A theory of local contagion

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Abstract

Prices are usually assumed to adjust so that any dynamic characteristics of a market are independent across regions within a market. This paper studies dynamics and interaction on a local scale. I argue that the micro-structure affects macro-phenomena in the sense that contagion between regions alters the dynamic properties of aggregate measures. This analysis uses both algebraic models and numerical simulation.

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This paper considers an economic process that lies between static equilibrium models, typified by Rosen (1974), and dynamic models intended to explain the time series properties of the average or median price for an entire city, typified by Capozza, Hendershott and Mack (2004). It is also motivated by the increasing availability of data sets which report on measures of market conditions involving subaggregates, such as neighborhoods. I consider the implications of the idea that, over time, changes in prices of one group of houses change the behavior of either buyers or sellers in ways that affect the prices of other groups of houses in the future. For example, an increase in prices in one neighborhood may cause buyers to look at other neighborhoods.

Spatial correlation models (e.g. Anselin 1988, 2001; LeSage and Pace 2004; Elhorst, 2003; see also Giacomini and Granger, 2004) are often used to improve the fit of a static hedonic price model since it is not unreasonable to suppose that many of the important omitted variables vary over space. This paper seeks to add a time dimension in two senses. First, that prices at a point in time in one neighborhood may depend on prices in other neighborhoods. Second, that even though many papers characterize a city by a single market price, disaggregating adds a new dimension to the time series properties. By noting certain types of behavior that have not been studied very carefully, this paper raises the question of whether the statistical errors that can be removed by refining an estimator are more important than errors due to misspecification.

It may be true that few have tried to characterize spatial correlation because another theory has been extremely useful. Rosen's (1974) equilibrium model predicts that prices should not have to keep adjusting for differences between locations without a continuing series of shocks. It is also not unreasonable to suppose that a marginal trader can arbitrage price differences over space in ways that are not possible to do, or are more risky to do, over time (Coulson, 1993; Colwell, 1993). Temporal arbitrage should be based on the value of time which should not vary across locations. This body of work suggests, but does not prove, that prices in different regions should move together over the long periods of time. A growing literature provides evidence to the contrary (Seslen, Wheaton and Pollakowski, 2005; Gyourko, Mayer and Sinai, 2006).

The following section offers a simple example to characterize disaggregated price dynamics. The example leads to a more general theory and a key step in the argument is finding a fairly general representation that can be used to construct simple examples and counter-examples. It would be nice to claim that the representation is fully general but I note that some of the desirable characteristics require restrictions. The analysis uses some simple and not so simple matrix algebra. The theory justifies a system of difference equations where the dynamics depend on two parameters, representing persistence and contagion.

To supplement an analysis based on the eigenvalues and eigenvectors of this simple system, some numerical simulations indicate the magnitudes involved. The simulations add a new correlation parameter to the analysis and considers the effects of random shocks. I show that the effects can be surprisingly large for a small degree of contagion. The effects seem most dramatic when the other parts of the model suggest prices have a strong cyclical component.

A Simple Model

I start by illustrating the ideas with a simple model of a city with three regions or nodes: one central (#2) and two on the periphery (#1 and #3), as shown in Figure 1. Later, this model will be modified and simulated numerically.

Figure 1: Nodes in a City

1 ----- 2 ----- 3

In this city, heterogenous buyers enter. Each buyer searches for an appropriate house located in a node that is appropriate for them and then bargain over the price. The search procedure takes time and, because buyers may react to excessive demands of sellers in a high priced area, it is not unreasonable to suppose that high prices in an area may affect prices in other areas with a lag. This story[1] suggests that that the time path of prices in each node can be described by

$$p^1_t = \mu^1 + \tau p^1_{t-1} + (\alpha/2)\,((p^2_{t-1} - \mu^2) - (p^1_{t-1} - \mu^1)) \quad (1)$$
$$p^2_t = \mu^2 + \tau p^2_{t-1} + (\alpha/2)\,((p^1_{t-1} - \mu^1) - (p^2_{t-1} - \mu^2)) + (\alpha/2)\,((p^3_{t-1} - \mu^3) - (p^2_{t-1} - \mu^2))$$
$$p^3_t = \mu^3 + \tau p^3_{t-1} + (\alpha/2)\,((p^2_{t-1} - \mu^2) - (p^3_{t-1} - \mu^3))$$

