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## Appendix Developing Attribute Importance and Value System

The main purpose of the following notes is to provide the intuition and mechanics of creating a value system for a reasonable product. This is accomplished by using several illustrative examples. In the first example, various steps involved in deriving a value system is provided. For the next two examples, the reader is asked to complete series of tasks to derive his / her own value system. There are four steps to derive a value system when a ranking and / or rating task is to be completed. These are

1. Rank (Rate) the provided profiles. One would expect that each reader will provide a different set of ranking in this task. Consequently, resulting value system will be different for each reader.
2. Transfer the above ranks to the scoring form.
3. Compute the averages and relative importance scores.
4. Interpret and assess the resulting value system.

When responses are in the form of ratings, derivation and interpretation of a value system is the same as that of rank order responses. That is, although rating responses might be easy to gather, evidence in literature indicates that ranking and rating responses may be treated with the same statistical operations. When responses are in the form of choices, intuition and interpretation associated with the value system are similar to ranking or rating forms of responses. However, precise derivation of a value system is complicated and mathematically demanding. Consequently, in the example dealing with choices, we sacrifice mathematical precision and explain and explore nature of value system derivation.

### Travel Radio - Complete Example

Suppose you are interested in buying a travel radio. A mall has 16 alternative radios. Each alternative is described in terms of brand name, price (\$24.95 to \$34.95), warranty (1 year or 3 years) and type of tuner (analog or digital). Rank order these products from most likely to buy (indicated by a "16") to least likely to buy ("1"). Following handwritten ranks were provided by one of our student at the University of Guelph.

- A . 6 Goldstar, price \$24.95, 1 year warranty, analog tuner.
- B . 15 Panasonic, price \$24.95, 3 year warranty, digital tuner.
- C . 7 Sony, price \$27.95, 3 year warranty, analog tuner.
- D . 2 General Electric (GE), price \$30.95, 1 year warranty, analog tuner.
- E . 14 Panasonic, price \$30.95, 1 year warranty, digital tuner.
- F . 8 Panasonic, price \$27.95, 1 year warranty, analog tuner.
- G . 12 Goldstar, price \$34.95, 1 year warranty, digital tuner.
- H . 9 Sony, price \$34.95, 1 year warranty, analog tuner.
- I . 16 Sony, price \$24.95, 1 year warranty, digital tuner.
- J . 10 GE, price \$27.95, 1 year warranty, digital tuner.
- K . 4 Panasonic, price \$34.95, 3 year warranty, analog tuner.
- L . 11 Goldstar, price \$27.95, 1 year warranty, digital tuner.
- M . 13 Sony, price \$30.95, 3 year warranty, digital tuner.
- N . 5 GE, price \$24.95, 3 year warranty, digital tuner.
- O . 1 GE, price \$34.95, 3 year warranty, analog tuner.
- P . 3 Goldstar, price \$30.95, 3 year warranty, analog tuner.

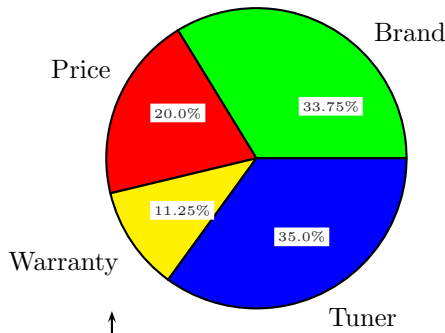
**Scoring Form for Travel Radio Value System**

