Contracting with Private Rewards*

René Kirkegaard
Department of Economics and Finance
University of Guelph
rkirkega@uoguelph.ca.

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Abstract

I extend the canonical moral hazard model to allow the agent to face endogenous and non-contractible uncertainty. The agent works for the principal and simultaneously pursues private rewards. I establish conditions under which the first-order approach remains valid. The model adds to the literature on intrinsic versus extrinsic motivation. Specifically, to induce higher effort at work the contract may offer higher rewards but flatter incentives. The contract change makes the agent reevaluate his "work-life balance". Larger employment rewards lessen the incentive to pursue private rewards. The greater reliance on labor income then necessitates weaker explicit incentives to induce high effort.

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1 Introduction

The principal-agent model has been tremendously influential in economics. However, the canonical model essentially assumes that the principal-agent relationship takes place in a perfect vacuum – there are no (non-contractible) outside random disturbances. For instance, the only payoff-relevant risk the agent faces is due to the uncertainty embodied in the incentive scheme offered by the principal.

In reality, however, it is easier to think of examples in which the agent faces some non-contractible outside uncertainty than examples in which this is not the case. Indeed, such uncertainty is often endogenous. That is, the agent pursues a host of potentially rewarding activities that are not directly observable (nor necessarily directly relevant) to the principal. Even seemingly mundane activities may in reality entail significant rewards. For instance, when the busy young professional tolerates dinner with her parents, she may hope to join the "27 percent of those purchasing a home for the first time [who] received a cash gift from relatives or friends to come up with a down payment."[1] When her older brother moves his family closer to their parents at the cost of a longer commute, he may be motivated by the fact that "by the time the average youngster reaches school age, they will have been babysat by their grandparents for more than 5,610 hours."[2] The "rewards" the parents bestow upon their children are most likely not observable to employers; they are non-contractible.

There are a plethora of other examples in which the agent directly receives a reward from a third party. Although the waiter has an employment contract with the restaurant owner, a significant part of her income often comes in the form of tips from the diner, despite the fact that there is no explicit contract between the two (nor is there an explicit contract between the parents and offspring in the previous paragraph). In other cases, the agent is in a formal contractual relationship with more than one principal, a situation known as common agency. Thus, developing an understanding of contracting with private rewards is a necessary

[1] The data is for the U.S, in 2013. See www.bloomberg.com/news/2014-09-19/mom-and-dad-banks-step-up-aid-to-first-time-home-buyers.html

[2] The data is for the U.K. The estimated monetary value of this amount of child care is £21,654.60. See http://www.dailymail.co.uk/femail/article-2263843/The-21-000-grandma-Grandparents-babysitting-duties-reduce-cost-childcare-whopping-4-300-EVERY-YEAR.html

first step towards analyzing common agency environments in which principals do not have access to the same information.[3]

Examples involving potentially large non-monetary rewards include the agent's health status as impacted by life-style choices, his social status in his peer group, the quality of his match on the marriage market as affected by his search intensity, and so on. The satisfaction from mastering a second language, or any other hobby, is another example. Even the agent's "job-satisfaction" may be endogenous, influenced by the enthusiasm with which he interacts with colleagues.

The aim of this paper is to analyze the consequences of endogenous "private rewards" on optimal contracting. The standard principal-agent model is amended to allow the agent to work on two tasks. The first "task" captures the effort the agent expends working on behalf of the principal. The second task describes the effort devoted to pursuing private rewards (which the principal may or may not directly care about). Thus, the agent is multi-tasking. However, only the first task produces a contractible signal.[4]

The formal contract offered by the principal combines with the promise of external rewards to form a mixed stew of incentives that ultimately determines how hard the agent works on both tasks. Now, in a recent survey of behavioral contract theory, Kőszegi (2014) singles out "the literature on the interaction between extrinsic and intrinsic motivation [as] one of the most exciting and productive in behavioral contract theory." In this literature, intrinsic motivation refers to non-monetary reasons why the agent would work hard on behalf of the principal. The call for more research is accompanied by the observation that "unlike extrinsic motivation, intrinsic motivation is a complex multifaceted phenomenon that is poorly understood." In the current paper, it is also the case that the contract does not capture all that is payoff-relevant to the agent. Thus, one "facet" of what looks like "intrinsic motivation" to the outsider may be that the agent has to evaluate how rewards on the job interact with private rewards.

[3] The existing literature on common agency in moral hazard problems typically employs Holmström and Milgrom's (1987, 1991) Linear-Exponential-Normal (LEN) model; see e.g. Holmström and Milgrom (1988) and Maier and Ottaviani (2009). As explained below, the model used in the current paper produces different and richer predictions.

[4] In the following, the term "action" refers to the *pair* of efforts devoted to the two different tasks. Conversely, a "task" describes one particular dimension of the action.

Indeed, the model can be interpreted as endogenizing the agent's pursuit of "work-life balance." The standard single-task model essentially focuses on the work dimension. In that model, the cost function may capture foregone leisure. However, given the separability that is usually assumed, the value of "leisure" is determined solely by effort at work but is otherwise independent of the contract. The current model, however, allows the agent to simultaneously invest in both dimensions – "work" and "life" – while recognizing that the contract may influence both decisions. Stated differently, in the standard model no consideration is given to how exactly the agent spends his time when he is not working; leisure is no more than a black-box residual. Here, in contrast, the agent can decide how intensely or actively he utilizes his leisure time. If the agent is paid poorly at work – rewards from labor are low – he may decide to seek rewards elsewhere, by e.g. investing more heavily in a hobby. The implied multi-tasking turns out to alter some key predictions of standard contract theory.

The dominant method for analyzing moral hazard is the first-order approach (FOA). The FOA has been justified in a class of multi-tasking problems only very recently; see Kirkegaard (2015a).[5] In this paper, I build on this work to extend the FOA to handle private rewards. Although rewards are assumed to be stochastically independent, the model allows for interdependencies in two ways. First, effort costs may be non-separable in the two tasks. Second, rewards from the two different sources may be substitutes in the agent's utility function.

Thus, the first contribution of the paper is to present a tractable model of contracting in the presence of private rewards. The second contribution – justifying the FOA – is methodological in nature. That is, I provide a solution technique that can be used in future research on contracting with private rewards. Third, I further specialize the model to obtain insights into how the terms of the contract interact with the agent's incentive to pursue private rewards. In other words, I use the model to provide a new perspective on intrinsic versus extrinsic motivation. Specifically, contracts that may seem to have "flatter" incentives – yielding

[5] Ábrahám et al (2011) justify the FOA in a model with hidden savings. The agent privately earns a return on his savings, but this is both deterministic and monetary. See Section 3 for a discussion. Ligon and Thistle (2013) introduce exogenous background risk into the standard model. In their setting, the agent's action is one-dimensional and can take only one of two values, thereby obviating the need for the FOA.

a smaller return to a marginal increase in on-the-job effort – may induce the agent to work harder on the job. Although this finding is at odds with the conventional wisdom, Kőszegi (2014) reviews several "behavioral economics" models in which results in this vein are obtained. Here, I identify a new mechanism, centered on considerations of "work-life balance", which is responsible for the result.

Englmaier and Leider (2012) note that if the agent has reciprocal preferences, the principal can "generate intrinsic motivation" by giving the agent higher base utility. The agent reciprocates by maintaining high effort even if explicit incentives are weakened. In the simplest version of Bénabou and Tirole's (2003) model, the agent derives utility (a source of intrinsic motivation) if he performs well on the job. However, the agent only has an imperfect signal about the cost of effort. If the principal knows that effort is very costly, he may be worried that the agent has received a bad signal. Consequently, he is more likely to offer steeper explicit incentives to partially compensate, yet that may not be enough to prevent the probability of high effort from declining. Bénabou and Tirole (2003) note that if these considerations are not taken into account, the "outside observer might actually underestimate the power of these incentives [and] conclude that rewards are negative reinforcers."

The current paper offers a different explanation, as follows. First, recall that labor income and private rewards are assumed to be substitutes. This implies that when the agent earns higher utility on the job, his incentive to pursue private rewards is lessened. As labor income then plays a more significant role in the agent's overall well-being, weaker or flatter incentives are sufficient to induce him to maintain the same effort on the job. In this way, qualitatively different contracts can incentivize constant effort on the job by manipulating how hard the agent is working to obtain private rewards. To an outside observer who fails to realize that the pursuit of private rewards is endogenous, it might seem that the principal generates intrinsic motivation by offering higher base utility and flatter incentives and in so doing manages to induce unchanged effort on the job. However, the outside observer overlooks the fact that the agent changes how hard he works off the job; the agent's work-life balance adjusts in the background.

In the specialized version of the model, it turns out that incentive compatibility on its own is so restrictive that there are implementable actions for which the participation constraint is slack. Indeed, when the principal takes an interest in both tasks, it may be optimal to implement precisely such an action. Note that the agent's rents are not due to the outside rewards per se, but rather to the fact that they are private. After all, if the outside rewards are contractible, the principal can effectively appropriate their monetary value by making the agent's pay contingent on both signals.[6]

The most popular multi-tasking model is due to Holmström and Milgrom (1987, 1991). Owing to the specific functional forms that are imposed it is often referred to as the Linear-Exponential-Normal (LEN) model. For instance, contracts are restricted to be linear. Holmström and Milgrom (1991) examine certain settings in which the agent receives (deterministic) private rewards from pursuing tasks that do not yield contractible signals. While they discuss optimal contract design in these settings, they stop short of discussing intrinsic versus extrinsic motivation. In fact, it is arguably the case that the vast amount of structure that makes the LEN model so famously tractable inherently limits its suitability for studying some of the intricacies of private rewards. The slope of the linear contract is *uniquely* characterized by the incentive compatibility constraint that incentivizes the correct effort on the job. Given that this coefficient is typically interpreted as measuring the strength of incentives, the LEN model thus cannot deliver the more nuanced explanation of intrinsic motivation identified here. In fact, how hard the agent pursues private rewards is uniquely determined by the effort on the job that the principal implements. Finally, the LEN model is not rich enough to explain why the agent may earn economic rents.

Moreover, the LEN model overlooks some implications of common agency, as described in a companion paper, Kirkegaard (2015b). Indeed, Kirkegaard (2015a) documents that the LEN model's predictions are not robust even when there are no private rewards. Together, this trilogy of papers thus aims to contribute to an understanding of multi-tasking outside the confines of the LEN model. The two competing models of multi-tasking are briefly contrasted in Section 5.1.

[6]There may be other reasons why the agent earns more than his reservation utility. This may occur if the agent is protected by limited liability. Moreover, Laffont and Martimort (2002, Section 5.3) explains how the agent may earn rents when his utility function is non-separable in income and effort. In this case the participation constraint is not redundant – it is just not optimal to make it binding. Here, I follow most of the literature by assuming separability.

2 An example

Actions and outcomes are continuous in the general model, but the following example assumes binary outcomes. The agent exerts effort a_1 on the job and a_2 on pursuing private rewards. The former produces a failure with probability $p_1(a_1)$ and a success with probability $1-p_1(a_1)$. The contract, $(\underline{w}, \overline{w})$, specifies pay in the two cases. Independently, a_2 yields a small private reward with probability $p_2(a_2)$ and a large reward with probability $1-p_2(a_2)$. The agent's utility is $m(w)$ if a small private reward is realized, where $m(w)$ is a negative, increasing, and concave function of labor income. The agent's utility is zero regardless of pay if a large private reward is realized. This models an extreme situation where the private reward is either worthless or so large that the agent is satiated no matter what his pay is. This assumption is for simplicity only and serves to simplify some of the equilibrium expressions and derivations in the example. What is important is that the two sources of utility are substitutes; an increase in the private reward is less important the higher labor income is, and vice versa.

