

The Phantom Made Me Jump! Preemptive Jump Bidding in (Takeover) Auctions*

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Abstract

It is well known that jump bidding can be used by buyers to communicate with each other. However, in this paper it is shown that jump bidding can also be utilized to conceal information from the auctioneer. This is useful when the auctioneer is suspected of submitting bids himself (phantom bidding), or of intending to renegotiate after the auction concludes, as is sometimes the case in takeover auctions. When taking jump bidding into account, the auctioneer's revenue diminishes, and efficiency may increase or decrease. The auctioneer may or may not gain by committing to an auction with no phantom bidding.

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1 Introduction

A *phantom bid* in an ongoing auction is a bid submitted, illegally, by the auctioneer himself.¹ In order to win the auction, a prospective buyer has to top the phantom bid. Hence, phantom bidding can be compared to a reserve price that the auctioneer can update as the auction progresses. This is advantageous for the auctioneer insofar as observing the auction unfold allows him to update his beliefs regarding the buyers, thus letting him continuously adjust his belief about what the optimal reserve price should be.

However, if buyers suspect the auctioneer is phantom bidding, it may be in their interest to try to diffuse the information that can be gathered by observing the auction. The aim of this paper is to show that *jump bidding* can be rationalized as a way for buyers to *conceal information from the auctioneer*. A jump bid is simply a bid that exceeds the current high bid by a “significant” amount (i.e. by more than the smallest allowable bid increment). This explanation of jump bidding is in marked contrast with the existing explanations in the literature, which focus, in one way or another, on jump bidding as a mechanism for buyers to *reveal information to other buyers*.²

The use of phantom bidding as a way for auctioneers to take advantage of information gathered by observing the auction was first noted by Graham et al. (1990).³ In their model, buyers are asymmetric and have independent private valuations for the object being sold, but the auctioneer is unable to distinguish between the buyers. By observing where bidding stagnates, the auctioneer is able to infer the valuation of the runner-up, which allows him to update his belief about the identity of the remaining active buyer. This is useful, because the optimal reserve price depends on the identity of the

¹See Cassady (1967) for examples. For more recent examples from online auctions, see, for instance, www.consumeraffairs.com/news04/shills.html. Ockenfels et al. (2006) survey the literature on online auctions, including that pertaining to phantom bidding. Online, *Yahoo!* permits the buyer two ways of bidding. The buyer can submit a bid that can be any order of magnitude higher than the standing high bid, or he can instruct *Yahoo!* to bid on his behalf, which corresponds to incremental bidding. In contrast, on *eBay* only the latter option is now available.

²The aim of jump bidding in these models is to coordinate on an equilibrium in the remainder of the auction, or to signal the buyer’s identity or valuation. For more details, see Avery (1998), Daniel and Hirshleifer (1998), Gunderson and Wang (1998), and Fishman (1988).

³See also Graham and Marshall (1987).

buyer (since buyers are asymmetric).⁴

As Graham et al. (1990) briefly allude to, and Bag et al. (2000) consider in detail, phantom bidding can also be explained if buyers are symmetric (causing buyer identities to become irrelevant), but valuations, though private, are affiliated or correlated. In this case, observing where the runner-up loses interest provides an opportunity for the auctioneer to update his belief about the distribution of the valuation of the active buyer (in excess of the fact that the active buyer's valuation must be to the right of the valuation of the runner-up).

Both of these explanations of phantom bidding are based on the fact that auctioneers update beliefs by observing how bidding progress in the auction, and both assume that valuations are private. While this will be the point of departure in this paper as well, there are other circumstances in which the auctioneer has an incentive to phantom bid. Chakraborty and Kosmopoulou (2004) recognize that there is an incentive for the seller to disguise as a buyer and bid in the auction if the object has an unknown common value, since buyers interpret more participation as evidence that the common value is more widely held to be high.⁵ In other words, participation in the auction is intended to mislead buyers, or manipulate their beliefs, rather than exploiting information.

Although the approach is very different, some of the results in this paper are not dissimilar from those in Chakraborty and Kosmopoulou (2004). In their model, rational buyers must realize that the seller has an incentive to participate. This leads them to be more skeptical when observing a lot of participation in the auction, and they duly scale down their bidding, to the detriment of the seller.⁶ Similarly, in a model with independent private values, McAdams and Schwarz (2006) find that the seller is worse off when phantom or shill bidding is possible, but costly.

It is interesting to notice that phantom bidding can be compared to an attempt by the auctioneer to renegotiate with the winner of the auction. In such an event, the rational auctioneer will presumably exploit whatever infor-

⁴Even if identities are known, phantom bidding can be useful as a way of discriminating between buyers. See Izmalkov (2004).

⁵Chakraborty and Kosmopoulou (2004) focus on bidding in the auction by the *seller*, rather than the auctioneer, a practise that is referred to as *shill bidding*. In the model in Section 2, the seller and the auctioneer is assumed to be one and the same.

⁶Kosmopoulou and De Silva (2006) find evidence in experiments to confirms this prediction.

mation is gleaned by observing the auction unfolding. According to Cramton and Schwartz (1991), renegotiation often takes place in *takeover auctions*.⁷ Rather than thinking of the phantom bid as a continuously updated reserve price, one can think of it as a take-it-or-leave-it offer after the auction ends, reflecting the updated beliefs.

Interestingly, a takeover auction is initiated by a prospective buyer. As described by Cramton (1988), the auction starts when a buyer submits a serious bid for the firm being taken over. Hence, the model in this paper combines two distinguishing features of takeover auctions, the initial (jump) bid and the likely renegotiation of the auction price.

Renegotiation may also take place in procurement auctions. Wang (2000) points out that the U.S. Department of Defence often renegotiates contracts, and proceeds to develop a model to study renegotiation in the context of first price auctions.

As mentioned, the purpose of this paper is to illustrate that buyers by jump bidding can bring about the erosion of the information that the auctioneer can gather by observing the auction, and that buyers benefit from doing so. To make this point, we present and analyze a deliberately simple model of correlated private values, in which the auctioneer has an incentive to phantom bid.⁸ In particular, in the benchmark model without jump bidding, the auctioneer gathers sufficient information to drive buyers' rent to zero. When jump bidding is permitted, buyers are able to retain rent.

While the auctioneer is worse off when buyers jump bid, buyers are never worse off, and the auction may be more or less efficient than without jump bidding. For the auctioneer, the problem is that buyers, anticipating phantom bidding, may manipulate his beliefs by jump bidding.⁹ We discuss the steps that the auctioneer can take to prevent the buyer from such manipulation. Moreover, we establish that the auctioneer *may or may not* benefit from committing to an auction with an entry fee or a reserve price but no

⁷Specifically, Cramton and Schwartz (1991) state that “target boards use strategies that are prohibited in the classic English auction, such as refusing to sell to the auction winner without further negotiation respecting price.”

