

Capacity Investments under Demand Uncertainty*

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Abstract

Capacity constraints often bind in network industries such as electricity markets and hot spot markets, and investment becomes most valuable in these markets. Equilibrium characterization with binding capacity constraints is particularly important in these industries in which uncertainty and time-to-build constraints play a key role in determining investment behavior. We study a dynamic Cournot game with capacity accumulation under demand uncertainty, in which the investment is divisible, irreversible, and productive with a lag. We characterize equilibrium investments under open-loop, Markov perfect and closed-loop information structures. Contrary to what is established in the deterministic games literature, we find that firms may invest at a higher level in the open-loop equilibrium than in the closed-loop Nash equilibrium. The rankings of the investment levels obtained in three equilibria depend on the initial capacities and on the degree of asymmetry.

Key Words: Capacity Investment, Dynamic Game, Open-loop Equilibrium, Markov perfect Equilibrium, Closed-loop Equilibrium.

1 Introduction

In many industries capital or capacity investments are made under uncertainty. Uncertainty may stem from the nature of production characteristics, demand, cost and macroeconomic conditions. Some uncertainties are industry specific and the degree of uncertainty may vary from industry to industry. Production capacity investments under uncertainty have been studied extensively in the literature. The recent studies revisit and extend the early contributions to incorporate different demand models and behavioral assumptions to study the new capital intensive markets including, e.g., restructured electric power generation, natural gas transportation, ethanol, and hot spot industries. The main objectives of these articles are to provide insights for equilibrium investment behavior, entry-exit decisions and explain policy relevant topics such as effects of mergers, the role of excess market capacity on market power and price caps on equilibrium predictions. However, the capacity competition over time, in which capacity is subject to a time-to-build constraint and firms face demand uncertainties over time, has not been adequately analyzed.¹ In particular, how firms

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¹Time-to-build decision is empirically observable. Koeva (2000) measures average time-to-build in some industries and finds that it ranges from 13 to 86 months.

would adjust their incremental capacity investments over time under different behavioral assumptions (precommitment versus no commitment, or open-loop versus Markov perfect or closed-loop) is an important question to be investigated. For example, in the electricity production industry competing power generation firms can invest incrementally in some technologies under demand uncertainty either using some precommitment policies or using some state-dependent policies.

We study a two-period duopoly game with capacity accumulation under demand uncertainty. Investment is not productive instantly, and there is a lag between investment and production. We characterize and compare open-loop, Markov perfect and closed-loop Nash equilibrium investments. There is a significant literature in dynamic games focusing on the comparison of closed-loop and open-loop strategies.² In the capacity expansion literature, to which this article naturally belongs, it has been established in Reynolds (1987), in an infinite horizon differential game, that Markov strategies increase competition, i.e., Markov-perfect equilibrium investments exceed the open-loop ones. The same qualitative result has been obtained in different articles (and topics), e.g., Dockner (1992), Driskill and McCafferty (1989), Long et al. (1999), and Driskill (2001), whereas other articles find that Markov behavior softens competition (see, e.g., van der Ploeg and de Zeeuw (1990), Melese and Michel (1991), Piga (1998), Figuères (2002)). In a model where investment is reversible (a crucial assumption), Figuères (2009) shows that these contradictory findings are related to the concept of strategic substitutability and complementarity. A common feature in this literature is that the model is deterministic. Some recent papers, such as Genc et al. (2007) and Garcia and Stacchetti (forthcoming), studied capacity investment games under various assumptions including demand uncertainty. However, these articles do not focus on the role of uncertainty on the different equilibrium types (or information structures), nor do they explicitly study the role of state variables on equilibrium predictions. A common feature in this literature is that the models are deterministic. Because of its analytical tractability many models on capacity accumulation games assume a linear-quadratic (LQ) framework in optimization settings without uncertainty and capacity constraints. They analyze the commitment value of capital investments. We, however, examine a more realistic model taking into account of uncertainty, capacity constraints and time-to-build constraints, and find that strategic considerations may not lead to larger market shares.

We compare open-loop and closed-loop equilibria due to two reasons. First, the difference between closed-loop and open-loop outcomes represents the strategic effect. Second, this comparison has been common in the literature, and is not only suitable as a technical exercise, but also becomes an interesting question for mechanism and auction designers. Precommitting to an investment profile (open-loop concept) could be an optimum strategy for a firm if its rival chooses its investment strategy at the outset of the game. Similar to the open-loop concept, in wholesale electricity markets electricity traders regularly employ fixed-mix investment strategies for power portfolio optimization (see Sen et al. (2006)) in which short or long positions are guided by a fixed proportion of the difference between expected demand and expected production capacity. We will use the open-loop concept as a benchmark case to differentiate the strategic value of investment that is observed under the closed-loop behavior. Also, we note that open-loop equilibria can be used in a moving-horizon approach to approximate a closed-loop equilibrium. The principle is simple: At each period t , the players determine the open-loop Nash-equilibrium strategies for a given T -period planning horizon. However, only the initial control action is implemented. At period $t + 1$, the players again compute the equilibrium strategies for the next T periods, implement the (new) initial action, and so on. The resulting moving-horizon equilibrium trajectories constitute an approximation of the closed-loop

²Indeed, many papers have dealt with the comparison of open-loop and closed-loop strategies and equilibria in different areas. See, e.g., Dockner et al. (2000) and Figuères (2002, 2009) for capital accumulation games, Kossioris et al. (2008) and Long et al. (1999) for examples in environmental and resource economics, and Piga (1998) and Breton et al. (2006) for examples of advertising investments.

equilibrium trajectories that would have been obtained at the outset of the dynamic game. Further, some studies found that open-loop equilibria have some empirical support. For instance, Haurie and Zaccour (2004) and Pineau et al. (2011) compared the predicted open-loop equilibrium strategies to realizations in the European gas market and the Finnish Electricity industry, respectively, and found that they are close.

We initially start by considering the simplest possible setting, that is a two-stage deterministic model involving one investment decision. We find that open-loop, Markov perfect and closed-loop Nash equilibria coincide. This unsurprising result holds because state vectors match at each stage for all equilibrium types. Departing from this setting by assuming that demand in the second period is uncertain, allows (i) to show that the three equilibrium concepts do not coincide any longer; and, interestingly, (ii) to characterize the observed differences in investment strategies in terms of the differences in initial capacities of the players (or their degree of asymmetry). More specifically, we find that when firms are symmetric in terms of initial capacities and costs, open-loop equilibrium investment, that coincides with that of Markov perfect, exceeds closed-loop investments. The intuition for the result is that due to the binding capacity constraint a player cannot affect rival output through its investment under both Markov perfect and open-loop equilibria. Under the closed-loop equilibrium a firm's output in the following period is increasing in the rival firm's investment. This creates collusive-like behavior in which when a firm reduces its investment the rival also decreases its investment. Hence closed-loop investment levels will be lower than the open-loop counterparts. Whenever firms are asymmetric and the larger firm does not make any investment in equilibrium then the small firm's investment under closed-loop information equilibrium, which corresponds to the Markov perfect one, is higher than the open-loop counterpart. The reason for this result is that small firm's investment is a decreasing function of rival firm's output under the closed-loop structure. It invests strategically and preemptively, hence its investment is higher under the closed-loop structure.