Cost of effort is linear, taking the form $c_1a_1 + c_2a_2$, with $c_1, c_2 > 0$. The agent's expected utility given the contract $(\underline{w}, \overline{w})$ and action (a_1, a_2) is

$$[m(\overline{w}) - (m(\overline{w}) - m(\underline{w}))\, p_1(a_1)]\, p_2(a_2) - c_1a_1 - c_2a_2. \tag{1}$$

Assume that $p_1(a_1)$ and $p_2(a_2)$ are strictly decreasing and strictly log-convex. Then, $p_1(a_1)p_2(a_2)$ is convex in (a_1, a_2).[7] With this assumption, the agent's problem is concave whenever $m(\overline{w}) \geq m(\underline{w})$, or $\overline{w} \geq \underline{w}$. However, $(\underline{w}, \overline{w})$ is endogenous. Now fix some interior (a_1, a_2) action that is to be induced. Incentive compatibility requires that the agent's expected utility is maximized at (a_1, a_2). The necessary first order condition with respect to a_1 is

$$m(\overline{w}) - m(\underline{w}) = -\frac{c_1}{p_1'(a_1)p_2(a_2)}, \tag{2}$$

which is strictly positive since $p_1'(a_1) < 0$. Hence, as expected, $m(\overline{w}) > m(\underline{w})$

[7] Assume that $\ln p_i(a_i)$ is convex in a_i, $i = 1, 2$. Then, $\ln p_i(a_i)$ is convex in (a_1, a_2). Turning to $p_1(a_1)p_2(a_2)$, the concave transformation $\ln[p_1(a_1)p_2(a_2)] = \ln p_1(a_1) + \ln p_2(a_2)$ must now be convex in (a_1, a_2) since it is a sum of convex functions. Hence, $p_1(a_1)p_2(a_2)$ must be convex.

is necessary to induce the agent to work on the job. This confirms that the agent's problem is concave. Hence, the first-order conditions are sufficient. Stated differently, the FOA is valid. Sections 3 and 4 generalize these arguments to justify the FOA in richer environments. Technical extensions are in Section 6.

Section 5 discusses implications for intrinsic and extrinsic motivation. The model is Section 5 is a specialized version of the model in Sections 3 and 4. The example in the current section is, however, a special case. Thus, the example can be used to illustrate some of the main insights, as follows. The reader who is primarily interesting in this application can skip Sections 3 and 4.

Since $p_1(a_1)$ is convex, (2) implies that the bonus (as measured in utils) is increasing in a_1. That is, steeper incentives are required to induce higher effort on the job. However, the bonus is also increasing in a_2. The reason is that the agent's labor income matters relatively less for his utility when he is induced to work harder in the pursuit of private rewards. Hence, he must be promised a larger bonus to maintain a constant a_1.

The first order condition with respect to a_2 can be written as

$$m(\overline{w}) - (m(\overline{w}) - m(\underline{w}))\, p_1(a_1) = \frac{c_2}{p_2'(a_2)}, \tag{3}$$

where the left hand side can be interpreted as the agent's utility from labor income. Since $p_2(a_2)$ is convex, this is decreasing in a_2. By lowering utility from labor income, private rewards matter more for the agent. In this manner the agent is incentivized to deliver higher a_2. Thus, holding a_1 fixed, increases in a_2 reduces utility from labor income but increases the bonus.[8] Hence, to an outsider who does not realize that a_2 is being manipulated, it may seem as if wage levels and "explicit incentives" can be made to move in opposite directions while inducing the agent to maintain the same effort on the job.

Note that the incentive compatibility constraints in (2) and (3) determine the two endogenous variables, $\underline{w}$ and $\overline{w}$. Thus, the contract may or may not satisfy the participation constraint. It follows that not all actions are implementable in general, and some can be implemented only by awarding rent to the agent in

[8] The first part implies that either $\underline{w}$ or $\overline{w}$ or both must decrease with a_2. In fact, it can be shown that the assumed log-convexity of $p_2(a_2)$ implies that both $\underline{w}$ and $\overline{w}$ decreases in a_2.

excess of his reservation utility. In fact, note that the agent's expected utility from labor income must be *independent* of a_1 since it is used as a tool to incentivize a_2, as evidenced by (3). Since cost of effort increases in a_1, the agent is thus worse off the higher a_1 is, for a given a_2. Increasing a_2, on the other hand, entails conflicting effects. As described previously, utility from labor income decreases in a_2, while utility from private rewards naturally increases. However, (3) implies that expected utility from both sources of rewards can be reduced to $\frac{c_2 p_2(a_2)}{p_2'(a_2)}$. Since $p_2(a_2)$ is log-convex, this is decreasing in a_2. Consequently, the agent is worse off the harder he is induced to work on either task. The participation constraint is thus satisfied only when a_1 and a_2 are not too large.

To illustrate, assume that $p_i(a_i)$ is log-linear, $i = 1, 2$. Let $k_i = \frac{p_i'(a_i)}{p_i(a_i)} < 0$ denote the constant slope of $\ln p_i(a_i)$. Expected utility from rewards can then be written as $\frac{c_2 p_2(a_2)}{p_2'(a_2)} = \frac{c_2}{k_2}$. Now, since wages are decreasing in a_2, the cheapest way to implement any given a_1 is to induce the highest possible a_2 value that satisfies the participation constraint. The latter will thus bind if the principal does not directly care about a_2. In this case, when the target level of a_1 changes, a_2 optimally adjusts to satisfy $\frac{c_2}{k_2} - c_1 a_1 - c_2 a_2 = \overline{u}$, where $\overline{u}$ is the agent's reservation utility. Consequently, $\frac{da_2}{da_1} = -\frac{c_1}{c_2} < 0$; a_2 must be made to move in the opposite direction as a_1. Hence, the effect on the bonus is ambiguous, by (2). However, it is readily verified that the optimal bonus (again measured in utils) declines when a larger a_1 is induced if $\frac{k_2}{c_2} < \frac{k_1}{c_1}$, i.e. if it is cheaper for the agent to lower $\ln p_2(a_2)$ than $\ln p_1(a_1)$. In conclusion, the agent may be offered a smaller bonus the harder he is induced to work on the job. The reason is that he is at the same time induced to pursue private rewards less intensively.

Note that the previous argument relies on a_2 not directly impacting the principal's utility. However, if he derives large disutility from a_2 it may be preferable to induce a lower a_2. In this case, the participation constraint may be slack.

3 The agent's problem

Before describing the model I briefly preview some of the key steps to justifying the FOA with private rewards. Kirkegaard's (2015a) multi-task justifications of the FOA is a good starting point. In particular, one of his justifications apply to environments in which the optimal contract turns out to be monotonic and such that the outcomes of the two tasks are substitutes from the agent's point of view. That is, a marginal improvement in the performance of task 2 is worth less to the agent if he performed extremely well on task 1. However, in Kirkegaard (2015a) the principal rewards both tasks. On the other hand, in the present setting it is quite natural to assume that labor income and private rewards are substitutes. That is, private rewards yields substitutability essentially for free. However, it turns out to be substantially harder to establish monotonicity.

To establish monotonicity, a main challenge is to sign the multipliers of the incentive compatibility constraints. This is accomplished by extending a classic argument by Rogerson (1985), involving a doubly-relaxed maximization problem. In essence, Rogerson (1985) shows that it is sufficient to prevent the agent from working less hard than intended. In the present setting, however, there are two tasks. As is perhaps intuitive, it turns out to be sufficient to simultaneously prevent the agent from shirking on job and working too hard on the private task.

Thus, the following assumptions on the primitives (technology and preferences) are used to either establish monotonicity and substitutability or to prove that the FOA is valid whenever the candidate contract takes such a form.

I consider a relatively simple model of a principal-agent relationship with endogenous private rewards. The agent performs two "tasks", a_1 and a_2, each of which belong to a compact interval, $a_i \in [\underline{a}_i, \overline{a}_i]$, $i = 1, 2$. The first task captures the agent's effort on the job, as a result of which a contractible signal, x_1, is produced. The signal's marginal distribution is $G^1(x_1|a_1)$. The second "task" reflects the agent's pursuit of a private reward. The agent receives a (possibly non-monetary) reward, x_2, which is determined by the marginal distribution function $G^2(x_2|a_2)$. Here, a_2 could measure life-style choices and x_2 the health outcome. Assume x_i belongs to a compact interval, $[\underline{x}_i, \overline{x}_i]$, which is independent of a_i. Assume G^1 and G^2 are continuously differentiable in both variables to the requisite degree. Let $g^1(x_1|a_1)$ and $g^2(x_2|a_2)$ denote the respective densities. Assume $g^i(x_i|a_i) > 0$ for all $x_i \in [\underline{x}_i, \overline{x}_i]$ and all $a_i \in [\underline{a}_i, \overline{a}_i]$.[9] Note that

[9] Throughout, all exogenous functions are assumed continosly differentiable to the requisite degree. For brevity, statements to that effect are omitted from the numbered assumptions.

each marginal distribution depends only on one task.[10] This property is further strengthened by assuming that x_1 and x_2 are independent.

Assumption A1 (Independence): *Outcomes are independent*, i.e. given a_1 and a_2, the joint distribution is given by

$$F(x_1, x_2|a_1, a_2) = G^1(x_1|a_1)G^2(x_2|a_2). \tag{4}$$

More structure is required on the components of the joint distribution function. Thus, define $l^i(x_i|a_i) = \ln g^i(x_i|a_i)$ and let $l^i_{a_i}(x_i|a_i)$ denote the likelihood-ratio, i.e. the derivative of $l^i(x_i|a_i)$ with respect to a_i, $i = 1, 2$.

Assumption A2 (MLRP): The marginal distributions have the *monotone likelihood ratio property*, i.e. for all $a_i \in [\underline{a}_i, \overline{a}_i]$ it holds that

$$\frac{\partial}{\partial x_i}\left(l^i_{a_i}(x_i|a_i)\right) = \frac{\partial^2 \ln g^i(x_i|a_i)}{\partial a_i \partial x_i} \geq 0 \text{ for all } x_i \in [\underline{x}_i, \overline{x}_i], \tag{5}$$

with strict inequality on a subset of strictly positive measure, $i = 1, 2$.

Assumption A2 implies that $G^i_{a_i}(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$.[11] The interpretation is that when the agent works harder, bad outcomes are less likely. In particular, if $a'_i > a''_i$ then $G^i(x_i|a'_i)$ first order stochastically dominates $G^i(x_i|a'_i)$.[12]

It is assumed that x_1 and x_2 are realized at the same time. In an important paper, Rogerson (1985) justifies the FOA in a one-signal, one-task model. He assumes the distribution function satisfies MLRP and that it is convex in the (one-dimensional) action. Rogerson (1985) refers to the latter as the convexity of distribution function condition (CDFC). Kirkegaard (2015a) extends the justification of the FOA to allow multiple tasks and signals. He shows that a natural

[10] This is somewhat less restrictive than it appears at first glance. For instance, assume G_i is a *one-parameter* distribution, and that a_1 and a_2 both influence the parameter. That is, G_i can be written $G_i(x_i|t_i(a_1, a_2))$. In this case, the problem can simply be reformulated to make t_1 and t_2 the two choice variables. However, the possibility that a_1 and a_2 influence different parameters of one or both of the marginal distributions is ruled out.

[11] To see this, recall first that the expected value of $l^i_{a_i}(x_i|a_i)$ is zero. Assumption A2 therefore implies that $l^i_{a_i}(\underline{x}_i|a_i) < 0 < l^i_{a_i}(\overline{x}_i|a_i)$. Since $G^i_{a_i}(\underline{x}_i|a_i) = G^i_{a_i}(\overline{x}_i|a_i) = 0$, it follows that $G^i_{a_i}(x_i|a_i) = \int_{\underline{x}_i}^{x_i} l^i_{a_i}(z_i|a_i)g^i(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$.

[12] The model would reduce to the standard single-task, one-signal model if $G^2(x_2|a_2)$ was degenerate and independent of a_2.

extension of the CDFC is to assume that the distribution function is convex in the (now many-dimensional) action. The same assumption is imposed here.

Assumption A3 (LOCC): $F(x_1, x_2|a_1, a_2)$ satisfies the *lower orthant convexity condition*; $F(x_1, x_2|a_1, a_2)$ is weakly convex in (a_1, a_2) for all (x_1, x_2) and (a_1, a_2).

Assumption A3 necessitates that G^i is convex in a_i, $i = 1, 2$. In fact, it implies that $G^i_{a_i a_i}(x_i|a_i) > 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$.[13] A sufficient condition for LOCC is that G^1 and G^2 are both log-convex. Kirkegaard (2015a) lists several examples. See also Ábrahám et al (2011), discussed in more detail at the end of this section. Alternatively, fix some G^1 that is strictly convex in a_1, but not necessarily log-convex. Then, there is always some "sufficiently convex" G^2 function that ensures that Assumption A3 is satisfied. For example, a non-negative function $h(z)$ is said to be ρ-convex if $h(z)^\rho/\rho$ is convex, or $h''(z)h(z)/h'(z)^2 \geq 1 - \rho$ for all z. Thus, a ρ-convex function is log-convex if and only if $\rho \leq 0$ (and convex if and only if $\rho \leq 1$). It is easy to see that if $G^2(x_2|a_2)$ satisfies Assumption A2 and is ρ-convex in a_2 (for all x_2) for some small enough ρ (i.e. ρ is negative, but numerically large), then Assumption A3 is satisfied.[14] To reiterate, as long as G^1 satisfies a strict version of CDFC there are G^2 functions that will permit the FOA to be justified even when allowing for private rewards.

