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Abstract

We perform a large-scale analysis to evaluate the performance of traditional and Markov-switching GARCH models for the volatility of 292 cryptocurrencies. For each cryptocurrency, we estimate a total of 27 alternative GARCH specifications. We consider models that allow up to three different regimes. First, the models are compared in terms of goodness-of-fit using the Deviance Information Criterion and the Bayesian Predictive Information Criterion. Next, we evaluate the ability of the models in forecasting one-day ahead conditional volatility and Value-at-Risk. The results indicate that for a wide range of cryptocurrencies, time-varying models outperform traditional ones.

Keywords: Bitcoin, Cryptocurrency, Volatility, GARCH, Markov-switching, Information criteria.

JEL Codes: C12, C13, C15, C22

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1 Introduction

The introduction of Bitcoin (BTC) Nakamoto (2009) spurred the creation of new cryptocurrencies, with their current number exceeding 5000. Digital currencies facilitate electronic payments without the need of a bank (or other third party) intermediation.[1] According to Yermack (2013), Bitcoin cannot function as money, due to its nearly fixed supply, as Bitcoin mining is an energy consuming activity. However, there are similarities with exhaustible commodity resources (see the discussion in Gronwald (2019)). Furthermore, Bitcoin and digital currencies have additional features, such as hedging, diversifying and safe haven capabilities that make them appealing as an asset, Dyhrberg (2016a) and Dyhrberg (2016b); Bouri et al. (2017c).

Bitcoin like many financial assets exhibit volatility clustering and structural breaks in their volatility dynamics. Ignoring these features could negatively impact on the precision of volatility forecasts; see Lamoureux and Lastrapes (1990) and Bauwens et al. (2014b). Since cryptocurrencies share similarities with other assets, one could expect that the volatility dynamics of cryptocurrencies also suffer from structural breaks. Recent studies examine the time-varying behaviour of Bitcoin returns and their volatility. Evidence from these reveals the presence of regime changes in the volatility of Bitcoin returns.[2] For example, Ardia et al. (2019b) argue that Markov-switching GARCH (MS-GARCH) models outperform single-regime specifications at predicting the one-day ahead Value at Risk (VaR).

The existing literature focuses on the analysis of Bitcoin and a limited number of major cryptocurrencies. This is, to some degree, justifiable since Bitcoin is the most traded digital currency in terms of market capitalisation. There are , however, other digital currencies that play an important role as speculative assets. The differences between cryptocurrencies in terms of market capitalisation and investors' attention suggests that different modelling approaches should be employed in each case. In

[1]Bitcoin also offers anonymity and low transactions fees, Panagiotidis et al. (2018).

[2]The next section offers a review of the related literature.

this paper, we perform an in-depth analysis of the volatility dynamics across a whole tranche of cryptocurrencies.

We examine the presence of regime changes in the volatility of various cryptocurrencies using MSGARCH models. We consider a total of 292 cryptocurrencies including Bitcoin, Ethereum, Ripple and Tether which dominate the market of cryptocurrencies and capture the main bulge of the total market capitalisation.[3] We employ three traditional GARCH-type models, the GARCH, TGARCH (threshold GARCH) and EGARCH (exponential GARCH). For each model we consider three cases depending on the number of regimes (one, two and three regimes) and three cases depending on the specification of the conditional distribution (the Normal distribution, the Generalised Error Distribution (GED) and the Student's-t distribution). In total, for each currency, we estimate 27 alternative specifications of MSGARCH-type models. First, we compare the models in terms of goodness-of-fit using the Deviance Information Criterion (DIC) and the Bayesian Predictive Information Criterion (BPIC). Next, we asses the out-of-sample one-step-ahead volatility and density risk forecasting performance of the models. We evaluate the models performance using the mean squared (MSE), the mean absolute (MAE) errors (for volatility forecasts), the conditional coverage test (CC) of Christoffersen (1998) and the dynamic quantile test (DQ) of Engle and Manganelli (2004) (for risk forecasts). The models are estimated using Bayesian Markov chain Monte Carlo (MCMC) procedures. We choose the Bayesian approach since assessment of financial risk requires employing state-of-the-art econometric methodologies. We aim to compare the performance of MSGARCH models to their single-regime counterparts and to identify patterns for the cryptocurrencies based on the models that best describe them.

The general conclusion that emerges from the analysis is that there is no one-model-fits-all solution in terms of modelling volatility cryptocurrencies as different GARCH-type models are found to be more suitable for different cryptocurrencies. More specifically, our analysis provide three main findings. First, MSGARCH models

[3]Throughout the paper the term 'big cryptocurrency' is used in terms of market capitalisation, at the time the study was conducted.

provide better results compared to their traditional counterparts. In-sample analysis suggests that MSGARCH models provide better goodness-of-fit results more than 50% of the examined cryptocurrencies and out-of-sample analysis indicates that MSGARCH models provide better forecasts more than 60% (in some cases more than 70%) of times. Second, EGARCH models are selected less times by the criteria compared to GARCH and TGARCH models. More formally, for a given number of regimes and a given conditional distribution, we obtain worse results (both in- and out-of-sample) from the EGARCH model compared to the non-exponential models. Finally, the analysis of two asymmetric models (the TGARCH and the EGARCH) indicates the presence of inverse leverage effect in most of the examined cryptocurrencies. That is, positive past returns affect the volatility of cryptocurrencies more than negative past returns.

The rest of the paper proceeds as follows: Section 2 discusses the related literature, the third one describes the econometric methodology, section 4 presents the main findings and the last one concludes.

2 Literature review

Despite the large number of active cryptocurrencies, the existing literature focuses on Bitcoin and some highly traded cryptocurrencies. One of the few exceptions is the work of Chu et al. (2017) where the seven most popular cryptocurrencies are considered. For each cryptocurrency, twelve GARCH models were fitted and compared based on five criteria.

Polasik et al. (2015) divide the Bitcoin analysis in four categories, one of which focuses on economic and financial issues from both theoretical and empirical perspective.[4] From the perspective of finance, Urquhart (2016) studies the market efficiency of Bitcoin and finds that the results were affected by the sampling period. When examining the whole sample, the results indicate that the Bitcoin market is not weakly

[4]For a detailed review on the Bitcoin literature see Panagiotidis et al. (2019).

efficient. When the sample was split in two periods, the results reveal that informational inefficiency was caused by the first subsample. Bariviera (2017), Bouri et al. (2017b) and Nadarajah and Chu (2017) also find that Bitcoin is in line with the efficient market hypothesis.

Bitcoin is considered an asset which can used for speculative purposes, Baek and Elbeck (2015); Kristoufek (2014); Dyhrberg (2016b); Blau (2017) and Corbet et al. (2018). This can lead to extreme volatility and bubbles (see Fry and Cheah (2016)). These characteristics are closely related to speculation (Ahamed (2009), Reinhart and Rogoff (2009)). Williams (2014) shows that the volatility of Bitcoin price is seven times greater than that of gold. In addition, Baur et al. (2018) argue that returns, volatility and correlation characteristics of Bitcoin are distinctively different compared to gold and the US dollar.[5]

A large part of the related literature examines the volatility dynamics of Bitcoin. GARCH-type specifications provide the best in-sample performance according to Chu et al. (2017).[6] Glaser et al. (2014) employ the GARCH(1,1) model. Gronwald (2014) uses an autoregressive jump-intensity GARCH model and finds that Bitcoin prices are particularly marked by extreme price movements. Bouoiyour and Selmi (2015) examine the goodness-of-fit of GARCH model over two different periods, 2010 to 2015 and the first half of 2015. During the first interval, the threshold-GARCH estimates reveal the long duration of persistence. For the second period, the EGARCH is selected, displaying less volatility persistence. Klein et al. (2018) also focus on volatility persistence using the Fractionally Integrated APARCH (FIGARCH) to model volatility of BTC. A number of papers examine the leverage effect on BTC volatility. Bouri et al. (2017a) find a negative relationship between the past shocks and volatility of Bitcoin before the first bubble burst in 2013 and no significant relation after. Katsiampa (2017) evaluates the performance of six different GARCH models using information criteria and selects

[5]This paper is a replication and extension of Dyhrberg (2016a) who suggest that, in terms of volatility, the Bitcoin shares similarities with both gold and the dollar.

[6]HAR models are also used in Urquhart (2017).

the Asymmetric Component GARCH model as the most appropriate. Stavroyiannis (2018) implements a GJR-GARCH model to examine the VaR and related measures for the Bitcoin. The findings suggest that Bitcoin is a highly volatile currency which violates the VaR measures more than the other assets (i.e. gold). Finally, Ardia et al. (2019b) use MSGARCH models to investigate the presence of regime changes in the volatility dynamics of Bitcoin. They find that that asymmetric MSGARCH models perform better than their single-regime counterparts.

The work of Ardia et al. (2019b) is not the only one to employ regime-switching models to model volatility of digital currencies. Caporale and Zekokh (2019) fit more than 1000 MSGARCH models to four cryptocurrencies and forecast one-day ahead VaR and expected shortfall. Mensi et al. (2019) support the existence of dual long memory in returns and volatility of Bitcoin and Ethereum. Cheikh et al. (2020) implement a Smooth Transition GARCH model to study the volatility dynamics of four major cryptocurrencies. Their findings support the existence of inverse leverage effect and safe haven hypothesis. Ma et al. (2020) propose a Markov regime-switching mixed-data sampling which improves the prediction accuracy of the realised variance of Bitcoin. Finally, Maciel (2021) argue that MSGARCH models provide better expected shortfall and VaR forecasts compared to their single-regime counterparts.

At the same time, Bayesian approaches have been employed in financial risk modelling. Geweke and Amisano (2010) compare Bayesian predictive distributions from five alternative models (including an ARCH model). Bauwens et al. (2010) proposed a MSGARCH model, estimated using a Bayesian MCMC algorithm. Bauwens et al. (2014a) developed an estimation and forecasting method, based on a differential evolution MCMC method, for inference in GARCH models that allow for an unknown number of structural breaks at unknown dates. Balcombe and Fraser (2017) employ Bayesian Markov switching models to examine nine bubble containing series (including Bitcoin returns). Nearly all series appear to display strong regime switching. Ardia et al. (2017) compare the risk forecast performance of several GARCH-type models estimated via Maximum Likelihood (ML) and Bayesian techniques and find that Bayesian predictive densities improve the VaR backtest. Finally, Thies and Molnár (2018) utilise a Bayesian change point model to study structural breaks in average returns and volatility of Bitcoin. They find that regimes with higher volatility have higher average returns, however, the most volatile regime is the only regime with negative average returns.

3 Methodology

3.1 Estimation

Let y_t denote the returns of a cryptocurrency at day t. We assume $\mathbf{E}[y_t] = 0$ and $\mathbf{E}[y_t y_{t-l}] = 0$, that is the series has zero mean and are serially uncorrelated. We ensure that these assumptions are met by filtering the series with an AR(1) model. We follow Ardia et al. (2018b) and express the general MSGARCH specification as:

$$y_t|(s_t = k, I_{t-1}) \sim D(0, h_{k,t}, \xi_k), \quad (1)$$

where $D(0, h_{k,t}, \xi_k)$ denotes a continuous distribution with zero mean, time-varying conditional variance $h_{k,t}$ and additional shape parameters gathered in the vector ξ_k and I_{t-i} is the information set observed up to time t. The stochastic variable s_t defined on the discrete space $\{1, \ldots, K\}$ evolves according to an unobserved first-order ergodic homogeneous Markov chain with $K \times K$ transition probability matrix $\mathbf{P} = (p_{i,j})$, where $p_{i,j}$ denotes the probability of a transition from state $s_{t-1} = i$ to state $s_t = j$. In the empirical analysis we allow up to $K = 3$ regimes. The standardised innovations are defined as $\eta_{k,t} = y_t / h_{k,t}^{1/2} \sim D(0, 1, \xi_k)$.