The plan of the rest of the article is as follows. Section 2 introduces the model, and Section 3 states some general results along with the key findings. Section 5 briefly concludes.

2 Model

We study a two-period duopoly game, in which firms produce a homogeneous good. At time $t = 0$ for a given demand and capacity state vector, firms produce competitively and make capacity investments under demand uncertainty. The stochastic process we consider is random walk with two states- upstate and downstate. An investment made at time $t = 0$ will become productive in the following period. After the demand uncertainty is revealed at time $t = 1$, firms make production decisions simultaneously and independently.

There are two players, $\{i, -i\}$. The firms compete over two periods, $t = 0, 1$. In period 0, inverse demand is known to be $P_0(Q) = 1 - Q$, with Q the total output of the two firms. The stochastic inverse demand in period 1 is:

$$P_1(Q) = \begin{cases} 1 + \xi - Q & \text{with probability } p \\ 1 - \xi - Q & \text{with probability } 1 - p \end{cases} \quad (1)$$

The demand at initial period has two successors with $1u$, $1d$ standing for demand shifting up or down. Denote I_{i0} the investment in the production capacity of player i . Assuming away obsolescence and taking into account the one-period delay for investment to become productive, the capacity accumulation dynamics is then given by

$$K_{i1} = K_{i0} + I_{i0}. \quad (2)$$

Each player must satisfy the production capacity constraint at each production node, i.e.,

$$q_{it} \leq K_{it}, \quad t = 0, 1. \quad (3)$$

In the sake of keeping the computations as simple as possible, we adopt a quadratic investment cost function and a linear production cost, i.e.,

$$F_i(I_i) = 1/2f I_i^2, \quad C_i(q_i) = cq_i,$$

where $f > 0$ and $c > 0$. Assuming profit maximization behavior, each player maximizes expected discounted payoff with a common discount factor $\delta \in (0, 1)$ subject to the above constraints.

3 Investment Dynamics with Time-to-Build and Competition

We report some general results pertaining to production and investment decisions at any time and demand state. We also show some relationships between output and investment decisions. These results provide some valuable insights for the characterization of open-loop, Markov perfect and closed-loop equilibria. Under the open-loop information structure, firms make decisions at the outset of the game. In the Markov perfect structure strategies are subgame perfect. The closed loop structure also considers history of decisions, i.e., it is a solution concept with memory.

Consider production decisions at any time. Because of the lag between investment and production, the investment decision is independent of the quantity decision at the same period. Each player chooses production quantities by solving the following problem,

$$\begin{aligned} \max P_t(Q) q_{it} - C_i(q_{it}), \\ \text{s.t.}, \quad 0 \leq q_{it} \leq K_{it}. \end{aligned}$$

Assume symmetric capacities, $K_{it} = K_{-it} \equiv K_t$. The solution of the problem produces three equilibrium candidates: (i) The interior Cournot solution, $q_{it} = q_{-it} = (1 + \hat{\xi} - c)/3$; (ii) The corner solution, $q_{it} = q_{-it} = K_t$; or (iii) The asymmetric solution with player i producing at full capacity, $q_{it} = K_t$ and the rival player $-i$ plays its best response strategy $q_{-it} = (1 + \hat{\xi} - c - K_t)/2$, where $\hat{\xi} \in \{0, \xi, -\xi\}$. The following lemma shows that the asymmetric solution is ruled out.

Lemma 1 *At any time and demand state, whenever capacities of the players are symmetric, Nash equilibrium outputs are unique and symmetric.*

Proof. See the Appendix. \square

The next lemma shows that it can never occur that a player's output in downstate $1d$ exceeds his production in upstate $1u$. Note that the result is independent of production capacities.

Lemma 2 *In period 1, $q_{i1d} \leq q_{i1u}$.*

Proof. See the Appendix. \square

The following lemma states that if a player invests at period 0, then this player will produce at full capacity in upstate.

Lemma 3 *If $I_{i0} > 0$, then player i produces at capacity in upstate, $q_{i1u} = K_{i1}$. Further, if $I_{i0} = 0$, then $q_{i1u} \leq K_{i1}$.*

Proof. See the Appendix. \square

Consider the simplest possible setting of rivalry investment decisions, where demand is known with certainty ($\tilde{\xi} = 0$). We have the following result.

Proposition 1 *In the absence of uncertainty, open-loop, Markov perfect and closed-loop Nash equilibrium investments coincide.*

Proof. See the Appendix. \square

The result holds because the closed-loop, Markov perfect and open-loop Nash equilibrium state vectors at each stage coincide, and the rollback solution is identical to the forward solution. Alternatively, as the investment cost is sunk for the second period and the effect of investment is to provide an upper bound for the production level, the equilibria coincide. Note that this result holds for any given initial production capacities. Further, as one can expect, total industry investment is lower than the welfare-maximizing level.³

Now consider stochastic demand ($\tilde{\xi} > 0$). The interior Cournot outputs will be $q_0^c = (1 - c)/3$, $q_u^c = (1 + \xi - c)/3$, and $q_d^c = (1 - \xi - c)/3$ for time zero, upstate and downstate demand, respectively. These outputs will be useful to characterize equilibrium investments in the following propositions. Depending on the model parameter values, different cases arise:

- Case 1 : $I_{i0} = 0$ and $q_{i1d} < q_{i1u} < K_{i1}$,
- Case 2 : $I_{i0} > 0$ and $q_{i1d} < q_{i1u} = K_{i1}$,
- Case 3 : $I_{i0} > 0$ and $q_{i1d} = q_{i1u} = K_{i1}$.

Case 1 occurs when the player's initial capacity is high, which happens when initial capacity is greater than Cournot output in high demand ($K_{i0} > (1 + \xi - c)/3$). Case 2 occurs (Proposition 2) when the initial capacity K_{i0} is large so that the capacity constraints do not always bind, but also low enough that firms have an incentive to invest in capacity ($I_{i0} > 0$). Case 3 occurs (Proposition 3) when the initial capacity is low and total capacity is binding in both states in period 1.