Assumptions A1–A3 describes the "technology". The next set of assumptions describes the agent's preferences. Given action (a_1, a_2), wage w, and private reward x_2, the agent's utility is assumed to take the form

$$v(w, x_2) - c(a_1, a_2),$$

where v is a benefit function and c a cost function. Both functions are assumed to be continuously differentiable in both their arguments to the requisite degree. The function $v(w, x_2)$ is strictly increasing and strictly concave in both arguments, $v_i > 0 > v_{ii}$, $i = 1, 2$, where subscripts denote derivatives. Note that it is

[13] LOCC necessitates that $G^i_{a_i a_i} \geq 0$ and $G^1 G^2 G^1_{a_1 a_1} G^2_{a_2 a_2} - \left(G^1_{a_1} G^2_{a_2}\right)^2 \geq 0$. At any interior (x_1, x_2), the last term is strictly positive, by A2. Thus, $G^1_{a_1 a_1} > 0$ and $G^2_{a_2 a_2} > 0$ are necessary.

[14] The inequality in the previous footnote can be written $G^1 G^1_{a_1 a_1} (G^2 G^2_{a_2 a_2} / \left(G^2_{a_2}\right)^2) - \left(G^1_{a_1}\right)^2 \geq 0$, for interior (x_1, x_2). By ρ-convexity, the left hand side is greater than $G^1 G^1_{a_1 a_1} (1 - \rho) - \left(G^1_{a_1}\right)^2 \geq 0$. Hence, the inequality is satisfied if ρ is small enough.

not necessary for $v(w, x_2)$ to be jointly concave in (w, x_2). The cost function is likewise assumed to be strictly increasing. It is also assumed to be jointly convex in (a_1, a_2). While Assumption A1 imply that there is no stochastic interaction between a_1 and a_2, the cost function allows interaction between tasks.

Note that if the private reward, x_2, is income, then $v(w, x_2)$ could be written $v(w+x_2)$, in which case it is automatic that $v_{12} < 0$. That is, employment income and outside income are substitutes. Indeed, even when x_2 is not income it is natural to assume that w and x_2 are strict substitutes. Thus, it will be assumed that $v_{12} < 0$; the higher x_2 is, the lower is the marginal utility of additional employment income. I will also assume that a_1 and a_2 are weak substitutes in the cost function, or $c_{12} \geq 0$. That is, the marginal cost of increasing a_1 is higher the higher a_2 is. Assumption A4 summarizes these assumptions.

Assumption A4 (Substitutes): The agent's Bernoulli utility is $v(w, x_2) - c(a_1, a_2)$; $v(w, x_2)$ is strictly increasing and strictly concave in both w and x_2, while $c(a_1, a_2)$ is strictly increasing and weakly convex in (a_1, a_2). Rewards are strict substitutes; $v_{12}(w, x_2) < 0$. Tasks are weak substitutes; $c_{12}(a_1, a_2) \geq 0$.

The principal specifies a contract of the form $w(x_1)$. That is, the contract details the wage to the agent if the verifiable signal is x_1.[15] Upon taking action (a_1, a_2), the agent's expected payoff is then

$$EU(a_1, a_2) = \int \int v(w(x_1), x_2) g^1(x_1|a_1) g^2(x_2|a_2) dx_1 dx_2 - c(a_1, a_2).$$

For notational simplicity, $EU(a_1, a_2)$ suppresses the dependency on the contract. Imagine the principal's intention is to induce the agent to take action (a_1', a_2'). For the agent to comply, $EU(a_1, a_2)$ must be maximized at (a_1', a_2'). Assuming the action is interior, this at the very least necessitates that expected utility is at a stationary point at (a_1', a_2'), or $EU_1(a_1', a_2') = EU_2(a_1', a_2') = 0$. The FOA relies on the latter conditions being not only necessary but also sufficient for utility maximization. This is legitimate if $EU(a_1, a_2)$ can be shown to be concave.

[15] This ignores the possibility that the principal may ask the agent to report x_2 and then make the wage dependent upon both x_1 and the report. It can easily be verified that the principal can never gain from such a scheme in the special version of the model in Section 5.

If $w(x_1)$ is non-decreasing in x_1, then, given Assumption A4, $v(w(x_1), x_2)$ is increasing in x_1 and x_2, and submodular in the two. Now, Kirkegaard (2015a) proves that if the agent faces such a reward function, then the FOA is valid if Assumption A3 (LOCC) is satisfied as well.[16] In fact, $EU(a_1, a_2)$ is concave in (a_1, a_2). To see this, note that after integration by parts with respect to x_2,

$$EU(a_1, a_2) = \int \left(v(w(x_1), \overline{x}_2) - \int v_2(w(x_1), x_2) G^2(x_2|a_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c(a_1, a_2) \tag{6}$$

Assuming for simplicity that $w(x_1)$ is differentiable (it will later be established that the optimal contract is indeed differentiable), another round of integrating by parts, this time with respect to x_1, yields

$$\begin{aligned} EU(a_1, a_2) &= v(w(\overline{x}_1), \overline{x}_2) - \int v_1(w(x_1), \overline{x}_2) w'(x_1) G^1(x_1|a_1) dx_1 \\ &\quad + \int \int v_{12}(w(x_1), x_2) w'(x_1) G^1(x_1|a_1) G^2(x_2|a_2) dx_1 dx_2 \\ &\quad - \int v_2(w(\overline{x}_1), x_2) G^2(x_2|a_2) dx_2 - c(a_1, a_2). \end{aligned} \tag{7}$$

Recall that $v_1, v_2 > 0 > v_{12}$ while $G^1(x_1|a_1)$, $G^2(x_2|a_2)$, $G^1(x_1|a_1)G^2(x_2|a_2)$, and $c(a_1, a_2)$ are all convex in $(a_{1,}a_2)$. Thus, $EU(a_1, a_2)$ is a sum of concave functions if $w'(x_1) \geq 0$. The first Lemma records this fact and two other useful properties.

Lemma 1 *Assume $w'(x_1) \geq 0$ for all $x_1 \in [\underline{x}_1, \overline{x}_1]$ and that Assumptions A1–A4 hold. Then, the agent's expected utility, $EU(a_1, a_2)$, is jointly concave in (a_1, a_2). Moreover, $EU(a_1, a_2)$ is strictly concave in a_2, $EU_{22}(a_1, a_2) < 0$, and the two tasks are substitutes in the agent's expected utility, $EU_{12}(a_1, a_2) \leq 0$. If $w'(x_1) > 0$ on a subset of positive measure, then $EU_{12}(a_1, a_2) < 0$.*

Proof. The first part is explained in the text, after (7). Since $v_2 > 0 > v_{12}$ and $G^2_{a_2 a_2} > 0$, (7) also implies that $EU_{22}(a_1, a_2) < 0$. Likewise, from (7),

$$EU_{12}(a_1, a_2) = \int \int v_{12}(w(x_1), x_2) w'(x_1) G^1_{a_1}(x_1|a_1) G^2_{a_2}(x_2|a_2) dx_1 dx_2 - c_{12}(a_1, a_2).$$

[16] Jewitt (1988) and Conlon (2009) present two results with a similar flavour in a model with two signals but a single task. Kirkegaard's (2015a) characterization is more general as it extends to more signals and more tasks.

Recalling that $G^i_{a_i}(x_i|a_i) < 0$ for all $x_i \in (\underline{x}_i, \overline{x}_i)$, $i = 1, 2$, it is now clear that the last two parts of Assumption A4 (substitutes), $v_{12} < 0$ and $c_{12} \geq 0$, pull in the same direction, thus proving that $EU_{12}(a_1, a_2) \leq 0$. The inequality is strict except possibly if $w'(x_1) = 0$ almost always. Hence, if $w'(x_1) > 0$ on a subset of positive measure, then $EU_{12}(a_1, a_2) < 0$.[17] ■

Unfortunately, it is far from trivial to establish that $w(x_1)$ is non-decreasing. Thus, the analysis in the rest of this section and most of the next is primarily devoted to that particular problem.

To proceed, it is necessary to impose more specific assumptions on the agent's risk preferences over labor income, w. Thus, it will be assumed that the absolute risk aversion with respect to w is decreasing in x_2. In other words, the agent is less sensitive to risk in labor income the higher the private reward is.

Assumption A5 (Decreasing absolute risk aversion, DARA): The agent's absolute risk aversion over labor income is decreasing in x_2, or

$$\frac{\partial}{\partial x_2}\left(\frac{-v_{11}(w, x_2)}{v_1(w, x_2)}\right) \leq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2].$$

Assumption A5 is equivalent to assuming that $v_1(w, x_2)$ is log-supermodular in (w, x_2), or

$$\frac{\partial^2 \ln v_1(w, x_2)}{\partial w \partial x_2} \geq 0 \text{ for all } w \text{ and all } x_2 \in [\underline{x}_2, \overline{x}_2]. \tag{8}$$

Likewise, note that Assumption A2 (MLRP) is equivalent to the requirement that $g^i(x_i|a_i)$ is log-supermodular in (x_i, a_i), $i = 1, 2$.

A series of assumptions have now been imposed on the technology and the agent's Bernoulli utility function. The next step is to combine or aggregate these in order to understand how the agent is impacted by stochastic private rewards. Let

$$V(w, a_2) = \int v(w, x_2) g^2(x_2|a_2) dx_2, \tag{9}$$

[17] Note that Assumption A4 (substitutes) rules out that $v_{12} = c_{12} = 0$ (which would imply $EU_{12}(a_1, a_2) = 0$). However, this case seems relatively uninteresting, as it would imply that there is a fixed a_2 which is optimal for the agent regardless of the contract. It is then easy to show that the FOA is valid if $G^1(x_1|a_1)$ satisfies MLRP and CDFC.

denote the expected utility of a fixed labor income, w, given that the agent exerts effort a_2 towards obtaining private rewards. For future reference, note that

$$EU(a_1, a_2) = \int V(w(x_1), a_2) g^1(x_1|a_1) dx_1 - c(a_1, a_2).$$

Given (9),

$$\begin{aligned} V_1(w, a_2) &= \int v_1(w, x_2) g^2(x_2|a_2) dx_2 > 0, \text{ and} \\ V_{12}(w, a_2) &= \int v_1(w, x_2) g^2_{a_2}(x_2|a_2) dx_2 < 0. \end{aligned}$$

Here, $V_1(w, a_2)$ describes the expected marginal utility of additional labor income given the agent's effort on the private task is a_2. Of course, $V_{12}(w, a_2)$ captures how this expectation changes with a_2. Assumptions A2 (MLRP) and A4 (substitutes) together implies that $V_{12}(w, a_2) < 0$. Evidently, $V_1(w, a_2)$ is strictly decreasing in w, or $V_{11}(w, a_2) < 0$.

Moreover, the term under the integration sign in $V_1(w, a_2)$ is, by Assumptions A2 (MLRP) and A5 (DARA), log-supermodular in (w, x_2, a_2). As described by e.g. Athey (2002), log-supermodularity is preserved under integration. Thus, $V_1(w, a_2)$ is log-supermodular in (w, a_2). That is, the agent's decreasing absolute risk aversion aggregates, or carries over to the expected utility in (9). Hence,

$$\frac{\partial}{\partial a_2} \left(\frac{-V_{11}(w, a_2)}{V_1(w, a_2)} \right) \leq 0,$$

such that the agent is less sensitive to risk in labor income the better the distribution of private rewards is. Equivalently,

$$\frac{\partial}{\partial w} \left(\frac{-V_{12}(w, a_2)}{V_1(w, a_2)} \right) \leq 0. \tag{10}$$

This property is especially important for the results in the next section.