⁸An earlier version of the paper analyzed a model with asymmetric buyers, and reached the same conclusions. However, it is required that the auctioneer is unable to tell who is who.

⁹Hence, in this paper, as well as in Chakraborty and Kosmopoulou (2004), the driving force is that buyers anticipate phantom bidding, and therefore adjust their behavior. Deltas (1999) discusses an alternative model in which phantom bidding is also anticipated.

phantom bidding. In contrast, in the common values auction studied by Chakraborty and Kosmopoulou (2004), commitment is *always* preferable. The same is true in ¹⁰. In the latter paper, ruling out phantom bidding will not change the allocation, and the only difference is that the auctioneer saves on phantom bidding costs. In the current paper, phantom bidding affects the allocation.¹¹

The remainder of the paper is organized as follows. In Section 2, a simple model of jump and phantom bidding is presented, and the benchmark case in which jump bidding is ruled out is briefly considered. Section 3 analyzes the game, allowing for jump bidding. We focus on one type of equilibria in which jump bidding is used sparingly and which easily allows us to illustrate the fact that jump bidding can be used to conceal information. Section 4 contains a discussion of the ways in which the auctioneer can counteract jump bidding. Section 5 concludes.

2 A simple model of jump bidding

We consider a simple model of jump bidding, the timing of which can be described as follows.

0. Buyers learn their private valuations for the good being sold.
1. Buyer 1 submits a non-negative bid, p_1 . If the bid is strictly positive, $p_1 > 0$, we will refer to it as a *jump bid*.
2. Starting from p_1 , all buyers compete in an English auction. The closing price in this auction is denoted by p_2 , $p_2 \geq p_1$.
3. The auctioneer observes the prices p_1 and p_2 as well as who submitted the original bid, and who won the auction in stage 2. Given this information, he may submit a *phantom bid*, r , in excess of p_2 , or he

¹⁰As in Chakraborty and Kosmopoulou (2004), McAdams and Schwarz (2006) assume buyers anticipate phantom bidding and therefore adjust their bids.

¹¹Another difference is that the type space is finite in the current paper. This implies that even if the allocation was the same, revenue need not be the same. The reason is that the downwards incentive compatibility constraint may be slack in a model with a finite type space (this is not the case in a model with a continuous type space).

may choose not to do so, in which case we will let $r = p_2$.¹² Without a phantom bid, the winner of stage 2 wins the auction, and pays p_2 . With a phantom bid, the winner (and only the winner) of stage 2 decides whether to buy the good at the price r .

It should be noted that the game, as specified, contains a few simplifying assumptions. First, it is common knowledge that the auctioneer engages in phantom bidding, but if this was really the case, it is not unlikely that the auctioneer would be prosecuted.¹³ Second, it is assumed that when a buyer gives up in stage 2, it is impossible for him to reenter the auction in stage 3.

Furthermore, only buyer 1 is given the chance to jump bid, but this assumption is merely for expositional clarity and could easily be relaxed. In particular, it is unavoidable that the model will have multiple equilibria, but the equilibrium on which we will focus could readily be reproduced in a richer model, by selecting appropriate off-equilibrium beliefs. Likewise, any jump bid is modelled as a jump from a price of zero. Again, this is purely for simplicity.¹⁴

If there is any activity in stage 2, the auctioneer is able to infer the valuation of the runner-up by observing when he drops out of the auction, since it is a weakly dominant strategy to bid up to one's true valuation.¹⁵ This may be used to update beliefs about the winner. Importantly, however, if buyer 1, having submitted a jump bid in stage 1, is not challenged in stage 2, the auctioneer can conclude only that buyer 1's rivals have valuations below p_1 , but he is unable to infer the exact valuations. Hence, jump bidding conceals information.

¹²In a takeover auction, r can be interpreted as a post-auction take-it-or-leave-it offer, capturing the possibility of renegotiation.

¹³In Chakraborty and Kosmopoulou (2004), there is an exogenous probability that the seller can shill bid (or is dishonest). In takeover auctions, however, it is perfectly legal for the auctioneer (the target) to renegotiate the price.

¹⁴In the model we will consider, buyers' valuations are known to be strictly greater than zero. We could let bidding start from zero, and buyer 1 could then jump from any price (using either a pure or a mixed strategy) below the lowest possible valuation. This would produce an equilibrium essentially equivalent to the one examined in Section 3, but with the added feature that a jump may occur *after* bidding has been initiated. However, in the context of a takeover auction, the current way of modelling stage 1 captures the fact that a serious bid is needed to set off such an auction.

¹⁵In the equilibrium we will examine, buyers (other than buyer 1) participate in stage 2 whenever p_1 is *strictly* smaller than their own valuation. Hence, p_1 does not serve as a signal between buyers.

This concludes the discussion of the timing of the game, and we turn to the modelling of buyers' private information (stage 0). For simplicity, we assume there are only two buyers. We let θ_i denote buyer i 's private valuation of the object, $i = 1, 2$.

To model correlation between valuations, we assume there are two possible states of the world, A and B . The probability of state s is P_s , $s = A, B$. In state s , buyer i has valuation $\underline{\theta}_s$ with probability $\underline{\pi}_s$ and valuation $\bar{\theta}_s$ with probability $\bar{\pi}_s$, $\underline{\pi}_s + \bar{\pi}_s = 1$, $i = 1, 2$, $s = A, B$. The finite type space greatly simplifies the analysis. However, we stress the fact that the incentive to conceal information would also be present in a model with a continuous type space.¹⁶

We impose two assumptions on the parameters, the first of which pertains to how the two states compare, the second to how valuations compare within a given state. First, we assume that

$$\bar{\theta}_A > \bar{\theta}_B > \underline{\theta}_B > \underline{\theta}_A > 0, \quad (1)$$

such that we can interpret state A as a "risky" state of the world. Initially, the auctioneer does not know the state of the world or buyers' valuations. Buyers, on the other hand, know their own valuation but not the valuation of the rival.¹⁷ The two different states may reflect that buyers are better informed about the uses of the object than the auctioneer.

Moreover, we assume that

$$\bar{\pi}_s \bar{\theta}_s > \underline{\theta}_s, \quad s = A, B. \quad (2)$$

This assumption implies that if the auctioneer faces one buyer, and knows the state of the world is state s , the optimal phantom bid is $\bar{\theta}_s$. With a phantom bid of $\bar{\theta}_s$, the good is sold with probability $\bar{\pi}_s$, whereas, if the phantom bid is $\underline{\theta}_s$, the good is sold with probability one, implying the former is optimal (any other phantom bid is dominated).¹⁸

Assumptions (1) and (2) in concert are imposed to most easily illustrate how jump bidding can serve to conceal information. Jump bidding would

¹⁶If anything, it seems more important that there is a mass point at the lower end-point of the support than the distribution is discrete.