Proposition 2 *Assume $K_{i0} = K_{-i0} = K_0$, and initial capacity is high such that $k' < K_0 < q_u^c$, where $k' = q_d^c(1 + \delta p/f) - 2\delta p\xi/f$. Then, for $i = 1, 2$*

1. *Symmetric open-loop (OL), Markov perfect (MP) and closed-loop (CL) Nash equilibrium investments are,*

$$I_{i0}^{OL} = \frac{\delta p(1 + \xi - c - 3K_{i0})}{f + 3\delta p} = I_{i0}^{MP}, \quad I_{i0}^{CL} = \frac{\delta p(1 + \xi - c - 4K_{i0})}{f + 4\delta p}.$$

2. *Equilibrium quantities at time 1 are given by*

$$\begin{aligned} q_{i1u}^{CL} &= K_{i1}^{CL} = \frac{fK_{i0} + \delta p(1 + \xi - c)}{f + 4\delta p}, & q_{i1d}^{CL} &= \frac{1 - \xi - c}{3}, \\ q_{i1u}^{OL} &= K_{i1}^{OL} = \frac{fK_{i0} + \delta p(1 + \xi - c)}{f + 3\delta p} = q_{i1u}^{MP}, & q_{i1d}^{OL} &= \frac{1 - \xi - c}{3} = q_{i1d}^{MP}. \end{aligned}$$

³It is shown in the Appendix that the total investments made in the market will be $I_{i0} + I_{-i0} = \delta[2 - 3K_0 - 2c]/(f + 3\delta)$, where $K_0 = K_{i0} + K_{-i0}$. Welfare maximizing efficient investment would be $I_0 = \delta[1 - K_0 - c]/(f + \delta)$, obtained through the solution of the problem

$$\max\left\{\int(1 - q)dq - cq_0 - fI_0^2/2 + \delta\left[\int(1 - q)dq - cq_1\right] + \alpha_0(K_0 - q_0) + \alpha_1(K_0 + I_0 - q_1)\right\}.$$

Clearly $I_0 > I_{i0} + I_{-i0}$, that is duopoly underinvests relative to the efficient level.

3. *Equilibrium profits compare*

$$\pi_i^{OL} = \pi_i^{MP} < \pi_i^{CL}.$$

4. *Asymmetric equilibrium in investment strategies is not possible.*

Proof. See the Appendix. \square

Proposition 3 *Assume $K_{i0} = K_{-i0} = K_0$, and initial capacity is low such that $0 < K_0 < k''$, where $k'' = q_d^c - 2\delta p\xi/f$. Then, for $i = 1, 2$*

1. *Symmetric open-loop, Markov perfect and closed-loop Nash equilibrium investments are,*

$$I_{i0}^{OL} = \frac{\delta(1 - \xi - c - 3K_{i0} + 2p\xi)}{f + 3\delta} = I_{i0}^{MP}, \quad I_{i0}^{CL} = \frac{\delta(1 - \xi - c - 4K_{i0} + 2p\xi)}{f + 4\delta}.$$

2. *Equilibrium production quantities at time 1 are*

$$\begin{aligned} q_{i1u}^{CL} &= q_{i1d}^{CL} = K_{i1}^{CL} = \frac{fK_{i0} + \delta(1 + \xi - c + 2p\xi)}{f + 4\delta}, \\ q_{i1u}^{OL} &= q_{i1d}^{OL} = q_{i1u}^{MP} = q_{i1d}^{MP} = K_{i1}^{OL} = \frac{fK_{i0} + \delta(1 + \xi - c + 2p\xi)}{f + 3\delta} = K_{i1}^{MP}. \end{aligned}$$

3. *Equilibrium profits compare*

$$\pi_i^{OL} = \pi_i^{MP} < \pi_i^{CL}.$$

4. *Asymmetric equilibrium in investment strategies is not possible.*

Proof. See the Appendix. \square

Contrasting Propositions 2 and 3 with Proposition 1 confirms the known conclusion that under uncertainty the three information structures do not produce the same investments. Further, a simple comparison of the investment strategies in Proposition 2, as well as in Proposition 3, shows that $I_{i0}^{OL} = I_{i0}^{MP} > I_{i0}^{CL}$, and, therefore, open-loop Nash equilibrium capacity per firm exceeds its closed-loop Nash equilibrium counterpart. Interestingly, open-loop equilibrium is subgame perfect, which stems from the binding capacity constraints. Under the closed-loop structure firms, a firm's output in the upstate is increasing in the rival firm's investment. This creates collusive-like behavior in which when a firm reduces its investment the rival also decreases its investment. Hence closed-loop investment levels will be lower than the open-loop counterparts. Under the OLNE a firm's period zero investment does not have any strategic value because it does not have an impact on its rival's period one output choice. Finally, note that under the assumption in Proposition 2, there is no room for an asymmetric equilibrium in investments strategies. This holds true under all information structures.

As can be seen from optimal investment expressions in Proposition 2 (and Proposition 3), investment is a function of demand probabilities. It can be easily shown that investment expressions are increasing in probability p . When $p = 0$ investments are zero in Proposition 2. This implies that if upstate demand is not likely to unfold, no investment occurs.

For completeness of the analysis of this symmetric game, it is easy to check that if both players do not invest in capacity (this is the case when initial capacities are large enough to cover the next-period upstate demand), then open-loop, Markov perfect and closed-loop outputs coincide at each node, and are given by

$$\begin{aligned} q_{i1u}^{OL} &= q_{i1u}^{MP} = q_{i1u}^{CL} = \frac{1 + \xi - c}{3}, \quad i = 1, 2, \\ q_{i1d}^{OL} &= q_{i1d}^{MP} = q_{i1d}^{CL} = \frac{1 - \xi - c}{3}, \quad i = 1, 2. \end{aligned}$$

Consequently, individual profits are the same under all information structures.

Proposition 4 *Assume asymmetric initial capacities, $K_{i0} \neq K_{-i0}$, and the capacities satisfy $k < K_{i0} < q_u^c < K_{-i0}$, where $k = q_d^c - \delta p \xi / f$. Then, the asymmetric OLNE, MPE and CLNE investments are given by*

$$I_{i0}^{OL} = \frac{\delta p[1 + \xi - c - 3K_{i0}]}{2f + 3\delta p}, \quad I_{i0}^{MP} = I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 2K_{i0}]}{2f + 2\delta p}, \quad I_{-i0}^{OL} = 0 = I_{-i0}^{CL} = I_{-i0}^{MP}$$

Further,

$$\begin{aligned} \pi_i^{OL} &< \pi_i^{CL} = \pi_i^{MP}, \\ \pi_{-i}^{OL} &> \pi_{-i}^{CL} = \pi_{-i}^{MP}. \end{aligned}$$

Proof. See the Appendix. \square

In this proposition, duopolists start with different initial capacities and in equilibrium one duopolist makes positive investment and the other does not. Facing a rival firm with large capacity, a player will invest less and realize lower profit in OLNE than in MPE (or CLNE). The MPE capacity for firm i exceeds its OLNE. Note that, player i produces at full capacity in the upstate and player $-i$ produces less than his capacity.