In fact, the assumption that $g^2(x_2|a_2)$ is log–supermodular can be relaxed (although $G^2_{a_2} \leq 0$ is still required) if (8) is replaced by the assumption that (10) holds. For instance, if x_2 is income and $v(w, x_2) = -e^{-r(w+x_2)}$, $r > 0$, then the agent exhibits constant absolute risk aversion in total income (and its components). In this case, (10) is trivially satisfied for any $g^2(x_2|a_2)$.

At this point, it is instructive to compare the present set-up with the literature on hidden savings. Hidden savings is non-contractible. Ábrahám et al (2011) consider a situation where the agent works for the principal while simultaneously privately investing in a risk-free asset. There is thus no uncertainty concerning the return to the non-contractible action. Hence, performance on the job, x_1, is the only source of uncertainty. Ábrahám et al (2011) justify the FOA by assuming that the distribution of x_1 is log-convex in effort and that the agent has decreasing absolute risk aversion. These two assumptions can be thought of as special cases of Assumptions A3 (LOCC) and A5 (DARA) in the current paper. Here, however, both types of rewards are stochastic and possibly non-monetary.

To complete the description of the agent's problem, assume that the only constraint other than incentive compatibility is a participation constraint. That is, the agent must earn expected utility of at least $\overline{u}$ to sign the contract.

4 Contracts with private rewards

The previous section describes the problem from the agent's point of view. Consider now the principal's problem. First, assume that the principal is risk neutral. Let $B(a_1, a_2)$ denote the principal's direct benefit of the agent's action and assume that it is continuously differentiable. For instance, $B(a_1, a_2)$ could be the expected value of x_1, given a_1. As explained below, for technical reasons I assume that $B_2(a_1, a_2) \leq 0$, such that the principal prefers a_2 to be as small as possible. This assumption is of course satisfied if B is independent of a_2. In the more specialized model in Section 6, it is possible to allow $B_2(a_1, a_2) > 0$, however. Finally, let $E[w|a_1, a_2]$ denote the expected wage costs if the agent is induced to take action (a_1, a_2).

Assumption A6 (The principal's preferences): The principal is risk neutral, with expected utility $B(a_1, a_2) - E[w|a_1, a_2]$, where $B_2(a_1, a_2) \leq 0$ for all (a_1, a_2).

It is natural to assume that $B(a_1, a_2)$ is increasing in a_1. Indeed, Proposition 2 in Section 7 will establish that if it is optimal to implement an interior action, then $B_1 > 0$ at that point. However, B need not be globally increasing in a_1.

The principal's problem is to maximize $B(a_1, a_2)$ less wage costs, subject to individual rationality and incentive compatibility, or

$$\max_{a_1,a_2,w} B(a_1, a_2) - \int w(x_1)g^1(x_1|a_1)dx_1$$
$$st. \quad EU(a_1, a_2) \geq \overline{u}$$
$$(a_1, a_2) \in \arg \max_{(a_1',a_2') \in [\underline{a}_1, \overline{a}_1] \times [\underline{a}_2, \overline{a}_2]} EU(a_1', a_2')$$

Any action (if one exists) that solves the problem is henceforth referred to as a second-best action.

It is important to realize that the contract indirectly determines not only how hard the agent works for the principal, but also how hard he works on the private task. From the agent's point of view, the function $v(w(x_1), x_2)$ is crucial to the decision of how much effort to devote to each task. It is immaterial that the reward x_2 happens to be not paid by the principal.

Assume the principal wishes to induce an interior action. Then, as mentioned previously, it is necessary that EU achieves a stationary point at the targeted action, or $EU_1(a_1, a_2) = 0 = EU_2(a_1, a_2)$. These constraints are referred to as the "local" incentive compatibility constraints. As in the existing FOA literature, the main objective of this part of the paper is to establish conditions under which the local constraints are in fact sufficient for "global" incentive compatibility.

Consider the following *relaxed problem*, so named because the incentive compatibility constraint in the original problem has been relaxed,

$$\max_{a_1,a_2,w} B(a_1, a_2) - \int w(x_1)g_1(x_1|a_1)dx_1$$
$$st. \quad EU(a_1, a_2) \geq \overline{u}$$
$$EU_1(a_1, a_2) = 0,$$
$$EU_2(a_1, a_2) = 0$$

The FOA is said to be valid if the solution to the relaxed problem also solves the unrelaxed (original) problem.

Let $\lambda \geq 0$ denote the multiplier to the participation constraint, and μ_1 and

μ_2 denote the multipliers to the two local incentive compatibility constraints in the relaxed problem. Assuming interior wages, the optimal wage if x_1 is observed is implicitly characterized by the necessary first order condition,

$$V_1(w, a_2)\left[\lambda + \mu_1 l^1_{a_1}(x_1|a_1)\right] + \mu_2 V_{12}(w, a_2) = 1 \tag{11}$$

or

$$\lambda + \mu_1 l^1_{a_1}(x_1|a_1) = \frac{1}{V_1(w, a_2)} - \mu_2 \frac{V_{12}(w, a_2)}{V_1(w, a_2)}. \tag{12}$$

Qualitatively, the solution almost certainly depends on the sign of the two multipliers μ_1 and μ_2. Indeed, it is not even clear that there is a unique solution. However, later arguments will establish that it is sufficient to focus on multipliers for which $\mu_1 \geq 0 \geq \mu_2$. Then, the contract $w(x_1)$ is well-behaved.

Lemma 2 *Given Assumptions A1-A6 and interior wages, $w(x_1)$ as defined in (12) is unique whenever $\mu_1 \geq 0 \geq \mu_2$. Moreover, the solution is differentiable, with $w'(x_1) \geq 0$ for all $x_1 \in [\underline{x}_1, \overline{x}_1]$. If $\mu_1 > 0 \geq \mu_2$, then $w'(x_1) > 0$ for a subset of $[\underline{x}_1, \overline{x}_1]$ of strictly positive measure.*

Proof. Given $\mu_2 \leq 0$, $V_{11} < 0$ and (10) imply that the right hand side of (12) is strictly increasing in w (the derivative is strictly positive). Thus, for each x_1 there is at most one solution to (12), $w(x_1)$. Differentiability now follows from the differentiability of all the components in (12) and the fact that the right hand side is strictly increasing in w. Since $\mu_1 \geq 0$, Assumption A2 (MLRP) implies that the left hand side is non-decreasing in x_1. Hence, $w(x_1)$ is non-decreasing in x_1. The last part likewise follows from Assumption A2 (MLRP). ■

The next step utilizes Rogerson's (1985) idea of considering a *doubly-relaxed* problem. In Rogerson's one-task model, the relaxed incentive compatibility constraint, $EU_1 = 0$, is replaced with the even weaker constraint that $EU_1 \geq 0$. In the current multi-task model, the appropriate doubly-relaxed problem assumes $EU_1 \geq 0$ and $EU_2 \leq 0$. The set of feasible contracts (i.e. the constraint set) is obviously larger in the doubly-relaxed problem than in the relaxed problem. For interior actions, any contract that is incentive compatible (i.e. feasible in the unrelaxed problem) is also feasible in both the relaxed and doubly-relaxed problems. However, this does not hold for all incentive compatible contracts that induce boundary actions. For this reason, extra care must be taken in dealing with corner solutions.

Conveniently, $\mu_1 \geq 0 \geq \mu_2$ must hold in the doubly-relaxed problem. As in Rogerson, assume there is a solution to the doubly-relaxed problem. In particular, this requires the constraint set to be non-empty. Likewise, for simplicity, it will be assumed that wages are interior. Rogerson imposes assumptions directly on the utility functions (see his assumption A3–A4 and A6–A7) to achieve this.

Assumption A7 (The doubly-relaxed problem): A solution to the doubly-relaxed problem exists. Any solution involves only wages in the interior of the domain of $v(w, x_2)$.

Any solution to the doubly-relaxed problem must take the form in (12). By Lemma 2, any solution thus features non-decreasing wages. By Lemma 1, the agent's problem is concave. The contract is then incentive compatible if $EU_1 = EU_2 = 0$ at the intended action, which holds if $\mu_1 > 0 > \mu_2$. Let (a_1^*, a_2^*) denote an action that forms part of a solution to the double-relaxed problem.

Lemma 3 *Given Assumptions A1–A7, any solution to the doubly-relaxed problem is incentive compatible and thus feasible in the unrelaxed problem. Moreover, if* $a_2^* > \underline{a}_2$ *then it is also feasible in the relaxed problem, which it also solves.*

Proof. Wages are constant if $\mu_1 = 0$. Then, $EU_1 = -c_1(a_1, a_2) < 0$, which violates the doubly-relaxed constraints. Hence, $\mu_1 > 0$ and so $EU_1(a_1^*, a_2^*) = 0$.[18] Now, if a_2^* is interior it must satisfy the first-order condition that

$$[B_2(a_1^*, a_2^*) + \lambda EU_2(a_1^*, a_2^*) + \mu_1 EU_{12}(a_1^*, a_2^*)] + \mu_2 EU_{22}(a_1^*, a_2^*) = 0. \tag{13}$$

By Assumption A6, $B_2(a_1, a_2) \leq 0$. By Lemma 1, it holds that $EU_{12}(a_1^*, a_2^*) < 0$ given the properties of $w(x_1)$ described in Lemma 2 when $\mu_1 > 0 \geq \mu_2$. Since $\lambda EU_2(a_1^*, a_2^*) \leq 0$, the term in the bracket in (13) is thus strictly negative. As $EU_{22}(a_1^*, a_2^*) < 0$, it is therefore necessary that $\mu_2 < 0$. Hence, $EU_2(a_1^*, a_2^*) = 0$. A similar argument applies if $a_2^* = \overline{a}_2$. Thus, both incentive constraints are

[18] Note that wages are constant if $\mu_1 = 0$ only because the principal is assumed to be risk neutral. In contrast, Rogerson (1985) allows the principal to be risk averse.

binding and, by concavity, the agent's utility is maximized at (a_1^*, a_2^*). That is, the contract is incentive compatible. If $a_2^* = \underline{a}_2$, it cannot be ruled out that $EU_2(a_1^*, a_2^*) < 0$. Nevertheless, by concavity, such a solution on the boundary is still incentive compatible. This completes the proof of the first part of the Lemma. Finally, note that when $a_2^* > \underline{a}_2$, the solution is feasible in the relaxed problem. Since the constraint set is smaller in the relaxed problem, the last part of the Lemma follows. ■

Lemma 3 is key to establishing the validity of the FOA. As explained after Lemma 2, the possibility of a corner solution gives rise to some complications. Thus, following the literature, assume the second-best action is interior. Then, the solution to the unrelaxed problem must satisfy $EU_1 = EU_2 = 0$, which implies that it is feasible in the doubly-relaxed problem. By Lemma 3, however, the solution to the doubly-relaxed problem is in turn feasible in the unrelaxed problem. Hence, the solutions to the unrelaxed and doubly-relaxed problems coincide. Finally, Lemma 3 implies that as the solution to the doubly-relaxed problem involves an interior action, the relaxed problem identifies the exact same solution.

Theorem 1 *Assume any second-best action* (a_1, a_2) *is interior. Then, given Assumptions A1–A7, the FOA is valid.*

It is perhaps natural to question the assumption that it is optimal to induce the agent to work on the private task (a_2 is interior). However, note that

$$EU_2(a_1, \underline{a}_2) = \int \left(\int v(w(x_1), x_2) g^2_{a_2}(x_2|\underline{a}_2) dx_2 \right) g^1(x_1|a_1) dx_1 - c_2(a_1, \underline{a}_2).$$

The inner integral is strictly positive regardless of the contract. Thus, if $c_2(a_1, \underline{a}_2) = 0$, then $EU_2(a_1, \underline{a}_2) > 0$ for any contract. In this case, it is impossible to persuade the agent to not pursue private rewards. Moreover, in the more specialized model in the next section, it is established that inducing an interior a_2 is optimal when $B_2(a_1, a_2) = 0$, even if $c_2(a_1, \underline{a}_2) > 0$.

5 Intrinsic and extrinsic motivation

This section further specializes the environment by imposing more structure on the agent's payoff function. Specifically, assume that the reward function is multiplicative, or that

$$v(w, x_2) = -m(w)n(x_2), \tag{14}$$

where m and n are strictly *negative* functions that are strictly increasing and strictly concave on their domain. Note that Assumptions A5 (DARA) is trivially satisfied, as is the part of Assumption A4 (substitutes) that pertains to $v(w, x_2)$. An obvious example that satisfies (14) is $m(w) = -e^{-rw}$ and $n(x_2) = -e^{-rx_2}$, for any $r > 0$. Then, $v(w, x_2) = -e^{-r(w+x_2)}$. Here, x_2 can be interpreted as income, and the agent exhibits constant absolute risk aversion (CARA). The example in Section 2 fits (14) as well, with $n(x_2)$ taking the value -1 or 0.

