Preferential Treatment may Hurt: Another Application of the All-Pay Auction*

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Abstract

In many contests a subset of contestants is granted preferential treatment which is presumably intended to be advantageous. Examples include affirmative action and biased procurement policies. In this paper, however, I show that some of the supposed beneficiaries may in fact become worse off when the favored group is diverse. The reason is that the other favored contestants become more aggressive, which may outweigh the advantage that is gained over contestants who do not receive preferential treatment. The contest is modelled as an incomplete-information all-pay auction in which contestants have heterogenous and non-linear cost functions. A source of uncertainty, such as incomplete information, is crucial for the results.

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1 Introduction

There are many examples of contests in which a subset of contestants receive "preferential treatment". On the labor market, affirmative action may influence which job applicant wins the prize, in this case the job. The same is true in the contest to win admission into university. Internal applicants are sometimes given preference over external applicants when a firm seeks to fill a senior position. In public procurement, domestic firms may be given preferential treatment over foreign firms, and so on.

In all these examples it is a *diverse* group of contestants who are favored. Affirmative action apply to individuals with different backgrounds, internal applicants for senior positions are likely to be heterogeneous, and domestic firms may have different technologies. Another feature of the examples is that the prize is not awarded based on the identities of the contestants alone, but also on the qualifications of the contestants in question. The investment in these qualifications – obtaining an education before applying for a job, preparing for the SAT, working hard to prove one's worth to the company, or building up expertise prior to seeking a procurement contract – may entail very significant costs. Importantly, the size of this investment is endogenous; it is likely to depend on the perceived strength of the competition and on whether the contestant is given preferential treatment. Since a given contestant may not have complete information regarding the skills, costs, or preferences of his rivals, asymmetric information may also play a role in determining the magnitude of a contestant's investments.

The objective of this paper is to study the consequences of preferential treatment in contests that are characterized by *within-group diversity* and *incomplete information*. I will show that the combination of these realistic features may, somewhat perversely, produce outcomes that are arguably opposite of what intuition would suggests. Specifically, the main result is that if the group of contestants who are given preferential treatment is diverse, a subset of them may participate less often, win less often, and overall be worse off when preferential treatment is introduced. This outcome is unlikely to be what is intended with policies that give preferential treatment to select contestants. Thus, the current paper serves as a note of caution; rather than "leveling the playing field", preferential treatment may, in principle, increase the severity of the problems or inequalities it was intended to minimize.[1]

[1] Sowell (2004), for example, argues that there are other, behavioral reasons affirmative action may not be advantageous in the real world. Specifically, affirmative action may breed resentment, or it may trigger discrimination within the diverse group that is given preferential treatment. In contrast, the driving force in this paper is the strategic response by the contestants themselves to the changes in the rules of the game. While Sowell (2004) points out that there may be an incentive to lower investments, he considers this problematic only insofar as it leads to a population with lower qualifications. Sowell's (2004) main objective is to empirically evaluate and question the actual consequences of affirmative action. The current

The intention of the paper is not to dismiss preferential treatment in its many forms, but rather to challenge the common intuition in a formal model and invite more research into the complex interaction of heterogenous contestants. Thus, this paper is concerned with contests in general, rather than specifically with affirmative action per se. There are several theoretical papers that address the specific question of affirmative action on the labor market or in university admission. See De Fraja (2005), Hickman (2010), Moro and Norman (2003), or Fang and Norman (2006) for examples. The last two papers also reach surprising conclusions about the effects of affirmative action, but for very different reasons than those presented here. These four papers, and others, consider contests that are in some sense "large", i.e. have many contestants and many prizes. Of these, Hickman's (2010) model is the one that is closest to the model studied here (for more, see below). The chief difference is that the current model is formally a model of medium-sized contests, where strategic considerations are perhaps particularly important.

The contest is modelled as a deterministic contest or, more formally, an all-pay auction, in which heterogenous contestants have private information about costs.[2] Thus, the paper is related to the literature on auctions with heterogenous participants. The papers by Lebrun (1999), Kirkegaard (2009), and Parreiras and Rubinchik (2010) are among the few that consider specific auction formats with more than two heterogenous participants. However, due to the technical challenges, most papers that compare different auction formats or the consequences of changes to the auction design assume there are exactly two heterogeneous participants.[3] Clearly, since the purpose of the current paper is to consider a setting with two groups, at least one of which is diverse, a model with only two contestants is not adequate.

In this paper, I take a first step towards a more general analysis. One of the consequences is to cast into doubt the robustness of results that are based on models with just two participants, at least for all-pay auctions. For example, in a contest with just two contestants, the sole beneficiary of preferential treatment is unambiguously made better off. Concerning the analysis itself, a measure of traction is obtained by explicitly engineering the set-up of

paper complements Sowell (2004) by pointing out that theoretical predictions – even in a simple model – do not necessarily support the received wisdom either. See Fryer and Loury (2005) for a brief discussion of "affirmative action and its mythology".

[2]The contest is deterministic in the sense that the contestant with the largest investment wins with probability one (in the absence of preferential treatment). This simplifying assumption facilitates the inclusion of private information in the model. Due to private information, a contestant cannot be sure of the actual investment of his rivals and on whether his investment will be sufficient to win.

[3]Recent prominent examples include Maskin and Riley's (2000) seminal comparison of first-price and second-price auctions, Hafalir and Krishna's (2008, 2009) analysis of such auctions with resale, Hörner and Sahuguet's (2007) analysis of jump bidding, and Goeree and Offerman's (2004) study of the "Amsterdam auction". In the context of all-pay auctions with private information, Clark and Riis (2000) and Kirkegaard (2010) consider the revenue effects of various forms of preferential treatment.

the problem to minimize the complications that arise from having several contestants and instead maximizing the use of insights from two-player contests. In particular, the reaction to preferential treatment in a contest with two participants is used to infer the main result. The analysis is outlined next.

Consider a contest with a "strong", a "weak", and a "very weak" contestant, and assume the two weaker contestants are given preferential treatment. As a consequence, they have less to fear from the strong contestant. However, that does not necessarily mean that they will work less hard to obtain qualifications. In fact, the "weak" contestant may push his newfound advantage by investing more aggressively. From the point of view of the "very weak" contestant, one rival has become less of a threat, but the other more of a threat. I show, under mild assumptions, that the very weak contestant would be less likely to win the prize with a small investment when he and the other weak bidder are given preferential treatment. The second step is to show that there are cost structures for which a monotonic equilibrium exists in which the "very weak" contestant wins the prize with probability zero and earns zero payoff, but that such an equilibrium does not exist without preferential treatment. The cost function of the very weak bidder must have the right amount of "curvature", not too much and not too little. Moreover, the cost functions of the weak and very weak contestants are generally not ordered (they cross). These assumptions are discussed in Section 4.