4 Conclusions

Capacity constraints often bind in network industries such as electricity markets and hot spot markets. Hence equilibrium characterization with binding capacity constraints becomes an important issue in competition settings in which uncertainty and time-to-build constraint play a key role in determining investment behavior. Our main contribution is to offer equilibrium characterization under different solution concepts (open-loop, Markov perfect, closed-loop) in the presence of uncertainty, capacity constraints, and the time-to-build. In our model firm capacities are endogenous and constraining production. Propositions 2 and 3 (and also Proposition 4) show the role of initial conditions and the degree of asymmetry on equilibrium predictions. In a fully symmetric game (and equilibrium), Proposition 2 indicates that each player invests more in the open-loop equilibrium than in its closed-loop counterpart. In Proposition 4, when we assume one firm has higher initial capacity than the other, and the firm with higher capacity does not make any investment, we find that the firm with low initial capacity invests and its investment would be higher under closed-loop information structure than under the open-loop structure. A first conclusion emerges from these propositions: when comparing investments made by a player under the two different information structures, our results show that everything can go either way, depending on initial capacity levels and their degree of asymmetry. However, in the deterministic capacity investments literature (e.g., Reynolds (1987)), it is observed that firms overinvest under the Markov perfect information structure relative to the open-loop structure. In that literature the intuition is based on “strategic investment”: a firm’s investment is a decreasing function of the rival’s output, and the investing firm seizes the rival firm’s capacity expansion. All players behave in the same manner, hence they overinvest. The second conclusion is that as shown in Proposition 4, player i who makes the strategic investment is better off by considering the role of its investment on rival player’s output choice and will realize a higher profit in closed-loop equilibrium than in open-loop one. Therefore, on top of being conceptually more appealing, this result provides a profit-grounded justification for the adoption of a state-dependent information structure. However, in terms of welfare, under open-loop behavior prices are lower and the production quantities are higher, and this benefits consumer surplus.

In this article we made, as in any modeling effort, a restrictive assumption on investment cost, that is worthwhile relaxing in future investigations to assess its impact on the equilibrium results. Following many contributions in the literature, we assumed quadratic investment cost. Adding a linear term, i.e., having a positive marginal cost for zero investment, would surely alter quantitatively and possibly qualitatively the results. Similarly, the addition of a fixed cost may have an interesting impact on the investment result obtained here.

APPENDIX

Proof of Lemma 1

Cases emerge depending on whether the capacity is greater or lower than the unconstrained Cournot equilibrium outputs. If $K \geq (1 + \xi - c)/3$ and player i plays $q_i = K$ then the best response of player $-i$ is $q_{-i} = (1 + \xi - c - K)/2$ by the profit maximization. In that case, $P(Q) = (1 + \xi + c - K)/2$, and the profit of player i is $\pi_i = (1 + \xi - c - K)K/2$. However, player i can do better, namely its best response to player $-i$ strategy q_{-i} is $q_i^* = (1 + \xi - c + K)/4$. Then, $P^*(Q) = (1 + \xi + 3c + K)/2$, and player i 's profit is $\pi_i^* = ((1 + \xi - c + K)/4)^2$. Then, clearly, $\pi_i^* \geq \pi_i$ if and only if $(1 + \xi - c - 3K)^2 \geq 0$, but this inequality holds because the production constraint must satisfy $q_i^* = (1 + \xi - c + K)/4 \leq K$. Hence, asymmetric outcomes are not part of the equilibrium. If $K < (1 + \xi - c)/3$ and player i plays K then the best response of player $-i$ is K , that is if the capacity K is lower than the symmetric Cournot level then the capacity constraints must be binding. If capacity K is greater than the symmetric Cournot outputs then the solution is the interior one. If K is equal to the Cournot outputs then the interior solution coincides with the corner solution. Therefore depending on the capacity level, the equilibrium will be unique.

Proof of Lemma 2

There are three possible cases. Case 1: For a player the output in the upstate is greater than the output in the downstate and for the other player the reverse applies. Case 2: Each player produces higher output in the downstate. Case 3, which is the claim of the proposition, that is, each player produces higher or equal outputs in the upstate than in the downstate. We will show that first two cases are not possible.

The Lagrangian of the profit maximization problem is

$$L_l = P_s(Q) q_{ls} - C_l(q_{ls}) + \lambda_{ls}(K_{ls} - q_{ls}).$$

The first order conditions (FOC) lead to

$$P_s(Q) + P'_s(Q) q_{ls} - C'_l(q_{ls}) - \lambda_{ls} = 0, \quad s = 1u, 1d, l = i, -i.$$

Case 1: Assume the supposition $q_{i1u} > q_{i1d}$ and $q_{-i1u} < q_{-i1d}$. Then the first inequality implies $\lambda_{i1d} = 0 \leq \lambda_{i1u}$. Linearity of the inverse demand curve and the convexity of the cost function together with $q_{i1u} > q_{i1d}$ and $\lambda_{i1d} = 0 \leq \lambda_{i1u}$ imply that the last 3 terms in the LHS of the FOC are more negative for the upstate than the downstate. So, by FOC we must have $P_{1u}(Q) > P_{1d}(Q)$. On the other hand, the inequality $q_{-i1u} < q_{-i1d}$ implies that $\lambda_{-i1d} \geq \lambda_{-i1u} = 0$. Linearity of the inverse demand curve, the convexity of the cost function along with the inequalities $q_{-i1u} < q_{-i1d}$ and $\lambda_{-i1d} \geq \lambda_{-i1u} = 0$, lead to $P_{1u}(Q) < P_{1d}(Q)$ by the FOC. These inequalities of price ranking is obviously a contradiction. Therefore, it cannot happen that one player produces higher output in the one state and the other produces higher output in the another state.

