Incomplete Information and Rent Dissipation in Deterministic Contests*

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Abstract

In a deterministic contest or all-pay auction, all rents are dissipated when information is complete and contestants are identical. As one contestant becomes "stronger", that is, values the prize more, total expenditures are known to decrease monotonically. Thus, asymmetry among contestants reduces competition and rent dissipation. Recently, this result has been shown to hold for other, non-deterministic, contest success functions as well, thereby suggesting a certain robustness. In this paper, however, the complete information assumption is shown to be crucial. I examine a tractable incomplete information model for which the complete information model is a special case. With incomplete information – regardless of how little – total expenditures in a deterministic two-player contest increase when one contestant becomes marginally stronger, starting from a symmetric contest. In fact, both contestants expend resources more aggressively; with complete information, neither of them do so. Thus, there is a "discontinuity" in the information structure.

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1 Introduction

Tullock's (1967, 1980) famous pair of papers have triggered a large literature on rent seeking. In large part, the focus of this literature is on the wasteful use of resources to contest rents. Applications thus include war, litigation, lobbying, and campaign spending, to mention but a few. Usually, complete information is assumed. That is, the value that each contestant place on winning the prize is common knowledge.[1] Simply put, the objective of this paper is to show that some of the important findings of this literature are not necessarily robust to the inclusion of incomplete information.

Consider a deterministic contest, or all-pay auction. In such a contest, the contestant who expends the most resources wins the prize with probability one. Assume there are two contestants, and that contestant i is known to value the prize at v_i, $i = 1, 2$. These rules and payoffs define a complete information game. The game has a unique Nash equilibrium in mixed strategies; contestants randomize to keep the opposition guessing. See Hillman and Riley (1989) and Baye, Kovenock, and de Vries (1996) for details. If $v_1 \geq v_2$, the expected expenditures of contestants 1 and 2 are, respectively,

$$EP_1^c = \frac{v_2}{2} \text{ and } EP_2^c = \frac{v_2^2}{2v_1}. \tag{1}$$

If the contest is symmetric, or $v_1 = v_2$, then all rents are dissipated since $EP_1^c + EP_2^c = v_2$. However, in many cases the contest is not symmetric. Hillman and Riley (1989) point out that an incumbent may value the prize more than a challenger. Nti (1999) lists a plethora of other examples. As v_1 increases beyond v_2, neither contestant becomes more aggressive. More precisely, contestant 1's expected expenditures are unchanged, while contestant 2's expected expenditures decline. As a corollary, total expenditures are strictly decreasing in v_1, for $v_1 \geq v_2$.

The conclusion is that asymmetries among contestants are detrimental to competition, and, as a result, there is less dissipation of rent in the fight for the prize.[2]

[1] For instance, in Konrad's (2009) textbook, incomplete information is discussed only sparingly. Likewise, in their collection of papers on rent seeking, Congleton, Hillman, and Konrad (2008) include only two papers, Wärneryd (2003) and Malueg and Yates (2004), which focus explicitly on incomplete information. Malueg and Yates (2004) concentrate on the Tullock contest; see Fey (2008) for a similar model. Wärneryd (2003) considers common value contests in which one contestant is uncertain of what that value is. Hurley and Shogren (1998) also examine one-sided incomplete information in a Tullock contest. Hillman and Riley (1989) consider two-sided incomplete information in a deterministic contest, assuming contestants are symmetric ex ante. Building on Amann and Leininger (1996), the current paper allows contestants to be asymmetric ex ante.

[2] As in much of the literature, the focus here is on the absolute level of rent dissipation, not the relative level. Moreover, this paper is concerned only with one-shot, simultaneous contests. See e.g. Amegashie (2006) and Konrad (2009) for discussions of dynamics in contests.

These comparative statics provide a key building block in other studies, most prominently in Baye, Kovenock, and de Vries' (1993) seminal paper on the "exclusion principle". The robustness of the results is therefore of interest.[3]

The preceding contest is extreme on at least two dimensions. First, the contest is deterministic; there is no room for luck or random factors to influence the outcome. However, the comparative statics have recently been shown to be robust to changes in the contest success function. Che and Gale (2000) and Alcalde and Dahm (2010) each consider a class of (mutually exclusive) contest success functions for which the deterministic contest is a limiting case. In both cases, there is an equilibrium for which expenditures are given by (1) whenever the contest success function is "sufficiently close" to the deterministic success function, in the sense that a contestant's winning probability increases dramatically enough when he unilaterally increases his expenditures, starting from a position of equal expenditures.[4]

Second, the contest assumes complete information. It is natural to ask whether the comparative statics are also robust to changes in the underlying information structure. While there are many ways to model incomplete information, I will pursue a particularly tractable incomplete information model, recently introduced by Kirkegaard (2011). The model shares many features with the complete information model and allows any bidder's strength to be captured by a single variable. In fact, the complete information model can be seen as a special case of the model examined here. Section 2 provides additional motivation for the model choice.

Starting from a symmetric contest, with incomplete information a marginal increase in bidder 1's strength causes the expected expenditures of both contestants to strictly *increase*. As a corollary, total expected expenditures strictly increase. Clearly, incomplete information gives rise to precisely the opposite comparative statics than complete information, at least when the asymmetry is small.