An equilibrium is not characterized in the absence of preferential treatment; it is merely shown that there is no equilibrium where the very weak contestant is inactive.[4] Hence, the approach is not unlike the one taken by Zhang and Wang (2009) in their study of dynamic elimination contests, which they also model as incomplete information all-pay auctions. They do not characterize equilibrium for all information revelation rules, but nevertheless conclude that an efficient equilibrium does not exist if all information is revealed between stages.

Siegel's (2010) paper nicely illustrates the technical difficulties involved in the study of asymmetric all-pay auctions. He characterizes equilibrium in certain complete information contests. However, in contests with just one prize, his characterization is generally valid only if there are exactly two contestants. For example, if there are three or more contestants with a common value of winning but non-ordered cost function, then it is generally not possible to characterize an equilibrium, even with complete information. Here, in contrast, I allow cost functions to be non-ordered and information to be incomplete.

Hickman (2010) studies incomplete information contests with many prizes. He assumes

[4] In a companion paper, Kirkegaard (2011), equilibrium is characterized in a simplified version of the model. There, the existence of a budget constraint or bidding cap introduces a discontinuity into the cost function. In that setting, if the group of contestants who are not given preferential treatment is diverse, a subset of them may participate more often, win more often, and overall be better off when preferential treatment is introduced.

there are two kinds of contestants, one of which is perceived as weak ex ante and the other as strong (interpreted as belonging to a minority group or non-minority group, respectively). Although each group may consist of many contestants that are heterogeneous ex post, there is no ex ante or observable within-group diversity. Nevertheless, as Hickman notes, the model is "analytically and computationally intractable" in the general case. As a remedy, he examines the "approximate equilibrium" which describes equilibrium in the model in the limiting case where the number of heterogeneous prizes and number of each kind of contestant goes to infinity. Thus, Hickman's (2010) model is a model of very large contests, with limited ex ante diversity. Formally, the model considered in the current paper is, in contrast, better thought of as a model of medium-sized contests. Roughly speaking, Hickman (2010) examines what happens when the model with a weak and a strong contestant is replicated ad infinitum. Of course, an additional complication in the current paper is that I also allow for a third kind of contestant (the very weak). By design, limiting arguments cannot be used either, since I examine medium-sized contests. Parreiras and Rubinchik (2010) also study medium-sized contests but they do not examine the consequences of changes to the contest design.

A related issue is the question of uniqueness of equilibrium. As mentioned, it is established that there is an equilibrium in which the very weak contestant drops out of the contest altogether after the introduction of preferential treatment. However, it is unclear whether this is the only equilibrium. Indeed, the papers that addresses uniqueness – Amann and Leininger (1996), Lizzeri and Persico (2000), and Siegel (2011) – explicitly and deliberately assume that there are only two contestants. It is evident from Parreiras and Rubinchik's (2010) analysis that the equilibria properties that are used to establish uniqueness in the two-player case do not carry over to the many-player case.

Using recent results by Siegel (2009) for complete information contests, it is shown that the assumption of incomplete information is critical for the results; weaker contestants are never hurt by preferential treatment in a deterministic complete information contest. This complements a finding by Kirkegaard (2010) that it may be profitable to handicap the weak contestant in a two-player contest when information is incomplete, but not when it is complete. Thus, the assumption of incomplete information adds an extra dimension and yields richer results. However, it is conceivable that other sources of uncertainty may have similar effects. Following Tullock (1980), complete information contests are often modeled as non-deterministic; random factors mean a contestant may win even if he is outperformed by a rival. In such a setting, it also cannot be ruled out without further study that preferential treatment may turn out to be disadvantageous to a subset of the intended beneficiaries.

The model is presented in Section 2 and analyzed in Section 3. Section 4 discusses modeling assumptions and the main result. Section 5 concludes. Proofs are in the Appendix.

2 A contest with preferential treatment

Consider a deterministic contest with n contestants and the following timing:

0. Contestants are informed about the rules of the contest and discover the value they place on winning the prize. These valuations are private information.

1. Contestants simultaneously invest effort or other resources into obtaining qualifications. The cost for contestant i of obtaining b units of qualifications is described by the twice continuously differentiable cost function $c_i(b)$, with $c_i(0) = 0$, $0 < c_i'(\cdot) < \infty$, and $c_i''(\cdot) \geq 0$, $i = 1, 2, ..., n$.[5]

2. The winner of the contest is the contestant with the highest *score*. Each contestant's score is a function (as specified in step 0) of his qualifications (b) and, possibly, his identity. Ties are broken with the toss of a fair coin.

The game described above is isomorphic to an all-pay auction with private information in which a set of bidders submit bids, but where the cost of bidding may be different from bidder to bidder, and where the winner is not necessarily the bidder who submitted the highest bid. The defining characteristic of the all-pay auction is that the cost of the bid is forfeited, whether or not the auction is won. The auction terminology is used in the remainder of the paper.

All bidders are, for now, assumed to be risk neutral, and to share the same value of not participating, which is normalized to zero. The consequences of risk aversion are discussed in Section 4.2. Each bidder has a privately known type, v, which captures how much he values winning the prize. Bidder i's type is distributed according to some strictly increasing and twice continuously differentiable distribution function, $F(v)$, with no mass points and support $[\underline{v}, \overline{v}]$, where $\overline{v} > \underline{v} > 0$, $i = 1, 2.., n$. Densities, denoted by f, are bounded above and below, away from zero. The following assumption is imposed.

Assumption A: The "average probability", $F(v)/v$, is strictly increasing in v.

Note that the "average probability", $F(v)/v$, is strictly increasing if "total probability", $F(v)$, is convex. For example, Assumption A is satisfied by the uniform distribution with support $[\underline{v}, \overline{v}]$ whenever $\underline{v} > 0$. The assumption that bidders are ex ante homogenous in

[5] The cost function may be identity dependent because the ability or access to obtain qualifications may differ, or because the same amount of training does not translate into the same perceived qualifications.

terms of their desire to win can be relaxed, but it serves to highlight that the important source of heterogeneity is differences in costs.[6,7]

Two different possibilities are considered for how scores are computed in stage two. The first possibility is that the auction is unbiased, in which case the score is simply identical to the bid. In the alternative specification, bidder 1 is handicapped. Formally, bidders other than bidder 1 obtain a score equal to their bid, as before, but bidder 1 must bid $h(s)$ to obtain a score of s. It is assumed that $h(s)$ is twice continuously differentiable, with $h(0) = 0$, $h(s) > s$ for all $s \in \mathbb{R}_{++}$, and $h'(s) > 0$ for all $s \in \mathbb{R}_+$.[8] The important point is that bidders $2, 3, ..., n$ are given the *same* kind of advantage, and, in particular, that there is no bias when the bids of bidders in this group are compared with each other. Nevertheless, the set of favoured bidders may be diverse, in the sense that they may have different cost functions.