¹⁷Buyers know the state of the world, or they can infer the state of the world from their own valuations.

¹⁸Indeed, when (2) is satisfied and the state is known to be state s , a reserve price, or phantom bid, of $\bar{\theta}_s$ implements the revenue maximizing mechanism.

function in a similar manner under other parameter configurations, but not all. For instance, if $\underline{\theta}_A > \bar{\pi}_A \bar{\theta}_A$ (contradicting (2)) it appears that there is no role for jump bidding as a means to meaningfully conceal information.

As a benchmark, it is useful to first examine the implications of these assumptions when *jump bidding is ruled out* (stage 1 is eliminated). Assuming buyers drop out of the English auction when the price reaches their valuation, the auctioneer can deduce that if bidding stops at $\underline{\theta}_s$, the state of the world is s (and the loser has valuation $\underline{\theta}_s$).¹⁹ Thus, without further knowledge about the winner, it is optimal for the auctioneer to submit a phantom bid of $\bar{\theta}_s$, by (2).

Importantly, (1) and (2) imply that the optimal phantom bid is higher in state A than in state B , or that the phantom bid is higher when $p_2 = \underline{\theta}_A$ is observed than when $p_2 = \underline{\theta}_B$ is observed. This creates an incentive for buyers in state A to manipulate the beliefs of the auctioneer.

With phantom bidding, and no jump bidding, expected revenue is

$$ER_P = P_A \bar{\theta}_A (1 - \underline{\pi}_A^2) + P_B \bar{\theta}_B (1 - \underline{\pi}_B^2), \quad (3)$$

since revenue is $\bar{\theta}_s$ if the state is s and at least one buyer has valuation $\bar{\theta}_s$.²⁰ Notice that buyers get zero surplus, and the good is not sold in state s if both buyers have valuation $\underline{\theta}_s$.

3 Equilibria with minimal jump bidding

Allowing for jump bidding, we will look for buyer strategies where jump bidding plays the smallest possible role, but which nevertheless permit positive rent to buyers.²¹ Hence, we assume that buyer 1 jumps to $\underline{\theta}_s$ if the state is $s = B$ or if the state is $s = A$ and his valuation is $\underline{\theta}_A$, $\underline{\theta}_s$ being the lowest price at which his competitor will ever allow him the good. Clearly, in state

¹⁹(1) implies full revelation of the state. However, in richer models with more types (or types which coincide across states), the auctioneer will also be able to update his beliefs, albeit less dramatically.

²⁰If both buyers have valuation $\bar{\theta}_s$, competition in the auction drives the price to $\bar{\theta}_s$. If exactly one buyer has valuation $\bar{\theta}_s$, phantom bidding will extract all rent from this buyer.

²¹Refraining from jump bidding ($p_1 = 0$) in stage 1 regardless of state and valuation can be supported as an equilibrium strategy for buyer 1, with revenue of ER_P and zero rents to buyers. However, the purpose of the paper is to prove that allowing for jump bidding leads to the existence of equilibria with positive rents to buyers.

A , if buyer 1 with valuation $\bar{\theta}_A$ does not mimic either a low valuation buyer (by jumping to $\underline{\theta}_A$) or a buyer in state B (by jumping to $\underline{\theta}_B$), he reveals his valuation, and thus, due to phantom bidding, will earn zero rent. Therefore, buyer 1 with valuation $\bar{\theta}_A$ jumps either to $\underline{\theta}_A$ or to $\underline{\theta}_B$. Let q denote the probability that he jumps to $\underline{\theta}_A$. We will refer to an equilibrium of this form, in which only buyer 1 with valuation $\bar{\theta}_A$ uses jump bidding for a strategic purpose, as an equilibrium with *minimal jump bidding*.

In an equilibrium with minimal jump bidding, the auctioneer observes one of four equilibrium paths in stages 1 and 2.

1. Buyer 1 jumps to $\underline{\theta}_A$, and he is not challenged in stage 2. Given the equilibrium strategies, this is consistent only with the event that the state is A , and that $\theta_2 = \underline{\theta}_A$.²² Depending on beliefs about buyer 1's valuation, the optimal phantom bid is $\underline{\theta}_A$ or $\bar{\theta}_A$. Let x denote the probability that the auctioneer chooses a phantom bid of $\underline{\theta}_A$.
2. Buyer 1 jumps to $\underline{\theta}_A$, and is challenged in stage 2. In this case, the state is A , and $\theta_2 = \bar{\theta}_A$. The auctioneer thus phantom bids $\bar{\theta}_A$.²³
3. Buyer 1 jumps to $\underline{\theta}_B$, and is challenged in stage 2. If buyer 1 does not respond to this challenge, we must infer that $\theta_1 = \underline{\theta}_B$. Hence, the state is B , and the optimal phantom bid is $\bar{\theta}_B$ (the challenge by buyer 2 reveals that $\theta_2 = \bar{\theta}_B$). If buyer 1 responds to the challenge, both buyers must have high valuation, and competition ensures that rent is driven to zero, implying that phantom bidding is unnecessary.
4. Buyer 1 jumps to $\underline{\theta}_B$, and is not challenged in stage 2. While buyer 2 is revealed to have a low valuation, the state is not revealed. However, we can rule out that $\theta_1 = \underline{\theta}_A$. Given (2), it is easily seen that a phantom bid of $\bar{\theta}_B$ dominates one of $\underline{\theta}_B$. Hence, the optimal phantom bid is either $\bar{\theta}_B$ or $\bar{\theta}_A$. Let y denote the probability that the auctioneer phantom bids $\bar{\theta}_B$.

²²We reiterate the assumption that buyer 2 enters stage 2 only if $p_1 < \theta_2$.

²³If buyer 1 does not respond to the challenge by buyer 2 (if $\theta_1 = \underline{\theta}_A$), a phantom bid of $\bar{\theta}_A$ is optimal to extract rent from buyer 2. If buyer 1 responds to the challenge, he has valuation $\bar{\theta}_A$, and competition drives the price to $\bar{\theta}_A$, in which case no further phantom bidding is needed.

In conclusion, an equilibrium with minimal jump bidding can be characterized by the ordered triple (q, x, y) .²⁴ Depending on the parameters, it will be shown in the following that there is either (i) a unique equilibrium with minimal jump bidding, or that (ii) the equilibrium outcome in any minimal jump bidding equilibrium is unique.