| Attributes & Levels |           | Ranks from previous page |       |       |       |       |       |       |       | Average rank |
|---------------------|-----------|--------------------------|-------|-------|-------|-------|-------|-------|-------|--------------|
| Brand               | Goldstar  | A. 6                     | G. 12 | L. 11 | P. 3  |       |       |       |       | 8.00         |
|                     | Panasonic | B. 15                    | E. 14 | F. 8  | K. 4  |       |       |       |       | 10.25        |
|                     | Sony      | C. 7                     | H. 9  | I. 16 | M. 13 |       |       |       |       | 11.25        |
|                     | GE        | D. 2                     | J. 10 | N. 5  | O. 1  |       |       |       |       | 4.50         |
| Price               | \$24.95   | A. 6                     | B. 15 | I. 16 | N. 5  |       |       |       |       | 10.50        |
|                     | \$27.95   | C. 7                     | F. 8  | J. 10 | L. 11 |       |       |       |       | 9.00         |
|                     | \$30.95   | D. 2                     | E. 14 | M. 13 | P. 3  |       |       |       |       | 8.00         |
|                     | \$34.95   | G. 12                    | H. 9  | K. 4  | O. 1  |       |       |       |       | 6.50         |
| Warranty            | 1 year    | A. 6                     | D. 2  | E. 14 | F. 8  | G. 12 | H. 9  | I. 16 | J. 10 | 9.63         |
|                     | 3 year    | B. 15                    | C. 7  | K. 4  | L. 11 | M. 13 | N. 5  | O. 1  | P. 3  | 7.38         |
| Tuner               | Digital   | B. 15                    | E. 14 | G. 12 | I. 16 | J. 10 | L. 11 | M. 13 | N. 5  | 12.00        |
|                     | Analog    | A. 6                     | C. 7  | D. 2  | F. 8  | H. 9  | K. 4  | O. 1  | P. 3  | 5.00         |

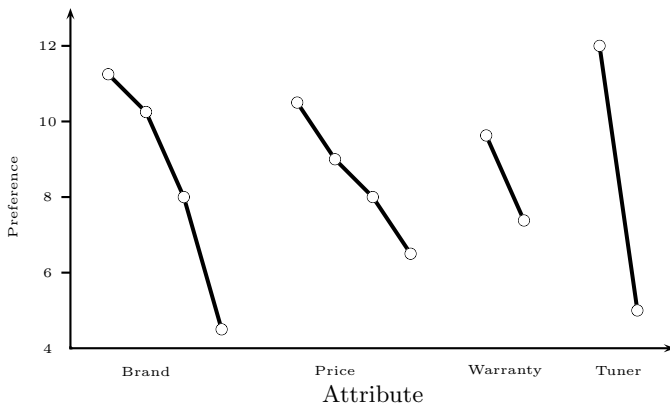
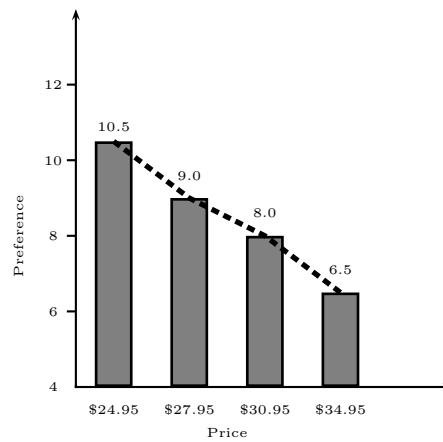
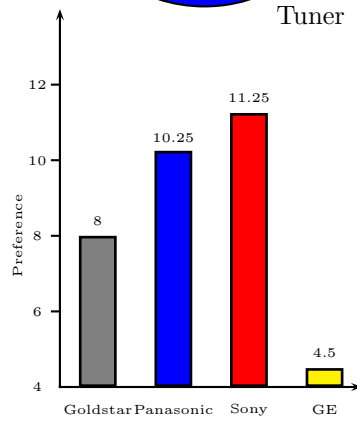
**Summary of Average Ranks**

| Measure                 | Attributes |        |          |        |
|-------------------------|------------|--------|----------|--------|
|                         | Brand      | Price  | Warranty | Tuner  |
| Minimum                 | 4.50       | 6.50   | 7.38     | 5.00   |
| Maximum                 | 11.25      | 10.50  | 9.63     | 12.00  |
| Difference              | 6.75       | 4.00   | 2.25     | 7.00   |
| Importance <sup>1</sup> | 33.75%     | 20.00% | 11.25%   | 35.00% |

<sup>1</sup>Importance obtained by summing all differences. In this case, the sum is 20. Then, divide each difference by the sum. For example,  $33.75 = \frac{6.75}{20} \times 100$ .