Note that handicapping bidder 1 is equivalent to giving preferential treatment to all other bidders. Assume that bidders $2, 3, ..., n$ obtain a score of $h(b) > b$ for any $b \in \mathbb{R}_{++}$, whereas bidder 1 only scores b when he bids b. This amounts to a simple "change of variables" compared to the model formulated in terms of handicaps. In particular, to tie with a rival bidder who scores s, bidder 1 has to bid $h(s)$, exactly as before. The difference between the two models amounts to an inconsequential rescaling or renaming of scores. More generally, if bidder i scores $H_i(b)$ with a bid of b, then the function $h(b) \equiv H_1^{-1}(H_2(b))$ measures the bid bidder 1 would have to submit in order to tie with bidder 2 if the latter bids b. The assumption in this paper is that bidder 1 is disadvantaged; $H_2(b) > H_1(b)$ or $h(b) > b$ for all $b \in \mathbb{R}_{++}$.

It is assumed that F and c_i are common knowledge among bidders. It is not necessarily assumed that the regulator of the contest knows the primitives of the game. The focus of the paper is not on determining what the "optimal" intervention may be, but merely on describing the actual consequences of changes to the game. Thus, it is also assumed that the regulator cannot or will not manipulate the number of prizes (which is assumed to be one), or their value.

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[6] All results hold if, for example, bidder 1's distribution first order stochastically dominates the distribution of bidder 2. See Amann and Leininger (1996) and Parreiras and Rubinchik (2010) for an analysis of all-pay auctions with two or more bidders, respectively, whose types are drawn from different distributions but where costs are linear.

[7] If bidder i with valuation v bids b and wins the auction with probability $q_i(b)$ then his expected payoff is $vq_i(b) - c_i(b)$. Thus, expected payoff is maximized where $q_i(v) - c_i(b)/v$ is maximized. Therefore, the model is isomorphic to one in which the value of the prize is known to be one, but where bidders' costs functions are unknown. It is known only that bidder i draws a cost function from the family $c_i(b)/v$. Obviously, the distribution of costs functions is determined by the distribution of v.

[8] For example, if the handicap is linear, by bidding b bidder 1 would obtain a score of $s = b/h$, $h \in (1, \infty)$, where h is the handicap. Thus, bidder 1's problem is equivalent to deciding which score, s, to obtain, given that to obtain a score of s he must bid $hs \geq s$.

Let Γ_n denote the auction involving bidders $1, 2, ..., n$ in which scores coincide with bids. Similarly, let Γ_n^h denote the game in which bidders $2, .., n$ are given preferential treatment. In the following, it is useful to think of bidders as choosing scores rather than bids. The set of actions is then $\{out\} \cup \mathbb{R}_+$, meaning that each bidder can choose to either stay out of the auction or to enter the auction and submit a non-negative score. When bidder 1 is handicapped, his cost of obtaining a score of s is $c_1^h(s) \equiv c_1(h(s))$.

It is assumed that the handicap increases bidder 1's marginal costs and that the cost function remains (weakly) convex. This is the case if, for example, $h(s)$ is linear or convex.

Assumption B: The handicap increases bidder 1's marginal costs; $c_1^{h\prime}(s) > c_1'(s)$ for all $s \in \mathbb{R}_+$.

To demonstrate the main point, it is sufficient to consider a situation with *three* bidders. In the games considered here, bidder 1, the bidder who is potentially handicapped, is the "strong" bidder in the absence of handicaps. Bidder 2 and bidder 3 are "weaker" bidders, although their weakness is manifested in different ways, as illustrated in Figure 1. Specifically, of all the bidders, bidder 2 is the one for whom small bids are the most expensive. On the other hand, bidder 3 is the bidder for whom large bids are the most expensive. Thus, for both bidders there are bids for which they would incur the highest cost of obtaining such bids. There are no bids with this property for bidder 1, the strong bidder. Although their cost functions cross, bidder 2 and bidder 3 will be referred to as "weak" and "very weak", respectively, because in a closely contested auction bidder 3's advantage over bidder 2 at low bids is less likely to be relevant. Alternatively, following Siegel (2009), define bidder i's *reach*, r_i, as the highest bid that the bidder would be willing to submit (in the absence of a handicap) even if he was guaranteed to win, such that $\overline{v} - c_i(r_i) = 0$ or $r_i = c_i^{-1}(\overline{v})$. No bid above r_i can be rationalized by bidder i. In Siegel's (2009) complete information contests (where types are common knowledge), the ranking of bidders' reaches is an important measure of strength (see also Che and Gale (2006)). This ranking, $r_1 > r_2 > r_3$, provides another justification for the strong, weak, and very weak terminology.

Following the previous discussion, the relationship between the cost functions of bidder 1 and bidder 2 is formalized by the next assumption. While Assumption B signifies that the handicap is detrimental to bidder 1, it is assumed that it is not big enough to completely negate the advantage he has over bidder 2.

Assumption C: Bidder 1 has a cost advantage over bidder 2, even after he is handicapped; $c_2'(s) > c_1^{h\prime}(s)$ for all $s \in [0, r_2]$.

At this point, no formal assumptions regarding c_3 is imposed. The reason is that it will be a *result* of this paper that there are c_3 functions with the general properties depicted in Figure 1 (where c_3 crosses c_2 from below) for which bidder 3 is worse off with preferential treatment. Thus, Figure 1 serves as a "preview" of the main result.

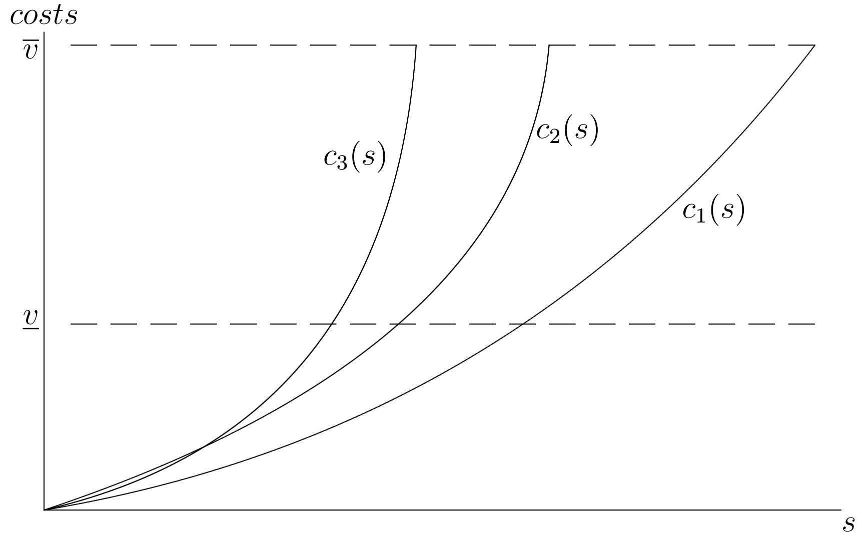

Figure 1: The strong, the weak, and the very weak.

3 Analysis

In this paper, I restrict attention to equilibria in increasing strategies. That is, each bidder has a cut-off type below which he scores zero or stays out of the auction, and above which he enters the auction with a score that is strictly increasing in his type. It follows from Athey's (2001, Theorem 7) more general analysis that an equilibrium of this nature exists.