The model described by (14) is from now on referred to as the multiplicative model. The term "multiplicative" may invoke thoughts of complementarity rather than substitutability. However, note that the product of the two (negative) functions is multiplied by -1. For this reason, w_1 and w_2 are substitutes.

As alluded to earlier, the multiplicative model strengthens Assumptions A4 and A5. In turn, this makes it possible to justify the FOA while weakening some of the other assumptions; see Section 6.2. For instance, it is not necessary to impose sign restrictions on c_{12} and B_2. For expositional simplicity, however, I maintain the assumption that $c_{12} \geq 0$ in the current section. On a technical note, Theorem 1 states that the FOA is a valid approach to use to characterize the optimal contract that implements the second-best action. In Section 6.2, it is established that in the context of the multiplicative model, the FOA can in fact be used to derive the optimal contract that implements any given action. Thus, implementation costs can be described for actions that are not necessarily second-best. As in Grossman and Hart (1983), one can then first characterize implementation costs for any (feasible) action, and then subsequently use this to derive the second-best action. In fact, it turns out to be sufficient to examine the relaxed problem; the doubly-relaxed problem is not needed.

Letting ω summarize the contract $w(x_1)$, $x_1 \in [\underline{x}_1, \overline{x}_1]$, define

$$M(a_1|\omega) = \int m(w(x_1))g^1(x_1|a_1)dx_1 < 0 \quad (15)$$

$$N(a_2) = -\int n(x_2)g^2(x_2|a_2)dx_2 > 0, \quad (16)$$

and let $M'(a_1|\omega)$ denote the derivative of $M(a_1|\omega)$ with respect to a_1, holding fixed the contract. An outside observer who does not realize that a_2 is endogenous might reasonably interpret $M(a_1|\omega)$ as measuring the agent's "base utility" at work and $M'(a_1|\omega)$ as measuring the intensity of the explicit or extrinsic incentives. After all, the agent's expected utility is

$$EU(a_1, a_2) = M(a_1|\omega)N(a_2) - c(a_1, a_2),$$

and an outsider who believes a_2 to be exogenously fixed would just think of $N(a_2) > 0$ as a constant.

To begin, fix some interior (a_1, a_2) action that the principal might like to induce. Then, $M(a_1|\omega)$ and $M'(a_1|\omega)$ are characterized completely by the local incentive compatibility constraints that $EU_1 = EU_2 = 0$, with

$$M'(a_1|\omega) = \frac{c_1(a_1, a_2)}{N(a_2)},\ M(a_1|\omega) = \frac{c_2(a_1, a_2)}{N'(a_2)}. \quad (17)$$

For ease of exposition, assume that for any action there exists a contract satisfying (17).[19] By Assumption A2 (MLRP), the expectation of $n(x_2)$ is strictly increasing in a_2. Likewise, Assumption A3 (LOCC) – and the weaker versions thereof presented in Section 6 – implies that the expectation is strictly concave in a_2 as well. Hence, $N(a_2)$ is positive, strictly decreasing, and strictly convex.

Note that $N(a_2)$ and $c(a_1, a_2)$ are exogenous, while $M(a_1|\omega)$ is determined by incentive compatibility. Thus, the agent's utility is already pinned down. Consequently, there may be actions for which the participation constraint is slack and others for which it is violated. The feasible set of actions is described momentarily. It is fruitful to first develop an understanding of how the contract as described

[19] Since $M(a_1|\omega)$ and $N'(a_2)$ are strictly negative, this necessitates that $c_2(a_1, a_2) > 0$. Thus, for future reference, it is assumed that $c_2(a_1, \underline{a}_2) > 0$.

by (17) depends on a_1 and a_2, assuming it is implementable. For brevity, I focus on interior actions, such that (17) holds. When needed, let $\omega(a_1, a_2)$ denote a contract that satisfies (17) for a given interior (a_1, a_2).

Consider first how the contract $\omega(a_1, a_2)$ depends on a_2,

$$\begin{aligned}\frac{\partial M'(a_1|\omega(a_1,a_2))}{\partial a_2} &= \frac{\partial}{\partial a_2}\left(\frac{c_1(a_1,a_2)}{N(a_2)}\right) = \frac{c_{12}(a_1,a_2)N(a_2) - c_1(a_1,a_2)N'(a_2)}{N(a_2)^2} > 0 \\ \frac{\partial M(a_1|\omega(a_1,a_2))}{\partial a_2} &= \frac{\partial}{\partial a_2}\left(\frac{c_2(a_1,a_2)}{N'(a_2)}\right) = \frac{c_{22}(a_1,a_2)N'(a_2) - c_2(a_1,a_2)N''(a_2)}{N'(a_2)^2} < 0\end{aligned}$$

since $N(a_2), N''(a_2) > 0 > N'(a_2)$. Holding a_1 fixed, it follows that $M(a_1|\omega)$ and $M'(a_1|\omega)$ are inversely related. Formally,

$$a_2'' > a_2' \Longrightarrow \begin{cases} M(a_1|\omega(a_1,a_2')) > M(a_1|\omega(a_1,a_2'')) \text{ and} \\ M'(a_1|\omega(a_1,a_2')) < M'(a_1|\omega(a_1,a_2'')). \end{cases} \tag{18}$$

The aforementioned outsider would reasonably conclude that $\omega(a_1, a_2')$ delivers higher "base utility" than $\omega(a_1, a_2'')$ but weaker explicit incentives. Nevertheless, the agent works equally hard on the job with either contract; a_1 is unchanged. Intuitively, higher "base utility" at work makes private rewards less important, thus lessening the agent's incentives to pursue such rewards. As intended, this reduces a_2. Labor income then contributes more significantly to the agent's overall utility. Hence, weaker explicit incentives are required to maintain a constant a_1 effort on the job.

Consider next the contract's dependence upon a_1. Here,

$$\begin{aligned}\frac{\partial M'(a_1|\omega(a_1,a_2))}{\partial a_1} &= \frac{\partial}{\partial a_1}\left(\frac{c_1(a_1,a_2)}{N(a_2)}\right) = \frac{c_{11}(a_1,a_2)}{N(a_2)} \geq 0 \\ \frac{\partial M(a_1|\omega(a_1,a_2))}{\partial a_1} &= \frac{\partial}{\partial a_1}\left(\frac{c_2(a_1,a_2)}{N'(a_2)}\right) = \frac{c_{12}(a_1,a_2)}{N'(a_2)} \leq 0.\end{aligned}$$

Holding a_2 fixed but varying a_1, it is once again the case that the agent's base utility from work is inversely related to the steepness of the incentives. That is,

$$a_1'' > a_1' \Longrightarrow \begin{cases} M(a_1'|\omega(a_1',a_2)) \geq M(a_1''|\omega(a_1'',a_2)) \text{ and} \\ M'(a_1'|\omega(a_1',a_2)) \leq M'(a_1''|\omega(a_1'',a_2)). \end{cases} \tag{19}$$

The second part is intuitive; steeper incentives are required to make the agent work harder on the job, other things equal. The first property is perhaps more surprising at first blush; the agent's reward at work is lower when he is induced to work harder. Contrary to the standard model, however, the agent's utility is not pegged down by the participation constraint but rather by incentive compatibility. When a_1 increases, the marginal cost of the private task, c_2, increases as well. To maintain unchanged incentives to pursue private rewards, these must be made more important in the agent's utility. Thus, the reward from work must decrease.

In summary, $M(a_1|\omega(a_1,a_2))$ is decreasing in both a_1 and a_2 while $M'(a_1|\omega(a_1,a_2))$ is increasing in both. See Figure 1.

The set of implementable actions is characterized next. Given (17), the participation constraint is satisfied if and only if

$$\frac{c_2(a_1,a_2)}{N'(a_2)}N(a_2) - c(a_1,a_2) \geq \overline{u} \tag{20}$$

when a_2 is interior. Simple differentiation shows that the left hand side is strictly decreasing in a_2. The reason is that $M(a_1|\omega(a_1,a_2))$ decreases enough to offset the increase in private rewards that come with an increase in a_2. Assuming (20) is satisfied at $\underline{a}_2$, define

$$t(a_1) = \max\left\{a_2 \in [\underline{a}_2, \overline{a}_2] \,|\, \frac{c_2(a_1,a_2)}{N'(a_2)}N(a_2) - c(a_1,a_2) \geq \overline{u}\right\}$$

as the threshold value of a_2 such that (20) holds for all a_2 below that value. Thus, (20) is satisfied if and only if $a_2 \in [\underline{a}_2, t(a_1)]$. In other words, only a_2 levels at or below the threshold $t(a_1)$ can be implemented.

Let $t(a_1) = \underline{a}_2$ if (20) is violated at $\underline{a}_2$. In this case, only $\underline{a}_2$ can be implemented.[20] As this case is less interesting, it will be ignored in the remainder. Assuming then that $t(a_1) > \underline{a}_2$, (20) must be slack for any $a_2 \in [\underline{a}_2, t(a_1))$, implying that the agent earns more than reservation utility.[21] However, the participation

[20] At $\underline{a}_2$, the incentive compatibility constraint that $EU_2 \leq 0$ is equivalent to $M(a_1|\omega) \geq \frac{c_2(a_1,\underline{a}_2)}{N'(\underline{a}_2)}$. Thus, even if (20) is violated at $\underline{a}_2$, $M(a_1|\omega)$ can be increased to induce participation without violating incentive compatibility. Hence, $\underline{a}_2$ is implementable as long as the assumption that $c_2(a_1,\underline{a}_2) > 0$ is satisfied.

[21] At the corner where $a_2 = \underline{a}_2$ the incentive compatibility constraint is that $EU_2(a_1,\underline{a}_2) \leq 0$,

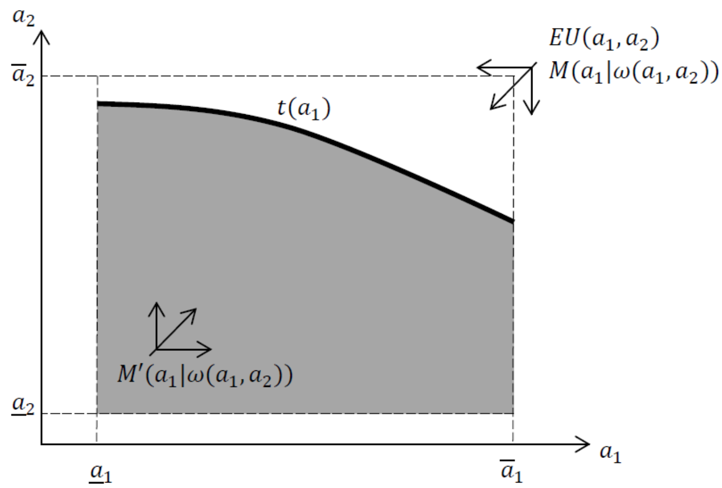

Figure 1: The feasible set and some comparative statics.

constraint binds at $a_2 = t(a_1)$, even if $t(a_1) = \overline{a}_2$.[22]

Note that the left hand side of (20) is strictly decreasing in a_1. The reason is that $M(a_1|\omega(a_1, a_2))$ decreases with a_1 and costs increase at the same time. Since (20) is strictly decreasing in both a_1 and a_2, $t(a_1)$ is decreasing in a_1, with $t'(a_1) < 0$ if $t(a_1)$ is interior. Hence, the larger a_1 is, the smaller is the set of implementable a_2 values. This completes the description of the feasible set of actions, illustrated as the shaded area in Figure 1. Note that the agent's expected utility is increasing towards the south-west (since the left hand side of (20) is decreasing in a_1 and a_2). As explained in the next subsection, a key reason the current model delivers richer predictions than the LEN model is that the feasible set in the latter is simply a curve. That is, for each a_1 there is a unique implementable a_2 value in the LEN model.