In examining the importance associated with the four attributes, the most important attributes for this individual is tuner followed by brand. Price is the third most important and warranty is the least important for this individual. Although one might be satisfied to know the relative ordering of these attributes, we are able to quantify the magnitude as well. Thus, tuner and brand attributes are very close in their importance. One might argue that such individual, would be indifferent to “poor brand” with digital tuner and “better brand” with analog tuner.



In examining the preference for alternative brands, Sony is the preferred brand followed (closely) by Panasonic. Note that GE is the least preferred for this individual. Consider a GE radio priced at \$24.95 compared to Goldstar priced at \$34.95. According to the value system derived here, we would conclude that GE is slightly preferred (preference score of  $10.5 + 4.5 = 15$ ) to Goldstar at a premium price (preference score of  $8 + 6.5 = 14.5$ ). Thus, GE is perceived as inferior to other brands for this individual. Note that \$10 discount (maximum price variation in this study) by GE is not enough to make this individual switch to Sony or Panasonic. It also should be noted that

for this individual that warranty rankings are in reverse order, suggesting that a longer term warranty is less preferred to a shorter term warranty. Such reversals often indicate that respondents may not have paid sufficient attention to such attributes.

There is an alternative approach to summarize attribute importance for all attributes which is provided in the figure above. It is clear from the figure that “tuner” is the most important attribute followed by brand and price.

### Which Pizza would you purchase?

Suppose you are interested in buying a Pizza. There are four stores located in this area and each store offers three toppings pizza. Each alternative is described in terms of brand name (Pizza Hut, Pizza Pizza, Dominos and Little Caesars), type of crust (thick or thin), price (\$8.99 to \$11.99), size (medium or large) and delivery time (20 or 30 minutes). Rank order these products from most likely to buy (indicated by a “16”) to least likely to buy (“1”). Enter these 16 ranks in the spaces to the left of each product description. Please do your best to make sure these ranks accurately reflect your preferences for these 16 product alternatives.

- A . \_\_\_\_\_ Pizza Hut, thick crust, medium size, price \$8.99, delivered in 20 mins.
- B . \_\_\_\_\_ Pizza Pizza, thick crust, large size, price \$8.99, delivered in 30 mins.
- C . \_\_\_\_\_ Dominos, thin crust, large size, price \$9.99, delivered in 20 mins.
- D . \_\_\_\_\_ Little Caesars, thick crust, medium size, price \$10.99, delivered in 20 mins.
- E . \_\_\_\_\_ Pizza Pizza, thin crust, medium size, price \$10.99, delivered in 30 mins.
- F . \_\_\_\_\_ Pizza Pizza, thick crust, medium size, price \$9.99, delivered in 20 mins.
- G . \_\_\_\_\_ Pizza Hut, thin crust, medium size, price \$11.99, delivered in 30 mins.
- H . \_\_\_\_\_ Dominos, thick crust, medium size, price \$11.99, delivered in 20 mins.
- I . \_\_\_\_\_ Dominos, thin crust, medium size, price \$8.99, delivered in 30 mins.
- J . \_\_\_\_\_ Little Caesars, thin crust, medium size, price \$9.99, delivered in 30 mins.
- K . \_\_\_\_\_ Pizza Pizza, thin crust, large size, price \$11.99, delivered in 20 mins.
- L . \_\_\_\_\_ Pizza Hut, thick crust, large size, price \$9.99, delivered in 30 mins.
- M . \_\_\_\_\_ Dominos, thick crust, large size, price \$10.99, delivered in 30 mins.
- N . \_\_\_\_\_ Little Caesars, thick crust, large size, price \$8.99, delivered in 30 mins.
- O . \_\_\_\_\_ Little Caesars, thin crust, large size, price \$11.99, delivered in 20 mins.
- P . \_\_\_\_\_ Pizza Hut, thin crust, large size, price \$10.99, delivered in 20 mins.