To determine q , consider buyer 1 with valuation $\bar{\theta}_A$, and notice that he can earn positive rent only in cases 1 and 4, i.e. if buyer 2 has valuation $\underline{\theta}_A$. Hence, buyer 1's strategy matters only if $\theta_2 = \underline{\theta}_A$. Then, it is easy to see that buyer 1 with valuation $\bar{\theta}_A$ cannot use a pure strategy, i.e. it must be the case that $q \in (0, 1)$.

Lemma 1 *In any minimal jump bidding equilibrium, $q \in (0, 1)$.*

Proof. By contradiction, assume buyer 1 with valuation $\bar{\theta}_A$ always jumps to $\underline{\theta}_A$, or $q = 1$. Then, if $\theta_2 = \underline{\theta}_A$, buyer 2 will not challenge buyer 1 in stage 2 (case 1, above). Since buyer 1 always jumps to $\underline{\theta}_A$ in state A , but never in state B , the state is revealed, and the optimal phantom bid is $\bar{\theta}_A$. However, by deviating to a jump of $\underline{\theta}_B$, buyer 1 would trick the auctioneer into believing the state is B , and he would consequently submit a lower phantom bid. Therefore, it must be the case that $q < 1$.

Similarly, assume that $q = 0$. Then, buyer 1 with valuation $\bar{\theta}_A$ is never supposed to submit a jump bid of $\underline{\theta}_A$. Thus, if the auctioneer observed an unchallenged jump bid to $\underline{\theta}_A$, he will refrain from further phantom bidding. It follows that it would be best for buyer 1 with valuation $\bar{\theta}_A$ to deviate, and we conclude that $q > 0$. ■

The fact that buyer 1 randomizes necessitates that he is indifferent between the pure strategies. This leads to the following condition.

²⁴The minimal jump bidding equilibrium is supported by the belief that if buyer 1 submits a jump bid other than $\underline{\theta}_A$ or $\underline{\theta}_B$, it signals $\theta_1 = \bar{\theta}_A$, and the auctioneer will therefore phantom bid $\bar{\theta}_A$ if buyer 1 wins stage 2. Thus, there is no incentive for buyer 1 to deviate to an off-equilibrium jump bid in stage 1. When the state is B , it is also necessary that buyer 1 does not have an incentive to bid $\underline{\theta}_A$ rather than $\underline{\theta}_B$ in stage 1. This is the case if, upon observing buyer 1 bidding $\underline{\theta}_A$ in stage 1 and winning stage 2 at a price of $\underline{\theta}_B$, the auctioneer believes that $\theta_1 = \bar{\theta}_B$ or $\theta_1 = \bar{\theta}_A$. For the purpose of ensuring that buyers do not deviate in stage 2, off-equilibrium beliefs in stage 2 are irrelevant, since if a buyer deviates, he either drops out too soon, and thereby loses the auction, or he stays in too long, thereby risking negative payoff.

Lemma 2 *In any minimal jump bidding equilibrium,*

$$x = \frac{\bar{\theta}_A - \bar{\theta}_B}{\bar{\theta}_A - \underline{\theta}_A} y. \quad (4)$$

Proof. Recall that the jump bid is relevant for payoff to buyer 1 with valuation $\bar{\theta}_A$ only if $\theta_2 = \underline{\theta}_A$. Contingent on this, expected payoff from jumping to $\underline{\theta}_A$ is $(\bar{\theta}_A - \underline{\theta}_A)x$, whereas expected payoff from jumping to $\underline{\theta}_B$ is $(\bar{\theta}_A - \bar{\theta}_B)y$, as payoff is zero if the auctioneer phantom bids $\bar{\theta}_A$. Since buyer 1 randomizes, $q \in (0, 1)$, we require that he is indifferent between jumping to $\underline{\theta}_A$ and $\underline{\theta}_B$, which gives rise to (4). ■

As for the auctioneer, there are two instances (cases 1 and 4) in which he randomizes between phantom bids. The auctioneer's best response clearly depends on q , and so we let $x(q)$ and $y(q)$ denote the best response correspondence in the two cases. First, consider the case where an unchallenged jump to $\underline{\theta}_A$ is observed.

Lemma 3 *If buyer 1 jumps to $\underline{\theta}_A$ and is not challenged in stage 2, the auctioneer randomizes between $\underline{\theta}_A$ (with probability x) and $\bar{\theta}_A$. The best response correspondence is*

$$x(q) = \begin{cases} 1 & \text{if } q < \tilde{q} \\ [0, 1] & \text{if } q = \tilde{q} \\ 0 & \text{if } q > \tilde{q} \end{cases}, \quad (5)$$

where

$$\tilde{q} \equiv \frac{\underline{\theta}_A \pi_A}{\bar{\pi}_A (\bar{\theta}_A - \underline{\theta}_A)} = \frac{\underline{\theta}_A \pi_A}{(\bar{\pi}_A \bar{\theta}_A - \underline{\theta}_A) + \underline{\theta}_A \pi_A} \in (0, 1). \quad (6)$$

Proof. If an unchallenged jump to $\underline{\theta}_A$ is observed, the auctioneer randomizes between phantom bids of $\underline{\theta}_A$ and $\bar{\theta}_A$. The former yields revenue of $\underline{\theta}_A$, while the latter produces expected revenue of

$$\bar{\theta}_A \frac{\bar{\pi}_A q}{\bar{\pi}_A q + \pi_A},$$

where the last term is the posterior belief that $\theta_1 = \bar{\theta}_A$.²⁵ Thus, the auctioneer is willing to randomize if, and only if, $q = \tilde{q}$. If $q > \tilde{q}$, the best response

²⁵Beliefs follow from the equilibrium strategies. If buyer 1 has valuation $\underline{\theta}_A$, he will submit a jump bid of $\underline{\theta}_A$ with probability one. On the other hand, if his valuation is $\bar{\theta}_A$, he submits a jump bid of $\underline{\theta}_A$ with probability q .

is to phantom bid $\bar{\theta}_A$ ($q = 0$), since a jump to $\underline{\theta}_A$ is relatively likely to be caused by a buyer with valuation $\bar{\theta}_A$. ■

However, in equilibrium, it must be the case that $x < 1$, since (4) would be violated otherwise, as y is at most one. We therefore require that

$$q \geq \tilde{q}. \quad (7)$$

Notice that \tilde{q} is independent of P_A (or P_B) since the state is perfectly revealed if the jump to $\underline{\theta}_A$ is not challenged. As the state is known to be A , the parameters relating to state B ($\underline{\theta}_B$, $\bar{\theta}_B$, $\bar{\pi}_B$, and $\underline{\pi}_B$) are also irrelevant.