Case 2: Assume the supposition $q_{l1u} < q_{l1d}$, $l = i, -i$. Then clearly $Q_{1u} < Q_{1d}$ must hold. Also $\lambda_{l1d} \geq \lambda_{l1u} = 0$ satisfies. By the properties of inverse demand and cost function and the supposition, the FOC results in

$$\begin{aligned} P_{1u}(Q) &= -P'_{1u}(Q) q_{l1u} + C'_l(q_{l1u}) \\ &< -P'_{1d}(Q) q_{l1d} + C'_l(q_{l1d}) \leq P_{1d}(Q), \end{aligned}$$

which implies that, due to the linearity of the inverse demand, $Q_{1u} > Q_{1d}$ which contradicts to $Q_{1u} < Q_{1d}$. Therefore, it cannot happen that both players produce higher outputs in the downstate.

Therefore, the result that firms do not produce lower outputs in the upstate than the downstate follows.

Proof of Lemma 3

Consider the optimization problem of player i in period 0, with the two successor states $1u$ and $1d$. It is straightforward to verify that the OL Nash equilibrium (OLNE) conditions include

$$\begin{aligned} \frac{\partial \pi_i}{\partial I_{i0}} &= -fI_{i0} + \lambda_{i1u} + \lambda_{i1d} = 0, \\ \frac{\partial \pi_i}{\partial q_{i1u}} &= \delta p [1 + \xi - 2q_{i1u} - q_{-i1u} - c] - \lambda_{i1u} = 0, \\ \lambda_{i1u} &\geq 0, \quad K_{i0} + I_{i0} - q_{i1u} \geq 0, \\ 0 &= \lambda_{i1u} [K_{i0} + I_{i0} - q_{i1u}], \\ \frac{\partial \pi_i}{\partial q_{i1d}} &= \delta (1 - p) [1 - \xi - 2q_{i1d} - q_{-i1d} - c] - \lambda_{i1d} = 0, \\ \lambda_{i1d} &\geq 0, \quad K_{i0} + I_{i0} - q_{i1d} \geq 0, \\ 0 &= \lambda_{i1d} [K_{i0} + I_{i0} - q_{i1d}]. \end{aligned}$$

For $I_{i0} > 0$, we have $\lambda_{i1u} + \lambda_{i1d} > 0$. We have the following possibilities

$$\begin{aligned} \lambda_{i1u} &> 0 \quad \text{and} \quad \lambda_{i1d} > 0, \\ \lambda_{i1u} &> 0 \quad \text{and} \quad \lambda_{i1d} = 0, \\ \lambda_{i1u} &= 0 \quad \text{and} \quad \lambda_{i1d} > 0. \end{aligned}$$

The last possibility is excluded because only the condition $q_{i1u} \leq K_{i1} = q_{i1d}$ implies this possibility. But this condition contradicts to the Lemma 2 which proves that $q_{i1u} \geq q_{i1d}$. Hence, in all events we have $\lambda_{i1u} > 0$, and from complementarity conditions we must have $q_{i1u} = K_{i0} + I_{i0}$.

For $I_{i0} = 0$, we have $\lambda_{i1u} + \lambda_{i1d} = 0$. The nonnegativity of the multipliers implies that $\lambda_{i1u} = \lambda_{i1d} = 0$, and hence $q_{i1u} \leq K_{i1}$ and $q_{i1d} \leq K_{i1}$ because of the first order and complementarity conditions.

The proof of the result for the Markov perfect and closed-loop Nash equilibria are also similar. The structure of the proof is available in the following propositions.

Proof of Proposition 1

In this deterministic case, there is only one state in each period and therefore there is no need to distinguish between periods and states. Consider first the open-loop case. Player i maximizes

$$L_i = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - fI_{i0}^2/2 + \delta[q_{i1}(1 - q_{i1} - q_{-i1}) - cq_{i1}] + \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{i1}(K_{i0} + I_{i0} - q_{i1}).$$

At time 0, the first order necessary conditions for production decisions (that are irrelevant of investment decisions) might yield several possibilities due to capacity constraints. It might produce interior Cournot solution: $q_{i0} = (1 - c)/3$ implying $\lambda_{i0} = 0$. Or, it might lead to one interior one corner solution: $q_{i0} = (1 - K_{-i0} - c)/2$ and $q_{-i0} = K_{-i0}$, yielding $\lambda_{i0} = 0$ and $\lambda_{-i0} > 0$. Or, both players are at the capacity: $q_{i0} = K_{i0}$, $l = i, -i$ implying $\lambda_{i0} \geq 0$ and $\lambda_{-i0} \geq 0$.

At time 1, the production quantities are the same as the ones above, except the state variable at that period might change with the possible capacity expansion made in earlier period. The optimum investment must solve the first order necessary conditions, which imply $I_{i0} = \lambda_{i1}/f$. Assuming positive investments by both firms means $\lambda_{i1} > 0$, which in turn implies, $K_{i1} + I_{i0} = q_{i1}$. The derivative of the objective function with respect to q_{i1} results in $\lambda_{i1} = \delta[1 - 2q_{i1} - q_{-i1} - c]$. Plugging this into the investment expression yields

$$fI_{i0} = \delta[1 - 2(K_{i0} + I_{i0}) - (K_{-i0} + I_{-i0}) - c],$$

The OLNE investment will satisfy this equality.

To characterize the Markov perfect equilibrium (MPE) investment levels we solve the problem backwards and start from the final stage. At time 1, the value function is

$$v_{i1} = q_{i1}(1 - q_{i1} - q_{-i1}) - cq_{i1} + \lambda_{i1}(K_{i0} + I_{i0} - q_{i1}).$$

The complementarity condition is, $\lambda_{i1}(K_{i0} + I_{i0} - q_{i1}) = 0$. Assuming that $\lambda_{i1} > 0$, we obtain the corner solution $q_{i1} = K_{i0} + I_{i0}$. Next we plug this expression into the value function and write the value function at time 0:

$$v_{i0} = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - fI_{i0}^2/2 + \delta v_{i1}(I_{i0}) + \lambda_{i0}(K_{i0} - q_{i0}).$$

Taking the derivative with respect to the investment results in, assuming positive investments by both firms,

$$fI_{i0} = \delta[1 - 2(K_{i0} + I_{i0}) - (K_{-i0} + I_{-i0}) - c].$$

Clearly this expression is the same as the one obtained for OLNE. The computation of the closed-loop Nash equilibrium (CLNE) will also be same as the Markov perfect equilibrium in this certainty setting. Hence, investment levels coincide under all equilibrium concepts.