**Scoring Form for Pizza**

| Attributes & Levels |                | Enter your ranks from previous page |    |    |    |    |    |    |    | Average rank |
|---------------------|----------------|-------------------------------------|----|----|----|----|----|----|----|--------------|
| Brand               | Pizza Hut      | A.                                  | G. | L. | P. |    |    |    |    |              |
|                     | Pizza Pizza    | B.                                  | E. | F. | K. |    |    |    |    |              |
|                     | Dominos        | C.                                  | H. | I. | M. |    |    |    |    |              |
|                     | Little Caesars | D.                                  | J. | N. | O. |    |    |    |    |              |
| Crust               | thick          | A.                                  | B. | D. | F. | H. | L. | M. | N. |              |
|                     | thin           | C.                                  | E. | G. | I. | J. | K. | O. | P. |              |
| Size                | medium         | A.                                  | D. | E. | F. | G. | H. | I. | J. |              |
|                     | Large          | B.                                  | C. | K. | L. | M. | N. | O. | P. |              |
| Price               | \$8.99         | A.                                  | B. | I. | N. |    |    |    |    |              |
|                     | \$9.99         | C.                                  | F. | J. | L. |    |    |    |    |              |
|                     | \$10.99        | D.                                  | E. | M. | P. |    |    |    |    |              |
|                     | \$11.99        | G.                                  | H. | K. | O. |    |    |    |    |              |
| Delivery time       | 20 mins.       | A.                                  | C. | D. | F. | H. | K. | O. | P. |              |
|                     | 30 mins.       | B.                                  | E. | G. | I. | J. | L. | M. | N. |              |

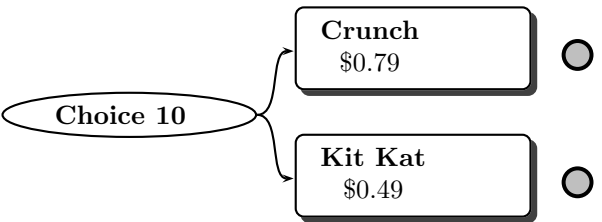
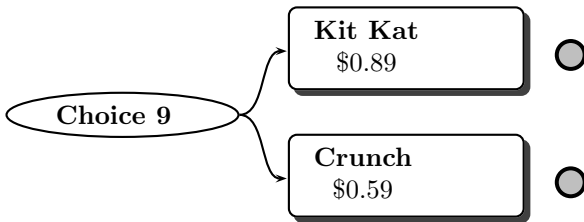
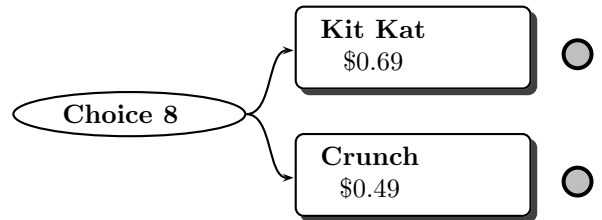
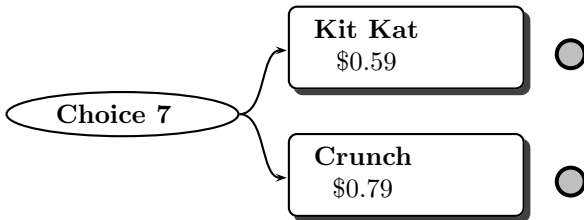
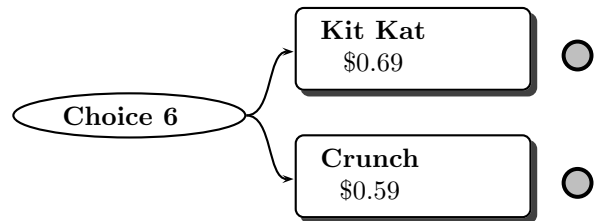
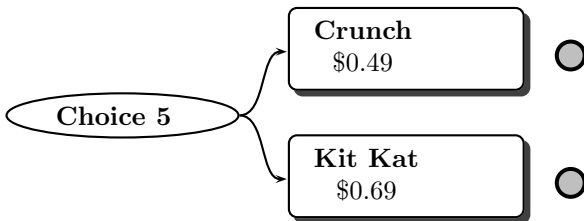
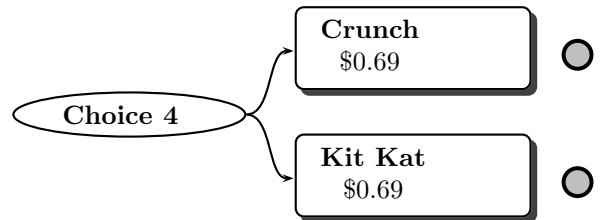
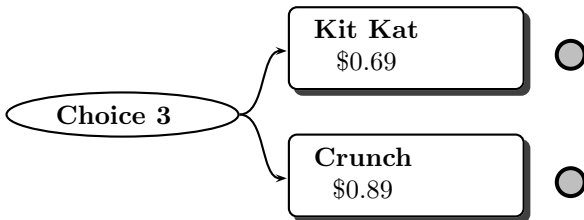
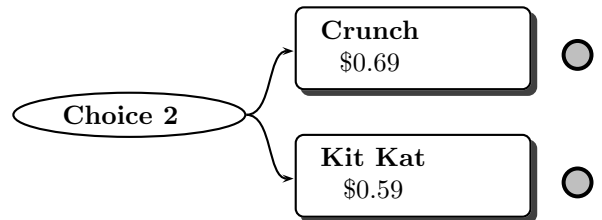
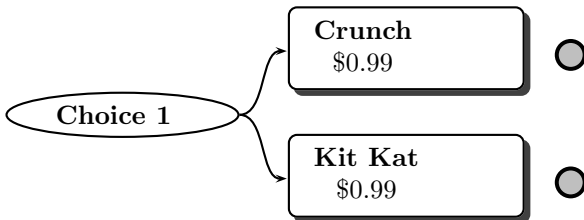
**Summary of Average Ranks**