The auctioneer may also randomize if an unchallenged jump to $\underline{\theta}_B$ is observed. In this case the state is not revealed, so both state A and state B parameters will enter into the auctioneer's considerations.

Lemma 4 *If buyer 1 jumps to $\underline{\theta}_B$ and is not challenged in stage 2, the auctioneer randomizes between $\bar{\theta}_B$ (with probability y) and $\bar{\theta}_A$. The best response correspondence is*

$$y(q) = \begin{cases} 0 & \text{if } q < \hat{q} \\ [0, 1] & \text{if } q = \hat{q} \\ 1 & \text{if } q > \hat{q} \end{cases}, \quad (8)$$

where

$$\hat{q} \equiv \frac{P_A \bar{\pi}_A \underline{\pi}_A (\bar{\theta}_A - \bar{\theta}_B) - P_B \bar{\pi}_B \underline{\pi}_B \bar{\theta}_B}{P_A \bar{\pi}_A \underline{\pi}_A (\bar{\theta}_A - \bar{\theta}_B)} = 1 - \frac{P_B \bar{\pi}_B \underline{\pi}_B \bar{\theta}_B}{P_A \bar{\pi}_A \underline{\pi}_A (\bar{\theta}_A - \bar{\theta}_B)} < 1. \quad (9)$$

Proof. If an unchallenged jump to $\underline{\theta}_B$ is observed, the auctioneer randomizes between $\bar{\theta}_B$ and $\bar{\theta}_A$.²⁶ The former, which appeals to buyer 1 with a high valuation in both states, yields expected revenue of

$$\bar{\theta}_B \frac{P_A \bar{\pi}_A (1 - q) \underline{\pi}_A + P_B \bar{\pi}_B \underline{\pi}_B}{P_A \bar{\pi}_A (1 - q) \underline{\pi}_A + P_B \bar{\pi}_B \underline{\pi}_B + P_B \underline{\pi}_B^2},$$

and the latter, which is only accepted in state A , generates expected revenue of

$$\bar{\theta}_A \frac{P_A \bar{\pi}_A (1 - q) \underline{\pi}_A}{P_A \bar{\pi}_A (1 - q) \underline{\pi}_A + P_B \bar{\pi}_B \underline{\pi}_B + P_B \underline{\pi}_B^2}. \quad ^{27}$$

²⁶A phantom bid of $\underline{\theta}_B$ is dominated by a phantom bid of $\bar{\theta}_B$.

²⁷Beliefs are derived as follows. An unchallenged jump to $\underline{\theta}_B$ is observed if the state is A , buyer 1 has a high valuation and happens to jump to $\underline{\theta}_B$, and buyer 2 has a low valuation. An unchallenged jump to $\underline{\theta}_B$ is also observed if the state is B and buyer 2 has a low valuation. A phantom bid of $\bar{\theta}_B$ is accepted in either state if buyer 1 has a high valuation, but a phantom bid of $\bar{\theta}_A$ is accepted only if the state is A .

Hence, the auctioneer is willing to randomize between $\bar{\theta}_B$ and $\bar{\theta}_A$ if, and only if, $q = \hat{q}$. If $q > \hat{q}$, the best response is a phantom bid of $\bar{\theta}_B$ ($y = 1$), since a jump bid of $\underline{\theta}_B$ is most likely indicative of the state B as there is high probability of a low jump bid in state A . ■

In conclusion, the auctioneer's best response depends on how q , the strategy of buyer 1, compares to \tilde{q} and \hat{q} . Generically, the two are different, and it is not difficult to construct examples where $\tilde{q} > \hat{q}$, or where $\tilde{q} < \hat{q}$.²⁸

Proposition 1 *If $\tilde{q} > \hat{q}$, there is a unique minimal jump bidding equilibrium, described by*

$$q = \tilde{q}, x = \frac{\bar{\theta}_A - \bar{\theta}_B}{\bar{\theta}_A - \underline{\theta}_A}, y = 1.$$

If $\tilde{q} < \hat{q}$, any minimal jump bidding equilibrium satisfies

$$q \in [\tilde{q}, \hat{q}], x = 0, y = 0.$$

and the minimal jump bidding equilibrium outcome is unique.

Proof. If $\tilde{q} > \hat{q}$, (7) implies that $q \geq \tilde{q} > \hat{q}$, meaning, by (8), that $y = 1$. Then, x follows from (4), and the fact that $x \in (0, 1)$ necessitates that $q = \tilde{q}$, by (5).

If $\tilde{q} < \hat{q}$, we start by ruling out that $q > \hat{q} > \tilde{q}$. By (5) and (8), this would imply that $x = 0, y = 1$, violating (4). Likewise, (7) rules out the possibility that $q < \tilde{q}$. However, any $q \in [\tilde{q}, \hat{q}]$ is part of an equilibrium. For any such q , q is either larger than \tilde{q} and/or smaller than \hat{q} , implying that at least one of x and y is zero. By (4), both must be zero, such that buyer 1 is willing to randomize between jumping to $\underline{\theta}_A$ or $\underline{\theta}_B$. Regardless of q , $q \in [\tilde{q}, \hat{q}]$, the equilibrium outcome is unique, because the auctioneer responds the same way to an unchallenged jump bid, regardless of what that jump bid is. ■

Obviously, there are two types of minimal jump bidding equilibria. If $\tilde{q} < \hat{q}$, any equilibrium involves very aggressive phantom bidding. That is, if the state is not revealed perfectly, the auctioneer submits a phantom bid of $\bar{\theta}_A$. On the other hand, jump bidding pays off if $\tilde{q} > \hat{q}$, as the auctioneer submits low phantom bids on average.

²⁸Specifically, \tilde{q} is independent of P_A , and is somewhere between 0 and 1, whereas \hat{q} can be made to vary between $-\infty$ and 1, by varying P_A .

Recall that \tilde{q} is independent of the state B parameters, $\underline{\theta}_B, \bar{\theta}_B, \bar{\pi}_B, \underline{\pi}_B$, and P_B . On the other hand, \hat{q} depends on these parameters (with the exception of $\underline{\theta}_B$).²⁹ For instance, \hat{q} is decreasing in P_B , meaning that \tilde{q} is more likely to exceed \hat{q} when there is a high probability the state is B . Clearly, if P_B is high, the auctioneer will be more inclined, other things being equal, to think that an unchallenged jump bid of $\underline{\theta}_B$ is an indication of the state being B , in which case a moderate phantom bid (of $\bar{\theta}_B$) is the best response. It follows that phantom bidding must also be more measured if an unchallenged jump bid of $\underline{\theta}_A$ is observed, lest buyer 1 should be too tempted to cheat. Likewise, the advantages of phantom bidding $\bar{\theta}_B$ in response to a jump bid of $\underline{\theta}_B$ are larger the larger $\bar{\theta}_B$ is.