Conditionally on regime k, the conditional variance $h_{k,t} = h(y_{t-1}, h_{k,t-1}, \theta_k)$ follows a GARCH-type model, as in Haas et al. (2004).[7] We consider three specifications. The GARCH Bollerslev (1986), TGARCH Zakoian (1994) and EGARCH Nelson (1991). The

[7] θ_k is the vector of additional regime-dependent parameters.

models are described by the following equations:

$$\text{GARCH:} \quad h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_k h_{k,t-1}, \tag{2}$$

$$\text{EGARCH:} \quad \ln(h_{k,t}) = \alpha_{0,k} + \alpha_{1,k}(|\eta_{k,t-1}| - \mathbf{E}[|\eta_{k,t-1}|]) + \alpha_{2,k}\eta_{t-1} + \beta_k \ln(h_{k,t-1}), \tag{3}$$

$$\text{TGARCH:} \quad h_{k,t}^{1/2} = \alpha_{0,k} + \alpha_{1,k} y_{t-1} \mathbb{I}\{y_{t-1} \geq 0\} + \alpha_{2,k} y_{t-1} \mathbb{I}\{y_{t-1} < 0\} + \beta_k h_{k,t-1}^{1/2}, \tag{4}$$

where $\mathbb{I}$ is the indicator function. For the conditional distribution we use the Normal, the GED and the Student's-t distribution.[8] Positivity and covariance stationarity conditions apply for all models. To ensure positivity in the GARCH and TGARCH models, we require that all coefficients are positive. No constraints are necessary in the EGARCH model. Covariance stationarity is obtained for the GARCH model by requiring that $\alpha_{1,k} + \beta_k < 1$. For the TGARCH model the constraint is $\alpha_{1,k}^2 + \beta_k^2 - 2\beta_k(\alpha_{1,k} + \alpha_{2,k}\mathbf{E}[\eta_{k,t}\mathbb{I}\{\eta_{k,t} < 0\}] - \alpha_{1,k}^2 - \alpha_{2,k}^2)\mathbf{E}[\eta_{k,t}^2\mathbb{I}\{\eta_{k,t} < 0\}] < 1$, see Francq and Zakoian (2010). Covariance stationarity for the EGARCH models requires that $\beta_k < 1$. The required constraints do not prevent the asymmetric models (TGARCH and EGARCH) from capturing the (inverse) leverage effect. In equation (3), ($\alpha_{2,k} > 0$) $\alpha_{2,k} < 0$ indicates the presence of (inverse) leverage effect. To examine the sign of asymmetry in equation (4), we examine whether the quantity $\alpha_{1,k} - \alpha_{2,k}$ is positive or not. In the first case, there is leverage effect while in the second cases there is inverse leverage effect.

The models are estimated using a Bayesian MCMC approach. We denote $\mathbf{\Psi} = \{\xi_1, \theta_1, \ldots, \xi_k, \theta_k, \mathbf{P}\}$ the vector of model parameters, $f(y_t|I_{t-1})$ the density of y_t given the past observations, I_{t-1} . The likelihood function is then written as:

$$\mathcal{L}(\mathbf{\Psi}|I_T) = \prod_{t=1}^{T} f(y_t|\mathbf{\Psi}, I_{t-1}). \tag{5}$$

The conditional density of y_t is:

[8] The PDF of the GED is given by: $f(\eta;\nu) = \frac{\nu e^{-\frac{1}{2}|\eta/\lambda|^{\nu})}}{\lambda 2^{(1+1/\nu)}\Gamma(1/\nu)}$, where $\lambda = \left(\frac{\Gamma(1/\nu}{4^{1/\nu}\Gamma(3/\nu)}\right)^{1/2}$, $\eta \in \mathbb{R}$ and the shape parameter ν is positive.

$$f(y_t|\mathbf{\Psi}, I_{t-1}) = \sum_{i=1}^{K}\sum_{j=1}^{K} p_{i,j} z_{i,t-1} f_D(y_t|s_t = j, \mathbf{\Psi}, I_{t-1}),$$

where $z_{i,t-1} = P[s_{t-1} = i|\mathbf{\Psi}, I_{t-1}]$ represents the filtered probability of state i at time $t-1$ obtained via the filter of Hamilton (1989) and f_D is the conditional density of y_t given $\mathbf{\Psi}$ and I_{t-1}.

Following Ardia (2008), the likelihood function is combined with a prior $f(\mathbf{\Psi})$ to build the kernel of posterior distribution $f(\mathbf{\Psi}|I_T)$. The prior is built from independent diffuse priors as follows:

$$\begin{aligned}
f(\mathbf{\Psi}) =& f(\boldsymbol{\theta}_1, \boldsymbol{\xi}_1) \cdots f(\boldsymbol{\theta}_k, \boldsymbol{\xi}_k) f(\boldsymbol{P}), \\
f(\boldsymbol{\theta}_k, \boldsymbol{\xi}_k) \propto& f(\boldsymbol{\theta}_k) f(\boldsymbol{\xi}_k) \mathbb{I}\{\boldsymbol{\theta}_k, \boldsymbol{\xi}_k \in \mathcal{CSC}_k\}, \\
f(\boldsymbol{\theta}_k) \propto& f_{\mathcal{N}}(\boldsymbol{\theta}_k; \boldsymbol{\mu}_{\boldsymbol{\theta}_k}, \text{diag}(\boldsymbol{\sigma}^2_{\boldsymbol{\theta_k}})) \mathbb{I}\{\boldsymbol{\theta}_k \in \mathcal{PC}_k\}, \\
f(\boldsymbol{\xi}_k) \propto& f_{\mathcal{N}}(\boldsymbol{\xi}_k; \boldsymbol{\mu}_{\boldsymbol{\xi}_k}, \text{diag}(\boldsymbol{\sigma}^2_{\boldsymbol{\xi_k}})) \mathbb{I}\{\xi_{k,1} > 0\}, \\
f(\boldsymbol{P}) \propto& \prod_{i=1}^{K} \left(\prod_{j=1}^{K} p_{i,j} \right) \mathbb{I}\{0 < p_{i,i} < 1\},
\end{aligned}$$

where $k = 1, \ldots, K$, $\mathcal{CSC}_k$ and $\mathcal{PC}_k$ denote the covariance stationarity condition and positivity condition in regime k (see Trottier and Ardia (2016)). $\xi_{k,1}$ is the asymmetry parameter in regime k. $f_{\mathcal{N}}(, \mu, \Sigma)$ denotes the multivariate Normal density with mean vector μ and covariance matrix Σ. The entries of prior means, $\mu_{\bullet}$ and variances, $\sigma^2_{\bullet}$ vectors are set to zero and 1000 respectively. The posterior must be approximated by simulation techniques, since is of unknown form. MCMC draws from the posterior are generated using the adaptive random walk Metropolis sampler of Vihola (2012). We set the number of the burn in draws to 5000. We then build a posterior sample of 5000 from 50000 draws by keeping every 10th draw.

3.2 Deviance and Bayesian predictive information criteria

The goodness-of-fit of each estimated model is evaluated using the Deviance (DIC) and Bayesian Predictive (BPIC) information criteria. DIC is a hierarchical modelling generalisation of the Akaike Information Criterion used in Bayesian model selection.[9] Deviance is defined as $D(\mathbf{\Psi}) = -2\ln(\mathcal{L}(\mathbf{\Psi}|I_T)) + C$, with C a constant that cancels out in all calculations that compare different models, and which therefore does not need to be known. The effective number of parameters of the models, p_D is calculated as in Spiegelhalter et al. (2002): $p_D = \overline{D(\mathbf{\Psi})} - D(\overline{\mathbf{\Psi}})$ where $\overline{\mathbf{\Psi}}$ is the expectation of Ψ and $\overline{D(\mathbf{\Psi})}$ is computed as the average of $D(\mathbf{\Psi})$ over the samples of $\mathbf{\Psi}$. DIC is then calculated as: DIC $= p_D + \overline{D(\mathbf{\Psi})}$. BPIC is an extension of the DIC, suggested by Ando (2008) to avoid over-fitting problems of DIC. BPIC is calculated as BPIC $= -2\mathbf{E}^{\mathbf{\Psi}}[\ln(\mathcal{L}(\mathbf{\Psi}|I_T)) + 2p_D]$

3.3 Forecasting volatility

For the one-step-ahead volatility forecast, we employ half of the total number of observations of y_t for estimation and test the performance of the models over the same number of out-of-sample observations. All models are estimated on a rolling window basis with the parameters being updated every every 10 observations. Since volatility itself is unobservable, the comparison of volatility forecasts relies on an observable proxy for the latent volatility process. We assess the forecasting performance of the different GARCH types using employ two criteria, the Mean Square Error (MSE) and the Mean Absolute Error (MAE).

3.4 Density risk forecasting

The one-step-ahead Value-at-Risk forecast (VaR) is estimated as a quantile of the predictive density function (CDF), F, by numerically inverting the predictive CDF. Specifically, for a given risk level α,

[9] Brooks and Burke (2003) argue that unmodified traditional information criteria cannot be used for order determination of conditional heteroscedastic models.

$$\text{VaR}^{\alpha}_{T+1} = \inf\{y_{T+1} \in \mathbb{R} | F(y_{T+1}|I_T) = \alpha\}. \tag{6}$$

To compute the CDF, we first note that the one-step-ahead conditional probability density function (PDF) of y_{T+1} is a mixture of K regime-dependent distributions:

$$f(y_{T+1}|\mathbf{\Psi}, I_T) = \sum_{k=1}^{K} \pi_{k,T+1} \times f_{\mathcal{D}}(y_{T+1}|s_{T+1} = k, \mathbf{\Psi}, I_T),$$

with mixing weights $\pi_{k,T+1} = \sum_{i=1}^{K} p_{i,k} z_{i,T}$, where $z_{i,T} = \mathbb{P}[s_T = i|\mathbf{\Psi}, I_T]$, for $i = 1, \ldots, K$ are the filtered probabilities at time T. The CDF is then given by:

$$F(y_{T+1}|\mathbf{\Psi}, I_T) = \int_{-\inf}^{y_{T+1}} f(z|\mathbf{\Psi}, I_T)\mathrm{d}z.$$

Given a posterior sample $\{\mathbf{\Psi}^{[m]}, m = 1, \ldots, M\}$, the predictive PDF is obtained as:[10]

$$f(y_{T+1}|I_T) = \int_{\mathbf{\Psi}} f(y_{T+1}|\mathbf{\Psi}, I_T) f(\mathbf{\Psi}|I_T)\mathrm{d}\mathbf{\Psi} \approx \frac{1}{M} \sum_{1=m}^{M} f(y_{T+1}|\mathbf{\Psi}^{[\mathbf{m}]}, I_T),$$

and the predictive CDF is given by:

$$F(y_{T+1}|I_T) = \int_{-\inf}^{y_{T+1}} f(z|I_T)\mathrm{d}z$$

We test the accuracy of the VaR predictions using the conditional coverage (CC) and dynamic quantile (DQ) backtesting procedures. To investigate CC, Christoffersen (1998) proposed a series of VaR exceedance $d_t, t = S, \ldots, S+H$, where $d_t^{\alpha} = I\{r_t < \text{VaR}_t(a)\}$, usually referred to as the hitting series. Specifically, if correct CC is achieved by the model, VaR exceedances should be independently distributed over time.

The DQ test by Engle and Manganelli (2004) assesses the joint hypothesis that $\mathbb{E}[\delta_t^{\alpha}] = \alpha$ and that the hit variables are distributed independently. The implementation of the test involves the de-meaned process, $\text{Hit}_t^{\alpha} = \delta_t^{\alpha} - \alpha$. Under the correct model

[10]By integrating out the parameter uncertainty.

specification, unconditionally and conditionally, Hit_t^α has zero mean and is serially uncorrelated. The DQ test is then the traditional Wald test of the joint nullity of all coefficients in the following linear regression:

$$\text{Hit}_t^\alpha = \delta_0 + \sum_{l=1}^{L} \text{Hit}_{t-l}^\alpha + \delta_{L+1}\text{VaR}_{t-1}^\alpha + \eta_t.$$

Under the null hypothesis of correct unconditional and conditional coverage, we have that the Wald test statistic is asymptotically $\mathcal{X}^2$ distributed with $L+2$ degrees of freedom.[11] Similar to volatility forecasts, we employ half of the total number of observations of the time-series for estimation and test the performance of the models over the same number of out-of-sample observations. The model parameters are estimated for every 10 observations. Ardia and Hoogerheide (2014) and Ardia et al. (2018b) show, in the context of GARCH models, that the performance of VaR forecasts is not affected significantly when the updating frequency is low.