| Measure                 | Attributes |       |      |       |           |
|-------------------------|------------|-------|------|-------|-----------|
|                         | Brand      | Crust | Size | Price | Del. Time |
| Minimum                 |            |       |      |       |           |
| Maximum                 |            |       |      |       |           |
| Difference              |            |       |      |       |           |
| Importance <sup>1</sup> |            |       |      |       |           |

<sup>1</sup>Importance obtained by summing all differences and then dividing each difference by the sum.

### Which brand of chocolate bar would you choose?

You are about to buy a chocolate bar for a snack. The place you are in offers two major brands. Your task is to choose a brand in each of the following situations. You should note at the outset that the price of the chocolate bars varies from a low of \$0.49 to a high of \$0.99. In each of ten situations, choose only one alternative and check mark your chosen alternative.



Suppose you are asked to make choices for above experiment. Please write down your responses before we examine how these may be used to derive the value system.

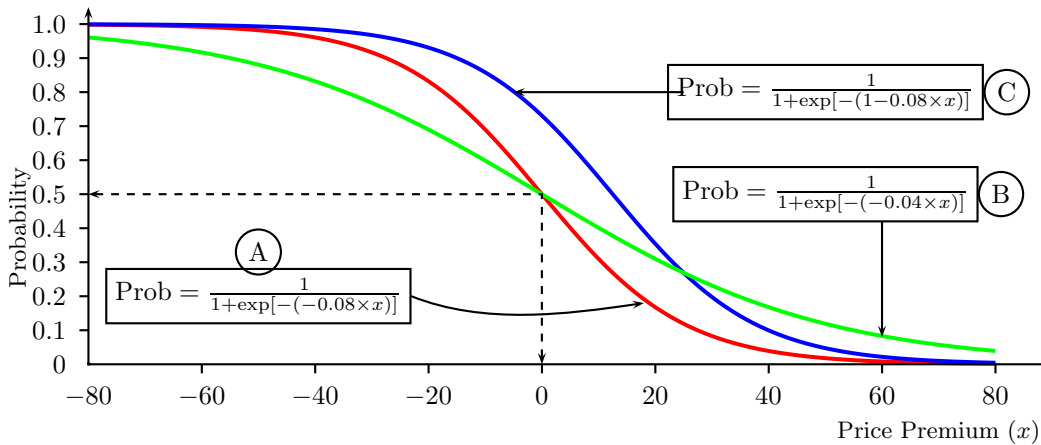
**Scoring Form for Chocolate Bar**

|                      | Choice Situation with Discount            |   |   |   |   |   | Choice Situation with Premium             |   |   |    |
|----------------------|---|---|---|---|---|---|---|---|---|----|
|                      | 1   | 4 | 5 | 6 | 8 | 9 | 2   | 3 | 7 | 10 |
| Crunch Chosen        |   |   |   |   |   |   |   |   |   |    |
| Total                | $n_c =$ out of 6                          |   |   |   |   |   | $n_d =$ out of 4                          |   |   |    |
| Compute <sup>1</sup> | $\log \left[ \frac{n_c}{6 - n_c} \right]$ |   |   |   |   |   | $\log \left[ \frac{n_c}{4 - n_c} \right]$ |   |   |    |