Similarly, moderate phantom bidding is more likely to be the best response to jump bidding when $\bar{\pi}_B \underline{\pi}_B$ is high. The reason is that an unchallenged jump to $\underline{\theta}_B$ occurs in state B only if buyer 2 has valuation $\underline{\theta}_B$, and that a phantom bid of $\bar{\theta}_B$ will yield a sale only if buyer 1 has valuation $\bar{\theta}_B$. Together, given the state is B , the probability that $\theta_1 = \bar{\theta}_B$ and $\theta_2 = \underline{\theta}_B$ is $\bar{\pi}_B \underline{\pi}_B$. Thus, a phantom bid of $\bar{\theta}_B$ is more likely to be optimal the higher $\bar{\pi}_B \underline{\pi}_B$ is. Of course, this term is maximized at $\bar{\pi}_B = \underline{\pi}_B = .5$, i.e. if the probability of having a high valuation in state B equals the probability of having a low valuation. Similar comparative statics can be performed for the state A parameters.³⁰

In the following we examine the efficiency of the auction with both phantom and jump bidding as well as the revenue effects of jump bidding. Thereafter, we discuss how the auctioneer may respond to jump bidding.

3.1 Efficiency

We start by observing that buyer 2 always gets zero surplus in equilibrium, as does buyer 1 in state B . The same is true for buyer 1 with valuation $\underline{\theta}_A$, but when $\tilde{q} > \hat{q}$ he may actually win the good. Indeed, when $\tilde{q} > \hat{q}$, the good

²⁹ $\underline{\theta}_B$ is irrelevant because (2) ensures that it is never the optimal phantom bid.

³⁰Moderate phantom bidding is more likely to be optimal if $\underline{\theta}_A$ is high, since the profit of a phantom bid of $\underline{\theta}_A$ is higher in this case. Aggressive phantom bidding is more probable if $\bar{\theta}_A$ is high, for the obvious reason. More interestingly, an increase in $\bar{\pi}_A$ has ambiguous effects. While aggressive phantom bidding may seem more profitable when $\bar{\pi}_A$ is high, the counteracting fact is that if buyers are likely to have high valuations in state A , an unchallenged jump bid of $\underline{\theta}_B$ suggests the state is B (since buyer 2 would probably challenge if the state was A).

is sold *more* often when buyers can submit jump bids (and, if it is sold, it is sold to the buyer with the highest valuation).³¹ Additionally, buyer 1 is better off if his valuation is $\bar{\theta}_A$, since he wins just as often as he does without the ability to jump bid, but he may end up paying a lower price than $\bar{\theta}_A$.

However, when $\tilde{q} < \hat{q}$, the good is sold *less* often, since, in state B , it is sold only if buyer 2 has valuation $\bar{\theta}_B$, in which case buyer 2 challenges buyer 1 (case 3 on the list of equilibrium paths). Notice also that buyers have zero surplus, regardless of identity, valuation, and state. In this case, then, buyer 1 is just as well off as without the ability to jump bid.

Corollary 1 *The ability of buyer 1 to submit jump bids may increase or decrease efficiency.*

Overall, efficiency may increase, or decrease, when allowing jump bidding to take place. At the same time, buyers are not worse off, and buyer 1 may be better off when he has valuation $\bar{\theta}_A$. Next, we consider the auctioneer's payoff in more detail.

3.2 Revenue

When buyers jump bid, the information available to the auctioneer is limited, and he is therefore worse off. Depending on whether $\tilde{q} > \hat{q}$ or $\tilde{q} < \hat{q}$, expected revenue from phantom bidding when buyers are jump bidding is, respectively,

$$ER_{PJ}^{\tilde{q} > \hat{q}} = P_A \left(\bar{\theta}_A \bar{\pi}_A + \bar{\theta}_B \bar{\pi}_A \underline{\pi}_A + \underline{\pi}_A^2 \frac{\bar{\theta}_A - \bar{\theta}_B}{\bar{\theta}_A - \underline{\theta}_A} \underline{\theta}_A \right) + P_B \bar{\theta}_B (1 - \underline{\pi}_B^2), \quad (10)$$

$$ER_{PJ}^{\tilde{q} < \hat{q}} = P_A \bar{\theta}_A (1 - \underline{\pi}_A^2) + P_B \bar{\theta}_B \bar{\pi}_B. \quad (11)$$

If $\tilde{q} > \hat{q}$, in state A revenue is $\bar{\theta}_A$ if $\theta_2 = \bar{\theta}_A$, in which case buyer 2 challenges buyer 1 in stage 2. If $(\theta_1, \theta_2) = (\bar{\theta}_A, \underline{\theta}_A)$, expected revenue is $\bar{\theta}_B$ since in this case buyer 1 will pay on average $\bar{\theta}_B$ regardless of his jump in stage 1. If $(\theta_1, \theta_2) = (\underline{\theta}_A, \underline{\theta}_A)$, an unchallenged jump bid to $\underline{\theta}_A$ is observed, and the good is sold if the auctioneer responds with a phantom bid of $\underline{\theta}_A$, which happens with probability x , as stated in Proposition 1. In state B , revenue is $\bar{\theta}_B$ if at least one buyer has a high valuation.

³¹If buyers cannot jump ($q = 1$), the best response is $x = 0, y = 1$, so phantom bidding is *less* aggressive if the buyers can jump, and $\tilde{q} > \hat{q}$.

Revenue can be calculated in a similar manner if $\tilde{q} < \hat{q}$. In particular, the good is sold in state B only if buyer 2 challenges buyer 1, necessitating that $\theta_2 = \bar{\theta}_B$.

Since the highest possible revenue in state s is $\bar{\theta}_s(1 - \underline{\pi}_s^2)$, by (2), it follows that while revenue is maximized in one state, jump bidding leads it to be diminished in the other state.³²

Corollary 2 *Compared to the situation with no jump bidding, the auctioneer is strictly worse off in a minimal jump bidding equilibrium.*

4 The auctioneer's response

Corollary 2 leads to the question of how the auctioneer can take steps to minimize the damage from bidders' manipulation of information. Most crudely, the auctioneer may simply disallow jump bidding.³³ For instance jump bidding is impossible in the Japanese auction, a variation of the English auction in which the price progresses on a clock until all but one buyer have dropped out. However, it is not necessarily possible for the auctioneer to dictate the auction format. As mentioned, takeover auctions are set off by a serious bid by a potential buyer, and so do not fit the "ascending clock" format.