The analysis is initiated by considering two games, Γ_2 and Γ_2^h. In these games, bidder 3 is ignored. Then, the larger games in which bidder 3 is potentially active, Γ_3 and Γ_3^h, are examined. Here, the central question is whether bidder 3 would select to be active if bidders 1 and 2 continue to play the increasing strategies from Γ_2 and Γ_2^h, respectively.

3.1 The strong and the weak bidder; Γ_2 and Γ_2^h

Consider the game Γ_2^h, in which bidder 3 is not present. To begin, note that it cannot be the case that both bidders stay out of the auction with strictly positive probability. The reason is that it would pay to enter the auction with a very small score, in order to win with a non-trivial probability. Let $\varphi_i^h(s)$ denote bidder i's inverse bidding strategy among the set of types who participate, such that bidder i scores s if his type is φ_i^h, $i = 1, 2$.

Amann and Leininger (1996) and Lizzeri and Persico (2000) analyze two-bidder all-pay auctions. Amann and Leininger (1996) assume that the cost function is identical (and linear) for the two bidders, but that types are drawn from different distribution functions. However, their arguments also apply to the model in the current paper. Thus, equilibrium strategies have the same properties as identified in Amann and Leininger (1996, Lemmas 1-5). In particular, $F_1^h(s) \equiv F(\varphi_1^h(s))$ and $F_2^h(s) \equiv F(\varphi_2^h(s))$ have a common support of the form $[0, \overline{s}^h]$, where $\overline{s}^h$ is the common maximal score. Although a mass of types may stay out, for those types that participate $F_i^h(s)$ has no atoms, no gaps, and is strictly increasing.[9] Since strategies are monotonic, they are differentiable almost everywhere and ties occur with probability zero. Hence, upon entry, bidder 1 with type v seeks to maximize $vF(\varphi_2^h(s)) - c_1^h(s)$, where $F(\varphi_2^h(s))$ is the probability that he outscores bidder 2 (and wins) with a score of s. Thus, any interior solution to bidder 1's problem must satisfy the first order condition,

$$v\frac{dF(\varphi_2^h(s))}{ds} - c_1^{h\prime}(s) = 0.$$

The first order condition for bidder 2 is obtained in similar fashion.

In equilibrium, bidder 1 scores s if his type is $v = \varphi_1^h(s)$ while bidder 2 scores s if his type is $\varphi_2^h(s)$. Substituting these into the first order conditions gives the following pair of conditions:

$$\frac{dF(\varphi_1^h(s))}{ds} = \frac{c_2'(s)}{\varphi_2^h(s)}, \quad \frac{dF(\varphi_2^h(s))}{ds} = \frac{c_1^{h\prime}(s)}{\varphi_1^h(s)} \tag{1}$$

or

$$\varphi_1^{h\prime}(s) = \frac{c_2'(s)}{f(\varphi_1^h(s))\varphi_2^h(s)}, \quad \varphi_2^{h\prime}(s) = \frac{c_1^{h\prime}(s)}{f(\varphi_2^h(s))\varphi_1^h(s)} \tag{2}$$

Since $\overline{s}^h$ is the common maximal bid the system of differential equations must satisfy the boundary condition $F(\varphi_1^h(\overline{s}^h)) = F(\varphi_2^h(\overline{s}^h)) = 1$. However, $\overline{s}^h$ is endogenous.

By assumption, the right hand side of the equations in (2) are continuously differentiable in φ_1^h, φ_2^h, and s. Likewise, the assumption that types are strictly positive, or $\underline{v} > 0$, implies that $F(\varphi_i^h(s))$ has finite slope everywhere and that the system is Lipschitz. Thus, as in Lizzeri and Persico (2000, Section 3), for any given guess on the value of $\overline{s}^h$, the system in (1) or (2) takes a unique path as s approaches zero ("shooting backwards" from $\overline{s}^h$ to 0); see e.g. Hirsch and Smale (1974, Theorem 1, p. 297). Verifying whether the outcome is consistent with an equilibrium then helps to pinpoint the values of $\overline{s}^h$ that are equilibrium

[9] In particular, no bidder scores or bids zero for a mass of types. Otherwise, the rival bidder with valuation $\underline{v} > 0$ should not score zero, but rather marginally above zero, in order to dramatically increase the probability of winning by ruling out a tie. In contrast, Amann and Leininger (1996) assume $\underline{v} = 0$, in which case the argument does not preclude a mass of types from scoring zero.

candidates.[10]

To this end, note that since strategies are monotonic and at least one bidder participates with probability one, either $F(\varphi_1^h(0)) = 0$ or $F(\varphi_2^h(0)) = 0$ (or both). By using this condition and the requirement of a common support, $[0, \overline{s}^h]$, it will be shown that there is a unique equilibrium in increasing strategies (see Proposition 1, below). That is, there is one, and only one, value of $\overline{s}^h$ for which (1) produces an equilibrium of the game.

In the following, let φ_i denote the strategies and let $\overline{s}$ denote the maximum equilibrium score when there is no handicap. The differential equations for this case are analogous to (1). Given Assumptions B and C, the strong bidder scores more aggressively than the weak bidder whether or not he is handicapped. Moreover, the weak bidder stays out of the auction with positive probability, whereas the strong bidder always participates. However, the strong bidder scores less aggressively when he is handicapped (although his bid may be higher). In response, the weak bidder becomes *more* aggressive, at least in the sense that he is now more likely to participate.

Proposition 1 (Equilibrium Properties) *There is a unique equilibrium in increasing strategies in Γ_2^h. In this equilibrium, the weak bidder stays out with strictly positive probability and is more likely to submit low bids than the strong bidder, $F(\varphi_1^h(s)) < F(\varphi_2^h(s))$ for all $s \in [0, \overline{s}^h)$, with $F(\varphi_1^h(0)) = 0 < F(\varphi_2^h(0))$.[11] The same properties hold for Γ_2.*

Proof. See the Appendix. ■

Proposition 2 (Comparative Statics) *The unique equilibrium in increasing strategies of Γ_2 compares with its counterpart in Γ_2^h as follows:*

1. *Scores are more compressed and the strong bidder scores less aggressively in Γ_2^h than in Γ_2: $\overline{s}^h < \overline{s}$, and $F(\varphi_1^h(s)) > F(\varphi_1(s))$ for all $s \in (0, \overline{s}^h]$.*

2. *The weak bidder participates more often in Γ_2^h than in Γ_2: $0 < F(\varphi_2^h(0)) < F(\varphi_2(0))$.*

Proof. See the Appendix. ■

Note that the strong bidder becomes less of a threat to the weak bidder when he is handicapped. For a fixed score, the weak bidder is more likely to win. Consequently, he is more likely to participate, and, if he participates, he is better off. Thus, depending on his

[10] Note that if φ_1^h and φ_2^h satisfy (2) then φ_i^h is increasing in s (the right hand side is strictly positive). Fixing v, the first derivative of bidder 1's payoff with respect to the score is $\left(v/\varphi_1^h(s) - 1\right) c_1^{h\prime}(s)$, which is positive when s is small (such that $\varphi_1^h(s) < v$) and negative when s is large (and $\varphi_1^h(s) > v$). Consequently, payoff is single peaked in s, and the first order conditions are sufficient if φ_1^h and φ_2^h satisfy (1).