Having now described the feasible set it is time to turn to the costs of implementation. Thus, fix a feasible (a_1, a_2) pair, and let $C(a_1, a_2)$ denote the

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or $M(a_1|\omega) \geq \frac{c_2(a_1, \underline{a}_2)}{N'(\underline{a}_2)}$. Note, however, that if the inequality is strict, then the principal can reduce $m(w(x_1))$ by the same amount in every state without affecting the $EU_1(a_1, \underline{a}_2) = 0$ constraint. Since this would reduce implementation costs, the conclusion must be that the optimal contract satisfies $EU_2(a_1, \underline{a}_2) = 0$ even at the corner. Since expected utility as calculated in (20) assumes $EU_2(a_1, \underline{a}_2) = 0$, it now follows that the participation constraint must be slack.

[22] At $\overline{a}_2$, the incentive compatibility constraint that $EU_2 \geq 0$ is equivalent to $M(a_1|\omega) \leq \frac{c_2(a_1, \underline{a}_2)}{N'(\underline{a}_2)}$. Thus, $M(a_1|\omega)$ can be lowered to make the participation constraint bind.

implementation costs. The first step is to examine how $C(a_1, a_2)$ depends on a_2. Intuitively, since a_1 and a_2 are substitutes, an increase in a_2 makes it harder to satisfy the incentive compatibility constraint that $EU_1 = 0$. On the other hand, the constraint that $EU_2 = 0$ is easier to satisfy, since the agent's problem is concave in a_2. Thus, whether implementation costs increase or decrease depends on which constraint is more costly to satisfy. In the proof of the following Proposition 1, I thus first bound the multiplier μ_1 relative to the multiplier μ_2. Using this bound, it can then be shown that $C(a_1, a_2)$ is decreasing in a_2 on $[\underline{a}_2, t(a_1)]$ if

$$\frac{\partial}{\partial a_2}\left(\frac{c_2(a_1,a_2)}{c_1(a_1,a_2)}\frac{N(a_2)}{N'(a_2)}\right) \leq 0 \tag{21}$$

for all (a_1, a_2). This is a relatively mild assumption. It is satisfied if c_{12} is close to zero and $N(a_2)$ is log-convex. The latter property holds if $G^2(x_2|a_2)$ is log-convex in a_2.[23] Recall that Assumption A3 (LOCC) holds if $G^1(x_1|a_1)$ and $G^2(x_2|a_2)$ are log-convex in a_1 and a_2, respectively. In the example in Section 2, $c_{12} = 0$ and $N(a_2) = p_2(a_2)$ is log-convex. The result relies on the validity of the FOA. Two further justifications of the FOA, Theorems 2 and 3, are presented in the extensions in Section 6.

Proposition 1 *Assume utility from rewards are multiplicative, (21) holds, and that the assumptions in one of Theorems 1, 2, or 3 hold. Assume wages are interior regardless of which feasible (a_1, a_2) pair the principal seeks to implement. Then, for any $a_1 \in (\underline{a}_1, \overline{a}_1)$ for which $t(a_1) > \underline{a}_2$, $C(a_1, a_2)$ is strictly decreasing in a_2 on $[\underline{a}_2, t(a_1)]$.*

Proof. In the Appendix. ■

Assume in the remainder of the section that (21) holds. As before, let $B(a_1, a_2)$ denote the principal's direct utility from the agent's action. However, assume for now that the principal does not directly obtain utility or disutility from a_2, i.e. $B_2(a_1, a_2) = 0$. Proposition 1 then signifies that for any (interior) a_1, the optimal a_2 to induce is on the boundary of the feasible set, $a_2 = t(a_1)$. The same conclusion of course holds if $B_2(a_1, a_2) \geq 0$. Consequently, the participation

[23] Indeed, it is sufficient that the antiderivative (with respect to x_2) of $G^2(x_2|a_2)$ is log-convex in a_2. The justifications of the FOA presented in Section 6 involve this antiderivative.

constraint is binding regardless of which a_1 the principal seeks to implement. Since $t(a_1)$ is decreasing it follows that if the principal desires the agent to work harder on the job, then it is optimal to simultaneously induce the agent to work less hard in the pursuit of private rewards.

Given a_1 is to be implemented, the agent's total costs is thus $c(a_1, t(a_1))$. To proceed, assume that $t(a_1)$ is interior. Although a_1 and $t(a_1)$ move in opposite directions, (21) implies that

$$\frac{\partial c(a_1, t(a_1))}{\partial a_1} \geq 0. \tag{22}$$

That is, the agent's total cost of effort increases when he is induced to work harder on the job, even though he works less intensively on the other task.[24]

Compare the cheapest way to implement two different a_1 levels, a_1'' and a_1', with $a_1'' > a_1'$. It has just been established that the agent's effort costs are higher when a_1'' is implemented than when a_1' is implemented and that the participation constraint binds in either case. Thus, the agent must earn higher rewards from the two activities combined when a_1'' is implemented. However, since $t(a_1'') < t(a_1')$, the agent earns lower private rewards when a_1'' is implemented. Consequently, it is necessary that rewards from labor income are higher, or

$$a_1'' > a_1' \Longrightarrow M(a_1'|\omega(a_1', t(a_1'))) < M(a_1''|\omega(a_1'', t(a_1''))). \tag{23}$$

Now, to illustrate the main point in the simplest possible way, assume that marginal costs of effort on the job are constant, or $c_1(a_1, a_2) = c_1$. To induce a given a_1 level, the contract must thus offer the same marginal improvement in the utility from both sources of rewards regardless of what a_1 is. Formally, the incentive compatibility condition that $EU_1(a_1, t(a_1)) = 0$ implies

$$M'(a_1|\omega(a_1, t(a_1))N(t(a_1)) = c_1$$

However, the expected private rewards are larger when a_1' is implemented than when a_1'' is implemented – or $N(t(a_1'')) > N(t(a_1'))$ (recall the change in sign from

[24]From (21), $t'(a_1) \geq -\frac{c_1(a_1, t(a_1))}{c_2(a_1, t(a_1))}$ which in turn yields (22).

$n(x_2)$ to $N(a_2)$). Since labor income is then less important to the agent, it is necessary to promise a higher marginal change in the utility of labor income when a_1' is to be implemented. In other words, when (21) holds and $c_1(a_1, a_2) = c_1$,

$$a_1'' > a_1' \Longrightarrow M'(a_1'|\omega(a_1', t(a_1'))) > M'(a_1''|\omega(a_1'', t(a_1''))). \tag{24}$$

Thus, from (23), the agent earns higher utility from labor income the harder he is induced to work on the job. However, from (24), an outsider who fails to take into account that a_2 is endogenous would conclude that the marginal return to extra effort is *lower* the harder the agent is induced to work.[25] In other words, it looks as if the agent works harder when given weaker explicit incentives but higher base utility. Corollary 1 summarizes.

Corollary 1 *Assume that (21) holds and that $c_1(a_1, a_2) = c_1$ is constant. Then, the cost-minimizing way of inducing a_1 entails inducing $a_2 = t(a_1)$ at the same time. Assume a_1 and $t(a_1)$ are interior for $a_1 = \{a_1', a_1''\}$, with $a_1'' > a_1'$. Then,*

$$a_1'' > a_1' \Longrightarrow \begin{cases} M(a_1'|\omega(a_1', t(a_1'))) < M(a_1''|\omega(a_1'', t(a_1''))) \text{ and} \\ M'(a_1'|\omega(a_1', t(a_1'))) > M'(a_1''|\omega(a_1'', t(a_1''))). \end{cases} \tag{25}$$

The signs in (25) are reversed compared to those in (19). The reason is that (25) takes into account that a_2 changes along with a_1. Since a_1 and a_2 are inversely related, (18) works in the opposite direction compared to (19). Under the assumptions in Corollary 1, the changes due to a_2 dominate those due to a_1.

Next, consider the possibility that the principal cares directly about a_2 as well, with $B_2 < 0$. Since cost minimization is then no longer the only concern relevant to which a_2 to induce, the second-best action may now move into the interior of the feasible set.[26] Hence, the participation constraint may be slack.

Corollary 2 *The agent may earn economic rents if $B_2 < 0$.*

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[25] However, if $c_1(a_1, a_2)$ is not constant but instead sufficiently steep in a_1, then the conclusion that $M'(a_1'|\omega(a_1', t(a_1'))) < M'(a_1''|\omega(a_1'', t(a_1'')))$ is obtained.

[26] Note that as long as $t(a_1)$ is interior, wage costs are continuous on $[\underline{a}_2, t(a_1)]$; see the proof of Proposition 1. Then, the principal would prefer to induce some $a_2 < t(a_1)$ if $B_2(a_1, t(a_1))$ is sufficiently small (i.e. negative).

The agent accrues rent due to what can be thought of as "incomplete" contracting as x_2 is non-verifiable. Although Corollary 2 might seem less surprising in this light, it is interesting to note that the participation constraint is always binding in the popular LEN model, even when allowing for private rewards. The next subsection contrasts the model in the current paper with the LEN model.

There are many situations in which the non-contractible uncertainty is payoff relevant to both the agent and the principal, as assumed in Corollary 2. For example, Holmström and Milgrom (1991) point out that a "contractor's performance (such as courtesy, attention to detail, or helpful advice) are unmeasurable but are enhanced by attention [...] spent on that activity".[27] Another example they give is when one wants to motivate "teachers to teach both basic skills and higher-order thinking skills, but [...] higher-order thinking skills cannot be measured". Likewise, consider a salesman who represents several companies and who invests effort a_i into understanding firm i's product line. The quantity of firm i's products that he manages to sell depends not only on a_i but also on a_j, i.e. on how well he presents competing products.[28] A similar story might hold for real estate agents. Note that the last two examples involve common agency.

5.1 Comparison with the LEN model

The Linear-Exponential-Normal (LEN) model is due to Holmström and Milgrom (1987, 1991). The agent is assumed to exhibit constant absolute risk aversion. Importantly, costs are assumed to be monetary. Thus, the agent's Bernoulli utility is $u(w, a_1, a_2) = -e^{-r(w-c(a_1,a_2))}$ when he is paid w and his action is (a_1, a_2). Signals are assumed to be jointly normally distributed. In one interpretation of the model, the agent's action reduces to picking the means of these signals (the covariance matrix is beyond control). Finally, contracts are restricted to be linear in the signals. Holmström and Milgrom (1987) develop a dynamic micro-

[27] Giving helpful advice may be time-consuming and thus carry with it the opportunity cost of giving up time that could be spent pursuing private rewards.

[28] Although firm i's signal (the volume of sales) depends on a_j, the model does accommodate some such settings as described in footnote 10. For instance, let $t_i(a_1, a_2) = 2a_i - a_j$ be a parameter in the distribution of x_1, with $a_1, a_2 \in \mathbb{R}$. Thinking of t_1 and t_2 as the choice variables (such that $a_i = (2t_i + t_j)/3$), the cost function is $K(t_1, t_2) = c((2t_1 + t_2)/3, (2t_2 + t_1)/3)$. It can be verified that $K(t_1, t_2)$ is convex if $c(a_1, a_2)$ is convex.

foundation for this static model which applies in some cases.

Assume there are two signals, x_1 and x_2. Given a contract $w(x_1, x_2) = \beta + \alpha_1 x_1 + \alpha_2 x_2$, the agent's certainty equivalent can be shown to equal

$$CE(a_1, a_2|\beta, \alpha_1, \alpha_2) = \beta + \alpha_1 a_1 + \alpha_2 a_2 - c(a_1, a_2) + k(\alpha_1, \alpha_2), \qquad (26)$$

where a_i is the mean of x_i, which is controlled by the agent, and where $k(\alpha_1, \alpha_2)$ depends on the terms in the covariance matrix but not on a_1 and a_2. Private rewards that are monetary and stochastic can be incorporated by letting x_2 be non-contractible and letting $\alpha_2 > 0$ be an exogenous constant that measures how rewarding the activity is. On the other hand, deterministic but non-monetary private rewards can be modelled by fixing $\alpha_2 = 0$ and assuming that $c(a_1, a_2)$ is U-shaped in a_2. Holmström and Milgrom (1991) utilize the latter model. However, given the additive nature of the certainty equivalent in the LEN model, the two models are essentially isomorphic as one may simply think of $C(a_1, a_2) = c(a_1, a_2) - \alpha_2 a_2$ as a cost function that incorporates private rewards. Thus, assume from now on that α_2 is a non-negative exogenous constant (possibly zero). Assume that c is weakly convex such that the first order conditions are sufficient to identify an interior maximum of the CE. Assume that $c_2 = \alpha_2$ somewhere in the interior, and that $c_{12} > 0$. The case where $c_{12} < 0$ yields analogous results.