If  $n_c = 6$ , then set  $n_c = 5.999$ . Similarly, if  $n_d = 4$ , then set  $n_d = 3.999$ .

**Statistical Reasoning**

Consider a reader who chose Kit Kat (*K*) for the first choice and did not choose Crunch (*C*). The same reader chose *C, K, C, C, C, K, C, C* and *K* for the remaining nine choices. In examining above choices, the reader appears to prefer Crunch over Kit Kat because in six of 10 times, this individual has chosen Crunch. One may be tempted by the conclusion that such individual is loyal to Crunch. Note, however, this loyalty notion is conditional on a set of prices that were used in our experiment. One way to account of changing prices (over choice situations) is to look at price differences and then calibrate the logistic curve as shown below. Note that all curves indicate that the likelihood of choosing Crunch decreases as the premium associated with Crunch is increased. Three curves exhibit different types of respondent behaviour. Two curves that are symmetric about zero and intersect each other at zero (curves labeled (A) and (B)), indicate the behaviour of two individuals with no brand preference. Moreover, the one with the steeper slope has a moderate level of price sensitivity (price elasticity of  $-1.33$ ) and one flatter slope has a moderate level of price insensitivity (price elasticity of  $-0.55$  at a price premium of 20 cents). The curve labelled (C) illustrates an individual with moderate price sensitivity who is willing to pay a small price premium to buy Crunch.



As one might expect, calibrating such a curve for an individual is a complicated process, especially when the

number of attributes (price, weight and chocolate bar content) is also varied systematically. However, we can gain an intuition of such calibration, if we sort choices by price premium for buying Crunch.

**Price Premium for Crunch and Choices**

| Price Premium | Choice Situation | Chosen Crunch |
|---------------|------------------|---------------|
| -30           | 9                | Yes           |
| -20           | 5                | Yes           |
| -20           | 8                | Yes           |
| -10           | 6                | Yes           |
| 0             | 1                | No            |
| 0             | 4                | Yes           |
| +10           | 2                | Yes           |
| +20           | 3                | No            |
| +20           | 7                | No            |
| +30           | 10               | No            |

When we sorted our choices, reader responses appeared systematic and coherent with an exception<sup>1</sup> about choice situation 1. This reader, as we might expect prefers Crunch over Kit Kat only when the price premium for Crunch is less than or equal to 10 cents. Suppose, for illustrative purpose, we ignore premium amount and postulate that we want to calibrate the value system with two components, the likelihood of choosing Crunch when it is at par ( $p_c$ ) or discounted compared to Kit Kat. Then, the second component would be the likelihood of choosing Crunch when it is at a premium ( $p_d$ ). That is, we are interested in two numbers, one that indicates the consumer's brand preference and the other one indicates the consumer's price sensitivity. Unlike rating or ranking responses, we know that the probabilities must always range from zero to one. Consequently, we need to obtain these numbers such that

the resulting probabilities are always constrained. Intuitively, we would expect  $\hat{p}_c$  to be equal to  $\frac{5}{6} = 0.825$  and  $\hat{p}_d$  to  $\frac{1}{4} = 0.25$ . Note that if we ignore the price premiums, we would get the overall likelihood that Crunch is chosen ( $p$ ) which in this case is  $\hat{p} = 0.6$ . Let us see whether we get these results using idea of the maximum likelihood estimation. Let us denote joint likelihood that Crunch is chosen over 10 choice situation by ignoring price premiums by  $\mathcal{L}_0$  and the likelihood that Crunch is chosen and  $\mathcal{L}_1$  when we account for price premium. Note that

$$\begin{aligned} \mathcal{L}_0 &= (1-p) \times p \times (1-p) \times p \times p \times p \times (1-p) \times p \times p \times (1-p) \\ &= p^6 \times (1-p)^4 \quad \text{or in logarithms} \\ \mathcal{L}\mathcal{L}_0 &= 6 \times \log(p) + 4 \times \log(1-p) \end{aligned}$$