The analysis is conducted in the `R` language using the `MSGARCH` and `GAS` packages by Ardia et al. (2019a) and Ardia et al. (2019c), respectively.[12] `MSGARCH` is used for model estimation and forecasting. The package is implemented such that positivity and covariance stationarity are ensured in the estimation. The package also provides estimates for the two information criteria. The `GAS` package is used to implement the CC and DQ tests and compute the p-values of two hypothesis tests of coverage of the VaR.

4 Data

We download the closing prices for a total of 292 digital currencies from Yahoo Finance. Table A1 shows the symbols of the cryptocurrencies used in the empirical analysis. The downloaded time-series have different sample lengths based on the date of the release

[11]We follow the standard choice and set $L = 4$ lags.

[12]The `MSGARCH` package can be found in cran.r-project.org/MSGARCH and the `GAS` package in cran.r-project.org/GAS, see also Ardia et al. (2018a).

date of each cryptocurrency and the availability of the data. For all time-series the last observation is for 16/9/2020. The most extensive sample consists of 2583 observations (the first observation is on 1/10/2013). Data for Ripple (XRP) and Litecoin (LTC) are also available from 1/10/2013. In total, seventeen time-series have a sample length equal to 2583 observations.

We use the retrieved data and calculate the daily returns as the natural logarithmic differences of closing prices. Table 1 reports the summary statistics of the returns for thirty digital currencies with different size of market capitalisation (large, medium and small). We report the mean, the median, the standard deviation, the skewness and the kurtosis coefficients of the entire sample. For all three groups we observe similar results. Standard deviation ranges at similar levels and skewness is both positive and negative. Furthermore, in all groups we observe cases of extreme kurtosis (USDT, AE and IOC). In addition to the descriptive statistics with employ the Jarque-Bera and the ADF tests to examine the normality and stationarity of the returns. In all cases, the tests indicate that the series are stationary and do not follow the Normal distribution. These results are not reported and are available on request.

Figure 1 shows the closing prices for Bitcoin, Ethereum, Tether and Ripple properly scaled. Bitcoin and Ethereum are plotted against the left axis and Tether and Ripple against the right axis. We observe a burst in prices that starts in 2017 and lasts until the end of 2018. The next four subfigures plot the returns for the four cryptocurrencies where deviations from the mean are obvious. The means and standard deviations for the ten largest cryptocurrencies are presented in Figure 2 and respective values for all cryptocurrencies are presented in the Appendix, in Figures A1 and A2. The examination of the standard deviations of all cryptocurrencies suggests that digital currencies with smaller market capitalisation have a greater standard deviation. In addition, we observe cases with extreme standard deviation (compared to the rest of the cryptocurrencies) such as WICC, DEV and MOON.

5 Results

5.1 In-sample analysis

This section evaluates the performance of the models in terms of goodness-of-fit. Table 2 summarises the results regarding model selection through information criteria. Table 2 reports the absolute frequency at which each model is selected via DIC (Panel A) and BPIC (Panel B). The same results are also presented in the first subplot of Figure 3. According to the findings, DIC indicates single-regime models for 111 out 292 time-series, and regime-switching models for 181 time-series (119 two-regime and 62 three-regime models). On the contrary, BPIC selects single-regime models more often than regime-switching models. Specifically, BPIC indicates single-regime models for 162 digital currencies, two-regime for 91 and three-regime models for 39 models. Despite these differences, the two criteria select the same model in terms of goodness-of-fit for 192 out of 292 cryptocurrencies. If we confine the analysis to the set of cryptocurrencies for which both criteria indicate the same model, we observe that multi-regime models are preferred in more cases than single-regime models (100 over 92). In addition, we observe that the models with Student's-t conditional distribution outperform other models regardless of the GARCH-type and the number of regimes. These findings are consistent with the stylised empirical facts of financial time-series as discussed in Cont (2001) and Cont and Tankov (2004). Finally, we find that for a given number of regimes, the exponential model is outperformed by the other models.

For the three out of the four most traded cryptocurrencies, the two criteria indicate the same model as the most appropriate. For Bitcoin and Tether, the best model in terms of goodness-of-fit is the the two-regime TGARCH with Student's-t conditional distribution. In the case of Ethereum, the three-regime GARCH with Generelised distribution is selected. We also find evidence of regime changes in the volatility of Ripple, however the criteria indicate different models. In some cases, DIC and BPIC suggest models with different number of regimes for the same digital currency. Among these cryptocurrencies are Bitcoin Cash, Binance and Litecoin.

As discussed in the literature review, positive past returns have a greater effect on BTC volatility. Here, we study the behaviour of leverage effect for all 292 cryptocurrencies, using the two asymmetric models (EGARCH and TGARCH). Table 3 reports the number of cryptocurrencies for which each model indicates the presence of inverse leverage effect in at least one regime. Clearly, for the majority of the time-series, both models support the hypothesis of inverse leverage effect. For example the traditional EGARCH model suggests positive asymmetry in the volatility of 177 out of 292 digital currencies. In total, there are 10 cryptocurrencies in which inverse leverage effect is indicated by all models. These are LINK, XTZ, HIVE, XWC, PLC, ZNN, QBSR, QRK, SAPP, USNBT. On the contrary, there are 3 cases where according to EGARCH, negative past returns affect the volatility more than positive past returns. These are CTC, DUN, XMC. For all cryptocurrencies, at least one specification of TGARCH indicates inverse leverage effect. Considering the biggest cryptocurrencies, we observe that most models indicate inverse leverage effect in at least one regime. For BTC only the single-regime models with normal conditional distribution do not support the hypothesis of inverse leverage effect.

In the last part of the in-sample analysis, we calculate the mean and standard deviation of conditional volatility of each estimated model. The results for the four major cryptocurrencies are presented in Figure A3. In all cases, the mean and the standard deviation values are close for all models. With the exception of Ripple, single-regime models exhibit a greater mean than MSGARCH models. For example, in the case of Ethereum, the two models with the higher mean values of the conditional volatility are the TGARCH and EGARCH with Student's-t distribution. Comparing the different digital currencies, we find that for most models the Bitcoin exhibits greater volatility. Similar plots for all examined cryptocurrencies are presented in the Appendix. From the examination of these figures it occurs that we can divide digital currencies in three groups. The first group contains the cryptocurrencies like BTC and Ethereum for which all mean and standard deviation do not exhibit substantial differences for all models. The second group contains the cryptocurencies where the mean and standard deviation of few models differs from the rest. Tether and Ripple belong in the second group. The last group consists of the cryptocurrencies for which we cannot identify a patter for the calculated means and standard deviations, i.e. NMC and NPC. For most ctyptocurrencies and models we observe a mean higher than the standard deviation. There are however, a few exceptions such as Ethereum, NPCoim, TenX and Veritaseum.

5.2 Out-of-sample analysis

We now turn to the out-of-sample analysis. First, we compare the ability of the 27 models to correctly forecast the one-day ahead conditional volatility. We evaluate the performance of the forecasts using MSE and MAE loss functions. The main results are presented in Table 4 and the middle subfigure of Figure 3. Overall, two-regime models outperform both traditional and three-regime models. According to the MSE, two-regime models perform better than the rest of the models 79% of the time. That percentage is a little lower when MAE is used, approximately 73%. According to both criteria, TGARCH outperforms both GARCH and EGARCH models, regardless of the number of regimes or the conditional distribution. In addition, we obtain the most accurate forecasts when we fit TGARCH-type models to the biggest cryptocurrencies. For 218 out of 292 examined cryptocurrencies, both MSE and MAE agree on the model selection. Similar to the in-sample analysis, EGARCH models are outperformed by the other models. We obtain similar results if we focus only on these models (Panel C of 4).

We focus on the major digital currencies such as Bitcoin, Ehtereum, Ripple and Tether. For each one of these four cryptocurrencies both MSE and MAE indicate the same model as the best one. In the case of Bitcoin and Ethereum, single-regime models are chosen, the TGARCH with Normal distribution and the GARCH with Generalised distribution, respectively. In the case of Tether and Ripple, the two-regime TGARCH model is chosen (with Student's-t and generalised error distribution, respectively). Overall, MSGARCH models seem to perform better than their single-regime counterparts. In addition, asymmetric models are preferred to symmetric ones (for a given number or regimes). These findings also apply to the major cryptocurrency of the dataset such as Bitcoin, Tether and Bitcoin Cash where models that account for leverage effect appears as the most appropriate.

Next, we evaluate the performance of the models in forecasting one-day ahead VaR. Similar to conditional volatility predictions, all models are compared against each other in VaR forecasting based on the p-value of the CC and DQ tests. Table 5 presents the number of times each model outperforms all other models at forecasting the one-step-ahead VaR (these results are presented in the last panel of Figure 3). According to CC, the two-regime TGARCH model with Normal distribution is the best model in predicting VaR. Specifically, CC indicates this model as the best for 72 cryptocurrencies. Similar to volatility forecasts, MSGARCH models outperform single-regime models. The results are different when the DQ test is used. First, there is no model specification which clearly outperforms all others. Both the three-regime GARCH model with Student's t conditional distribution and the two-regime TGARCH with Normal distribution perform equally well (27 and 26 out of 292, respectively). Second, the GARCH models performs better than TGARCH models. Despite these differences, the EGARCH produces the worst results compared to the other models. The two test indicate the same model to provide the most accurate VaR forecasts only for 74 time-series. In this case, the results are qualitatively the same with the ones we obtained using the CC test.

Considering the major cryptocurrencies, we observe that a different GARCH specification should be used in predicting one-step-ahead VaR. In the case of Bitcoin, the three-regime GARCH model is selected. Regime-switching models are also suitable for Ripple. CC indicates a two-regime GARCH model with Normal distribution and DQ a three-regime GARCH model with Student's-t conditional distribution. The best model for forecasting the VaR of Ethereum and Tether is the traditional GARCH model. For the remaining popular digital currencies the asymmetric models (TGARCH) are preferred.

Furthermore, we report the percentages of cryptocurrencies for which the null hypothesis of correct unconditional and conditional coverage is rejected at the 5% significance level (Table 6). For all models the failure rate is less than 8%. Based on the CC test, the model with the highest failure rate is the traditional GARCH model with Normal conditional distribution and based on the DQ test, the two-regime EGARCH model with Student's t conditional distribution. For both tests, the model with the lowest rejection frequency is the TGARCH model with three regimes and Student's t conditional distribution. Overall, the rejection level decreases as the number of regimes increases. In addition, for a given number of regimes, the TGARCH models outperform the other models.

6 Conclusions

Cryptocurrencies have flourished over the last decade. Since the introduction of Bitcoin, hundreds of digital currencies have been created. Studies that examined Bitcoin and other cryptocurrencies have classified the cryptocurrencies as an asset used mostly for speculative purposes. This is an explanation for the extreme volatility of the returns of Bitcoin. Despite their similarities, there are significant differences between cryptocurrencies. For example, the level of market capitalisation between digital currencies and the attention each digital currency receives from investors are distinct. In this paper, we perform an extensive analysis of the volatility of the cryptocurrency markets. We examine the volatility of the returns for a total of 292 different cryptocurrencies. For each one we estimate 27 GARCH-type models using Bayesian methods.

First, we perform an in-sample analysis. We evaluate the performance of each model in terms of goodness-of-fit using DIC and BPIC and investigate the presence of leverage effect. While DIC selects MSGARCH models more and BPIC selects single-regime models more, the two criteria agree on the model selection approximately 66% of cryptocurrencies. In this case, MSGARCH models are preferred in 52% of the cases. The examination of leverage effect suggests that most cryptocurrencies returns exhibit inverse leverage effect and respond more strongly to positive past returns than negative past returns. Next, we perform the out-of-sample analysis and consider two forecast exercises. First, we evaluate the ability of the models to predict the one-step-ahead volatility. We find that two-regime models produce better forecasts than the rest of the models. Next, we assess the ability of the models to predict the one-step-ahead VaR based on conditional and unconditional convergence tests. For both tests the three-regime TGARCH models have the lowest rejection frequencies. Both from in-sample and out-of-sample analysis, we conclude that the EGARCH models are outperformed both by the GARCH and the TGARCH models.

Our study can be expanded in various ways. First, the analysis can be repeated, considering other digital currencies not included in this one. Second, it would be of interest to see whether including skewed versions of the conditional distributions would affect the results. Finally, we do not consider the GJR-GARCH model in our analysis. The positivity constraints, regarding the GJR-GARCH model, requires that the leverage coefficient is positive. However, this rules out the possibility of inverse leverage effect which is often observed in cryptocurrencies. To consider the GJR-GARCH model, we should ensure positivity as in Ardia et al. (2019b).