A superscript denotes the node and a subscript denotes the time period. τ represents the persistence in a region with the simplest possible time series process: AR(1).

α represents the degree of contagion between nodes and the contagion takes a particular form. First, the effect depends on the difference in prices, adjusted for differences in the amenities in each node (i.e. μ^i). This representation suggests that the gain for one node is a loss to another node. Second, the price in a node affects its neighbor(s) with a lag of one period. Given the relationships shown in Figure 1, the price in node 1 can affect the price in node 2 but not in node 3. The price in node 2 can affect the price in both nodes 1 and node. Third, the contagion parameter is equal for both nodes 1 and 3.

If $\alpha = 0$ and there were only one node, then this would be simple time series model. If $\tau = 0$, then any dynamic features would be caused by contagion. This story is very simple and future work should refine the parameters since the proposed behavior depends on many things, including the distribution of buyer types, their willingness to travel, the degree to which a buyer is informed about prices and characteristics of houses in other nodes, and the willingness to participate. This story also invokes simplistic ideas of seller bargaining behavior to generate the resulting price in each period.

The simplest type of solution is a steady state solution. The only steady state solution to this model is

$$(p^{1s}_t, p^{2s}_t, p^{3s}_t) = (\mu^1, \mu^2, \mu^3). \quad (2)$$

This solution acts a reference point. Conditional on historic information and information

[1] This paper focuses on the dynamics and does not derive a behavioral model of the law of motion described in equation (1). The concluding section offers some thoughts on the foundations of a behavioral model.

from nearby nodes, deviations from a steady state are possible. It is also interesting to solve for the time path explicitly if p^i_t were perturbed from the steady state. (p^1_t, p^2_t, p^3_t) evolves according to a relatively straightforward difference equation:

$$\begin{bmatrix} p_t^1 - \mu^1 \\ p_t^2 - \mu^2 \\ p_t^3 - \mu^3 \end{bmatrix} = \begin{bmatrix} \tau + \alpha/2 & -\alpha/2 & 0 \\ -\alpha/2 & \tau + \alpha & -\alpha/2 \\ 0 & -\alpha/2 & \tau + \alpha/2 \end{bmatrix} \begin{bmatrix} p_{t-1}^1 - \mu^1 \\ p_{t-1}^2 - \mu^2 \\ p_{t-1}^3 - \mu^3 \end{bmatrix} \tag{3}$$

Since this is a linear difference equation, the general solution for the price path (Hoy et al, 2001, Section 24.3) is

$$(p^1_t, p^2_t, p^3_t)' = a_1 (\lambda_1)^t e_1 + a_2 (\lambda_2)^t e_2 + a_3 (\lambda_3)^t e_3 \tag{4}$$

where λ_i represents an eigenvalue of the 3x 3 matrix, e_i represents an eigenvector and (a_1, a_2, a_3) is determined by the initial conditions. For this problem, the needed eigenvalues and eigenvectors are:

Eigenvalues	Eigenvectors
$\lambda_1 = \tau - 3\alpha/2$	$e_1 = [\ 1, -2, 1]'$
$\lambda_2 = \tau - \alpha/2$	$e_2 = [-1, \ 0, 1]'$
$\lambda_3 = \tau$	$e_3 = [\ 1, \ 1, 1]'$.