Proof of Proposition 2

First we characterize *open-loop Nash equilibrium* investments. We write the objective function to be maximized by firms $i = 1, 2$,

$$\begin{aligned} z_{i0} &= q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - fI_{i0}^2/2 + \delta p[q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] \\ &\quad + \delta(1 - p)[q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] \\ &\quad + \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{i1u}(K_{i0} + I_{i0} - q_{i1u}) + \lambda_{i1d}(K_{i0} + I_{i0} - q_{i1d}). \end{aligned}$$

Under the assumption $k' < K_0 < q_u^c$ it is clear that $\lambda_{i1u} > \lambda_{i1d} = 0$. It follows that $q_{i1u} = K_{i1u}$, $q_{i1d} = (1 - c - \xi)/3$. In period 1, capacity constraints only bind when demand is high, and the investment has an impact in the high demand state.

Taking the derivative of the above objective function (z_{i0}) with respect to the investment will yield to $I_{i0} = (\lambda_{i1u} + \lambda_{i1d})/f$, and the multipliers satisfy

$$\lambda_{i1u} = \delta p[1 + \xi - c - 2q_{i1u} - q_{-i1u}] = \delta p[1 + \xi - c - 3(K_{i0} + I_{i0})],$$

and $\lambda_{i1d} = 0$. Then, the OLNE strategy as a function of the model parameters is

$$I_{i0}^{OL} = \frac{\delta p[1 + \xi - c - 3K_{i0}]}{f + 3\delta p}, i = 1, 2.$$

Next we characterize *Markov perfect equilibrium* investments. Under the assumption $k' < K_0 < q_u^c$, investment only benefits in the upstate demand. At time 1 on state $1u$ player i maximizes

$$v_{iu} = [q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] + \lambda_{i1u}(K_{i1u} - q_{i1u}),$$

where $K_{i1u} = I_{i0} + K_{i0}$. The optimum output will satisfy $q_{i1u} = K_{i1u}$ because of the assumption.

At time 1 on state $1d$ player i maximizes

$$v_{id} = [q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] + \lambda_{i1d}(K_{i1d} - q_{i1d}).$$

The optimum output will satisfy $q_{i1d} < K_{i1d}$ because of the assumption, where $K_{i1d} = K_{i1u}$ as up and down states share the same root/history.

At initial node, player i maximizes

$$v_{i0} = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f I_{i0}^2/2 + \delta p w_{iu}(K_{i1u}, K_{-i1u}) + \delta(1 - p)w_{id}(\cdot) + \lambda_{i0}(K_{i0} - q_{i0}),$$

where $w_{iu}(K_{i1u}, K_{-i1u}) = [(I_{i0} + K_{i0})(1 + \xi - I_{i0} - K_{i0} - I_{-i0} - K_{-i0} - c)]$ is the profit for player i at state $1u$ in period 1 when it has capacity of $K_{i1u} = I_{i0} + K_{i0}$ and the rival has the capacity of $K_{-i1u} = I_{-i0} + K_{-i0}$. Also $w_{id}(\cdot) = q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}$ is the profit for player i at state $1d$ in period 1. The optimal investment must satisfy the first order condition

$$-f I_{i0} + p\delta \frac{\partial w_{iu}}{\partial K_{i1u}} \frac{\partial K_{i1u}}{\partial I_{i0}} = 0,$$

or

$$-f I_{i0} + p\delta[1 + \xi - c - q_{-i1u} - 2K_{i1u} - K_{i1u} \frac{\partial q_{-i1u}}{\partial I_{i0}}] = 0. \quad (1)$$

$$-f I_{-i0} + p\delta[1 + \xi - c - q_{i1u} - 2K_{-i1u} - K_{-i1u} \frac{\partial q_{i1u}}{\partial I_{-i0}}] = 0. \quad (2)$$

for players i and $-i$ respectively.

At time 1 optimality conditions are $q_{i1u} = I_{i0} + K_0$ and $q_{-i1u} = I_{-i0} + K_0$. Plugging these outputs into (1) and (2) and driving $\frac{\partial q_{-i1u}}{\partial I_{i0}} = 0 = \frac{\partial q_{i1u}}{\partial I_{-i0}}$ at time 1 and solving (1) and (2) we have

$$I_{i0}^{MP} = \frac{\delta p[1 + \xi - c - 3K_0]}{f + 3\delta p}, i = 1, 2.$$

The equilibrium production quantities at time 1 will satisfy $q_{1u} = (K_{i1u}, K_{-i1u})$ at the upstate demand, and $q_{1d} = ((1 - \xi - c)/3, (1 - \xi - c)/3)$ at the downstate demand. Since Markov perfect investment coincides with the open-loop one, they will produce the identical outputs and profits.

Next we characterize *closed-loop Nash equilibrium (with memory)* investments. Players still solve the problem backwards like they do in the computation of Markov perfect equilibrium. The only difference is the information structure. Namely, players remember the past decisions and take them into account of current decisions.

Under the assumption $k' < K_0 < q_u^c$, investment only benefits in the upstate demand. At time 1 at state $1u$ player i maximizes

$$v_{iu} = [q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] + \lambda_{i1u}(K_{i1u} - q_{i1u}),$$

where $K_{i1u} = I_{i0} + K_{i0}$. The optimum output will satisfy $q_{i1u} = K_{i1u}$ because of the assumption.

At time 1 at state d player i maximizes

$$v_{id} = [q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] + \lambda_{i1d}(K_{i1u} - q_{i1d}).$$

The optimum output will satisfy $q_{i1d} < K_{i1d}$, where $K_{i1d} = K_{i1u}$, because of the assumption.

At initial node, player i maximizes

$$v_{i0} = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - fI_{i0}^2/2 + \delta p w_{iu}(K_{i1u}, K_{-i1u}) + \delta(1 - p)w_{id}(\cdot) + \lambda_{i0}(K_{i0} - q_{i0}),$$

where $w_{iu}(K_{i1u}, K_{-i1u})$ is the profit for player i at state $1u$ in period 1 when it has capacity of $K_{i1u} = I_{i0} + K_{i0}$ and the rival has the capacity of $K_{-i1u} = I_{-i0} + K_{-i0}$. Also $w_{id}(\cdot) = q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}$ is the profit for player i at state $1d$ in period 1. The optimal investment must satisfy

$$-fI_{i0} + p\delta \frac{\partial w_{iu}}{\partial K_{iu}} \frac{\partial K_{iu}}{\partial I_{i0}} = 0,$$

or

$$-fI_{i0} + p\delta[1 + \xi - c - q_{-i1u} - 2K_{iu} - K_{iu} \frac{\partial q_{-i1u}}{\partial I_{i0}}] = 0. \quad (3)$$

$$-fI_{-i0} + p\delta[1 + \xi - c - q_{i1u} - 2K_{-iu} - K_{-iu} \frac{\partial q_{i1u}}{\partial I_{-i0}}] = 0. \quad (4)$$

for players i and $-i$ respectively.

