-1}.



...... (2)
($-\frac{1}{2}$ each error)

(Question continued on next page)

(c) Let $f(x) = x^2$. Describe how the rules of transformations are applied to the graph of f to obtain the graph of

$$g(x) = 1 - 2(x-3)^2.$$

step 1:
Shift the graph of f to the *right* by $\mathbf{3}$ unit, i.e.
$f(x) \longrightarrow f(x-3) = (x-3)^2$ $\cdots\cdots$ (1/2)

step 2:
Reflect the graph from step 1 in the x-axis, i.e.
$f(x-3) \longrightarrow -f(x-3) = -(x-3)^2$ $\cdots\cdots$ (1/2)

step 3:
Stretch the graph of from step 2 in the y-direction by a factor of $\mathbf{2}$, i.e.
$-f(x-3) \longrightarrow -2f(x-3) = -2(x-3)^2$ $\cdots\cdots$ (1/2)

step 4:
Shift the graph of from step 3 *up* by $\mathbf{1}$ unit. i.e.
$-2f(x-3) \longrightarrow 1 - 2f(x-3) = 1 - 2(x-3)^2$ $\cdots\cdots$ (1/2)

END OF TEST

TA Initials (print) ________ Total: / **31**