Depending on the values of τ and α, the price paths have different dynamic properties. The most familiar solution arises when $\tau > 0$ and α is small enough that $|\tau - 3\alpha/2| < \tau < 1$. Under these conditions, the largest eigenvalue is $\lambda_3 = \tau$ and its effect will dominate in the long term. (p^1_t, p^2_t, p^3_t) will converge to the steady state along the vector (1, 1, 1): all prices move together. The time path in the short term is determined by the other eigenvalues and eigenvectors: prices in the peripheral nodes move in opposite directions and prices in the center and periphery move in opposite directions. The relative importance of these two forces varies with the initial conditions. Thus, the short term path of prices is not representative of the long term trends.

If $\tau < 0$ and $\alpha > 0$, or $\tau > 0$ and α large, then the dynamic properties are very different. The eigenvalue whose effects will dominate long term adjustment is $\lambda_1 = \tau - 3\alpha/2$. Since $\lambda_1 < 0$ under these conditions, the time path will cycle. If α is sufficiently large, then the time path could be explosive because the absolute value of the eigenvalue could exceed 1. The relevant eigenvector is e_1 where the center and periphery move in equal and opposite directions. Under the stated conditions, $\lambda_3 = \tau$ is the eigenvalue with the smallest absolute value and its effect dissipates fastest. This case also illustrates how the price of houses in a node can vary over time in ways that are important to a one buyer buying one house in one node even if there is no persistence, or apparent risk, in aggregated prices.

A General Model and General Results

Consider a city defined by a set of nodes, I, which is indexed by i. The effect on the price in node i during period t, $p^i(t)$, of a change in the price in node j during period t-1, $p^j(t-1)$, is governed by the parameter α^{ij}. Since one node may have many neighbors, the aggregation of these effects is represented by

$$p^i(t) = \mu^i + \sum_{j \varepsilon I} \alpha^{ij} \left((p^j(t-1) - \mu^j) - (p^i(t-1) - \mu^i) \right) \qquad (5)$$

Using a linear representation simplifies the analysis and enables the use of matrix methods:

$$P(t) = M + A\,(P(t-1) - M) \qquad (6)$$

where P(t) is the vector of node's prices in period t, $M = (\mu^1, \mu^2, \ldots, \mu^I)'$ and the (i, j)th element of A is α^{ij}.

This notation is intended to be fairly general. It can represent the fixed relationship between neighbors, as estimated by most parametric models of spatial econometrics. The index set of nodes, I, is also sufficiently general that it need not be one dimensional.[2]

An especially simple special case is a city with isolated nodes, i.e. A is a diagonal matrix. The price in each node would evolve independently according to its local persistence parameter. A more exciting special case is

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

[2] With only a little effort, it may be possible to extend this analysis of an arbitrarily fine but discrete representation in space to a continuous representation. I choose not to do so because the mathematical tools required, especially those involving "eigenfunctions", are not well known and because I expect few new results.

Arithmetic shows that, with this matrix, any deviation from the steady state in node 1 is completely transmitted to region 2 after one period and any deviation from the steady state in node 2 is transmitted to node 3 after one period and so on. The time path of an equally-weighted aggregate price would display no cycle because each node has a local cycle that repeats every three periods and travels around the city in a circle forever without diminishing. To consider (or test the restrictiveness of) different ways of aggregating nodes, such as when aggregating individual blocks of houses into a neighborhood or a district, the interaction can be restricted into equivalence classes: i.e. $\alpha^{ij}= \alpha^{i'j'}$ if i and i' belong to the same neighborhood and j and j' belong to the same neighborhood.

An important restriction on A is that the sum of the elements for each row be constant, and this constant will be compared to τ from the previous section. In part, this restriction is a normalization in the sense that the diagonal elements of the matrix should be interpreted as the effect of persistence within a node and the off diagonal elements indicate contagion between nodes. But, I wish to maintain the hypothesis that the persistence parameter for each node is also a persistence parameter that applies to the city as a whole. To the extent that contagion is important, a buyer who buys in one node is not buying in another. Thus, the restriction represents an adding up restriction. The conclusion will note that this representation of the adding up restriction imposes other conditions also. A similar normalization is commonly used in models of spatial econometrics.