At time 1 optimality conditions are $q_{i1u} = I_{i0} + K_{i0}$ and $q_{-i1u} = I_{-i0} + K_{-i0}$. At time 0 both players can derive the optimality conditions (3) and (4). They observe that these expressions are symmetric and at period 1 they will produce at the capacity. Therefore, the same investment level must solve (3) and (4) simultaneously. Hence, period 1 outputs must be identical. That is, $q_{i1u} = I_{i0} + K_{i0} = q_{-i1u} = I_{-i0} + K_{-i0}$. And then, $\frac{\partial q_{-i1u}}{\partial I_{i0}} = 1 = \frac{\partial q_{i1u}}{\partial I_{-i0}}$ must satisfy. Given these conditions, we solve (3) and (4) to obtain

$$I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 4K_0]}{f + 4\delta p}, \quad i = 1, 2.$$

The equilibrium production quantities at time 1 will satisfy $q_{1u} = (K_{i1u}, K_{-i1u})$ at the upstate demand, and $q_{1d} = ((1 - \xi - c)/3, (1 - \xi - c)/3)$ at the downstate demand.

Next we obtain the lower bound of initial capacity, k' , that entails non-binding capacity at the downstate demand by solving $K_{i0} + I_{i0} > q_d^c$. When we insert I_{i0}^{OL} into this inequality, we obtain that $K_{i0} > q_d^c - 2\delta\xi p/f$. When we insert I_{i0}^{CL} into this inequality we obtain $K_{i0} > q_d^c(1 + \delta p/f) - 2\delta\xi p/f$. To make the investment levels comparable in these regions we take the maximum of these bounds; $\max[q_d^c - 2\delta\xi p/f, q_d^c(1 + \delta p/f) - 2\delta\xi p/f] = k'$.

We now show that $\pi_i^{CL} > \pi_i^{OL}$. The CLNE and OLNE profits at initial node and node d in period 1 are clearly the same. Therefore, we need to compare the profits at node u in period 1. The difference in profits is given by

$$\pi_i^{OL} - \pi_i^{CL} = A + B,$$

where, dropping the player index,

$$\begin{aligned} A &= -f((I^{OL})^2 - (I^{CL})^2)/2 = -f(I^{OL} - I^{CL})(I^{OL} + I^{CL})/2, \\ B &= \delta p[(K_0 + I^{OL})(1 + \xi - 2(K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - 2(K_0 + I^{CL}) - c)]. \end{aligned}$$

Because $I^{OL} > I^{CL}$, A is negative. If the sign of B is negative, then we are done. Otherwise, we need to determine the sign of $|A| - B$. We have

$$\begin{aligned}
B &= \delta p[(K_0 + I^{OL})(1 + \xi - 2(K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - 2(K_0 + I^{CL}) - c)] \\
&= \delta p[-2K_0 I^{OL} + 2K_0 I^{CL} + I^{OL}(1 + \xi - 2(K_0 + I^{OL}) - c) - I^{CL}(1 + \xi - 2(K_0 + I^{CL}) - c)] \\
&= \delta p[2K_0(I^{CL} - I^{OL}) + (1 + \xi - 2K_0 - c)(I^{OL} - I^{CL}) - 2(I^{OL})^2 + 2(I^{CL})^2] \\
&= \delta p[(I^{OL} - I^{CL})(1 + \xi - 4K_0 - c) - 2(I^{OL} - I^{CL})(I^{OL} + I^{CL})] \\
&= \delta p[(I^{OL} - I^{CL})(1 + \xi - c - 4K_0 - 2(I^{OL} + I^{CL}))].
\end{aligned}$$

In the expression $\pi_i^{OL} - \pi_i^{CL} = A + B$, we will show that $|A| > B$. Indeed,

$$\begin{aligned}
|A| - B &= (I^{OL} - I^{CL})\left[\frac{f(I^{OL} + I^{CL})}{2} - \delta p(1 + \xi - c - 4K_0 - 2(I^{OL} + I^{CL}))\right] \\
&= (I^{OL} - I^{CL})[(I^{OL} + I^{CL})(2\delta p + f/2) - \delta p(1 + \xi - c - 4K_0)] \\
&= (I^{OL} - I^{CL})[(I^{OL} + I^{CL})(2\delta p + f/2) - I^{CL}(f + 4\delta p)] \\
&= (I^{OL} - I^{CL})[(I^{OL}(2\delta p + f/2) - I^{CL}(2\delta p + f/2))] \\
&= (I^{OL} - I^{CL})^2(2\delta p + f/2) > 0.
\end{aligned}$$

Hence, $\pi_i^{OL} - \pi_i^{CL} < 0$.

Next we show that asymmetric equilibrium in investment strategies is not possible. That is whenever $K_{i0} = K_0 = K_{-i0}$ and investment is positive then $I_{i0}^{CL} = I_{-i0}^{CL}$, and $I_{i0}^{OL} = I_{-i0}^{OL}$. To see this in the OLNE we look at the investment expression, $I_{i0} = \lambda_{i1u}/f$, where

$$\lambda_{i1u} = \delta p[1 + \xi - 2q_{i1u} - q_{-i1u}] = \delta p[1 + \xi - c - 2(K_0 + I_{i0}) - (K_0 + I_{-i0})].$$

Then, we will have

$$\begin{aligned}
I_{i0}^{OL} &= \frac{\delta p[1 + \xi - c - 3K_0 - 2I_{i0}^{OL} - I_{-i0}^{OL}]}{f}, \\
I_{-i0}^{OL} &= \frac{\delta p[1 + \xi - c - 3K_0 - 2I_{-i0}^{OL} - I_{i0}^{OL}]}{f},
\end{aligned}$$

which are clearly symmetric expressions and the only solution is $I_{i0}^{OL} = I_{j0}^{OL}$.

In the CLNE at initial node player i maximizes

$$v_{i0} = v - f I_{i0}^2/2 + \delta p[(K_0 + I_{i0})(1 + \xi - c - 2K_0 - I_{i0} - I_{-i0})],$$

where v is the portion of the profit not involving the investment term. Taking the derivative of this expression with respect to I_{i0} and equating it to zero yield

$$I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 3K_0 - I_{-i0}^{CL}]}{f + 2\delta p}.$$

Similarly, for player j we obtain

$$I_{-i0}^{CL} = \frac{\delta p[1 + d - c - 3K_0 - I_{i0}^{CL}]}{f + 2\delta p}.$$

Clearly these best response functions admit a unique symmetric solution. Hence $I_{i0}^{CL} = I_{-i0}^{CL}$.