[11] More precisely, the equilibrium is "essentially unique" because it does not matter whether bidder 2 with type $\varphi_2^h(0)$ scores zero or stays out.

type, v, he is either indifferent or strictly better off when he is given preferential treatment. Ex ante (before his type is known), he must therefore be strictly better off.

Corollary 1 *With just two bidders, bidder 2 is weakly better off regardless of his type if he is given preferential treatment, and strictly better of ex ante.*

3.2 The very weak bidder; Γ_3 and Γ_3^h

Consider now bidder 3, the very weak bidder. If bidder 1 and bidder 2 compete as described above, does bidder 3 have an incentive to become active in the auction? Figure 2 illustrates the response of bidders 1 and 2 to the handicap (assuming bidder 3 stays out), as described in Proposition 2. If bidder 3 enters with a small bid after preferential treatment is extended to the weak bidders, he is *more likely* to beat the strong bidder, but *less likely* to beat the weak bidder compared to the situation before preferential treatment. Of course, bidder 3 is concerned with outscoring both bidders, the probability of which is

$$q_3^h(s) \equiv F(\varphi_1^h(s))F(\varphi_2^h(s)) \tag{3}$$

when he submits a score of s.

For small scores, both $F(\varphi_1^h(s))$ and $F(\varphi_2^h(s))$ are steeper than $F(\varphi_1(s))$ and $F(\varphi_2(s))$, which perhaps suggests a greater return to submitting a small score for bidder 3. However, this is counteracted by the fact that bidder 2 is more likely to participate. Given (1), the derivative of $q_3^h(s)$ is

$$q_3^{h\prime}(s) = \frac{F(\varphi_1^h(s))}{\varphi_1^h(s)} c_1^{h\prime}(s) + \frac{F(\varphi_2^h(s))}{\varphi_2^h(s)} c_2^{\prime}(s) \, . \tag{4}$$

Since marginal costs are increasing, Assumption A implies that the right hand side is increasing in s. In other words, $q_3^h(s)$ is strictly convex. Since $F(\varphi_1^h(0)) = 0$ the derivative at $s = 0$ is

$$q_3^{h\prime}(0) = \frac{F(\varphi_2^h(0))}{\varphi_2^h(0)} c_2^{\prime}(0) \, . \tag{5}$$

The main result follows from (5). In particular, $\varphi_2^h(0) < \varphi_2(0)$ (Proposition 2) and Assumption A together imply that $q_3^h(s)$ is *flatter* than $q_3(s) \equiv F(\varphi_1(s))F(\varphi_2(s))$ near $s = 0$, though each individual term is steeper. Since $q_3^h(0) = q_3(0) = 0$, $q_3^h(s)$ must be *below* $q_3(s)$ for small s; after bidder 3 is given preferential treatment (along with bidder 2), he faces a worse distribution of rival scores at the bottom.

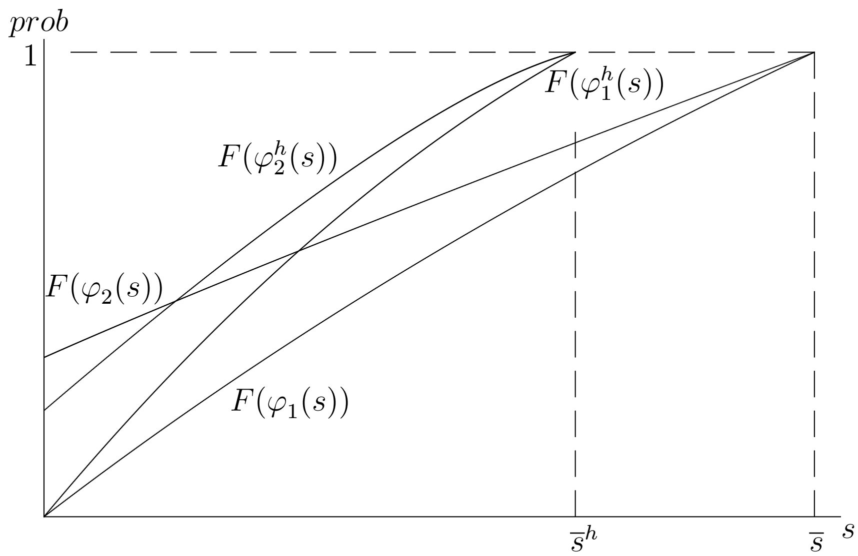

Figure 2: Individual distributions of scores.

The incentive to enter the auction is strongest for bidder 3 if his type is $\overline{v}$, in which case he maximizes $\overline{v}q_3^h(s) - c_3(s)$, or, equivalently,

$$q_3^h(s) - \frac{c_3(s)}{\overline{v}}. \tag{6}$$

For bidder 3 to find entry profitable, there must be a score, $s > 0$, for which $q_3^h(s) \geq c_3(s)/\overline{v}$.

Figure 3 depicts $q_3^h(s)$ and $q_3(s)$. The important properties are: (i) $q_3^h(s)$ is flatter than $q_3(s)$ for low s, and (ii) $\overline{s}^h < \overline{s}$. Thus, $q_3^h(s)$ and $q_3(s)$ must cross. Now, with the cost function c_3 depicted in Figure 3, the very weak bidder should enter the auction with a strictly positive bid (and earn positive payoff) if there is no preferential treatment, but he should stay out if bidder 1 is handicapped. More precisely, it is not an equilibrium for bidder 3 to be inactive in the absence of preferential treatment, but there is an equilibrium in which he stays out after he and the other weak bidder is given preferential treatment.[12] Equilibrium is not characterized in the former case, but it follows from Athey (2001) that one exists.

Theorem 1 *There exists a strictly increasing and strictly convex cost function, c_3, for which there is no equilibrium in increasing strategies of Γ_3 where bidder 3 wins with probability zero, but for which there is an equilibrium in increasing strategies of Γ_3^h in which bidder 3 wins with probability zero.*

Proof. See the Appendix. ■

[12] The proof of Theorem 1 describes an entire class of cost-functions (which depends on $F(\cdot)$, $c_1(\cdot)$, and $c_2(\cdot)$) for which this result holds. The opposite is also possible. Specifically, there are other c_3 functions for which bidder 3 would be inactive without preferential treatment, but active with preferential treatment.