References

Ahamed, L. (2009). *Lords of finance: The bankers who broke the world*. Penguin Press.

Ando, T. (2008). "Bayesian predictive information criterion for the evaluation of hierarchical Bayesian and empirical Bayes models". In: *Biometrika* 94.2, pp. 443–458.

Ardia, D. (2008). *Financial risk management with Bayesian estimation of GARCH Models: Theory and applications*. Vol. 612, Lecture Notes in Economics and Mathematical Systems. Berlin Heidelberg: Springer-Verlag.

Ardia, D. et al. (2019a). "Markov-switching GARCH models in R: The **MSGARCH** package". In: *Journal of Statistical Software* 91.4, pp. 1–38.

Ardia, D., K. Bluteau, and M. Ruede (2019b). "Regime changes in Bitcoin GARCH volatility dynamics". In: *Finance Research Letters* 29.3, pp. 266–271.

Ardia, D., K. Boudt, and L. Catania (2018a). "Downside risk evaluation with the R package **GAS**". In: *The R Journal* 10.2, pp. 410–421.

Ardia, D., K. Boudt, and L. Catania (2019c). "Generalized Autoregressive Score Models in R: The **GAS** Package". In: *Journal of Statistical Software* 88.6, pp. 1–28.

Ardia, D. and L. Hoogerheide (2014). "GARCH models for daily stock returns: Impact of estimation frequency on Value-at-Risk and Expected Shortfall forecasts". In: *Economics Letters* 123.2, pp. 187–190.

Ardia, D., J. Kolly, and D.-A. Trottier (2017). "The impact of parameter and model uncertainty on market risk predictions from GARCH-type models". In: *Journal of Forecasting* 36.7, pp. 808–823.

Ardia, D. et al. (2018b). "Forecasting risk with Markov-switching GARCH models: A large-scale performance study". In: *International Journal of Forecasting* 34.4, pp. 733–747.

Baek, C. and M. Elbeck (2015). "Bitcoins as an investment or speculative vehicle? A first look". In: *Applied Economics Letters* 22.1, pp. 30–34.

Balcombe, K. and I. Fraser (2017). "Do bubbles have an explosive signature in markov switching models?" In: *Economic Modelling* 66, pp. 81–100. ISSN: 0264-9993.

Bariviera, A. F. (2017). "The inefficiency of Bitcoin revisited: A dynamic approach". In: *Economics Letters* 161, pp. 1–4.

Baur, D., T. Dimpfl, and K. Kuck (2018). "Bitcoin, gold and the US dollar A replication and extension". In: *Finance Research Letters* 25, pp. 103–110. ISSN: 1544-6123.

Bauwens, L., B. De Backer, and A. Dufays (2014a). "A Bayesian Method of Change-point Estimation with Recurrent Regimes: Application to GARCH Models". In: *Journal of Empirical Finance* 29, pp. 207–229.

Bauwens, L., A. Dufays, and J. V. Rombouts (2014b). "Marginal likelihood for Markov-switching and change-point GARCH models". In: *Journal of Econometrics* 178, pp. 508–522. ISSN: 0304-4076.

Bauwens, L., A. Preminger, and J. Rombouts (2010). "Theory and inference for a Markov switching GARCH model". In: *The Econometrics Journal* 13.2, pp. 218–244.

Blau, B. (2017). "Price dynamics and speculative trading in bitcoin". In: *Research in International Business and Finance* 41.3, pp. 493–499.

Bollerslev, T. (1986). "Generalized autoregressive conditional heteroskedasticity". In: *Journal of Econometrics* 31.3, pp. 307–327.

Bouoiyour, J. and R. Selmi (2015). *Bitcoin Price: Is it really that New Round of Volatility can be on way?* Tech. rep. MPRA Paper 65580, University Library of Munich, Germany.

Bouri, E., G. Azzi, and A. Dyhrberg (2017a). "On the Return-volatility Relationship in the Bitcoin Market Around the Price Crash of 2013". In: *Economics E-Journal* 11, pp. 1–16.

Bouri, E. et al. (2017b). "Does Bitcoin hedge global uncertainty? Evidence from wavelet-based quantile-in-quantile regressions". In: *Finance Research Letters* 23, pp. 87–95. ISSN: 1544-6123.

Bouri, E. et al. (2017c). "On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier?" In: *Finance Research Letters* 20, pp. 192–198. ISSN: 1544-6123.

Brooks, C. and S. Burke (2003). “Information criteria for GARCH model selection”. In: *European Journal of Finance* 9.6, pp. 557–580.

Caporale, G. M. and T. Zekokh (2019). “Modelling volatility of cryptocurrencies using Markov-Switching GARCH models”. In: *Research in International Business and Finance* 48, pp. 143–155.

Cheikh, N. B., Y. B. Zaied, and J. Chevallier (2020). “Asymmetric volatility in cryptocurrency markets: New evidence from smooth transition GARCH models”. In: *Finance Research Letters* 35, p. 101293. ISSN: 1544-6123.

Christoffersen, P. (1998). “Evaluating Interval Forecasts”. In: *International Economic Review* 39.4, pp. 841–862.

Chu, J. et al. (2017). “GARCH Modelling of Cryptocurrencies”. In: *Journal of Risk and Financial Management* 10, p. 17.

Cont, R. (2001). “Empirical properties of asset returns: stylized facts and statistical issues”. In: *Quantitative Finance* 1.2, pp. 223–236.

Cont, R. and P. Tankov (2004). *Financial Modelling With Jump Processes*. Boca Raton London New York Washington D. C.: Chapman and Hall/CRC.

Corbet, S., B. Lucey, and L. Yarovya (2018). “Datestamping the Bitcoin and Ethereum Bubbles”. In: *Finance Research Letters* 26, pp. 81–88.

Dyhrberg, A. H. (2016a). “Bitcoin, gold and the dollar - A GARCH volatility analysis”. In: *International Review of Financial Analysis* 16, pp. 85–92.

Dyhrberg, A. H. (2016b). “Hedging capabilities of bitcoin. Is it the virtual gold?” In: *Finance Research Letters* 16.C, pp. 139–144.

Engle, R. and S. Manganelli (2004). “CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles”. In: *Journal of Business and Economic Statistics* 22.4, pp. 367–381.

Francq, C. and J.-M. Zakoian (2010). *GARCH Models: zStructure, Statistical Inference and Financial Applications*. John Wiley & Sons.

Fry, J. and E.-T. Cheah (2016). “Negative bubbles and shocks in cryptocurrency markets”. In: *International Review of Financial Analysis* 47, pp. 343–352.

Geweke, J. and G. Amisano (2010). “Comparing and evaluating Bayesian predictive distributions of asset returns”. In: *International Journal of Forecasting* 26.2. Special Issue: Bayesian Forecasting in Economics, pp. 216–230. ISSN: 0169-2070.

Glaser, F. et al. (2014). *Bitcoin - Asset or currency? Revealing users' hidden intentions*. Tech. rep. ECIS 2014 Proceedings - 22nd European Conference on Information Systems.

Gronwald, M. (2014). *The Economics of Bitcoins - Market Characteristics and Price Jumps*. Tech. rep. CESifo Working Paper Series 5121, CESifo.

Gronwald, M. (2019). “Is Bitcoin a Commodity? On price jumps, demand shocks, and certainty of supply”. In: *Journal of International Money and Finance* 97, pp. 86–92. ISSN: 0261-5606.

Haas, M., S. Mittnik, and M. Paolella (2004). “A new approach to Markov-switching GARCH models”. In: *Journal of Financial Econometrics* 2, pp. 493–530.

Hamilton, J. (1989). “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle”. In: *Econometrica* 57.2, pp. 357–384. ISSN: 00129682, 14680262.

Katsiampa, P. (2017). “Volatility estimation for Bitcoin: A comparison of GARCH models”. In: *Economics Letters* 158.3, pp. 3–6.

Klein, T., H. P. Thu, and T. Walther (2018). “Bitcoin is not the New Gold - A comparison of volatility, correlation, and portfolio performance”. In: *International Review of Financial Analysis* 59, pp. 105–116.

Kristoufek, L. (2014). *What are the main drivers of the Bitcoin price? Evidence from wavelet coherence analysis*. FinMaP-Working Papers 23. Collaborative EU Project FinMaP - Financial Distortions, Macroeconomic Performance: Expectations, Constraints, and Interaction of Agents.

Lamoureux, C. G. and W. D. Lastrapes (1990). "Persistence in Variance, Structural Change, and the GARCH Model". In: *Journal of Business & Economic Statistics* 8.2, pp. 225–234. ISSN: 07350015.

Ma, F. et al. (2020). "Cryptocurrency volatility forecasting: A Markov regime-switching MIDAS approach". In: *Journal of Forecasting* 39.8, pp. 1277–1290.

Maciel, L. (2021). "Cryptocurrencies value-at-risk and expected shortfall: Do regime-switching volatility models improve forecasting?" In: *International Journal of Finance & Economics* 26.3, pp. 4840–4855.

Mensi, W., K. H. Al-Yahyaee, and S. H. Kang (2019). "Structural breaks and double long memory of cryptocurrency prices: A comparative analysis from Bitcoin and Ethereum". In: *Finance Research Letters* 29, pp. 222–230. ISSN: 1544-6123.

Nadarajah, S. and J. Chu (2017). "On the inefficiency of Bitcoin". In: *Economics Letters* 150.3, pp. 6–9.

Nakamoto, S. (2009). "Bitcoin: A Peer-to-Peer Electronic Cash System". Available at: https://bitcoin.org/bitcoin.pdf.

Nelson, D. (1991). "Conditional Heteroskedasticity in Asset Returns: A New Approach". In: *Econometrica* 59.2, pp. 347–370.

Panagiotidis, T., T. Stengos, and O. Vravosinos (2018). "On the determinants of bitcoin returns: a LASSO approach". In: *Finance Research Letters* 27, pp. 235–240.

Panagiotidis, T., T. Stengos, and O. Vravosinos (2019). "The effects of markets, uncertainty and search intensity on Bitcoin returns". In: *International Review of Financial Analysis* 63, pp. 220–242. ISSN: 1057-5219.

Polasik, M. et al. (2015). "Price Fluctuations and the Use of Bitcoin: An Empirical Inquiry". In: *International Journal of Electronic Commerce* 20, pp. 9–49.

Reinhart, C. and K. Rogoff (2009). "The Aftermath of Financial Crises". In: *American Economic Review* 99.2, pp. 466–72.

Spiegelhalter, D. et al. (2002). “Bayesian measures of model complexity and fit”. In: *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 64.4, pp. 583–639.

Stavroyiannis, S. (2018). “Value-at-risk and related measures for the Bitcoin”. In: *Journal of Risk Finance* 19.2, pp. 127–136.

Thies, S. and P. Molnár (2018). “Bayesian change point analysis of Bitcoin returns”. In: *Finance Research Letters* 27, pp. 223–227. ISSN: 1544-6123.

Trottier, D.-A. and D. Ardia (2016). “Moments of standardized FernandezSteel skewed distributions: Applications to the estimation of GARCH-type models”. In: *Finance Research Letters* 18.8, pp. 311–316. ISSN: 1544-6123.

Urquhart, A. (2016). “The inefficiency of Bitcoin”. In: *Economics Letters* 148, pp. 80–82. ISSN: 0165-1765.

Urquhart, A. (2017). *The Volatility of Bitcoin*. Tech. rep. SSRN Electronic Journal.

Vihola, M. (2012). “Robust adaptive Metropolis algorithm with coerced acceptance rate”. In: *Statistics and Computating* 22.5, pp. 997–1008.

Williams, M. (2014). “Virtual currencies - Bitcoin risk”. In: *World Bank Conference*. Washington, DC. No. 21.

Yermack, D. (2013). *Is Bitcoin a Real Currency? An economic appraisal*. NBER Working Papers 19747. National Bureau of Economic Research, Inc.

Zakoian, J.-M. (1994). “Threshold heteroskedastic models”. In: *Journal of Economic Dynamics and Control* 18.5, pp. 931–955.

Table 1: Descriptive statistics of return prices for 30 cryptocurrenies.