The following example demonstrates the main point. Assume bidder i, $i = 1, 2$, draws his valuation from the distribution function

$$F_i(v) = \varepsilon \frac{v}{\overline{v}_i} + (1-\varepsilon)\left(\frac{v}{\overline{v}_i}\right)^{\frac{1}{\varepsilon}}, v \in [0, \overline{v}_i],$$

[3]This is not the first paper to question the robustness of these results. Anderson et al (1990) state that asymmetries "can lead to unintuitive comparative statics in a Nash equilibrium." They then develop a model with bounded rationality which exhibits very different comparative statics. For instance, a contestant who becomes stronger will respond by becoming more aggressive, in equilibrium. In contrast, contestants are rational in the current paper.

[4]However, the comparative statics are different when the contests success function is not too discriminating, see e.g. Nti (1999) and Baik (1994). Fang (2002) demonstrates that the exclusion principle does not hold in this case either. Riley (1999) examines the comparative statics in deterministic contests in which not all effort is a sunk cost.

where $\overline{v}_1 \geq \overline{v}_2$ and ε is a small but positive number, $\varepsilon \in (0,1]$. Ex ante, bidders differ only with respect to $\overline{v}_i$, which can then be used as a measure of their strength. The density is strictly positive everywhere, as long as $\varepsilon > 0$. As $\varepsilon \to 0$, $F(v) \to 0$ for all $v \in [0, \overline{v}_i)$; hence, all mass is concentrated close to $v = \overline{v}_i$. If $\varepsilon = 0$, the distribution is degenerate (information is complete). The point is that the comparative statics for $\varepsilon = 0$ are the opposite of those arising for *any* strictly positive ε. Thus, in this model there is an intriguing "discontinuity" with respect to the information structure. Complete information is a knife-edge.[5]

2 Model

The contest is assumed to take the form of an all-pay auction. Hence, the auction terminology will be used. Two risk neutral bidders (contestants) are vying for a prize by submitting bids (committing effort to the contest). The highest bid wins, but both bidders pay their respective bids. A bidder is characterized by his type, which measures the monetary value he places on winning the prize. If a bidder with type v bids b, his payoff is $v - b$ if he wins and $-b$ if he loses.

Bidder i's type is assumed to be drawn from a twice continuously differentiable distribution function F_i with support $[0, \overline{v}_i]$, $i = 1, 2$. The distribution function is assumed to have no mass points and to have strictly positive density, denoted by f_i, everywhere. The assumption that the lower end-point of the support is zero is deliberate and important. Its implications are discussed in Section 3.

Amann and Leininger (1996, Lemmata 1 – 5) provide an implicit characterization of equilibrium in this general model.[6] Both bidders submit strictly positive bids with probability one (i.e. whenever their type is non-zero) and they both employ pure strategies that are strictly increasing in type.[7] The equilibrium is unique (Amann and Leininger (1996, Theorem 1)).

However, more structure will be imposed on F_1 and F_2 in the current paper, in order to facilitate comparison with the complete information model. The following model was first introduced by Kirkegaard (2011). However, he considers F_1 and F_2 to be fixed, and instead asks whether the auctioneer would profit from handicapping

[5] However, there are many other ways incomplete information could be modeled. Thus, to be accurate, this paper merely establishes *an* environment in which incomplete information leads to different comparative statics than complete information.

[6] Technically, Amann and Leininger (1996) assume that $\overline{v}_1 = \overline{v}_2$. However, none of their results rely on this assumption.

[7] Incomplete information purifies the strategies. The working paper compares the strategic effects of increased asymmetry in the complete and incomplete information models more fully.

one of the bidders. The comparative statics examined in this paper are thus along a different dimension.[8] Assume that

$$F_i(v) = F\left(\frac{v}{\overline{v}_i}\right),\ v \in [0, \overline{v}_i], \tag{2}$$

where F is a distribution function with the aforementioned properties on its support, $[0,1]$. Let $f > 0$ denote its density. The example in the introduction fits this model. Note that the complete information model can be seen as a special case, although F is of course degenerate in that case.[9]

For future reference, let

$$v_i^s = \frac{v}{\overline{v}_i}$$

denote bidder i's "scale-adjusted" type. By (2), the two bidders' scale-adjusted types have the same distribution. Without loss of generality, assume $\overline{v}_1 \geq \overline{v}_2$. The bidders are symmetric if and only if $\overline{v}_1 = \overline{v}_2$. The distribution F_2 is a "scaled-down" version of F_1 if $\overline{v}_1 > \overline{v}_2$; the two distributions have the exact same shape, but are of different scale to fit onto their respective supports. This can also be seen from the relationship

$$F_2(v) = F_1\left(\frac{\overline{v}_1}{\overline{v}_2}v\right),\ v \in [0, \overline{v}_2].$$

Note that F_1 first order stochastically dominates F_2 whenever $\overline{v}_1 > \overline{v}_2$. That is, bidder 1 is less likely to have a low type than bidder 2; he is perceived to be stronger.