Clearly, β does not influence incentives at all, beyond participation. One conceptual advantage of the LEN model is that there is a simple measure of the strength of incentives, namely α_1, whereas I had to resort to using $M'(a_1|\omega)$ in the multiplicative model. On the other hand, the principal has only one instrument at his disposal when attempting to induce some (a_1, a_2) pair in the LEN model.

To complete the analysis of the LEN model, assume for simplicity that $c_{11}c_{22} - c_{12}^2 > 0$ and $c_{12} > 0$.[29] Then, there is a unique (a_1, a_2) pair which satisfies the first order conditions for any fixed (α_1, α_2). Moreover, a_1 is strictly increasing in α_1, while a_2 is strictly decreasing in α_1.[30] Two important conclusions follow:

[29] Holmström and Milgrom (1991, Section 3) consider a version of the model where the cost function takes the form $c(a_1 + a_2)$, in which case $c_{11}c_{22} - c_{12}^2 = 0$. The two main properties of the LEN model identified below also hold with this specification.

[30] An exception (which violates the above assumptions) occurs when $c(a_1, a_2) = c(a_1) + \gamma a_2$.

1. for any a_1 the principal might want to induce, there is exactly one implementable a_2 value, and
2. there is a unique α_1 coefficient that implements that particular a_1 value (and its a_2 companion).

The first property does not generally hold in the current paper. This is of course particularly significant when the principal has preferences over a_2 as well. In this case, the agent may earn economic rent in the multiplicative model, but never in the LEN model where β is adjusted to absorb all the rent. Note also that in the LEN model, a_1 and a_2 are competing in the sense that they must necessarily move in opposite directions. In contrast, the multiplicative model exemplifies that there are other contracting environments in which the principal has greater freedom to influence all dimensions of the agent's action. In particular, it is conceivable that a change in $B(a_1, a_2)$ will lead the principal to induce the agent to work less hard on both tasks, something that is impossible in the LEN model. In fact, combining the two properties of the LEN model reveals that for any desired a_1, there is a unique (linear) contract that can be rationalized – α_1 is determined by incentive compatibility, β by the participation constraint. In the multiplicative model, however, the optimal contract that implements a given a_1 depends on the principal's preferences over a_2.

The second property in particular is pertinent to the discussion of intrinsic and extrinsic motivation. Specifically, the LEN model can not reproduce the result that the same a_1 can be implemented with different contracts that vary in the strength of extrinsic incentives, at least when these are measured by the coefficient α_1. An important insight of the multiplicative model is that the agent's effort on the job is generally not exclusively determined by the steepness of the contract; the level of compensation matter too as it influences the agent's trade-off between work on the job and in the private activity.

Here, the agent's choice of a_2 is independent of the contract whenever $\gamma \neq \alpha_2$. If $\gamma = \alpha_2$, the agent is indifferent between all values of a_2.

6 Extensions: Relaxing LOCC

The stringency of Rogerson's CDFC is the main source of criticism of the FOA in the standard model. As mentioned earlier, Assumption A3 (LOCC) generalizes CDFC to allow multi-tasking. In this section, I consider two possible ways of relaxing Assumption A3. The first requires mild assumptions on the marginal utility of labor income. The second extension explicitly assumes that the multiplicative model in (14) applies.

6.1 Supermodular marginal utility

Jewitt (1988) was first to relax the CDFC. In the one-signal, one-task case, he replaces the CDFC assumption with the assumption that

$$\int_{\underline{x}_1}^{x_1} G^1(y_1|a_1)dy_1 \tag{27}$$

is convex in a_1 for all x_1. Jewitt (1988) shows that this condition is sufficient to justify the FOA provided that the agent's utility is increasing and concave in x_1. Concavity must thus be established. In Jewitt's setting, this turns out to require that the likelihood ratio is increasing and concave. Kirkegaard (2015a) proves that this type of justification of the FOA can be extended to many signals and many tasks, as long as the tasks are independent. In the case with two signals and two tasks, the appropriate assumption is that the antiderivative of $F(x_1, x_2|a_1, a_2)$ is weakly convex in (a_1, a_2), as described in the next assumption. Kirkegaard (2015a) terms this condition the *cumulative lower orthant convexity condition* (CLOCC).

Assumption A3' (CLOCC): $F(x_1, x_2|a_1, a_2)$ satisfies the *cumulative lower orthant convexity condition (CLOCC)*, i.e.

$$\int_{\underline{x}_1}^{x_1}\int_{\underline{x}_2}^{x_2} G^1(y_1|a_1)G^2(y_2|a_2)dy_2dy_1 = \int_{\underline{x}_1}^{x_1} G^1(y_1|a_1)dy_1 \int_{\underline{x}_2}^{x_2} G^2(y_2|a_2)dy_2 \tag{28}$$

is weakly convex in (a_1, a_2) for all (x_1, x_2) and all (a_1, a_2).

Unfortunately, it is not possible to sign the second derivative of $v(w(x_1), x_2)$

with respect to x_1 in general. The next subsection considers a special case where it can be done, and where CLOCC may serve to justify the FOA.

However, it is straightforward to sign the second derivative of $v(w(x_1), x_2)$ with respect to x_2. In fact, by assumption, the agent's utility is concave in x_2. This observation opens the door for a simpler relaxation of Assumption A3 (LOCC). In particular, I will exploit that $v(w(x_1), x_2)$ is increasing in x_1 and increasing and concave in x_2. Thus, a hybrid of LOCC and CLOCC is called for.

ASSUMPTION A3'' (HOCC): $F(x_1, x_2|a_1, a_2)$ satisfies the *hybrid orthant convexity condition (HOCC)*, i.e.

$$G^1(x_1|a_1) \int_{\underline{x}_2}^{x_2} G^2(y_2|a_2) dy_2 \tag{29}$$

is weakly convex in (a_1, a_2) for all (x_1, x_2) and all (a_1, a_2).

Assumptions A3, A3', and A3" can be ordered according to how restrictive they are. Specifically, Assumption A3 (LOCC) implies A3" (HOCC), which in turn implies A3' (CLOCC). As was the case for Assumption A3, Assumptions A3' and A3" both imply that each factor in (28) and (29), respectively, must be strictly convex in a_i for interior x_i. As before, HOCC is satisfied if e.g. the two factors are log-convex in a_1 and a_2, respectively. Although HOCC is weaker than LOCC, it is easy to see that it remains the case that $EU(a_1, a_2)$ is strictly concave in a_2, or $EU_{22}(a_1, a_2) < 0$, regardless of the contract. Formally, this can be established by using integration by parts twice and invoking the assumption that $v_{22} < 0$. Likewise, as before, $EU(a_1, a_2)$ is concave in a_1 whenever the contract is monotonic. While the agent's expected utility is thus concave in each task, it remains to show that it is jointly concave in (a_1, a_2).

As explained in Kirkegaard (2015a), multi-signal justifications of the FOA that rely on LOCC or CLOCC also require one to sign certain cross-partial derivatives; recall that $v_{12} < 0$ was invoked to prove Lemma 1. Given HOCC, it turns out to be sufficient to add the mild assumption that $v_{122} \geq 0$. In other words, the price of weakening Assumption A3 by replacing it with Assumption A3" is that $v_{122} \geq 0$ must be assumed. However, note that if x_2 is income, then $v_{122} \geq 0$ is implied by Assumption A5 (DARA). In this case, then, there is no cost of replacing A3 by A3". More generally, in view of Assumption 5, $v_{122} \geq 0$ is satisfied if e.g. $v_2(w, x_2)$ is log-supermodular in (w, x_2), i.e. if the agent's risk aversion with respect to the private rewards is decreasing with labor income.

Assumption A8 (Supermodular marginal utility): The agent's marginal utility of the private reward is supermodular. That is, $v_2(w, x_2)$ is supermodular in (w, x_2), or $v_{122}(w, x_2) \geq 0$.

Theorem 2 proves that the FOA remains valid once LOCC is replaced by HOCC, provided that Assumption A8 is imposed as well.

Theorem 2 *Assume any second-best action (a_1, a_2) is interior. Then, given Assumptions A1,A2,A3", and A4–A8, the FOA is valid.*

Proof. The argument that $w(x_1)$ is increasing in x_1 remains unchanged as Assumption A3 is replaced by A3". Using integration by parts repeatedly yields

$$\begin{aligned} EU(a_1, a_2) &= v(w(\overline{x}_1), \overline{x}_2) - v_2(w(\overline{x}_1), \overline{x}_2) \int G^2(x_2|a_2)dx_2 \\ &+ \int v_{22}(w(\overline{x}_1), x_2) \int_{\underline{x}_2}^{x_2} G^2(y_2|a_2)dy_2 dx_2 \\ &- \int v_1(w(x_1), \overline{x}_2) w'(x_1) G^1(x_1|a_1)dx_1 \\ &+ \int v_{12}(w(x_1), \overline{x}_2) w'(x_1) \left(\int G^2(x_2|a_2)dx_2 G^1(x_1|a_1) \right) dx_1 \\ &- \int \int v_{122}(w(x_1), x_2) w'(x_1) \left(\int_{\underline{x}_2}^{x_2} G^2(y_2|a_2)dy_2 G^1(x_1|a_1) \right) dx_2 dx_1 \\ &- c(a_1, a_2). \end{aligned}$$

By assumption, $v_1, v_2 > 0 > v_{12}, v_{22}$ and $v_{112} \geq 0$. Thus, by Assumption A3", $EU(a_1, a_2)$ is the sum of functions that are concave in (a_1, a_2). Hence, the agent's utility is concave. The theorem now follows by the same arguments that established Theorem 1. ■

6.2 Multiplicative rewards

In the following I reexamine the special case in which the agent's Bernoulli utility takes the form in (14). Then, (12) can be simplified to yield

$$(\lambda N(a_2) + \mu_2 N'(a_2)) + (\mu_1 N(a_2))\, l^1_{a_1}(x_1|a_1) = \frac{1}{m'(w)}, \qquad (30)$$

or, by renaming the terms in the parentheses,

$$\widehat{\lambda} + \widehat{\mu} l^1_{a_1}(x_1|a_1) = \frac{1}{m'(w)}$$

precisely as in the usual model with no private rewards. Standard methods can now be used to prove that $\mu_1 > 0$ (or $\widehat{\mu} > 0$). Specifically, given MLRP, the contract would be non-increasing if $\mu_1 \leq 0$, thus implying that $EU_1 < 0$ in violation of the incentive-compatibility constraints. In other words, given only that $a_1 > \underline{a}_1$, it must hold that $\mu_1 > 0$ such that the optimal contract is non-decreasing in x_1 regardless of which (a_1, a_2) the principal seeks to implement. Note that as long as wages are interior, this argument applies to all (a_1, a_2) pairs with $a_1 > \underline{a}_1$, not only the pair that turns out to be optimal. Note also that it is not necessary to sign μ_2. Thus, there is no need to consider the doubly-relaxed problem, and thus no need to invoke Lemma 3, which is the only place where the assumptions that $B_2(a_1, a_2) \leq 0$ and $c_{12}(a_1, a_2) \geq 0$ are utilized. Thus, in the multiplicative model, part of Assumptions A4 and A6 can be relaxed. As only the relaxed problem is considered, Assumption A7 is modified as well.

Assumption A6' (The principal's preferences): The principal is risk neutral, with expected utility $B(a_1, a_2) - E[w|a_1, a_2]$.

Assumption A7' (The relaxed problem): A solution to the relaxed problem exists. Any solution involves only wages in the interior of the domain of $v(w, x_2)$.

Jewitt's (1988) proof that $m(w(x_1))$ may be concave applies to the current setting as well. He proves that this property holds if $l^1_{a_1}(x_1|a_1)$ is increasing and concave and

$$\frac{d}{dw}\left(\frac{-m''(w)}{m'(w)^3}\right) \geq 0.^{31} \tag{31}$$

The latter condition is satisfied if $m(w) = -e^{-rw}$, $r > 0$, as in the CARA example. Note that if $m(w(x_1))$ is increasing and concave in x_1, then so is $v(w(x_1), x_2)$. It is for this reason that Assumption A3' (CLOCC) from Section 6.1 will prove to be sufficient to justify the FOA. However, to use Jewitt's argument, it is evidently necessary to strengthen Assumptions A2 and parts of A4. Recall that $c_{12}(a_1, a_2) \geq 0$ is no longer required.