Symbol	Mean	Median	Std	Skewness	Kurtosis	Max	Min
Large market-cap							
BTC	1.57e-03	1.94e-03	0.03	-0.94	13.48	0.22	-0.46
ETH	2.63e-03	3.12e-05	0.06	-3.47	72.03	0.41	-1.30
USDT	-9.15e-05	0.00	0.01	-13.11	970.62	0.50	-0.68
XRP	-4.43e-04	-2.66e-03	0.07	0.23	10.26	0.43	-0.56
DOT	1.73e-03	-2.13e-03	0.06	2.80	45.66	1.02	-0.61
ADA	4.60e-03	7.89e-04	0.07	0.95	15.30	0.67	-0.54
LTC	3.61e-03	-1.05e-03	0.07	0.19	6.82	0.48	-0.61
BCH	1.22e-03	9.88e-05	0.07	2.31	26.89	0.86	-0.50
LINK	1.11e-03	-5.77e-04	0.05	0.41	14.71	0.51	-0.51
BNB	-1.89e-03	-4.17e-03	0.04	0.27	7.25	0.22	-0.18
Medium market-cap							
VSYS	-3.08e-03	-2.37e-03	0.05	-0.04	10.62	0.28	-0.25
SERO	6.26e-04	-3.52e-03	0.04	0.92	2.71	0.19	-0.12
MAID	8.44e-04	9.88e-04	0.06	-0.28	6.76	0.49	-0.57
TFUEL	9.09e-03	-8.19e-03	0.10	3.28	17.32	0.67	-0.25
AION	-1.43e-03	2.00e-03	0.05	-0.44	2.97	0.17	-0.27
AE	-1.56e-03	-5.49e-04	0.10	-2.39	34.73	0.62	-1.21
DEV	-2.70e-03	-4.16e-03	0.61	-0.23	31.49	4.42	-4.66
IOTX	5.96e-03	-3.43e-04	0.07	0.74	2.78	0.35	-0.18
XNC	3.76e-03	2.31e-03	0.02	0.85	6.87	0.13	-0.08
GXC	-1.39e-03	-2.29e-03	0.04	-1.12	6.87	0.11	-0.28
Large market-cap							
TUBE	-2.61e-03	-9.64e-03	0.08	0.76	2.10	0.36	-0.23
CURE	1.38e-04	-6.55e-03	0.05	0.97	3.32	0.24	-0.15
BST	-4.71e-03	-9.97e-03	0.24	-0.23	0.74	0.57	-0.83
NLC2	4.54e-04	-5.78e-03	0.16	1.12	37.65	2.03	-1.93
IOC	6.85e-04	-1.53e-03	0.17	0.07	276.73	4.03	-4.02
MGO	-5.02e-03	-1.68e-03	0.11	-0.25	4.55	0.62	-0.67
EDG	-1.57e-03	-6.09e-03	0.09	0.34	27.08	1.15	-1.02
SUB	-3.07e-03	-3.47e-03	0.09	-0.02	3.17	0.45	-0.56
ATB	-6.31e-03	-4.41e-03	0.12	0.07	8.42	0.71	-0.83
XUC	-3.94e-04	-1.28e-03	0.07	3.12	36.21	0.85	-0.40

Notes: The table reports the summary statistics for cryptocurrencies with the different level of market capitalisation.

Table 2: Model selection based on DIC and BPIC.

	$K=1$			$K=2$			$K=3$		
	N	Std	GED	N	Std	GED	N	Std	GED
	Panel A: DIC								
GARCH	1	29	21	5	28	7	6	17	6
EGARCH	0	14	7	10	13	6	0	7	4
TGARCH	0	30	9	10	34	6	3	15	4
	Panel B: BPIC								
GARCH	1	46	41	6	24	9	4	11	3
EGARCH	0	17	3	3	7	4	0	3	2
TGARCH	1	42	11	5	25	8	2	12	2
	Panel C: Counts of agreement between DIC and BPIC								
GARCH	1	27	21	3	18	5	4	9	3
EGARCH	0	8	3	3	6	4	0	3	2
TGARCH	0	26	6	4	22	2	1	9	2

Notes: The total number of cryptocurrencies is 292. The values denote the number of time-series for which the corresponding model is selected through DIC (panel A), BPIC (panel B) and both criteria (panel C). N, Std and GED denote the Normal, Student's-t and GED distributions, respectively. K refers to the number of regimes in the model.

Table 3: Inverse leverage effect.

	$K=1$			$K=2$			$K=3$		
	N	Std	GED	N	Std	GED	N	Std	GED
EGARCH	177	196	202	245	240	241	265	261	262
TGARCH	187	218	208	249	255	257	268	268	267

Notes: The total number of cryptocurrencies is 292. The values denote the number of time-series for which the corresponding model indicates the existence of inverse leverage effect in at least one regime. N, Std and GED denote the Normal, Student's-t and GED distributions, respectively. K refers to the number of regimes in the model.

Table 4: Predictive power of the models.

	$K=1$			$K=2$			$K=3$		
	N	Std	GED	N	Std	GED	N	Std	GED
	Panel A: MSE								
GARCH	9	5	7	22	20	27	2	2	2
EGARCH	3	1	1	9	10	5	0	1	0
TGARCH	11	6	3	50	33	55	4	1	3
	Panel A: MAE								
GARCH	11	5	9	22	17	23	4	2	3
EGARCH	2	0	3	10	16	2	3	1	0
TGARCH	10	9	5	39	31	54	5	2	4
	Panel C: Counts of agreement between MSE and MAE								
GARCH	7	4	6	15	12	19	2	2	1
EGARCH	1	0	1	6	9	1	0	1	0
TGARCH	9	5	3	35	27	45	4	1	2

Notes: The total number of cryptocurrencies is 292. The values denote the number of time-series for which the corresponding model is minimises the MSE (panel A), MAE (panel B) and both criteria (panel C). N, Std and GED denote the Normal, Student's-t and GED distributions, respectively. K refers to the number of regimes in the model.

Table 5: Model selection based on VaR forecasts.

	$K=1$			$K=2$			$K=3$		
	N	Std	GED	N	Std	GED	N	Std	GED
	Panel A: CC								
GARCH	15	7	2	17	17	4	9	8	3
EGARCH	2	2	0	19	16	3	1	2	1
TGARCH	15	13	7	72	30	11	3	10	3
	Panel B: DQ								
GARCH	18	16	4	24	13	13	17	27	8
EGARCH	3	3	3	11	8	7	0	1	0
TGARCH	12	17	6	26	19	10	8	14	4
	Panel C: Counts of agreement between CC and DQ								
GARCH	4	2	0	7	4	2	3	2	1
EGARCH	1	1	0	2	2	0	0	1	0
TGARCH	4	4	2	16	7	2	1	5	1

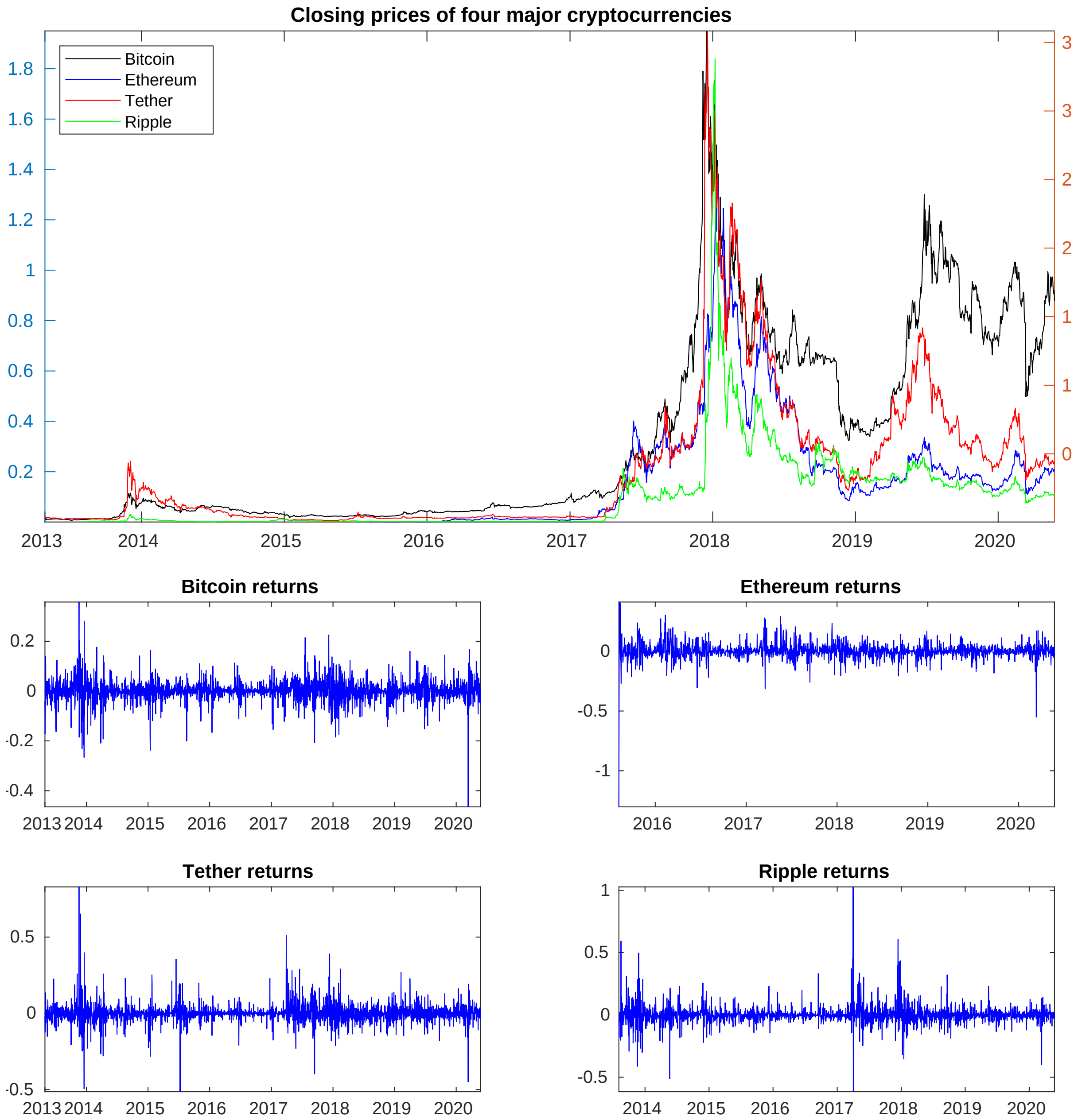
Notes: The total number of cryptocurrencies is 292. The values denote the number of time-series for which the corresponding model is selected through CC (panel A) and DQ (panel B). N, Std and GED denote the Normal, Student's-t and GED distributions, respectively. K refers to the number of regimes in the model.

Table 6: Percentage of cryptocurrencies for which the validity of the VaR predictions is rejected.

	CC			DQ		
No. of regimes	1	2	3	1	2	3
GARCH-N	7.12	6.02	5.13	6.74	6.02	5.41
GARCH-Std	6.57	6.71	6.67	7.12	7.05	6.91
GARCH-GED	6.95	3.93	3.01	6.98	4.04	3.21
EGARCH-N	6.95	4.96	3.18	6.47	5.17	3.21
EGARCH-Std	6.71	6.74	6.57	6.95	7.15	6.95
EGARCH-GED	6.88	3.42	2.08	6.84	3.21	2.15
TGARCH-N	4.41	1.43	0.30	3.93	0.85	0.17
TGARCH-Std	4.86	1.16	0.58	4.34	0.75	0.30
TGARCH-GED	4.38	0.95	0.47	3.73	0.61	0.34

Notes: The values denote the percentage of cryptocurrencies for which the validity of the VaR forecasts is rejected for each model. N, Std and GED denote the Normal, Student's-t and GED distributions, respectively.

Figure 1: Daily closing prices and returns of the Bitcoin, Ethereum, Tether and Ripple.



Notes: The first figure shows the closing prices of Bitcoin (black), Ethereum (blue), Tether (red) and Ripple (red) scaled appropriately. Bitcoin and Ethereum are plotted against the left axis and Tether and Ripple against the right axis. The next four figures plot the returns of the four cryptocurrencies.

Figure 2: Mean values for the returns (scaled to 10^3) and standard deviations for the ten major cryptocurrencies.