Let EV_i denote the expected value of v_i, $i = 1, 2$, and let EV_F denote the expected value of a draw from the distribution function F. Then, substituting the scale-adjusted type for the actual type (at the second-to-last equality) yields

$$EV_i = \int_0^{\overline{v}_i} v f_i(v) dv = \int_0^{\overline{v}_i} \frac{v}{\overline{v}_i} f\left(\frac{v}{\overline{v}_i}\right) dv = \overline{v}_i \int_0^1 v^s f(v^s) dv^s = \overline{v}_i EV_F. \tag{3}$$

Thus, for a *fixed* distribution, F, $\overline{v}_i$ is a natural measure of the (absolute) strength of bidder i, as evidenced by (2) and (3). Therefore, $\overline{v}_i$ will be used as the basis for comparative statics. Let

$$r_i \equiv \frac{\overline{v}_j}{\overline{v}_i}$$

[8] Kirkegaard (2011) is best compared to Fu (2006). The latter finds that the optimal handicap in a complete information model precisely nullifies the strong bidder's advantage. In the incomplete information model, however, Kirkegaard (2011) shows that the weak bidder should be "overcompensated" and thus wins more often than the strong bidder.

[9] Specifically, in the complete information model, $F_i(x) = 0$ if $\frac{x}{v_i} < 1$ and $F_i(x) = 1$ if $\frac{x}{v_i} \geq 1$.

denote bidder i's relative strength, $i = 1, 2$, $i \neq j$. The lower r_i is, the stronger bidder i is compared to bidder j.

Compared to the benchmark model with complete information, the partial overlap of the supports implies that bidder 2 may happen to have a higher type than bidder 1, even if $\overline{v}_1 > \overline{v}_2$. However, the probability of this event can be made arbitrarily small by concentrating most of the probability mass of F close to 1 (e.g. let F be "very convex"). See also the example in the Introduction.

The remainder of this section describes the equilibrium and the expected payments, for a fixed $(\overline{v}_1, \overline{v}_2)$ pair.

2.1 Equilibrium and expected payments

Since strategies are strictly increasing, bidder i with type v must submit the exact same bid as bidder j with some, possibly different, type, $k_{ij}(v)$, $i, j = 1, 2$, $i \neq j$. Thus, bidder i wins if bidder j's type is below $k_{ij}(v)$. Since strictly positive bids are submitted whenever $v > 0$, it must be the case that $k_{ij}(0) = 0$. Likewise, $k_{ij}(\overline{v}_i) = \overline{v}_j$ must also hold. Otherwise, one bidder would irrationally be submitting bids that are strictly higher than is required to win with certainty.

Amann and Leininger's (1996) contribution is to supply an implicit characterization of the function $k_{ij}(v)$ for all $v \in [0, \overline{v}_i]$.[10] The function is derived from the bidders' first order conditions, which form an autonomous system of differential equations. The boundary condition is $k_{ij}(\overline{v}_i) = \overline{v}_j$. In the present model, however, it is convenient to express the allocation in terms of the scale-adjusted types. Thus, define $k^s_{ij}(v^s)$ as the scale-adjusted type of bidder j that bidder i with scale-adjusted type v^s would tie with, $v^s \in [0, 1]$. Substituting the scale-adjusted type, as in (3), into Amann and Leininger's (1996) formula yields the relationship

$$\int_{k^s_{ij}(v^s)}^{1} \frac{f(x)}{x} dx = r_i \int_{v^s}^{1} \frac{f(x)}{x} dx, \tag{4}$$

for all $v^s \in (0, 1]$. The details are omitted from the present paper, but they can be found in Kirkegaard (2011, Section 5).

For a fixed distribution, F, $k^s_{ij}(v^s)$ is determined exclusively by the relative strength of bidder i, r_i. As the purpose of the analysis is to vary $\overline{v}_i$, the function will be written as $k^s_{ij}(v^s|r_i)$ whenever the reliance on the relative strength needs to be

[10] k_{ij} can be explicitly characterized only in special cases, such as when F is the uniform distribution. The implicit characterization I provide here, in (4), is obtained by integrating up equation (1) in Amann and Leininger (1996), as in their Lemma 6.

made explicit. As bidder i becomes stronger, or r_i falls, $k^s_{ij}(v^s)$ must strictly increase to maintain the equality in (4), for any $v^s \in (0,1)$. Thus, for a fixed scale-adjusted type, bidder i wins more often the stronger he is.

If $\overline{v}_1 > \overline{v}_2$ or $r_1 < 1$, $k^s_{12}(v^s) > v^s$ for all $v^s \in (0,1)$. In words, bidder 1 outbids a higher scale-adjusted type. Consequently, bidder 1's distribution of bids first order stochastically dominates that of bidder 2; bidder 1 is less likely to submit low bids. The same property holds in complete information all-pay auctions. As $\overline{v}_1 \to \infty$, (4) implies that $k^s_{12}(v^s) \to 1$ for all $v^s \in (0,1)$. In the limit, bidder 1 wins with probability one. This is also the case in the complete information all-pay auction. Lemma 1 summarizes these preliminary results.