Finally, A is assumed to be invertible. If A were not invertible then a continuum of steady state solutions would satisfy equation (6). This outcome may be reasonable from the short term perspective of equation (6) but is unreasonable from a long term perspective which considers buyer tastes and choice more broadly. To the extent that differences in μ^i represents differences in the value placed on a node by an average buyer, then having a continuum of possible solutions is unreasonable. In principle, it can also be tested.

Proposition 1

The unique steady state solution is $P^s = M$.

Proof

Letting $P(t) = P(t-1) = P^s = M$ in equation (6), shows that P^s is a steady state solution. Since A is assumed to an invertible, P^s is the unique solution. Q.E.D.

Proposition 2

Let $\tau = (\sum \alpha^{ij})/I$. τ is an eigenvalue of A and (1, …, 1)' is the associated eigenvector.

Proof

Since the sum of elements for each row of A is the same for all rows, τ is equal to that sum and calculation shows that

$$A\,(1, ..., 1)' = \tau \text{ Iden } (1, \ldots, 1)'. \tag{7}$$

where Iden is an identity matrix with I rows and I columns. Since there is a non-zero vector such that (A- τ Iden) times that vector is a zero vector, the determinant of (A- τ Iden) must be zero. Thus, τ satisfies the definition of being an eigenvalue of A, and (1, …, 1)' is the associated eigenvector. Q.E.D.

This Proposition depends critically on sum of the elements on each row adding up to a common number. If (1, …, 1)' is not an eigenvector of A, then even if $p^i(t-1) - \mu^i$ were equal in all nodes but not zero, then the time path of prices would not move as an aggregate. Some nodes would converge slower and others faster, so the time path of the aggregate price would depend on the relative importance of the different nodes within any aggregate. Estimating the time series properties of such an unbalanced system would be more difficult and the estimated coefficients are more likely to be unstable.

Proposition 3

Suppose that $x = (x_1, \ldots, x_I)'$ is an eigenvector of A whose eigenvalue differs from τ. If A is symmetric, i.e. $\alpha^{ij} = \alpha^{ji}$ for all i and j, then $\sum_i x_i = 0$.

Proof

Proposition 2 shows that (1, …, 1)' is one of the eigenvectors. Many books on matrix algebra (e.g. Hoy et al, 2001, p. 428) note that every symmetric matrix has a set of orthogonal eigenvectors. Thus, the result of multiplying the vectors (1, …, 1)' and (x_1, …, x_I)' must be 0: $\sum_i x_i = 0$. Q.E.D.

This Proposition generalizes the discussion of the previous section. Common movements in prices are defined by the eigenvector (1, …, 1)' and its eigenvalue, τ. $\sum_i x_i = 0$ implies that every other eigenvector must have some positive elements and some negative elements. Thus, short run fluctuations are necessarily characterized by opposing movements in different nodes whose aggregate value is 0.

Unfortunately, the premise of this proposition is not always true. Consider a city with a centre and a periphery. In the centre, a shift in local demand would tend to change the price but, in the periphery, a similar shift would tend to shift the quantity of housing. In my model, a change in another node's price would shift the demand shift the node-specific demand curve but the cross-effect on prices should differ across nodes according to the supply response. This result may be approximately true on a local scale where symmetry in the interaction among a group of nodes may dominate any effects due to interaction between that group and nodes in other parts of a city.

Its value as an approximation is open for empirical analysis since, even if A is not precisely symmetric, it may be close enough that the eigenvectors are nearly orthogonal: since the eigenvalues of a matrix A depend on its elements continuously and since the eigenvectors depend on A and the eigenvalues continuously, a small change in A has a small effect on the eigenvectors. Giacomini and Granger (2004) outline the advantages of setting parameters with small values equal to zero when estimating.