The maximum of such function does exist at point  $\hat{p} = 0.6$  and the maximum of  $\mathcal{L}\mathcal{L}_0$  is equal to  $-6.73$ . Let us now look at what would happen when we separate discount and premium choices.

$$\begin{aligned} \mathcal{L}_1 &= (1-p_c) \times p_d \times (1-p_d) \times p_c \times p_c \times p_c \times (1-p_d) \times p_c \times p_c \times (1-p_d) \\ &= p_c^5 \times (1-p_c) \times p_d \times (1-p_d)^3 \quad \text{or in logarithms} \\ \mathcal{L}\mathcal{L}_1 &= 5 \times \log(p_c) + \log(1-p_c) + \log(p_d) + 3 \times \log(1-p_d) \end{aligned}$$

The maximum of such function exists at point  $\hat{p}_c = 0.825$  and  $\hat{p}_d = 0.25$ . That is, when Crunch is offered with price discounts our respondent is likely to choose Crunch over Kit Kat. Furthermore, the maximum for the logarithm of likelihood function at such a point is  $-5.53$ . There is also a statistical test to determine whether adding separate parameter for price discount improves the fit of the model. This statistical test is given by:

$$\chi^2 = -2 \times (\mathcal{L}\mathcal{L}_0 - \mathcal{L}\mathcal{L}_k)$$

where  $k$  different estimates (one in our case). This statistic is distributed with  $\chi^2$  with  $k$  degrees of freedom. For our example,  $\chi^2$  is 2.4 and we would not reject the null hypothesis at a probability level of 0.05 that parameters are equal to zero for with / without discount (Critical value of  $\chi^2$  is 3.84 at a probability of 0.05 and 1 degree of freedom). Although the above described approach gives an idea of the value system, it does not calibrate

<sup>1</sup>Note that both brands are priced at \$0.99 and it may be that respondent does not prefer either alternatives.

the logistic function described above. One approach to directly calibrate the logistic function is to state the likelihoods,  $p$ ,  $p_c$  and  $p_d$  in terms of logarithms of odds ratio, that is,  $\log\left[\frac{p}{1-p}\right]$ . Thus, the log odds ratio when the likelihood is 0.6 is equal to 0.4055. Similarly, we can calibrate log odds ratios for other likelihoods. With some algebraic manipulations, it can be shown that

$$\text{prob}(\text{Crunch is chosen}) = \frac{1}{1 + \exp(-b)}$$

where  $b$  is the value of the log odds ratio (or 0.4055 in our case). The same equation with price premium (0 = No premium and 1 = Some premium) would be

$$\text{prob}(\text{Crunch is chosen}) = \frac{1}{1 + \exp[-(1.6094 - 2.7081 \times \text{Price Premium})]}.$$

Although the above expression looks complicated, the estimated numbers are the same as  $\log\left[\frac{p_c}{1-p_c}\right]$  and since  $p_c = 0.8333$ , we get the log odds ratio of 1.6094. Similarly,  $\log\left[\frac{p_d}{1-p_d}\right] = -1.0986$  and thus estimate associated with Price Premium is  $-1.6094 - 1.0986 = -2.7081$ . From this we would conclude that this particular respondent is price sensitive as well as one who prefers Crunch chocolate bar.

We noted in the beginning of this section that the above procedure to calibrate brand preference and price sensitivity parameters is mathematically imprecise, yet reasonable. If we were to calibrate the above parameter using statistical software, we would obtain the following:

$$\text{prob}(\text{Crunch is chosen}) = \frac{1}{1 + \exp[-(1.066 - 0.159 \times \text{Price Diff. in cents})]}.$$

One might be tempted to reject the intuitive approach as incorrect but that is not the case. Note that the indicator variable whether price premium is required or not is a variable that takes only two values. On the other hand, the price difference in cents varies between  $-30$  to  $+30$ . This change in range affects the calibrated estimates.