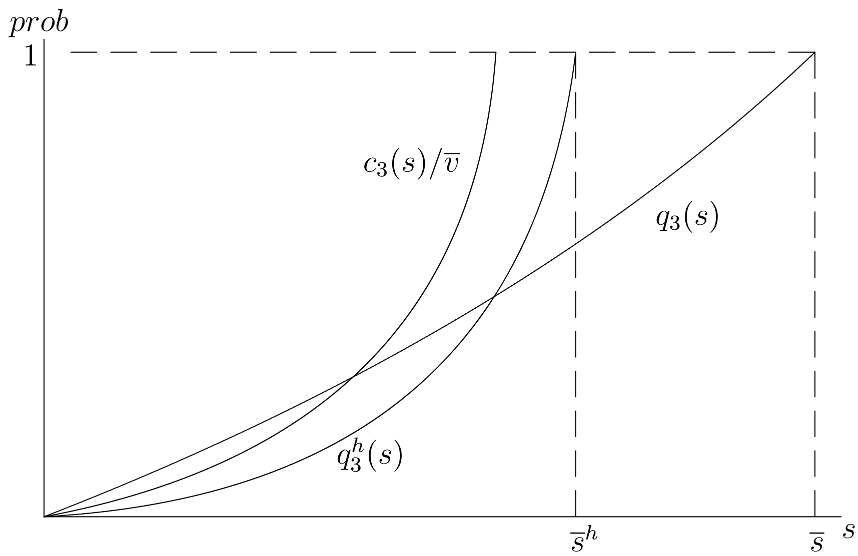

Figure 3: Distribution of the highest score and the cost function.

Consider now some c_3 function for which Theorem 1 is valid. Note that multiple equilibria in increasing strategies has not been ruled out, but Theorem 1 proves that bidder 3 wins with strictly positive probability in *any* such equilibrium of Γ_3. However, this is not true for Γ_3^h. Even if there are multiple equilibria of Γ_3^h, the introduction of preferential treatment may afford bidder 1 and bidder 2 the opportunity to "collude" and effectively exclude bidder 3 in situations where that would have been impossible without preferential treatment.

Since bidder 3 wins with positive probability for a mass of types in Γ_3, he has types for which his payoff is positive.[13] In this case, bidder 3 would go from participating with positive probability, winning with positive probability, and having positive payoff, to participating with probability zero, winning with probability zero, and having zero payoff.

Corollary 2 *Bidder 3 may be worse off when he and bidder 2 are given preferential treatment.*

Sowell (2004) provides a plethora of examples from around the world in which preferential treatment was initially intended for a relatively small and well-defined group of individuals only, but where, over time, it grew to encompass a larger group of people. Even if preferential treatment as initially envisioned was beneficial to the former group, the results in this paper caution that adding more people to the list may not only "dilute" the advantage of the first group, as is intuitive, but may in fact *reverse* the effect of preferential treatment, and ultimately leave the initial beneficiaries worse off than before preferential treatment.

[13] It follows from Myerson's (1981) analysis that in any mechanism where a bidder wins with positive probability for a mass of types, these types must earn strictly positive payoff, except possibly for the lowest participating type.

4 Discussion

The conditions under which Theorem 1 holds are discussed below.

4.1 Comparing bidders

Given Theorem 1, it is now possible to make more precise the sense in which bidder 3 is "very weak". For the cost function c_3 to satisfy Theorem 1, it is clearly necessary that a score of $\overline{s}^h$ is prohibitively expensive for bidder 3. On the other hand, it is not prohibitively expensive for bidders 1 and 2, since it is an equilibrium score. Hence, bidders 1 and 2 have an advantage over bidder 3 at high scores; their reaches are higher, $r_1 > r_1^h > r_2 > \overline{s}^h > r_3$, where r_1^h is bidder 1's reach when he is handicapped.

Next, consider low scores. Since bidder 3 is active when there is no handicap in place, there must be some profitable score if bidders 1 and 2 use their strategies from Γ_2. For instance, if bidder 3 profits by submitting a bid marginally above zero (as in the construct in the proof of Theorem 1, illustrated in Figure 3), then

$$0 < q_3'(0) - \frac{c_3'(0)}{\overline{v}} = \frac{F(\varphi_2(0))}{\varphi_2(0)} c_2'(0) - \frac{c_3'(0)}{\overline{v}} < \frac{c_2'(0)}{\overline{v}} - \frac{c_3'(0)}{\overline{v}},$$

where the first equality follows from (5) and the last inequality from Assumption A. In conclusion, $c_3'(0) < c_2'(0)$. Thus, bidder 3 has a cost advantage over bidder 2 for low scores, but bidder 2 has the advantage when scores are high. Figure 1 illustrates cost structures for which Theorem 1 is applicable.

Although it is tempting to make the mathematically expedient – and arguably more elegant – assumption that the bidders' cost functions can be ordered (that they do not cross), such an assumption may inadvertently cause the modeler to miss potentially important consequences of a policy intervention. Moreover, there is little reason to believe that real-world cost function can always be ranked in such a manner, and there may even be empirical evidence to suggest that cost functions cross in some contests.

For example, Fryer and Torelli (2010) study the relationship between academic achievement and social status among whites, blacks, and Hispanics in high-school. They find that social status is increasing at a fast rate in grades for whites (Fryer and Torelli (2010), Figure 1B). For blacks and Hispanics, the curve has an inverse U shape; it has a peak. The curve for blacks is relatively flat. For Hispanics, however, the curve is first increasing at a rate faster than the curve for blacks (but not as quickly as for whites), but it reaches its peak much sooner, after which it drops at a very fast rate.

Imagine now that obtaining higher grades involves two considerations on the costs side, incurring higher effort costs and experiencing an increase or decrease in one's social status. Assume the cost of effort is the same for all groups. Since whites experience a large increase in status from higher grades, their net costs are arguably lower than the overall costs of the other groups. Since the status of a Hispanic student at first rises faster in achievement than is the case for a black student, it could also be argued that blacks have the largest marginal costs at low achievement levels. However, because the curve drops so dramatically, and early, for Hispanic students, the overall costs of achieving high grades are very steep for Hispanics. The costs functions of whites, blacks, and Hispanics, may then resemble those of bidder 1, bidder 2, and bidder 3, respectively, in Figure 1. Of course, Fryer and Torelli's (2010) figure represents aggregates over large populations. In contrast, the formal model studied in the current paper assumes the population is not too large.

4.2 Risk aversion

So far, bidders have been assumed to be risk neutral. Assume now that bidder 3 is risk averse, that w is his initial wealth, and that v measures the monetary value of winning the auction. Any bid or score can then be viewed as producing a lottery with an outcome of $w + v - c_3(s)$ with probability $q_3^h(s)$ and an outcome of $w - c_3(s)$ with probability $1 - q_3^h(s)$. Similarly, any point in Figure 3 can be viewed as representing a lottery, with a function of costs on the horizontal axis and the win probability on the vertical axis. The curves $q_3(s)$ and $q_3^h(s)$ can be thought of as feasible sets; a score of s results in a win probability of $q_3(s)$ and $q_3^h(s)$ without and with a handicap, respectively.