This continuity also helps with a different question. The advantage of a general representation is that it is open to interpretation. The assumption that there are exactly I nodes at all times may be questioned, especially in a city with a growing population. It is possible to avoid this assumption by adding extra nodes but making them irrelevant by setting $\alpha^{ij} = 0$ for all j. Thus, the properties of a system change with extra nodes but they change only a little if the coefficients are small.

The previous section defined the time path of a vector of prices for a specific model. For the next Proposition, it is necessary to give a general definition. Using Hoy et al (2001, Section 24.3), the time path of P(t) which satisfies equation (6) is

$$P^*(t)= M+ \sum_i a_i (\lambda_i)^t e_i. \quad (8)$$

The relative importance of different eigenvalues depends on how the starting point P(0) determines the coefficients $a= (a_1, \ldots, a_I)$.[3]

The interaction between nodes can also affect the properties of aggregates such as the average price. Define the average price as

$$AP(t)= \sum_i \pi^i p^i(t) \quad (9)$$

for some constants π^i. For certain types of questions, i.e. shocks which have equal effects on all nodes, Proposition 4 i) shows when the time series properties can be parameterized by a single parameter, τ. When combined with previous propositions, Proposition 4 ii) and iii) distinguishes the characteristics of endogenous cycles from the characteristics of trends driven by forces external to a market.

Proposition 4

i) If $p^i(t-1)- \mu^i$ is the same in all nodes, then $AP(t)- \sum_i \pi^i \mu^i = \tau (AP(t-1)- \sum_i \pi^i \mu^i)$.

ii) If the system is in the steady state in period t then an increase in μ^i by 1 in period t+ 1 increases $p^j(t+1)$ by a^{ji}.

iii) Between steady states, an increase in μ^i by 1 has no effect on p^{js}.

Proof

i) Let $K= p^i(t-1)- \mu^i$ be the same in all nodes. Then, in equation (8), the coefficients on $e_i= (1, \ldots, 1)'$ would be K and the coefficient on all other eigenvector would be zero. Thus, $P^*(t)= M+ K \tau^t (1, \ldots, 1)'$. Therefore,

$$\begin{aligned} AP(t)- \sum_i \pi^i \mu^i &= \sum_i \pi^i (p^i(t) -\mu^i) \quad (10)\\ &= \sum_i \pi^i K \tau^t \\ &= \tau \sum_i \pi^i (p^i(t-1) -\mu^i) \\ &= \tau (AP(t-1)- \sum_i \pi^i \mu^i) \end{aligned}$$

ii) follows from equation (5)

[3] Formally, a must satisfy $P(0)= \sum_i a_i (\lambda_i)^0 e_i$.

iii) is an implication of Proposition 1.

Q.E.D.

The previous discussion emphasized how contagion affects the time series properties of a system. It is equally important to note that contagion has implications for our understanding of a static model. It is a common practice to treat locations using a series of dummy variables although more sophisticated datasets and software facilitate better spatial analysis. The comment that the relative prices of different nodes *can* change over time may be obvious but it is also not a constructive comment. A more constructive comment may be seen by considering a city with no location differences in terms of contagion.

Proposition 5

If α^{ij} is equal for all i and j then one of the eigenvalues is τ and all other eigenvalues must be 0.

Proof

Proposition 2 showed that one of the eigenvalues is τ and that its associated eigenvector is (1, …, 1)'. The premise of this proposition implies that the proposed matrix A is symmetric and Proposition 3 shows that all other eigenvectors would be orthogonal to (1, …, 1)'. Since each row of A is a scalar multiple of (1, …, 1)', the product of A and any eigenvector other than (1, …, 1)' equals (0, …, 0)'. Since A is a non-invertible matrix, the determinant of (A- 0 Iden) is zero and 0 is an eigenvalue of A. Since this argument applies to all eigenvectors other than (1, …, 1)', the eigenvalues associated with all other eigenvectors are 0.