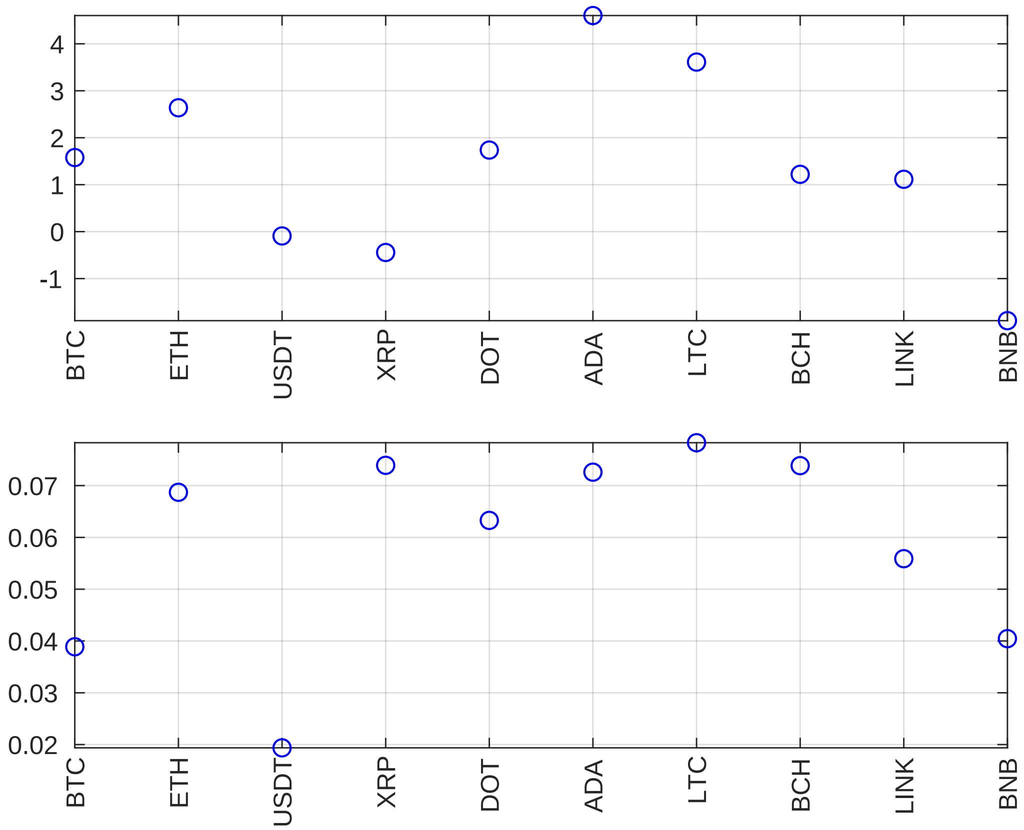
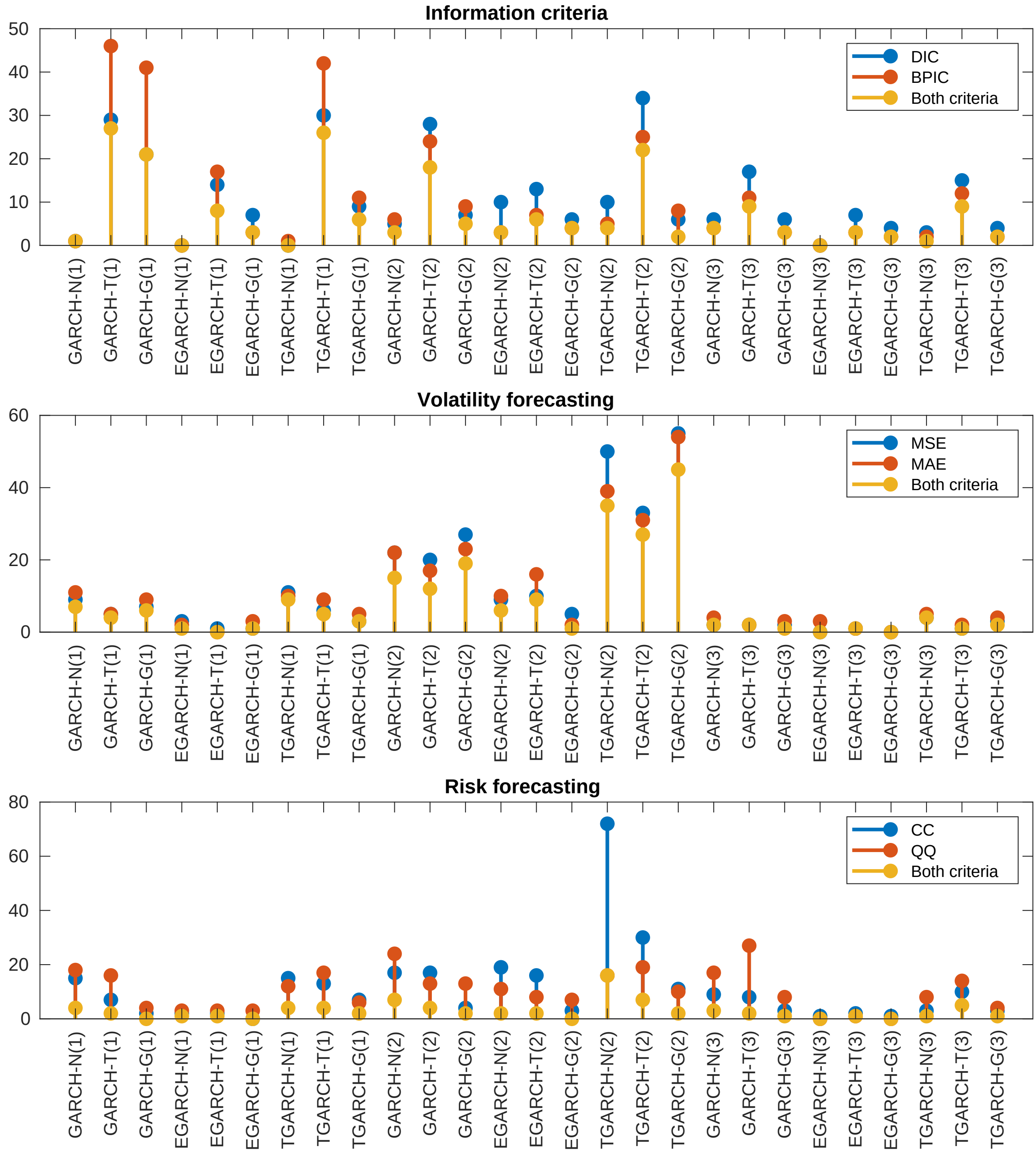


Figure 3: Absolute frequency of model selection based on in-sample and out-of-sample ananlysis.



Notes: Each figure presents the absolute frequency at which each model is selected. The total number of examined cryptocurrencies is 292. N, T and G denote the Normal, Student's-t and Generalised distributions. The number in parentheses denote the number of regimes.

A Complementary tables and figures

Table A1: List of symbols of cryptocurrencies used in the analysis.

ABBC	BONO	DERO	GAS	KMD	NAS	POLIS	SUB	XAS
ACT	BST	DEV	GBYTE	KNC	NAV	PPC	SYS	XCP
ADA	BSV	DGD	GHOST	KRT	NEBL	PPT	TERA	XDN
ADK	BTC	DIME	GNO	KSM	NEO	PZM	TFUEL	XEM
ADX	BTC2	DIVI	GNT	LCC	NIM	QASH	THETA	XHV
AE	BTG	DMD	GO	LINK	NKN	QRK	TOMO	XLM
AEON	BTM	DNT	GRC	LKK	NLC2	QTUM	TRUE	XLQ
AIB	BTS	DOGE	GRIN	LOKI	NLG	RADS	TT	XMC
AION	BTX	DTEP	GRN	LRC	NMC	RBTC	TUBE	XMY
ALGO	BURST	DUN	GRS	LRG	NPC	RBY	UBQ	XNC
AMB	CCA	DYN	GXC	LSK	NRG	RDD	UNO	XRC
ANT	CCXX	ECA	HBAR	LTC	NULS	REP	USDT	XRP
APL	CET	ECC	HC	MAID	NXS	RLC	USNBT	XSN
ARDR	CHI	EDC	HIVE	MAN	NXT	RSTR	VBK	XST
ARK	CKB	EDG	HNC	MBC	NYE	RVN	VERI	XTZ
ARRR	CLAM	ELA	HNS	MCO	NYZO	SALT	VET	XCC
ATB	CNX	EMC2	HPB	MED	OBSR	SAPP	VEX	XVG
AYA	COLX	ERK	HSS	META	OMG	SBD	VIA	XWC
BAT	COTI	ETC	HTDF	MGO	ONT	SC	VIN	XZC
BCA	CRW	ETH	HTML	MHC	OTO	SCP	VITAE	YOYOW
BCD	CSC	ETN	HYC	MIDAS	OURO	SERO	VITE	ZANO
BCH	CTC	ETP	ICX	MINT	OWC	SFT	VLX	ZEC
BCN	CTXC	EWT	ILC	MIOTA	PAC	SHIFT	VRA	ZEL
BDX	CURE	FCT	INSTAR	MIR	PAI	SKY	VSYS	ZEN
BEAM	CUT	FLASH	INT	MLN	PART	SLS	VTC	ZIL
BHD	CVC	FLO	IOC	MOAC	PAY	SMART	WAN	ZNN
BHP	DAG	FO	IOST	MONA	PCX	SNGLS	WAVES	ZRX
BIP	DASH	FRST	IOTX	MOON	PHR	SNM	WAXP	ZYN
BLK	DCN	FSN	IRIS	MRX	PI	SNT	WGR	
BLOCK	DCR	FTC	JDC	MTL	PIVX	STEEM	WICC	
BNB	DCY	FUN	JUL	MWC	PLC	STORJ	WINGS	
BNT	DDK	GAME	KIN	NANO	POA	STRAT	WTC	

Figure A1: Mean values for the returns of the cryptocurrencies.

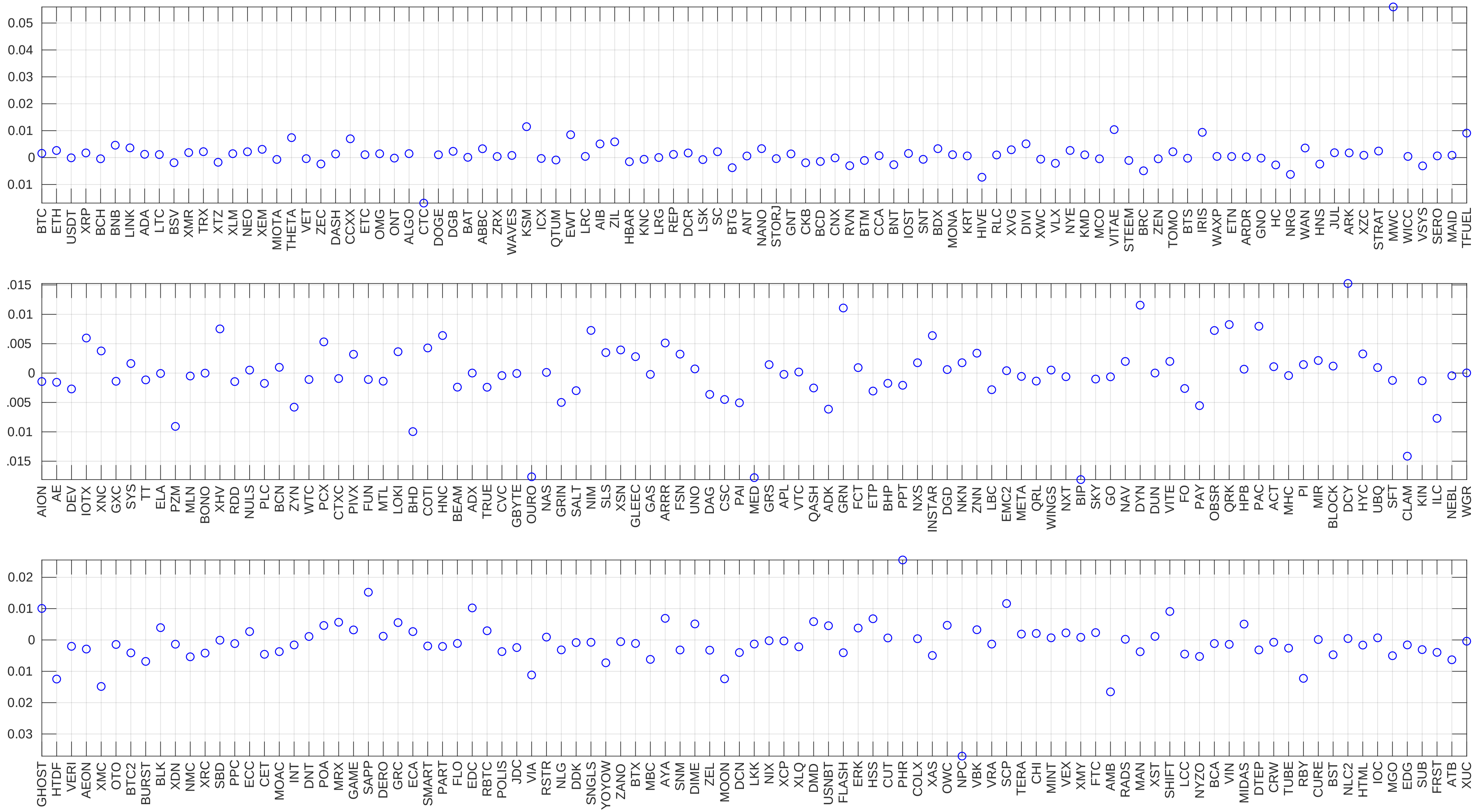


Figure A2: Standard deviations of the returns of the cryptocurrencies.