Lemma 1 *$k^s_{ij}(v^s|r_i)$ is strictly decreasing in r_i for all $v^s \in (0,1)$. As $r_i \to 0$, $k^s_{ij}(v^s|r_i) \to 1$ for all $v^s \in (0,1)$, and as $r_i \to \infty$, $k^s_{ij}(v^s|r_i) \to 0$ for all $v^s \in (0,1)$.*

Next, consider the expected bid or expected payment of bidder i, $EP_i(\overline{v}_i, \overline{v}_j)$. There are at least two ways of obtaining $EP_i(\overline{v}_i, \overline{v}_j)$. Amann and Leininger (1996) use k_{ij} to derive bidding strategies. The expected bid can then be calculated. The alternative is to use Myerson's (1981) insight that the allocation in any given mechanism permits $EP_i(\overline{v}_i, \overline{v}_j)$ to be calculated directly, without first having to deduce bidding strategies. Kirkegaard (2011) uses the second method to obtain the key property that $EP_i(\overline{v}_i, \overline{v}_j)$ is separable in the bidder's absolute strength, $\overline{v}_i$, and relative strength, r_i.[11] As will be demonstrated shortly, the complete information model has the same property. Lemma 2 summarizes Kirkegaard's (2011) results.

Lemma 2 *Bidder i's payment can be written*

$$EP_i(\overline{v}_i, \overline{v}_j) = \overline{v}_i \times EP^s(r_i), \tag{5}$$

where

$$EP^s(r_i) = r_i \int_0^1 k^s_{ij}(v^s|r_i)(1 - F(v^s))f(v^s)dv^s. \tag{6}$$

EP^s is differentiable in r_i and has the property that $EP^{s\prime}(1) > 0$.

Proof. Here it will be proven only that $EP_i(\overline{v}_i, \overline{v}_j)$ is separable, as stated in (5). Proving the remainder of the Lemma is more involved. For completeness, the Appendix reproduces the proof in Kirkegaard (2011, Lemma 3).

[11] In Kirkegaard (2011), however, the "relative strength" is in part determined by an endogenous handicap to one of the bidders. Roughly speaking, his comparative statics are based on keeping $\overline{v}_i$ constant and changing r_i in isolation. In the current paper, the point is to vary $\overline{v}_i$, which has an indirect effect on r_i.

Since a bidder with type zero earns zero rent, it follows from Myerson (1981) that bidder i's ex ante expected payment is

$$\begin{aligned} EP_i(\overline{v}_i, \overline{v}_j) &= \int_0^{\overline{v}_i} \left(v - \frac{1 - F_i(v)}{f_i(v)} \right) F_j(k_{ij}(v|r_i)) f_i(v) dv \\ &= \overline{v}_i \int_0^{\overline{v}_i} \left(\frac{v}{\overline{v}_i} - \frac{1 - F\left(\frac{v}{\overline{v}_i}\right)}{f\left(\frac{v}{\overline{v}_i}\right)} \right) F\left(\frac{k_{ij}(v|r_i)}{\overline{v}_j} \right) \frac{1}{\overline{v}_i} f\left(\frac{v}{\overline{v}_i}\right) dv. \end{aligned}$$

Substituting in the scale-adjusted type, $v^s = \frac{v}{\overline{v}_i}$, then yields

$$EP_i(\overline{v}_i, \overline{v}_j) = \overline{v}_i \int_0^1 \left(v^s - \frac{1 - F(v^s)}{f(v^s)} \right) F\left(k_{ij}^s(v^s|r_i)\right) f(v^s) dv,$$

which takes the form (5). Integration by parts produces (6). ■

Before examining the implications of Lemma 2, it is interesting to compare (5) and its counterpart in the complete information model, (1). The latter can also be written in a separable form,

$$EP_i^c(v_i, v_j) = v_i \times EP^c(r_i^c), \tag{7}$$

where $r_i^c \equiv v_j / v_i$, and

$$EP^c(r_i^c) = \begin{cases} \frac{1}{2} r_i^c & \text{if } r_i^c \leq 1 \\ \frac{1}{2} (r_i^c)^{-1} & \text{otherwise} \end{cases}. \tag{8}$$

Note that EP^c is not differentiable at $r_i^c = 1$, where it peaks. In contrast, EP^s is everywhere differentiable and does not peak at $r_i = 1$. This difference turns out to be instrumental in explaining the different results in the two models.

Inserting (6) into (5) and recalling that $\overline{v}_i r_i = \overline{v}_j$ yield

$$EP_i(\overline{v}_i, \overline{v}_j) = \overline{v}_j \int_0^1 k_{ij}^s(v^s|r_i)(1 - F(v^s)) f(v^s) dv^s. \tag{9}$$

It is sometimes emphasized that all rents are dissipated in a symmetric complete information all-pay auction, but not in a symmetric incomplete information all-pay auction. Bidders earn "information rent" in the latter. Nevertheless, from (9) observe that if $\overline{v}_i = \overline{v}_j$,

$$EP_i(\overline{v}_i, \overline{v}_i) = \frac{1}{2} \times \overline{v}_i \int_0^1 2v^s(1 - F(v^s)) f(v^s) dv^s. \tag{10}$$

Hence, as with complete information, each bidder pays half the expected value of the second highest type. The difference is that there is no wedge between the highest and second highest type in a symmetric complete information model.