Q.E.D.

If all nodes compete with all other nodes equally, then prices should adjust for any differences in general preferences represented by μ^i and α^{ij} should not vary with i and j. Combining Proposition 5 with equation (8) shows that the price dynamics would be extremely simple.

Simulations with Random Shocks

The model above ignores the effects of random shocks which have their own dynamic characteristics and change the smooth convergence described in the previous section. For this reason, and to demonstrate the magnitude of the effects discussed above, this section offers numerical simulations using the three-node model described in the beginning. The discussion focuses on the relative importance of adjustment, of persistence in shocks and of contagion to an explanation of observed data.

To equation (1), I add node-specific *random* shocks and remove all node-specific or general inflationary effects that may alter the steady state (i.e. $\mu^1 = \mu^2 = \mu^3 = 0$). Specifically, the prices in the nodes evolve according to

$$p^1_t = \tau p^1_{t-1} + \varepsilon^1_t + (\alpha/2)\,(p^2_{t-1} - p^1_{t-1}) \qquad (11)$$
$$p^2_t = \tau p^2_{t-1} + \varepsilon^2_t + (\alpha/2)\,(p^1_{t-1} - p^2_{t-1}) + (\alpha/2)\,(p^3_{t-1} - p^2_{t-1})$$
$$p^3_t = \tau p^3_{t-1} + \varepsilon^3_t + (\alpha/2)\,(p^2_{t-1} - p^3_{t-1})$$

where ε^i_t follows an AR(1) process which is independent between nodes,

$$\varepsilon^i_t = \rho\varepsilon^i_t + u^i_t, \qquad (12)$$

and u^i_t is i.i.d. Normal with a mean of 0. The important difference between ρ and α is that evidence for contagion can be found in other nodes whereas random effects are assumed to be node specific.

The tables below will consider how selected summary statistics vary with the values of ρ, τ and α. Since I wish to focus on the dynamic and contagion properties of the model, I look at correlations between prices in various times and nodes. I normalize the variance of u^i_t is 1; doubling this variance doubles each of the covariances reported below.

A problem with using simulations is that the reported results are, formally, only isolated examples. To ensure that the reported results represent results that can also be derived from a larger set of parameter values, it is necessary that the solution to a system vary "smoothly" with respect to the parameters.[4] The facts that equation (11) is a linear

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[4] Without loss of generality, consider the covariance between p^1_t and p^2_{t-1}. This covariance is a function of the parameters of the model, ρ τ and α. And this argument shows that the covariance depends continuously

system and that the eigenvalues of a matrix vary continuously with the parameter values ensures that the non-stochastic aspects of the solution are well-behaved.[5] Because of the difficulty in manipulating probability distributions, finding which results generalize from a model of three nodes arranged in a specific pattern to more than three or with an alternate pattern of interaction is beyond the scope of this paper. The concluding section offers some thought on this issue. The fact that the distribution of shocks is based on an AR(1) process combined with the Normal distribution should ensure that any results are not surprising for novel reasons.

To further ensure that the chosen parameter values are realistic, I use values that are broadly consistent[6] with those estimated by Capozza, Hendershott and Mack (2004). Using annual data, they found that the most common characterization of the time path of prices in a city was a cycle that converged to a steady state. The next most common characterization was a monotonic path that converged to a steady state. Thus, I consider a range of values that cover these cases: τ= -0.6 represents a case of a cycle with a slow convergence, τ= -0.2 represents a case of a cycle whose significance fades quickly and τ= 0.2 represents a case of slow monotonic convergence. Unfortunately, with only one parameter, the period and duration of the cycle cannot vary independently. I also consider the case of τ= 0.0 to isolate the effects of changes in α. Two values of ρ are considered, mostly to determine which features are sensitive to correlation in shocks to a node: ρ= 0.0 and 0.2.