Proof of Proposition 3

The proof is similar to the proof of Proposition 2 and is omitted. But we will derive the lower bound of initial capacity, k'' , that entails binding capacity at both upstate demand and the downstate demand by solving $K_{i0} + I_{i0} < q_d^c$. When we insert I_{i0}^{OL} into this inequality, we obtain that $K_{i0} < q_d^c - 2\delta\xi p/f$. When we insert I_{i0}^{CL} into this inequality we obtain $K_{i0} < q_d^c(1 + \delta/f) - 2\delta\xi p/f$. To make the investment levels comparable in these regions we take the minimum of these bounds;

$$\min[q_d^c - 2\delta\xi p/f, q_d^c(1 + \delta/f) - 2\delta\xi p/f] = k''.$$

Proof of Proposition 4

We write the objective function to be maximized by firm i :

$$\begin{aligned} v_{i0} = & q_{i0}(1 - q_{i0} - q_{-i0}) - cq_{i0} - fI_{i0}^2/2 + \delta p[q_{i1u}(1 + \xi - q_{i1u} - q_{-i1u}) - cq_{i1u}] + \\ & \delta(1 - p)[q_{i1d}(1 - \xi - q_{i1d} - q_{-i1d}) - cq_{i1d}] \\ & + \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{i1u}(K_{i0} + I_{i0} - q_{i1u}) + \lambda_{i1d}(K_{i0} + I_{i0} - q_{i1d}). \end{aligned}$$

Under the assumption $k < K_{i0} < q_u^c < K_{-i0}$, clearly firm $-i$ does not invest at all and firm i invests and its investment only provides benefits in the upstate demand.

First we characterize open-loop investment strategy. We optimize the above objective function and obtain that $I_{i0} = \lambda_{i1}/f$, and $\lambda_{i1} = \delta p[1 + \xi - c - 2q_{i1u} - q_{-i1u}]$, where $q_{i1u} = K_{i0} + I_{i0}$, and $q_{-i1u} = (1 + \xi - c - K_{i0} - I_{i0})/2$. Then the OLNE investment will be equal to $I_{i0}^{OL} = \frac{p\delta[1 + \xi - c - 3K_{i0}]}{2f + 3p\delta}$.

Next we characterize Markov perfect equilibrium investments. At the upstate demand $q_{i1u} = K_{i0} + I_{i0}$, and $q_{-i1u} = (1 + \xi - c - K_{i0} - I_{i0})/2$ will hold. At the downstate demand, we have $q_{i1d} = (1 - \xi - c)/3 = q_{-i1d}$. We plug these expressions into the above objective function and maximize with respect to I_{i0} for firm i . The Markov perfect equilibrium investment strategy will be equal to $I_{i0}^{MP} = \frac{p\delta[1 + \xi - c - 2K_{i0}]}{2f + 2p\delta}$. The closed-loop equilibrium investment level will be identical to the Markov perfect investment, because $\frac{\partial q_{-i1u}}{\partial I_{i0}} = -1/2$, $\frac{\partial q_{i1u}}{\partial I_{i0}} = 1$, and $\frac{\partial q_{i1d}}{\partial I_{i0}} = 0$. Clearly, $I_{i0}^{CL} = I_{i0}^{MP} > I_{i0}^{OL}$ holds.

We derive the lower bound of initial capacity, k , that entails binding capacity at the upstate demand for firm i by solving $K_{i0} + I_{i0} > q_d^c$. When we insert I_{i0}^{OL} into this inequality, we obtain that $K_{i0} > q_d^c - \delta\xi p/f$. When we insert I_{i0}^{CL} into this inequality we obtain $K_{i0} > q_d^c(1 - \delta p/2f) - \delta\xi p/f$. To make the investment levels comparable for firm i under both equilibrium concepts, we take the maximum of these bounds;

$$k = \max[q_d^c - \delta\xi p/f, q_d^c(1 - \delta p/2f) - \delta\xi p/f] = q_d^c - \delta\xi p/f.$$

We now show that $\pi_i^{CL} > \pi_i^{OL}$. We have

$$\begin{aligned} \pi_i^{CL} &= \Pi - f(I_{i0}^{CL})^2/2 + \delta p[(K_{i0} + I_{i0}^{CL})(1 + \xi - (K_{i0} + I_{i0}^{CL}) - (1 + \xi - c - K_{i0} - I_{i0}^{CL})/2 - c)], \\ \pi_i^{OL} &= \Pi - f(I_{i0}^{OL})^2/2 + \delta p[(K_{i0} + I_{i0}^{OL})(1 + \xi - (K_{i0} + I_{i0}^{OL}) - (1 + \xi - c - K_{i0} - I_{i0}^{OL})/2 - c)], \end{aligned}$$

where Π is the profit term involving initial node and node d in period 1. The profit difference is thus given by

$$\begin{aligned} \pi_i^{OL} - \pi_i^{CL} &= -f((I_{i0}^{OL})^2 - (I_{i0}^{CL})^2)/2 + \delta p[(K_{i0} + I_{i0}^{OL})(1 + \xi - (K_{i0} + I_{i0}^{OL}) - c) \\ &\quad - (K_{i0} + I_{i0}^{CL})(1 + \xi - (K_{i0} + I_{i0}^{CL}) - c)]/2. \end{aligned}$$

Let

$$\begin{aligned} A &= -f((I^{OL})^2 - (I^{CL})^2)/2, \\ B &= \delta p[(K_0 + I^{OL})(1 + \xi - (K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - (K_0 + I^{CL}) - c)]/2. \end{aligned}$$

A is positive because $(I^{OL} - I^{CL})(I^{OL} + I^{CL}) < 0$ because $I^{OL} < I^{CL}$. It is easy to check that B reduces to

$$B = \delta p[(I^{OL} - I^{CL})(1 + \xi - c - 2K_0 - (I^{OL} + I^{CL}))].$$

Now, note that

$$\begin{aligned} \pi_i^{OL} - \pi_i^{CL} &= A + B, \\ &= (I^{MP} - I^{OL})(I^{OL} - I^{CL})(\delta p/2 + f/2), \end{aligned}$$

which is negative, and hence $\pi_i^{OL} < \pi_i^{CL}$.

We next show that $\pi_{-i}^{CL} < \pi_{-i}^{OL}$ for player $-i$. Similar to the profit difference for player i , the profit difference for player $-i$ under both equilibria boils down to

$$\pi_{-i}^{OL} - \pi_{-i}^{CL} = (I^{CL} - I^{OL})(2 + 2\xi - 2c - 2K_0 + I^{CL} - I^{OL}).$$

Note that the investment levels I^{CL} , I^{OL} are the investments made by player $-i$. The difference is positive because both the first term and the second term on the right hand side are positive.

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