3 Comparative Statics

Consider the individual responses to changes in $\overline{v}_i$.

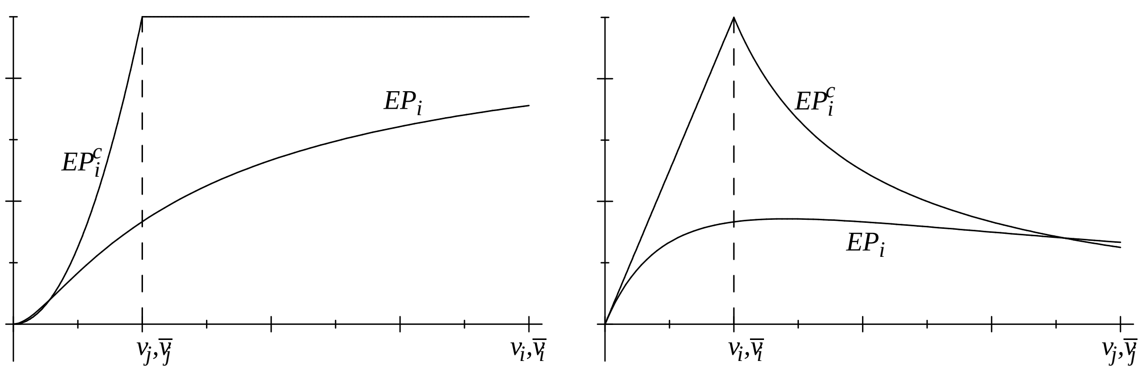
Proposition 1 *$EP_i(\overline{v}_i, \overline{v}_j)$ is strictly increasing in $\overline{v}_i$ and non-monotonic in $\overline{v}_j$, with*

$$\frac{\partial EP_i(\overline{v}_i, \overline{v}_j)}{\partial \overline{v}_j}_{|\overline{v}_j = \overline{v}_i} > 0.$$

Proof. By Lemma 1, $k^s_{ij}(v^s)$ is strictly increasing in $\overline{v}_i$, for all $v^s \in (0, 1)$. Equation (9) then implies that $EP_i(\overline{v}_i, \overline{v}_j)$ must be strictly increasing in $\overline{v}_i$ as well. From (5),

$$\frac{\partial EP_i(\overline{v}_i, \overline{v}_j)}{\partial \overline{v}_j} = \overline{v}_i \times EP^{s\prime}(r_i) \frac{1}{\overline{v}_i} = EP^{s\prime}(r_i).$$

By Lemma 2, this derivative is strictly positive when evaluated at $\overline{v}_j = \overline{v}_i$. At $\overline{v}_j = \overline{v}_i$, EP_i is strictly increasing in $\overline{v}_j$ and strictly positive. By Lemma 1, $k^s_{ij}(v^s) \to 0$ as $\overline{v}_j \to \infty$. It then follows from the last expression of $EP_i(\overline{v}_i, \overline{v}_j)$ in the proof of Lemma 2 that $EP_i(\overline{v}_i, \overline{v}_j) \to 0$ as $\overline{v}_j \to \infty$. Hence, $EP_i(\overline{v}_i, \overline{v}_j)$ cannot be monotonic in $\overline{v}_j$. ■



(a) EP_i^c, EP_i as functions of $v_i, \overline{v}_i$.

(b) EP_i^c, EP_i as functions of $v_j, \overline{v}_j$.

Figure 1: Comparative statics in complete and incomplete information models.[12]

In a complete information all-pay auction, EP_i^c is increasing in v_i as long as $v_i < v_j$. Thereafter, EP_i^c is unchanged if v_i increases further. In contrast, in an

[12] Formally, it has not been proven that EP_i in general is single-peaked in $\overline{v}_j$. Figure 1 depicts actual expected payments when F is a uniform distribution, in which case EP_i is in fact single-peaked in $\overline{v}_j$.

incomplete information contest, EP_i is globally strictly increasing in $\overline{v}_i$. EP_i^c is non-monotonic in v_j; increasing in v_j as long as $v_j < v_i$, and decreasing thereafter. While EP_i is also non-monotonic in $\overline{v}_j$ in an incomplete information contest, it does not attain its maximum at $\overline{v}_j = \overline{v}_i$. In particular, starting from a point of symmetry, $\overline{v}_j = \overline{v}_i$, bidder i actually becomes *more* aggressive (in expectation) when his rival becomes marginally stronger, which is precisely the opposite of what happens in the complete information all-pay auction. Figure 1 illustrates the difference between the two models. Note the "kinks" in the complete information model. One effect of incomplete information is to smooth out these kinks.

Since both bidders become more aggressive as one bidder becomes marginally stronger, starting from symmetry, it is an obvious corollary that total payments must increase as well.[13] Thus, in contrast to the complete information model, rent dissipation may be an increasing problem as bidders become asymmetric.[14]

To reconcile the differences in the two models' predictions, note that a change in the competitive balance may, in principle, have two distinct effects. The first and most obvious effect is that it may change the allocation. The second, and equally interesting effect, is that it may impact the amount of rent bidders are able to appropriate from the contest. However, a key difference between the complete and incomplete information models is the effectiveness with which the all-pay auction robs bidders of rent. In the complete information model, bidder 1 earns expected payoff of $v_1 - v_2$ when $v_1 \geq v_2$. His "participation constraint" is not binding. It becomes "more slack" as v_1 grows, meaning that the effectiveness of the all-pay auction at extracting rent diminishes as bidder 1 grows stronger. This opens a gap between the all-pay auction and the payment-maximizing mechanism with the same allocation. An all-pay auction that charges bidder 1 a fee to eliminate his rent would yield higher expenditures without affecting the allocation, and those expenditures would be increasing in v_1.