Furthermore, the incentive to mislead the auctioneer persists in the Japanese auction. Since jump bidding is impossible, the buyers may seek other ways of concealing information. For instance, the two buyers may collude, in the following way. If the valuations are $\theta_1 = \bar{\theta}_A$ and $\theta_2 = \underline{\theta}_A$ respectively, buyer 2 could drop out at a price of $\underline{\theta}_B$ rather than $\underline{\theta}_A$, thereby hiding the fact that the state is A . In this case, collusion leads to bidding that is initially *more* aggressive than without collusion.

As long as the auctioneer is suspected of phantom bidding or renegotiating the contract, the incentive for concealment of information exists. As another example, if the auction used is a sealed bid auction, buyers must use mixed

³²More formally, if $\tilde{q} > \hat{q}$, revenue in state A converges to $\bar{\theta}_A(1 - \underline{\pi}_A^2)$ from below as $\bar{\theta}_B$ increases towards $\bar{\theta}_A$. In the case where $\tilde{q} < \hat{q}$, notice that $\bar{\theta}_B \bar{\pi}_B$ is always below $\bar{\theta}_B(1 - \underline{\pi}_B^2)$, as the former involves a sale (at price $\bar{\theta}_B$) only if buyer 2 has valuation $\bar{\theta}_B$, and the latter involves a sale if *either* buyer has valuation $\bar{\theta}_B$.

³³In the specific model considered here, the problem is that a jump bid to $\underline{\theta}_B$ is possible. Disallowing this appears to solve the auctioneer's problem.

strategies lest their bid reveals all information.^{34,35} Hence, the auction may be inefficient.

Given it is difficult for the auctioneer to stop buyers from manipulating information, it becomes relevant to ask whether the auctioneer may benefit from *committing* to not use phantom bidding. If the auctioneer is able to commit in this manner, buyers no longer has an incentive to manipulate information.

In order to maximize revenue from an auction without phantom bidding the auctioneer requires another instrument to ration. In particular, (2) implies that any auction in which a buyer with a low valuation may win is not optimal. In the following, we endow the auctioneer with one instrument to ration, and provide two examples, one in which commitment is profitable for the auctioneer and one in which it is not. More specifically, the auctioneer may use *either* an entry fee or a reserve price to ration.³⁶

A reserve price is the most common way to ration, but since $\bar{\theta}_A > \bar{\theta}_B$ it will not be successful in maximizing expected revenue. If the reserve exceeds $\bar{\theta}_B$, the object will be withheld too often, and if the reserve equals (or is below) $\bar{\theta}_B$ then rent is left on the table for the buyer with a high valuation if the state is A .

Alternatively, the auctioneer could demand an entry fee. Typically, an entry fee will suffer from problems similar to those described above for a reserve price. However, there are exceptions. In particular, assume for now that

$$\bar{\theta}_A \underline{\pi}_A = \bar{\theta}_B \underline{\pi}_B = c. \quad (12)$$

In this case, an entry fee of c , coupled with an English auction and a commitment not to use phantom bidding, will generate the exact same revenue, (3), as in the benchmark model in Section 2. The reason is that regardless of what the state of the world is, only buyers with high valuations will enter

³⁴This is the case when the auctioneer has all the bargaining power after the auction concludes. Wang (2000) studies a model where renegotiation is costly to the auctioneer, and where both parties have bargaining power.

³⁵McAdams and Schwarz (2006) consider phantom or shill bidding in a second price sealed bid auction. They assume that a fake bid marginally below the highest bid is inserted, thereby forcing the winner to essentially pay his bid. This creates an incentive for buyers to shade their bid below their valuation in an effort to pay less. In contrast, the incentive to manipulate information would arise if the auctioneer tried to infer valuations from bids and use this information to renegotiate with the winner.

³⁶In the examples we consider, an entry fee is at least as profitable as a reserve price.

the auction, and they will earn zero rent. This is an equilibrium because the buyer with a high valuation in state s gains $\bar{\theta}_s$ upon entering the auction if, and only if, the other buyer has low valuation, and thus stays out of the auction. If the other buyer has a high valuation, and also enters the auction, competition drives the price to $\bar{\theta}_s$. Consequently, the high valuation buyer is willing to pay exactly c to enter the auction, regardless of the state.³⁷

Nevertheless, the auctioneer does have an incentive to renege on the commitment not to phantom bid. In the event that a single buyer entered the auction, the auctioneer infers that the buyer would be willing to pay at least $\bar{\theta}_B$ to obtain the good. Hence, at this point it would be rational for the auctioneer to “renegotiate”, demanding a price of $\bar{\theta}_A$ or $\bar{\theta}_B$. However, if the buyers doubt the commitment of the auctioneer, the equilibrium is for buyers *never* to enter the auction, given the entry fee is c .

In conclusion, if the auctioneer can credibly commit to not use a phantom bid, revenue can be maximized simply by augmenting the English auction with an entry fee, as long as (12) is satisfied. However, the stakes are high as even the smallest doubt concerning this commitment will reduce revenue to zero.

Moreover, when $\bar{\theta}_A \underline{\pi}_A$ and $\bar{\theta}_B \underline{\pi}_B$ differ, an auction with an entry fee will yield lower revenue than (3). However, if the difference between $\bar{\theta}_A \underline{\pi}_A$ and $\bar{\theta}_B \underline{\pi}_B$ is small, the difference between the revenues will also be small. In this case, committing to an auction with an entry fee (of $\min\{\bar{\theta}_A \underline{\pi}_A, \bar{\theta}_B \underline{\pi}_B\}$) and no phantom bidding is more profitable, by continuity, than conducting an auction with phantom bidding when buyers can jump bid. The reason is that the latter mechanism is not close to yielding the benchmark revenue of (3), whereas the former is.

However, a commitment to abstain from phantom bidding is not always preferable. For example, assume that

$$\underline{\pi}_A = \underline{\pi}_B = \underline{\pi}. \tag{13}$$

Consider first an entry fee of $\underline{\pi} \bar{\theta}_B$. The auctioneer would appropriate all rent if the state is B , but buyers retain rent if the state is A . To calculate revenue in state A , observe that the auctioneer receives the fee from any given buyer if that buyer has valuation $\bar{\theta}_A$. In addition, competition drives the price

³⁷It is unprofitable for a low valuation buyer to deviate and enter the auction, since his net pay-off from doing so would be $\theta_s \underline{\pi}_s - c < 0$.

to $\bar{\theta}_A$ if both buyers have valuation $\bar{\theta}_A$. Expected revenue is therefore

$$ER_c = P_A (2\bar{\pi} \times \underline{\pi}\bar{\theta}_B + \bar{\theta}_A\bar{\pi}^2) + P_B\bar{\theta}_B(1 - \underline{\pi}^2).$$

Before proceeding, notice that the entry fee of $\underline{\pi}\bar{\theta}_B$ is equivalent to a reserve price of $\bar{\theta}_B$, in the sense that the allocation and the payoffs to all agents would be the same with such a reserve price.