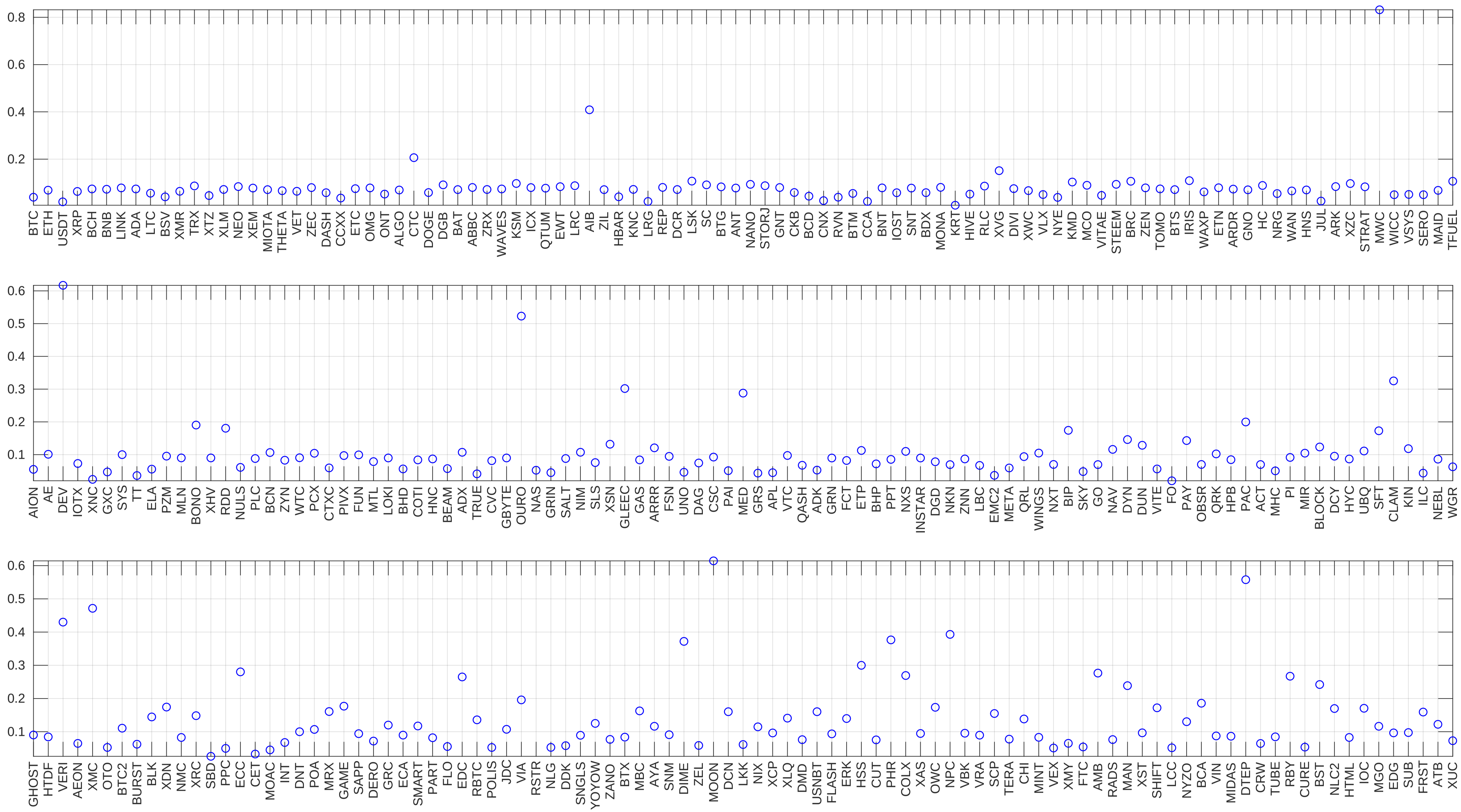
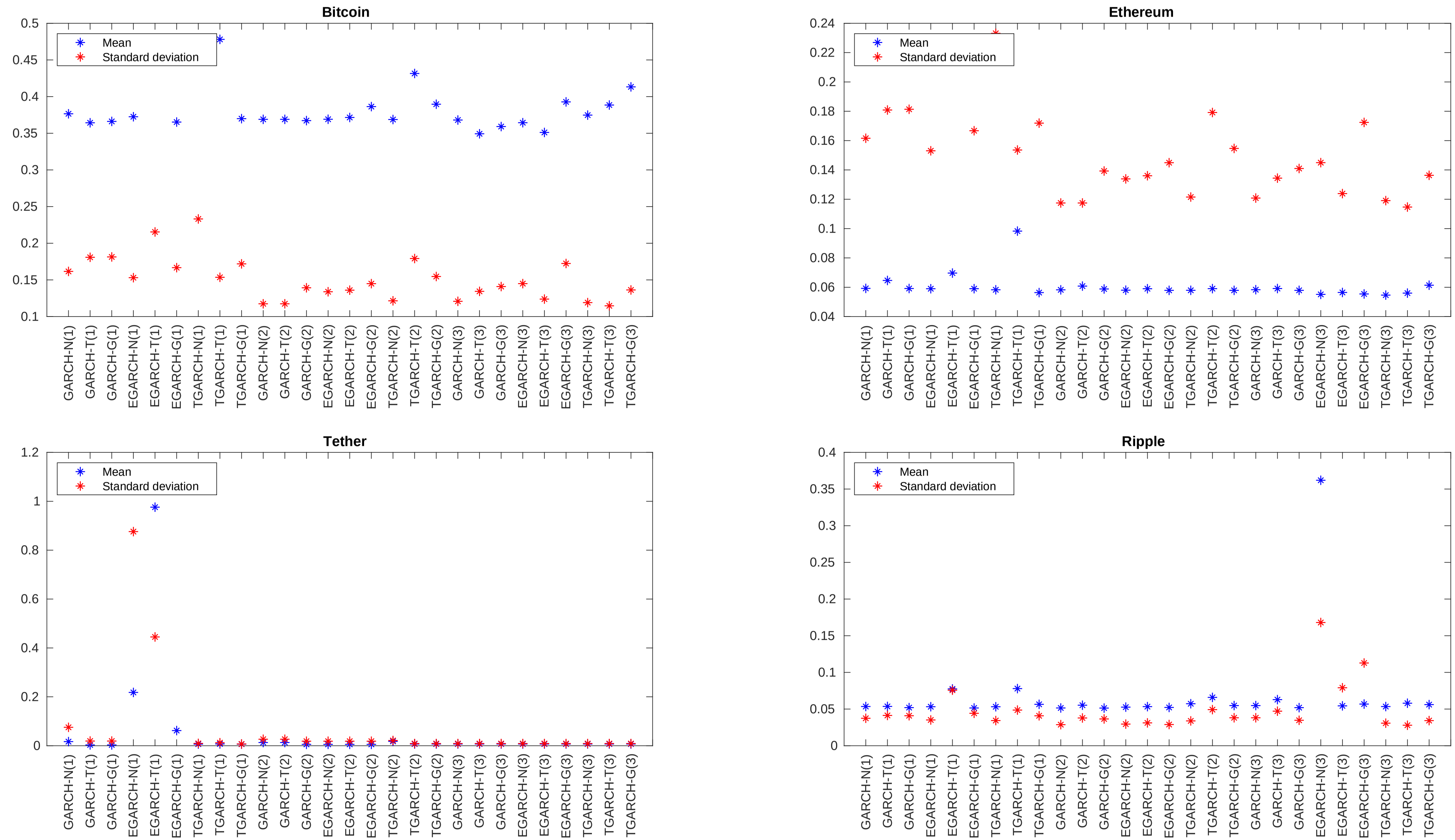
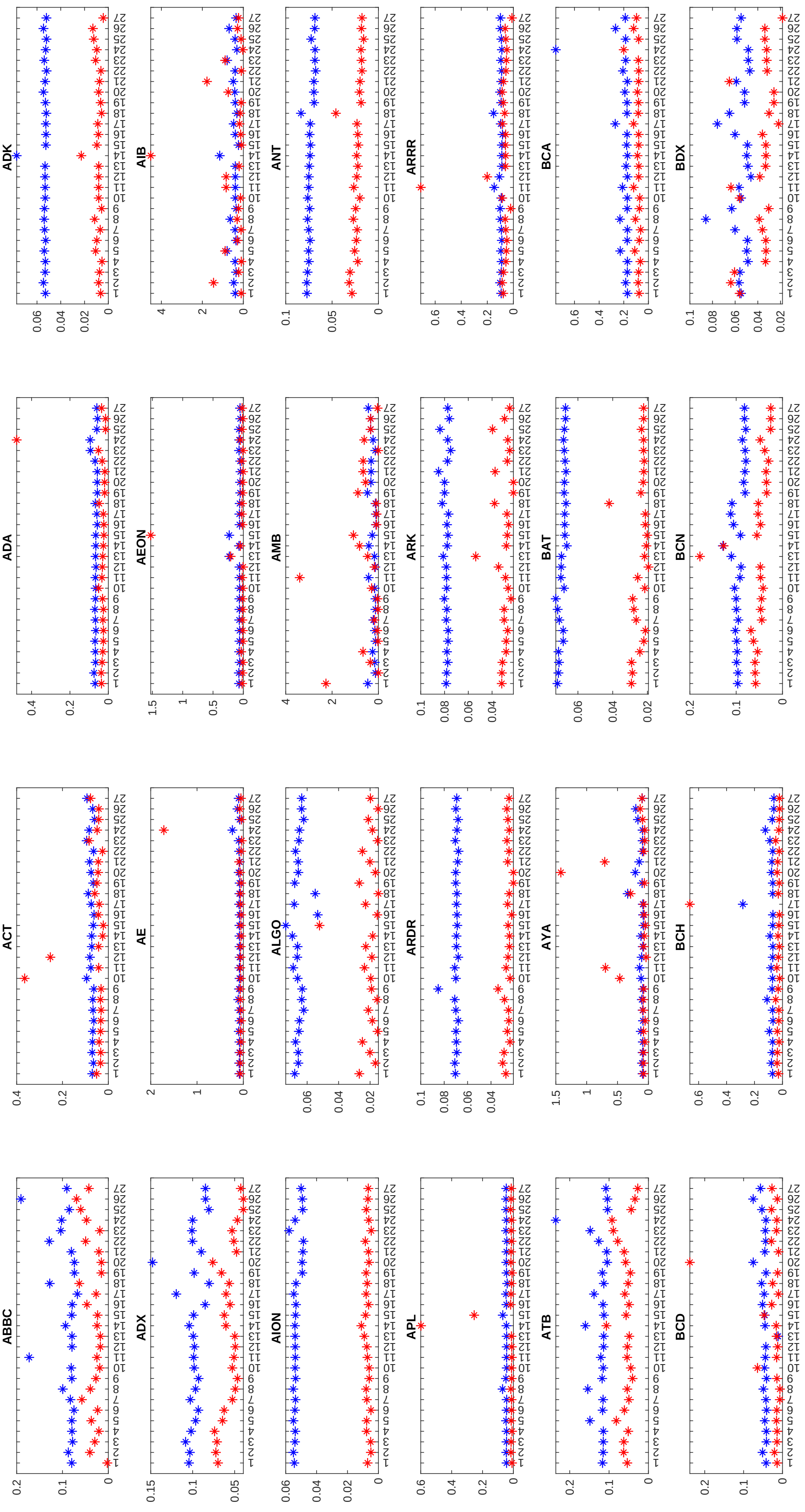


Figure A3: Mean and standard deviation of conditional volatility of cryprocurrencies.