In comparison, the participation constraints are always binding in the incomplete information model. This property is due to the deliberate assumption that the lowest type is zero. Hence, no other mechanism could extract more rent from bidders

[13]When F is uniform and $\overline{v}_2 = 1$, the total expected expenditures attains its maximum at $\overline{v}_1 \approx 12.3$. In other words, bidder 1 must be at least 12 times as strong as bidder 2 in order for further increases in $\overline{v}_1$ to reduce total payments. The working paper contains a more detailed discussion of large asymmetries.

[14]Baye et al (1993) prove their exclusion principle by utilizing the two properties that in a complete information model, (1) only the two strongest bidders participate, and (2) revenue is decreasing in the valuation of the strongest bidder. Parreiras and Rubinchik (2010) prove the first property does not necessarily hold in the incomplete information model. The results in the current paper reveals that the second property need not hold either.

without changing the allocation. Thus, the confounding effect mentioned in the previous paragraph is absent.[15]

Changes in $\overline{v}_1$ obviously also affect the allocation. These changes can be used to explain why total expenditures increase when bidder 1 becomes marginally stronger, starting from a position of symmetry. To do so, it is worthwhile to note that the change in $\overline{v}_1$ changes bidder 1's absolute strength and his relative strength, and to decompose the effects. Since $r_2 = r_1^{-1}$, total expected payments can be written as

$$ET(\overline{v}_1, \overline{v}_2) = \overline{v}_1 \times EP^s(r_1) + \overline{v}_2 \times EP^s\left(r_1^{-1}\right). \tag{11}$$

Now imagine keeping $\overline{v}_1$ fixed at $\overline{v}_1 = \overline{v}_2$, but differentiating (11) with respect to r_1. Evaluated at $r_1 = 1$, this derivative is zero in the incomplete information model. The interpretation is that, starting from symmetry, a small change in the *relative* strength of the bidders has no first order effect. In words, bidder 2's expected payment increases when he becomes relatively weaker, but that is precisely offset by the decrease in bidder 1's expected payments that is attributable to his increased relative strength.[16] Thus, when $\overline{v}_1$ increases marginally, starting from $\overline{v}_1 = \overline{v}_2$, it is only the change in bidder 1's *absolute* strength that is important. Total expected expenditures thus increase because bidder 1's willingness to pay is higher.

These observations can be used to understand other comparative statics that can also be meaningfully employed to shed light on the consequences of increased asymmetry. For instance, Konrad (2009) considers a complete information Tullock contest in which $v_1 = v + d$ and $v_2 = v - d$, $d \in [0, v]$. Applied to the all-pay auction, it is easy to see that total expenditures in the complete information model is *strictly* decreasing in d. However, this is not the case in the incomplete information model.

Proposition 2 *Consider the incomplete information model and assume that* $\overline{v}_1 = \overline{v} + d$*,* $\overline{v}_2 = \overline{v} - d$*,* $d \in [0, \overline{v}]$*. Then, small asymmetries are irrelevant, or*

$$\frac{\partial ET(\overline{v} + d, \overline{v} - d)}{\partial d}_{|d=0} = 0.$$

[15] However, it is conceivable that the incomplete information model would yield similar comparative statics to the complete information model if the lowest type in the support is sufficiently high. A related literature on auctions with small asymmetries show that total expenditures must increase if one bidder becomes marginally stronger, starting from symmetry; see Fibich et al (2004) and Lebrun (2009). However, that literature is silent on individual expenditures and technically assumes that the support of F_1 and F_2 are the same. It is also explicitly assumed that the participation constraint is binding for both bidders, which necessitates that the lowest type is zero.

[16] Bidder 2's diminished relative strength can be compared to a handicap. As in Kirkegaard (2011), a handicap spurs the bidder to compensate by becoming more aggressive; see Lemma 2.

Proof. The Proposition follows directly from

$$\begin{aligned} ET(\overline{v}+d,\overline{v}-d) &= (\overline{v}+d)\times EP^s\left(\frac{\overline{v}-d}{\overline{v}+d}\right)+(\overline{v}-d)\times EP^s\left(\frac{\overline{v}+d}{\overline{v}-d}\right) \\ &= \overline{v}\times\left(EP^s\left(\frac{\overline{v}-d}{\overline{v}+d}\right)+EP^s\left(\frac{\overline{v}+d}{\overline{v}-d}\right)\right)+d\left(EP^s\left(\frac{\overline{v}-d}{\overline{v}+d}\right)-EP^s\left(\frac{\overline{v}+d}{\overline{v}-d}\right)\right), \end{aligned}$$

given that the term in the first parenthesis has a stationary point at $d = 0$ and the term in the second parenthesis is zero at $d = 0$. ■

Once again, the "kinks" in the complete information model cause small changes to have large effects. In the incomplete information model, it has already been demonstrated that there are no first order effects of small changes in relative strength levels. The effects of the changes in the absolute strength of the two bidders exactly cancel out, at $d = 0$. In this sense, small asymmetries are inconsequential.