For the risk neutral case, the curve $q = c_3(s)/\overline{v}$ in Figure 3 captures an indifference curve; bidder 3 is indifferent between any combination of score (s) and win probability (q) on this curve. Any lottery to the north-west of this curve would generate higher expected utility than to not participate. Bidder 3 would "accept" such a lottery.

However, it is a standard result that the "acceptance set" diminishes as the agent becomes more risk averse (the indifference curve in Figure 3 shifts toward the north-west in the interior). Thus, the more risk averse bidder 3 is, the less likely any given score is to produce a lottery that bidder 3 would accept (compared to the risk-less alternative of not participating). In other words, he is less likely to participate in both Γ_3 and Γ_3^h. However, contingent on bidder 3 remaining active in Γ_3, the conclusion must be that handicapping bidder 1 is more likely to scare off bidder 3 the more risk averse he is; he was closer to giving up in Γ_3 and it takes less of a change to persuade him to stay out completely. The common assertion that risk aversion is diminishing in wealth then suggests that the intervention is more likely to deter bidder 3 the poorer he is.

4.3 Complete versus incomplete information

Siegel (2009) considers a very general class of contests that encompasses all-pay auctions with and without handicaps. While he allows bidders to be heterogenous in valuations and costs, it is assumed that information is complete. The implication of his analysis is that the bidder with the highest reach is the only bidder with strictly positive expected payoff. Thus, weak bidders earn zero payoff. Clearly, the two weak bidders cannot be worse off than this when they are given preferential treatment; preferential treatment cannot hurt the intended beneficiaries in a complete-information contest.[14] Consequently, the assumption of incomplete information imposed in this paper is as important as it is realistic.

5 Conclusion

This paper considered a contest with a number of realistic features: There are more than two contestants, contestants are heterogenous, and information is incomplete. In this environment, preferential treatment may have unintended consequences. Specifically, when a diverse group of contestants are given preferential treatment compared to the remaining contestants, a subset of the intended beneficiaries may become worse off. The reason is that the dynamics within the "favored" group changes. In particular, the stronger of the favored contestants may become more aggressive. From the point of view of the weaker of the favored contestants, this effect may outweigh the advantage that is gained over contestants who do not receive preferential treatment.

The possibility that preferential treatment may be disadvantageous was demonstrated in a setting with very specific cost structures, in which multiple equilibria were not ruled out. However, since the result is a negative result, the main point remains valid: Jumping to the conclusion that preferential treatment is unambiguously beneficial to the weaker contestants is not justified. The theory of contests with more than two heterogeneous contestants is underdeveloped, and more research is needed to better assess the consequences of manipulating or regulating the contest.

References

Amann, E. and Leininger, W., 1996, Asymmetric All-Pay Auctions with Incomplete Information: The Two-Player Case, *Games and Economic Behavior*, 14: 1-18.

[14] More generally, a handicap reduces the reach of the handicapped bidder. The other bidders cannot be made worse off as a consequence. Thus, if bidder i is handicapped, the other two bidder are not hurt, $i = 1, 2, 3$.

Athey, S., 2001, Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information, *Econometrica*, 69: 861-889.

Che, Y-K. and Gale, I.L., 2006, Caps on Political Lobbying: Reply, *American Economic Review*, 96 (4), 1355-1360.

Clark, D.J. and Riis, C., 2000, Allocation efficiency in a competitive bribery game, *Journal of Economic Behavior & Organization*, 42: 109-124.

De Fraja, G., 2005, Reverse Discrimination and Efficiency in Education, *International Economic Review*, 46 (3): 1009-1031,

Fang, H. and Norman, P., 2006, Government-Mandated Discriminatory Policies: Theory and Evidence, *International Economic Review*, 47 (2): 361-389.

Fryer, R.G., and Torelli, P., 2010, An Empirical Analysis of 'Acting White', *Journal of Public Economics*, 94: 380-396.

Fryer, R.G., and Loury, G.C., 2005, Affirmative Action and Its Mythology, *Journal of Economic Perspectives*, 19 (3): 147-162.

Goeree, J.K. and Offerman, T., 2004, The Amsterdam Auction, *Econometrica*, 72 (1): 281-294.

Hafalir, I. and Krishna, V., 2008, Asymmetric Auctions with Resale, *American Economic Review*, 98 (1): 87-112.

Hafalir, I. and Krishna, V., 2009, Revenue and Efficiency Effects of Resale in First-Price Auctions, *Journal of Mathematical Economics*, 45 (9-10): 589-602.

Hickman, B.R., 2010, Effort, Race Gaps, And Affirmative Action: A Game-Theoretic Analysis of College Admissions, mimeo.

Hirsch, M. and Smale, S., 1974, *Differential Equations, Dynamical Systems, and Linear Algebra*, Academic Press.

Hörner, J. and Sahuguet, N., 2007, Costly Signalling in Auctions, *Review of Economic Studies*, 74: 173-206.

Kirkegaard, R., 2009, Asymmetric First Price Auctions, *Journal of Economic Theory*, 144 (4): 1617-1635.

Kirkegaard, R., 2010, Favoritism in Asymmetric Contests: Head Starts and Handicaps, mimeo.

Kirkegaard, R., 2011, A Poor Bidder's Perspective on All-Pay Auctions: More Competitors, Please, mimeo.

Lebrun, B., 1999, First Price Auctions in the Asymmetric N Bidder Case, *International Economic Review*, 40, 125-142.

Lizzeri, A. and Persico, N., 2000, Uniqueness and Existence of Equilibrium in Auctions with a Reserve Price, *Games and Economic Behavior*, 30: 83-114.

Maskin, E. and Riley, J., 2000, Asymmetric Auctions, *Review of Economic Studies*, 67, 413-438.

Moro, A. and Norman, P., 2003, Affirmative Action in a Competitive Economy, *Journal of Public Economics*, 87: 567-594.

Myerson, R.B., 1981, Optimal Auction Design, *Mathematics of Operations Research*, 6: 58-73.

Parreiras, S. and Rubinchik, A., 2010, Contests with Three or More Heterogeneous Agents, *Games and Economic Behavior*, 68: 703-715.

Siegel, R., 2009, All-Pay Contests, *Econometrica*. 77(1): 71-92

Siegel, R., 2010, Asymmetric Contests with Conditional Investments, *American Economic Review*, 100: 2230-2260.

Siegel, R., 2011, Asymmetric Contests with Interdependent Valuations and Incomplete Information, mimeo.

Sowell, T., 2004, *Affirmative Action Around the World: An Empirical Study.* Yale University Press, New Haven and London.

Tullock, G., 1980, Efficient Rent Seeking, in *Toward a Theory of the Rent-Seeking Society*, edited by Buchanan, J.M., Tollison, R.D., and Tullock, G., Texas A&M University Press.

Zhang, J. and Wang, R., 2009, The Role of Information Revelation in Elimination Contests, *Economic Journal*, 119: 613-641.