It is easily seen that $ER_{P,J}^{\tilde{q} > \hat{q}} > ER_c$. It follows that if $\tilde{q} > \hat{q}$ (e.g. if P_A is small) committing to not phantom bidding is unprofitable, if a fee of $\underline{\pi}\bar{\theta}_B$ is charged. Indeed, lower fees (or reserve prices below $\bar{\theta}_B$) are even worse. First, expected revenue decreases in state B , as the fee of $\underline{\pi}\bar{\theta}_B$ maximized revenue in that state. Moreover, expected revenue also decreases in state A . As long as buyers with valuation $\underline{\theta}_A$ stay out, expected revenue must decrease if the fee decreases since high valuation buyers get more rent and the allocation is unchanged. If buyers with valuation $\underline{\theta}_A$ enter the auction the fee must be zero (or the reserve price at most $\underline{\theta}_A$). In this case, expected revenue in state A is

$$\underline{\theta}_A(1 - \bar{\pi}^2) + \bar{\theta}_A\bar{\pi}^2,$$

which is less than revenue would have been from a fee of $\underline{\pi}\bar{\theta}_B$.³⁸

It remains to consider entry fees higher than $\underline{\pi}\bar{\theta}_B$ and reserve prices higher than $\bar{\theta}_B$. First, a reserve price higher than $\bar{\theta}_B$ eliminates revenue in state B , and leads to revenue of at most $\bar{\theta}_A(1 - \underline{\pi}^2)$ in state A (if the reserve is set at $\bar{\theta}_A$). Hence, a high reserve price is therefore inferior to using aggressive phantom bidding, which would yield revenue of (11).

Finally, consider an entry fee in excess of $\underline{\pi}\bar{\theta}_B$. In state B , the only symmetric equilibrium involves buyers staying out of the auction if they have low valuations, and entering with some probability, $w_B < 1$, if they have high valuation. If the fee is larger than $\bar{\theta}_B$ then $w_B = 0$, but otherwise we require that

$$(1 - \bar{\pi}w_B)\bar{\theta}_B - c = 0,$$

in order to support the mixed strategy. Rearranging yields,

$$\bar{\pi}w_B = \frac{\bar{\theta}_B - c}{\bar{\theta}_B}. \quad (14)$$

³⁸Revenue from the fee of $\underline{\pi}\bar{\theta}_B$ is larger than revenue from no fee if $2\underline{\pi}\bar{\pi}\bar{\theta}_B > \underline{\theta}_A(1 - \bar{\pi}^2)$. This inequality holds since $2\underline{\pi}\bar{\pi}\bar{\theta}_B > 2\underline{\pi}\bar{\theta}_B > 2\underline{\pi}\underline{\theta}_A$, by (1) and (2), and the latter is larger than $\underline{\theta}_A(1 - \bar{\pi}^2)$.

Similarly, in state A buyers with high valuations enter with probability w_A . w_A is one as long as the fee is at most $\underline{\pi}\bar{\theta}_A$, but a fee in excess of $\underline{\pi}\bar{\theta}_A$ forces w_A to dip below one. Assuming the fee is below $\underline{\pi}\bar{\theta}_A$, expected revenue is

$$ER_c = P_A(2\bar{\pi}c + \bar{\pi}^2\bar{\theta}_A) + P_B(2\bar{\pi}w_Bc + (\bar{\pi}w_B)^2\bar{\theta}_B),$$

or, by using (14),

$$ER_c = P_A(2\bar{\pi}c + \bar{\pi}^2\bar{\theta}_A) + P_B\frac{\bar{\theta}_B^2 - c^2}{\bar{\theta}_B}.$$

As long as P_A is small (below $\underline{\pi}$) ER_c is decreasing on the domain on which (14) is valid, $c \in [\underline{\pi}\bar{\theta}_B, \min\{\bar{\theta}_B, \underline{\pi}\bar{\theta}_A\}]$. Hence, a fee of $\underline{\pi}\bar{\theta}_B$ dominates any other fee in this domain. To make matters simple, assume that $\underline{\pi}\bar{\theta}_A > \bar{\theta}_B$. Then, a fee above $\bar{\theta}_B$ eliminates revenue in state B , but may maximize revenue in state A (if the fee is exactly $\underline{\pi}\bar{\theta}_A$). However, the resulting revenue is inferior to the revenue from aggressive phantom bidding (11).

In conclusion, if (1) $\underline{\pi}_A = \underline{\pi}_B = \underline{\pi}$, (2) P_A is small (so that $\tilde{q} > \hat{q}$) and (3) $\underline{\pi}\bar{\theta}_A > \bar{\theta}_B$, then it is better to phantom bid than to commit not to phantom bid, even with the optimally chosen entry fee or reserve price.

The two examples prove that the auctioneer may or may not profit from a commitment not to phantom bid.

Corollary 3 *A commitment not to phantom bid combined with the optimal entry fee or reserve price may, or may not, be profitable for the auctioneer.*

We stress the fact that we have allowed the auctioneer only one way to ration. In general, if $\underline{\pi}_A \neq \underline{\pi}_B$, the auctioneer can achieve revenue of (3) by combining a reserve price, r , and an entry fee c , to satisfy

$$\begin{aligned}\underline{\pi}_A(\bar{\theta}_A - r) - c &= 0 \\ \underline{\pi}_B(\bar{\theta}_B - r) - c &= 0.\end{aligned}$$

With such a reserve price and entry fee, the high valuation buyers are willing to enter the auction, but will earn zero rent.³⁹ Notice that c might be negative (a subsidy) or that r might be negative if c is positive (meaning that some of the entry fee is reimbursed if the buyer is alone in entering). The two

³⁹To keep the low valuation buyers from entering the auction as well, we require that if a buyer is alone in entering the auction, he *must* purchase the good (at a price of r).

instruments are sufficient to maximize revenue since we have assumed only two states in the model. However, with more states the combination of a reserve price and an entry fee is not enough to maximize revenue.

Moreover, when (13) holds, $\underline{\pi}_A = \underline{\pi}_B$, it is not possible to maximize revenue by using a reserve price and/or an entry fee.

5 Conclusion

It is rational for an auctioneer, or any other seller, to make use of the information available to him regarding the potential buyers. Rational buyers, however, must foresee this incentive, and it may therefore be in their interest to take steps towards diffusing the information available to the auctioneer. In this paper, we used a stylized model to argue that the use of jump bids can serve this purpose. Anticipating this reaction by the buyers, the auctioneer may, or may not, benefit from committing not to phantom bid.

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