Assumption A2' (Concave likelihood-ratio): The marginal distributions satisfy Assumption A2 (MLRP). Moreover, $l^1_{a_1}(x_1|a_1)$ is weakly concave in x_1.

Assumption A4' (Multiplicative rewards): The agent's Bernoulli utility is $-m(w)n(x_2) - c(a_1, a_2)$. Costs, $c(a_1, a_2)$, are strictly increasing and weakly convex in (a_1, a_2). Moreover, m and n are strictly negative functions that are strictly increasing and strictly concave on their domain. Finally, m satisfies (31).

The FOA can now be justified in the multiplicative model.

Theorem 3 *Assume any second-best action* (a_1, a_2) *is interior. Then, given Assumptions A1,A2',A3',A4', A6', and A7', the FOA is valid.*

Proof. Starting from the expression of $EU(a_1, a_2)$ derived in Theorem 2, another round of integration by parts leads to a new expression that depends only on the terms in Assumption A3' (rather than A3" as in Theorem 2). Concavity then obtains if the derivatives of $u(x_1, x_2) = v(w(x_1), x_2)$ have the correct sign. It is required that $u_1, u_2 \geq 0 \geq u_{11}, u_{12}, u_{22}$ and $u_{112}, u_{122} \geq 0 \geq u_{1122}$. Assumptions A2 and A4 together implies that $u_{11} \leq 0$ (as $m(w(x_1))$ is concave in x_1). Given the multiplicative nature of u, it then follows that the second set of inequalities is also satisfied. Thus, the agent's utility is concave. ■

[31] Jewitt (1988) requires that a certain complicated composite function is concave. His equation (3.1) proves that this is equivalent to assuming (31).

7 Discussion

In the standard single-task model, $\mu_1 > 0$ implies that the principal is better off if the agent (by mistake) works marginally higher than a_1^* on the job. The same is true with private rewards, though it takes slightly more effort to establish it.

Proposition 2 *Assume the second-best action is in the interior. Then, given Assumptions A1–A7, the principal is (weakly) better off if the agent works marginally harder than a_1^*, or*

$$B_1(a_1^*, a_2^*) - \int w(x_1) g_{a_1}^1(x_1|a_1^*) dx_1 \geq 0. \tag{32}$$

Proof. Given $EU_1(a_1^*, a_2^*) = 0$, the adjoint equation for a_1 reduces to

$$B_1(a_1^*, a_2^*) - \int w(x_1) g_{a_1}^1(x_1|a_1^*) dx_1 + \mu_1 EU_{11}(a_1^*, a_2^*) + \mu_2 EU_{12}(a_1^*, a_2^*) = 0. \tag{33}$$

Now, substitute (13) into (33), to get

$$\left(B_1(a_1^*, a_2^*) - \int w(x_1) g_{a_1}^1(x_1|a_1^*) dx_1 \right) + \mu_1 \frac{EU_{11}(a_1^*, a_2^*) EU_{22}(a_1^*, a_2^*) - EU_{12}(a_1^*, a_2^*)^2}{EU_{22}(a_1^*, a_2^*)} = 0.$$

Recall that $EU_{22}(a_1^*, a_2^*) < 0$. Since $EU(a_1, a_2)$ is concave the numerator must be non-negative. Then, (32) follows from $\mu_1 > 0$. ■

Since the optimal contract is monotonic, the last term on the left hand side in (32) is strictly positive; wage costs are higher when the agent works harder on task a_1. Hence, (32) necessitates that $B_1(a_1^*, a_2^*) > 0$. That is, (a_1^*, a_2^*) can be optimal only if the principal's benefit function is increasing in a_1 at that point.

Although it is intuitive that $\mu_2 < 0$, it is instructive to consider a thought experiment where x_2 can be contracted upon. For simplicity, think of x_2 as outside income. Then, the principal can simply appropriate the private rewards. This is now a more or less standard moral hazard problem, in which case Kirkegaard (2015a) shows that both multipliers are strictly positive. Hence, with private rewards, the negative multiplier is not due to the outside rewards as such, but rather due entirely to the assumption that the rewards are secret. Recall that a similar observation explains why the participation constraint may be slack.

8 Conclusion

The current paper extends the canonical principal-agent model to allow the agent to pursue private, stochastic, and possibly non-monetary rewards.

However, a justification of the first-order approach (FOA) in this setting necessitates an understanding of the basic moral hazard problem with multi-tasking. Nevertheless, multi-tasking has been largely ignored in the literature until very recently (the LEN model being an exception). The justification of the FOA presented here thus builds upon Kirkegaard's (2015a) recent analysis. As explained there, the main technical cost of allowing multi-tasking is that outcomes from different tasks must be stochastically independent. With this restriction in place, however, the current paper establishes additional conditions under which Kirkegaard's (2015a) justifications extends to private rewards. Once the required assumptions on the technology have been made to handle multi-tasking, the economically significant assumptions are that the agent perceives outcomes and tasks to be substitutes and that his absolute risk aversion over labor income is decreasing in the private reward. It should be stressed that these assumptions appear to be rather mild. In other words, the costs of permitting private rewards are low.

The model of private rewards presented here abstracts away from a few potentially important complications. As just mentioned, a key assumption is that rewards are independent. However, it is not inconceivable that for example the gifts parents bestow on their children depend on the job held by the latter. Strictly speaking, the model does not allow the distribution of private rewards to be a direct function of the contract. However, the distribution could depend on the type of profession the agent is employed in, much in the same way that the outside option is likely to be a function of the agent's profession or level of education.

Although there are thus several directions in which the technical results could conceivably be extended in future research, the current model already identifies important economic insights. Conceptually, "unpacking" leisure by recognizing that rewards earned while not on the job are also endogenous reveals that the principal manipulates the agent's "work-life balance" through his contract design. Low base utility at work may entice the agent to focus more on pursuing private rewards and so steeper incentives on the job are required to compete for the agent's attention. Conversely, higher base utility at work reduces the incentive to pursue outside rewards. To an outside observer, the agent may now appear more "intrinsically motivated" as he can be induced to work hard on the job with flatter extrinsic incentives. In this respect, the model contributes to the behavioral contract theory literature by offering another perspective on intrinsic versus extrinsic motivation.

Space limitations prevent a fuller exploration of the model's potential to shed light on other economic phenomena. One question left for future research is whether differences in the emphasis placed on private rewards by observably different groups, like men and women, may explain why such groups are sometimes compensated in different ways despite only slight, if any, differences in performance. Having justified the FOA provides a method for researchers to explore this and other questions in the future. The multiplicative model in particular is quite tractable, yet richer than the LEN model. Finally, understanding contracting under private rewards represents a necessary first step in a longer-horizon endeavour to analyze common agency under various informational assumptions. Kirkegaard (2015b) explores some first implications of the model in this regard.

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Appendix

Proof of Proposition 1. Fix a feasible (a_1, a_2) pair where the participation constraint is slack, or $a_2 \in [\underline{a}_2, t(a_1))$. Formulate the cost-minimization problem that derives the cheapest contract that induces (a_1, a_2) subject only to the incentive compatibility constraints that $EU_1 = EU_2 = 0$, i.e.

$$\max_w \left(- \int w(x_1) g_1(x_1|a_1) dx_1 \right) \tag{34}$$
$$st. \quad EU_i(a_1, a_2) = 0,\ i = 1, 2.$$

By assumption, the optimal contract involves only interior wages. As established in Section 6.2, the contract is monotonic and the multiplier to the first constraint is strictly positive, $\mu_1 > 0$, even if (a_1, a_2) is not a second-best action (see the paragraph before Assumption A6'). Given the assumptions in any of Theorems 1–3, the agent's expected utility is thus concave. Thus, the local incentive compatibility constraints are sufficient. Hence, the above problem correctly identifies the cheapest contract that implements (a_1, a_2). That is, (34) identifies $-C(a_1, a_2)$.

The Envelope Theorem implies that a marginal change in a_2 causes $-C(a_1, a_2)$ to change by $\mu_1 EU_{12} + \mu_2 EU_{22}$, which depends on μ_1 and μ_2. Next, Jewitt's (1988) proof that $\mu_1 c_1 > 0$ in the standard single-task model is modified to establish that

$$\mu_1 c_1(a_1, a_2) + \mu_2 c_2(a_1, a_2) < 0 \tag{35}$$

in the multiplicative model, whenever the participation constraint is slack. The point is that this inequality bounds μ_1 relative to μ_2. To prove (35), start with the constraint that $EU_1(a_1, a_2) = 0$, or $M'(a_1|\omega)N(a_2) - c_1(a_1, a_2) = 0$. Thus,

$$\int m(w(x_1)) \mu_1 l^1_{a_1}(x_1|a_1) g^1(x_1|a_1) dx_1 N(a_2) = \mu_1 c_1(a_1, a_2).$$

Borrowing a trick from Jewitt (1988), solve (30) for $\mu_1 l^1_{a_1}$, keeping in mind that now $\lambda = 0$. Rewriting the above yields

$$\int m(w(x_1)) \left(\frac{1}{m'(w(x_1))} - \mu_2 N'(a_2) \right) g^1(x_1|a_1) dx_1 = \mu_1 c_1(a_1, a_2).$$

Since $EU_2(a_1, a_2) = 0$, or $N'(a_2) = \frac{c_2(a_1,a_2)}{M(a_1)}$,

$$\begin{aligned}\int \frac{m(w(x_1))}{m'(w(x_1))} g^1(x_1|a_1)dx_1 &= \mu_1 c_1(a_1, a_2) + \mu_2 \int m(w(x_1)) \frac{c_2(a_1, a_2)}{M(a_1|\omega)} g^1(x_1|a_1)dx_1 \\ &= \mu_1 c_1(a_1, a_2) + \mu_2 c_2(a_1, a_2).\end{aligned}$$

Since m is negative and increasing, the left hand side is negative. Thus, (35) holds. Equivalently,

$$\mu_2 c_2(a_1, a_2) < -\mu_1 c_1(a_1, a_2) < 0.$$

By the Envelope Theorem, a small increase in a_2 reduces costs by

$$\begin{aligned}\mu_1 EU_{12} + \mu_2 EU_{22} &= \mu_1 \left[M'(a_1)N'(a_2) - c_{12}\right] + \mu_2 \left[M(a_1)N''(a_2) - c_{22}\right] \\ &= \mu_1 \left[\frac{c_1}{N(a_2)} N'(a_2) - c_{12}\right] + \mu_2 \left[\frac{c_2}{N'(a_2)} N''(a_2) - c_{22}\right] \\ &= \mu_1 c_1 \left[\frac{N'(a_2)}{N(a_2)} - \frac{c_{12}}{c_1}\right] + \mu_2 c_2 \left[\frac{N''(a_2)}{N'(a_2)} - \frac{c_{22}}{c_2}\right] \\ &> \mu_1 c_1 \left[\frac{N'(a_2)}{N(a_2)} - \frac{c_{12}}{c_1}\right] - \mu_1 c_1 \left[\frac{N''(a_2)}{N'(a_2)} - \frac{c_{22}}{c_2}\right] \\ &= \mu_1 c_1(a_1, a_2) \frac{N'(a_2)}{N(a_2)} \frac{c_2(a_1, a_2)}{c_1(a_1, a_2)} \frac{\partial}{\partial a_2} \left(\frac{c_2(a_1, a_2)}{c_1(a_1, a_2)} \frac{N(a_2)}{N'(a_2)}\right) > 0,\end{aligned}$$

where the second equality uses $EU_1 = EU_2 = 0$. The first inequality uses the bound on $\mu_2 c_2$ derived earlier, combined with the fact that the term in the last bracket is negative. The second inequality invokes (21). Thus, costs are strictly decreasing on $a_2 \in [\underline{a}_2, t(a_1))$. The solution to the stated cost-minimization problem is continuous in a_2, and hence $a_2 = t(a_1)$ is the cheapest way of inducing a_1 on $[\underline{a}_2, t(a_1)]$. Incidentally, the solution to the stated cost-minimization problem may even over-estimate the cost at $a_2 = t(a_1)$. The reason is that if $t(a_1) = \overline{a}_2$, the constraint that $EU_2 = 0$ can be replaced by the weaker $EU_2 \geq 0$. In other words, implementation costs need not be continuous at $t(a_1)$. ■