Appendix: Proofs

Proof of Proposition 1. Consider the game Γ_2^h. The system described by (1) is monotonic in $\bar{s}^h$. If $\bar{s}^h$ is reduced to $\tilde{s}^h < \bar{s}^h$, it must be the case that $\varphi_i^h(\tilde{s}^h) = \bar{v}$ after the change, but that $\varphi_i(\tilde{s}^h) < \bar{v}$ before the change. Given (1), $F(\varphi_j^h(s))$ must therefore be strictly flatter than before at $s = \tilde{s}^h$. Continuing this argument as s is reduced implies that $F(\varphi_1^h(\cdot))$ and $F(\varphi_2^h(\cdot))$ shift up when $\bar{s}^h$ is reduced. Thus, there is precisely one value for which $\bar{s}^h$ and the resulting unique paths of $F(\varphi_1^h(s))$ and $F(\varphi_2^h(s))$ satisfy the equilibrium requirement that $F(\varphi_1^h(0)) = 0$ and $F(\varphi_2^h(0)) \geq 0$ or vice versa.

Next, note that if $F(\varphi_1^h(s)) = F(\varphi_2^h(s))$ then $\varphi_1^h(s) = \varphi_2^h(s)$, in which case Assumption C and (1) together imply that $F(\varphi_1^h(s))$ is strictly steeper than $F(\varphi_2^h(s))$. Thus, $F(\varphi_1^h(s))$ and $F(\varphi_2^h(s))$ coincide at most once. In fact, this occurs at $\bar{s}^h$, since $F(\varphi_1^h(\bar{s}^h)) = F(\varphi_2^h(\bar{s}^h)) = 1$. Since $F(\varphi_1^h(s))$ is strictly steeper than $F(\varphi_2^h(s))$ at $\bar{s}^h$, the implication is that $F(\varphi_1^h(s)) < F(\varphi_2^h(s))$ for all $s \in [0, \bar{s}^h)$, as claimed. These arguments also apply to Γ_2. ■

Proof of Proposition 2. Assume that $\bar{s}^h = \bar{s}$. In this case $\varphi_i^h(\bar{s}) = \varphi_i(\bar{s}) = \bar{v}$, $i = 1, 2$, and it follows from Assumption B that $F(\varphi_2^h(\cdot))$ is strictly steeper than $F(\varphi_2(\cdot))$ at $\bar{s}$. Hence, $\varphi_2^h < \varphi_2$ immediately to the left of $\bar{s}$, which implies that $F(\varphi_1^h(\cdot))$ is strictly steeper than $F(\varphi_1(\cdot))$, and which in turn implies that $\varphi_1^h < \varphi_1$. These arguments repeat themselves as s is reduced even further, and it follows that $F(\varphi_1^h(\cdot))$ becomes zero for some $s > 0$. As discussed earlier, this contradicts that $\bar{s}^h$ and φ_1^h, φ_2^h form an equilibrium. It is easily seen that $\bar{s}^h > \bar{s}$ would lead to a similar contradiction. Thus, $\bar{s}^h < \bar{s}$, and $F(\varphi_1^h(\bar{s}^h)) = 1 > F(\varphi_1(\bar{s}^h))$.

Moving to the left, consider the first $s \in (0, \bar{s}^h)$ to the left of $\bar{s}^h$, if it exists, for which $F(\varphi_1^h(\cdot))$ and $F(\varphi_1(\cdot))$ coincide, or $\varphi_1^h = \varphi_1$. Since $F(\varphi_1^h(\cdot)) > F(\varphi_1(\cdot))$ to the right of this point, by definition, $F(\varphi_1^h(\cdot))$ must be at least as steep as $F(\varphi_1(\cdot))$, which implies that $\varphi_2^h \leq \varphi_2$ or $F(\varphi_2^h) \leq F(\varphi_2)$. As $\varphi_1^h = \varphi_1$, it follows from Assumption B and (1) that $F(\varphi_2^h(\cdot))$ is strictly steeper than $F(\varphi_2(\cdot))$ at this point, which means that $\varphi_2^h < \varphi_2$ just to the left of this score. As before, this leads to the conclusion that $F(\varphi_1^h(\cdot))$ is steeper than $F(\varphi_1(\cdot))$ to the left of this point. Once again, this can be ruled out, because $F(\varphi_1^h(\cdot))$ becomes zero for some $s > 0$. Thus, by contradiction, $F(\varphi_1^h(s)) > F(\varphi_1(s))$ for all $s \in (0, \bar{s}^h]$.

Assumption B and $\varphi_1^h(0) = \varphi_1(0) = \underline{v}$ imply that $F(\varphi_2^h(\cdot))$ is strictly steeper than $F(\varphi_2(\cdot))$ at $s = 0$. Assume now that $F(\varphi_2^h(0)) \geq F(\varphi_2(0))$ or $\varphi_2^h(0) \geq \varphi_2(0)$. Then, $\varphi_2^h > \varphi_2$ for small, strictly positive s, implying that $F(\varphi_1^h(\cdot))$ is strictly flatter than $F(\varphi_1(\cdot))$. Consequently, $\varphi_1^h < \varphi_1$ for small s. However, this contradicts the property that bidder 1 is less aggressive when he is handicapped. ■

Proof of Theorem 1. To begin, note that q_3^h as defined in (3) is exogenous to the games Γ_3 and Γ_3^h, since it depends only on the strategies φ_1^h and φ_2^h from the game Γ_2^h. For similar reasons, $q_3(s) \equiv F(\varphi_1(s))F(\varphi_2(s))$ is also exogenous to the games Γ_3 and Γ_3^h. In particular, neither depends on bidder 3's characteristics, $c_3(\cdot)$, since he is assumed absent from Γ_2 and Γ_2^h.

So, assume bidder 3's cost function takes the form $c_3(s) = \alpha \bar{v} q_3^h(s)$, with $1 < \alpha < q_3'(0)/q_3^{h\prime}(0)$. This is a strictly increasing and strictly convex function. Since $\alpha > 1$, (6) reveals that there is no incentive for bidder 3 to become active in Γ_3^h when bidders 1 and 2 follow the increasing strategies from Γ_2^h. Given bidder 3 stays out with probability one, there is no incentive for bidders 1 and 2 to deviate from the increasing strategies in Γ_2^h. Thus, it is an equilibrium for bidder 3 to always stay out, and for bidders 1 and 2 to continue using the increasing strategies from Γ_2^h; all bidders are best responding given their belief, and beliefs are consistent with strategies. Clearly, bidder 3 wins the auction with probability zero.

Consider now the game without a handicap, Γ_3. If bidder 3 wins with probability zero, no strictly positive score or bid can be rationalized. Thus, bidder 3 must either stay out, or bid zero. The arguments leading to Proposition 1 still apply, and bidder 1 and bidder 2's increasing strategies from Γ_2 are the unique pair of candidates for their increasing equilibrium strategies. However, (6) and $\alpha < q_3'(0)/q_3^{h\prime}(0)$ imply that bidder 3 should deviate; there is no equilibrium in increasing strategies of Γ_3 where bidder 3 wins with probability 0. ■