4 Concluding remarks

The dominant assumption in the rent seeking literature is that information is complete. Among the advantages of such models are their analytical simplicity. Although the assumption of incomplete information is almost certainly more realistic, it is a challenge to build a tractable model which allows comparative statics. Such a model is presented in this paper. The model features a one-variable measure of strength, which means that comparative statics in the two models can easily be compared.

The comparative statics are qualitatively different when incomplete information is present. Indeed, in this model, *any* amount of incomplete information changes the conclusions. The main result is that ex ante asymmetries may make both contestants more aggressive and thus cause more rent to be dissipated in the pursuit of the prize, contrary to the conclusion that obtains in a complete information model. Thus, asymmetry does not necessarily dampen competition.

References

[1] Alcalde, J. and Dahm, M., 2010, Rent seeking and rent dissipation: A neutrality result, *Journal of Public Economics*, 94:1-7.

[2] Amann, E. and Leininger, W., 1996, Asymmetric All-Pay Auctions with Incomplete Information: The Two-Player Case, *Games and Economic Behavior*, 14: 1-18.

[3] Amegashie, J.A., 2006, Asymmetry and Collusion in Infinitely Repeated Contests, mimeo.

[4] Anderson, S.P., Goeree, J.K., and Holt, C.A., 1998, Rent Seeking with Bounded Rationality: An Analysis of the All-Pay Auction, *Journal of Political Economy*, 106 (4): 828-853.

[5] Baik, K.H., 1994, Effort Levels in Contests with Two Asymmetric Players, *Southern Economic Journal*, 61 (2): 367-378.

[6] Baye, M.R., Kovonock, D., and de Vries, C.G., 1993, Rigging the Lobbying Process: An Application of the All-Pay Auction, *American Economic Review*, 83 (1): 289-294.

[7] Baye, M.R., Kovonock, D., and de Vries, C.G., 1996, The all-pay auction with complete information, *Economic Theory*, 8: 291-305.

[8] Che, Y-K., and Gale, I., 2000, Difference-Form Contests and The Robustness of All-Pay Auctions, *Games and Economic Behavior*, 30: 22-43.

[9] Congleton, R.D., Hillman, A.L., and Konrad, K.A., 2008, *40 Years of Research on Rent Seeking*, Vols. 1 and 2, Springer.

[10] Fang, H., 2002, Lottery versus all-pay auction models of lobbying, *Public Choice*, 112: 351-371.

[11] Fey, M., 2008, Rent-seeking contests with incomplete information, *Public Choice*, 135: 225-236.

[12] Fibich, G., Gavious, A., and Sela, A., 2004, Revenue equivalence in asymmetric auctions, *Journal of Economic Theory*, 115: 309-321.

[13] Fu, Q., 2006, A Theory of Affirmative Action in College Admissions, *Economic Inquiry*, 44 (3): 420-428.

[14] Hillman, A.L., and Riley, J.G., 1989, Politically Contestable Rents and Transfers, *Economics and Politics*, 1 (1): 17-39.

[15] Hurley, T.M. and Shogren, J.F., 1998, Effort levels in a Cournot Nash contest with asymmetric information, *Journal of Public Economics*, 69: 195-210.

[16] Kirkegaard, R., 2011, Favoritism in Asymmetric Contests: Head Starts and Handicaps, mimeo.

[17] Konrad, K.A., 2009, *Strategy and Dynamics in Contests*, Oxford University Press.

[18] Lebrun, B., 2009, Auctions with almost homogeneous bidders, *Journal of Economic Theory*, 144: 1341-1351.

[19] Malueg, D.A., and Yates, A.J., 2004, Rent Seeking with Private Values, *Public Choice*, 119 (1-2): 161-78.

[20] Maskin, E. and Riley, J., 2000, Asymmetric Auctions, *Review of Economic Studies*, 67, 413-438.

[21] Myerson, R.B., 1981, Optimal Auction Design, *Mathematics of Operations Research*, 6: 58-73.

[22] Nti, K.O., 1999, Rent-seeking with asymmetric valuations, *Public Choice*, 98: 415-430.

[23] Parreiras, S.O. and Rubinchik, A., 2010, Contests with Three or More Heterogeneous Agents, *Games and Economic Behavior*, 68: 703-715.

[24] Riley, J.G., 1999, Asymmetric contests: a resolution of the Tullock paradox, in *Money, Markets and Method: Essays in Honour of Robert W. Clover*, edited by P. Howitt, E. de Antoni, and A. Leijonhufvud, Edward Elgar.

[25] Tullock, G., 1967, The Welfare Costs of Tariffs, Monopolies, and Theft, *Western Economic Journal*, 5: 224-232.

[26] Tullock, G., 1980, Efficient Rent Seeking, in: James Buchanan, Roger Tollison, and Gordon Tullock (eds.), *Towards a Theory of the Rent-Seeking Society*, Texas A&M University Press, 97-112.