I let α= 0.0, 0.05 and 0.1 and consider how observed values of certain covariances change. The variation in α is small in the sense that the value of the parameter is less than the others, in the sense that it would not change the properties based on the discussion in the second section, and, as shown later, in the sense that if τ= ρ= 0.0 then these covariances are usually small. More specifically, I consider autocorrelation in prices within the two types of nodes, the contagion from one node to another node in a

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on the parameters. Because of the difficulties in working with a system of stochastic variables, proving that the dependence is monotonic is beyond the scope of this paper.

[5] Non-linear systems can produce "chaotic" types of solutions.

[6] It is interesting to note that they consider two dynamic processes, serial correlation and mean reversion. I consider only the first since the concept of mean reversion in a disaggregated model is unclear: is the price in a node returning to the mean or is it returning to the mean price of nearby nodes?

later period and, to compare the characteristics of a solution for a node with a comparable aggregated measure, I consider $AP_t = (p^1_t + 2\, p^2_t + p^3_t)/4$.

The simulations were programmed using MATLAB. In each, the system was allowed to run for 100,000 iterations, with the first 10 percent being deleted to reduce the sensitivity to my choice of initial values. To further reduce the role of random differences between simulations, and to focus on the effects of different parameter values per se, each simulation uses the same series of random variables.

Table 1 shows that, when $\alpha = 0$, the reported covariance values are close to the true values of τ and ρ: i.e. the first and fourth row of numbers on the right hand panel plus the first and fourth line on the second last column. The exception to this rule occurs when $\tau = -0.6$. Each column shows that an increase in α decreases the apparent (first order) autocorrelation within a node. This decrease is about twice as large in the central node compared to the periphery node. The covariance sometimes exceeds the variance of the shocks (i.e. 1). The second order autocorrelation within a node is usually small, i.e. almost always positive but 0.05 or less, except for the case of $\tau = -0.6$ where it can be half of the first order autocorrelation or more. If α were larger, then the second order autocorrelation would become much more significant since it takes at least two periods for contagion to be reflected back to the original source.

Table 1: Autocorrelation within Nodes

	$\rho = 0.0$				$\rho = 0.2$			
Periphery (Node 1)	$\tau = -0.6$	-0.2	0.0	0.2	$\tau = -0.6$	-0.2	0.0	0.2
$\alpha = 0.0$	-0.924	-0.205	-0.002	0.209	-0.577	0.000	0.212	0.453
0.05	-1.016	-0.234	-0.026	0.181	-0.646	-0.027	0.185	0.419
0.1	-1.141	-0.265	-0.052	0.155	-0.738	-0.054	0.159	0.388
Central Node								
$\alpha = 0.0$	-0.945	-0.210	0.000	0.205	-0.577	-0.003	0.205	0.455
0.05	-1.147	-0.269	-0.050	0.151	-0.723	-0.056	0.152	0.389
0.1	-1.462	-0.337	-0.102	0.100	-0.948	-0.113	0.100	0.329

Table 2 shows the effects of contagion by considering how prices in one node are correlated with a price in another node at an earlier time. These effects increase with α. Surprisingly, given that a central node is interacting with twice as many nodes as a periphery node, the effect of a periphery node on a central node is about the same as the reverse effect. The magnitudes are usually small, in part because α is small, and seem to vary only a little with changes in τ or ρ, except when τ= -0.6.

Table 2: Contagion Effects between Regions over Time

	ρ= 0.0				ρ= 0.2			
Node 1→ Node 2; $Cov(p^1_{t-1}, p^2_t)$	τ= -0.6	-0.2	0.0	0.2	τ= -0.6	-0.2	0.0	0.2
α= 0.0	-0.008	0.000	-0.003	0.003	0.002	0.002	0.000	-0.003
0.05	0.090	0.029	0.022	0.031	0.075	0.029	0.027	0.030
0.1	0.244	0.063	0.048	0.056	0.188	0.057	0.053	0.060
Node 2 → Node 1; $cov(p^2_{t-1}, p^1_t)$								
α= 0.0	-0.002	0.001	0.005	-0.003	0.005	-0.001	-0.004	0.002
0.05	0.097	0.031	0.030	0.024	0.077	0.025	0.023	0.035
0.1	0.251	0.065	0.055	0.049	0.190	0.054	0.049	0.064

Finally, I compare the properties of an aggregate measure, similar to what would be reported by a city-wide real estate board, with those reported above for the individual nodes. The first order correlations are much smaller than those reported in Table 1. In part, this reduction is caused by the fact that the average price is a weighted average of three random variables. This fact may also explain why the magnitude of second order correlation within the aggregate is also much smaller, again with the exception of τ= -0.6.

Table 3: Autocorrelation of the Aggregated Price

	ρ= 0.0				ρ= 0.2			
R10	τ= -0.6	-0.2	0.0	0.2	τ= -0.6	-0.2	0.0	0.2
α= 0.0	-0.356	-0.078	0.001	0.077	-0.216	-0.001	0.076	0.171
0.05	-0.369	-0.082	-0.002	0.074	-0.225	-0.004	0.073	0.167
0.1	-0.389	-0.086	-0.005	0.070	-0.239	-0.008	0.069	0.164

I should note that the findings for the aggregate measure are not robust. An early experiment used $P_t = (p^1_t + p^2_t + p^3_t)/3$ and found that changes in α had *no* effect, to the sixth decimal place. After more thought, this finding makes sense given the eigenvectors: using this aggregation, the effects on the two periphery nodes offset each other or the combined effects of the periphery nodes offset the effect of the central node, or both. The fact that results are sensitive to the weighting is further evidence of the insights that can be obtained by disaggregating.

Conclusion

This paper was motivated by a desire to understand how relative prices of houses in different areas can vary, since nobody buys a property that is "representative" of an entire market. Thus, helping buyers and sellers in one node with local tactical advice would be valuable. This paper has also shown that a better understanding of local dynamics can inform the analysis of aggregate prices because contagion can affect the time series properties of prices in a city. The algebraic analysis shows that the evolution of prices in a node during the short term can be expected to have very different properties than long term evolution of prices in that node. The simulation analysis shows that a little bit of contagion can have a surprisingly large effect, especially in the case designed to mimic the case which Capozza, Hendershott and Mack (2004) identified as the most common in the US. These examples should be sufficient to convince the reader that contagion between nodes are enough to change the time series properties of aggregate measures, such as the average price.

Key parameters, especially α, should have a better behavioral foundation. For example, in the simple 3-node model, a gain for one node represents an equal loss for another node. A better model would recognize that the distribution of owner types across nodes does not necessarily represent the distribution of preference types for marginal buyers in each node. It is also true that the supply response to a given demand shock, induced by a given price change in another node, can differ between the central core of a city and undeveloped suburban areas. In areas where building new homes is a significant feature, any response could change the dynamic properties of the system permanently.

Matching theory offers an alternative to the perfectly competitive model of Rosen (1974) which recognizes the kinds of transaction costs that prevent instantaneous spatial arbitrage and make contagion more likely. But few have tried to extend these models to the kinds of complex environments proposed here. A more formal model could be derived from the literature on search and matching models, including papers such as Weitzman (1979), Arnott (1989), and the growing literature on directed search. Alternatively and instead of claiming that the price in one node reacts to prices in other nodes, one might consider that the price in a node reacts to local indicators of excess demand which may or may not react to prices in other nodes. A theory based on this idea would add new types of variables to the system proposed above but estimating such a system may be difficult if the increase in the number of variables eliminates the available degrees of freedom. The general theory presented in this paper may help to indicate the significance of restrictions intended to increase the degrees of freedom.

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