[27] Wärneryd, K., 2003, Information in conflicts, *Journal of Economic Theory*, 110: 121-136.

Appendix: Proof of the last part of Lemma 2

Equation (4) reveals how k_{ij}^s depends on r_i

$$\frac{\partial k_{ij}^s(v^s)}{\partial r_i} = -\frac{k_{ij}^s(v^s)}{f(k_{ij}^s(v^s))}\int_{v^s}^1 \frac{f(x)}{x}dx,$$

and it follows that

$$\begin{aligned}\frac{\partial EP_i^s(r_i)}{\partial r_i} &= \int_0^1 k_{ij}^s(v^s)(1-F(v^s))f(v^s)dv^s \\ &\quad -\int_0^1 (1-F(v^s))\left(r_i\frac{k_{ij}^s(v^s)f(v^s)}{f(k_{ij}^s(v^s))}\int_{v^s}^1 \frac{f(x)}{x}dx\right)dv^s.\end{aligned} \tag{12}$$

For notational simplicity, subscripts will be suppressed from now on. From (4),

$$r\frac{k^s(v^s)f(v^s)}{f(k^s(v^s))} = k^{s\prime}(v^s)v^s$$

meaning that (12) can be written

$$\frac{\partial EP^s(r)}{\partial r} = \int_0^1 k^s(v^s)(1-F(v^s))f(v^s)dv^s - \int_0^1 v^s(1-F(v^s))g(v^s)dv^s \tag{13}$$

where

$$g(v^s) = k^{s\prime}(v^s)\int_{v^s}^1 \frac{f(x)}{x}dx.$$

The antiderivative of g is

$$\begin{aligned}G(v^s) &= \int_0^{v^s} g(y)dy = \left[k^s(y)\int_y^1 \frac{f(x)}{x}dx\right]_0^{v^s} + \int_0^{v^s}\frac{k^s(y)}{y}f(y)dy \\ &= k^s(v^s)\int_{v^s}^1 \frac{f(x)}{x}dx + \int_0^{v^s}\frac{k^s(y)}{y}f(y)dy,\end{aligned}$$

since L'Hopital's rule can be used to show that $k^s(y)\int_y^1 f(x)/xdx$ converges to zero as y converges to zero. Let $D(r)$ denote the last term in (12) or (13). Using integration by parts,

$$D(r) = \int_0^1 v^s(1-F(v^s))g(v^s)dv^s = \int_0^1 G(v^s)\left(v^s f(v^s) - (1-F(v^s))\right)dv^s$$

or

$$\begin{aligned} D(r) &= \int_0^1 k^s(v^s)v^s \int_{v^s}^1 \frac{f(x)}{x} dx f(v^s) dv^s - \int_0^1 k^s(v^s)(1-F(v^s)) \int_{v^s}^1 \frac{f(x)}{x} dx dv^s \\ &\quad + \int_0^1 \left(\int_0^{v^s} \frac{k(x)}{x} f(x) dx \right) (v^s f(v^s) - (1-F(v^s))) \, dv^s. \end{aligned}$$

Rearranging yields

$$2D(r) + A(r) = \int_0^1 k^s(v^s)v^s \int_{v^s}^1 \frac{f(x)}{x} dx f(v^s) dv^s + \int_0^1 k^s(v^s)(1-F(v^s)) f(v^s) dv^s,$$

where integration by parts was used to obtain the last part on the right hand side and where

$$A(r) = \int_0^1 k^s(v^s)(1-F(v^s)) \int_{v^s}^1 \frac{f(x)}{x} dx \left(1 - \frac{rf(v^s)}{f(k^s(v^s))}\right) dv^s.$$

Thus,

$$D(r) = \frac{1}{2} \left(\int_0^1 k^s(v^s)v^s \int_{v^s}^1 \frac{f(x)}{x} dx f(v^s) dv^s + \int_0^1 k^s(v^s)(1-F(v^s)) f(v^s) dv^s - A(r) \right).$$

Inserting this back into (12) yields

$$\begin{aligned} \frac{\partial EP_2^s(r)}{\partial r} &= \frac{1}{2} \left(\int_0^1 k^s(v^s)(1-F(v^s)) f(v^s) dv^s - \int_0^1 k^s(v^s)v^s \int_{v^s}^1 \frac{f(x)}{x} dx f(v^s) dv^s + A(r) \right) \\ &= \frac{1}{2} \left(\int_0^1 k^s(v^s)v^s \left(\frac{1-F(v^s)}{v^s} - \int_{v^s}^1 \frac{f(x)}{x} dx \right) f(v^s) dv^s + A(r) \right) \\ &= \frac{1}{2} \left(\int_0^1 k^s(v^s)v^s \left(\int_{v^s}^1 \frac{f(x)}{v^s} dx - \int_{v^s}^1 \frac{f(x)}{x} dx \right) f(v^s) dv^s + A(r) \right). \end{aligned}$$

Note that the first term is strictly positive. If $r = 1$ then $k^s(v^s) = v^s$. Thus, $A(1) = 0$ and so $EP_2^{s\prime}(1) > 0$. ■