Notes: Each plot presents the mean and standard deviation of the conditional volatility for the different GARCH models. The total number of models is 27. N, T and G denote the Normal, Student's-t and Generalised distributions. The number in parentheses denote the number of regimes.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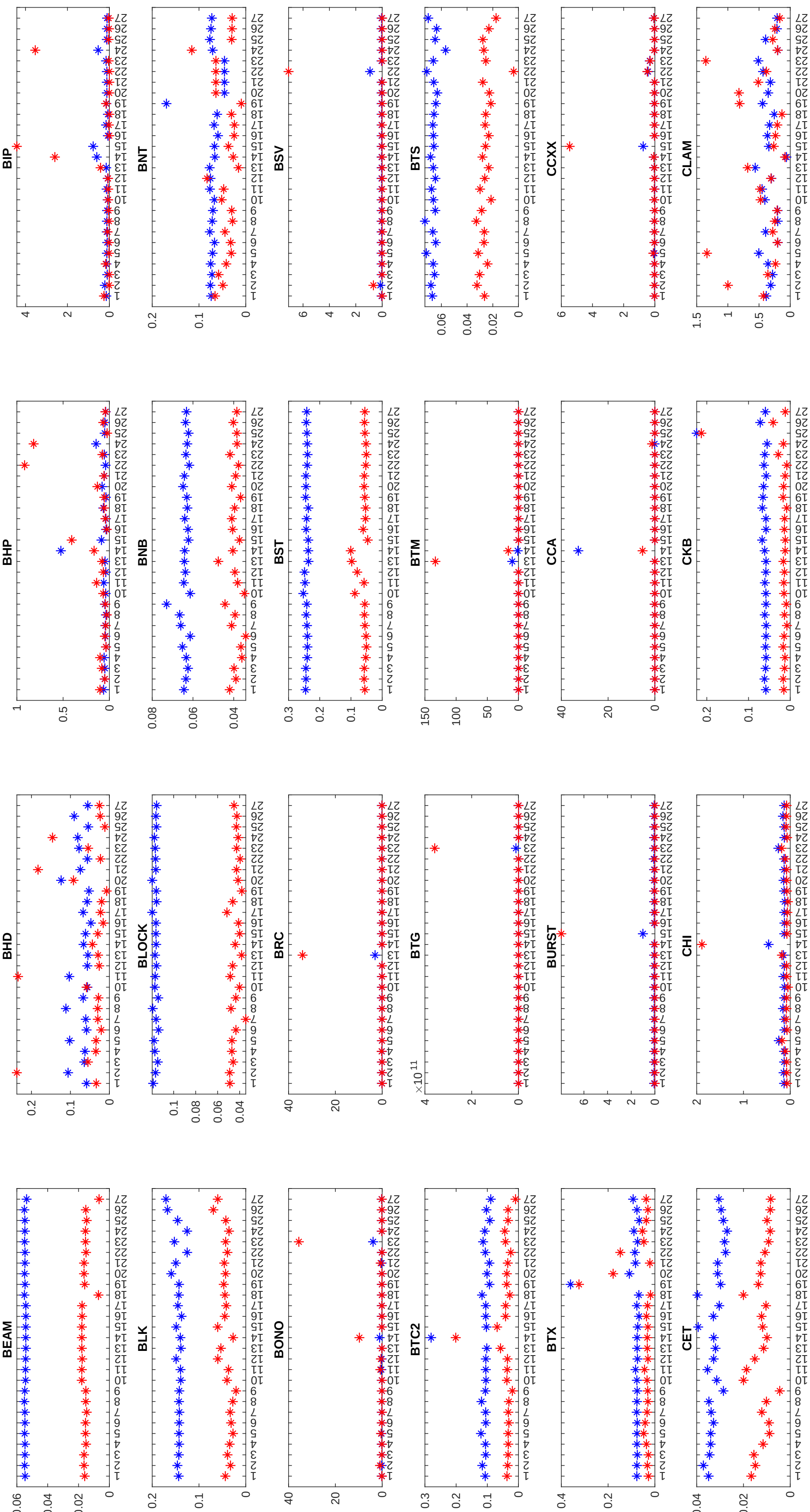
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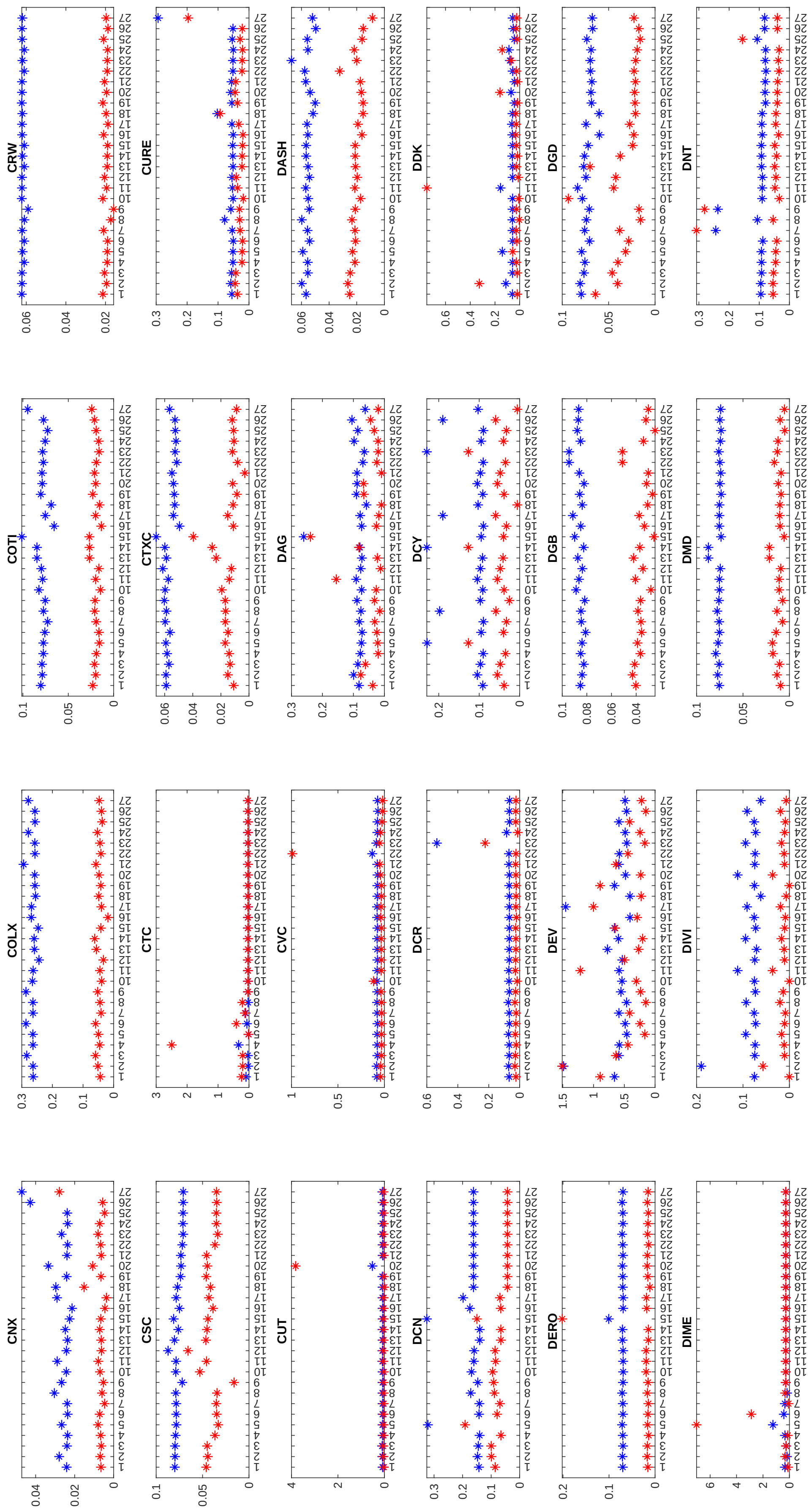

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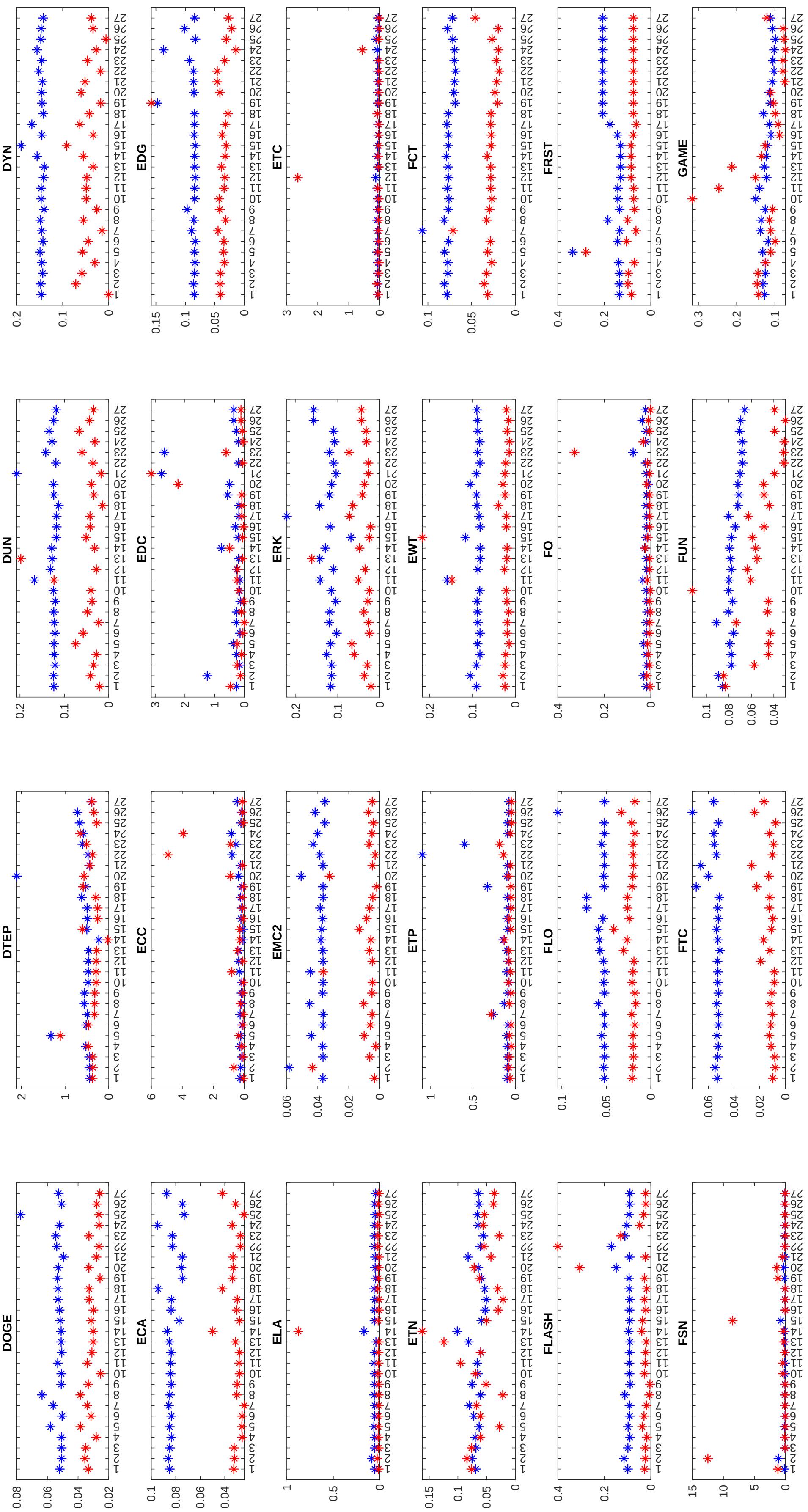
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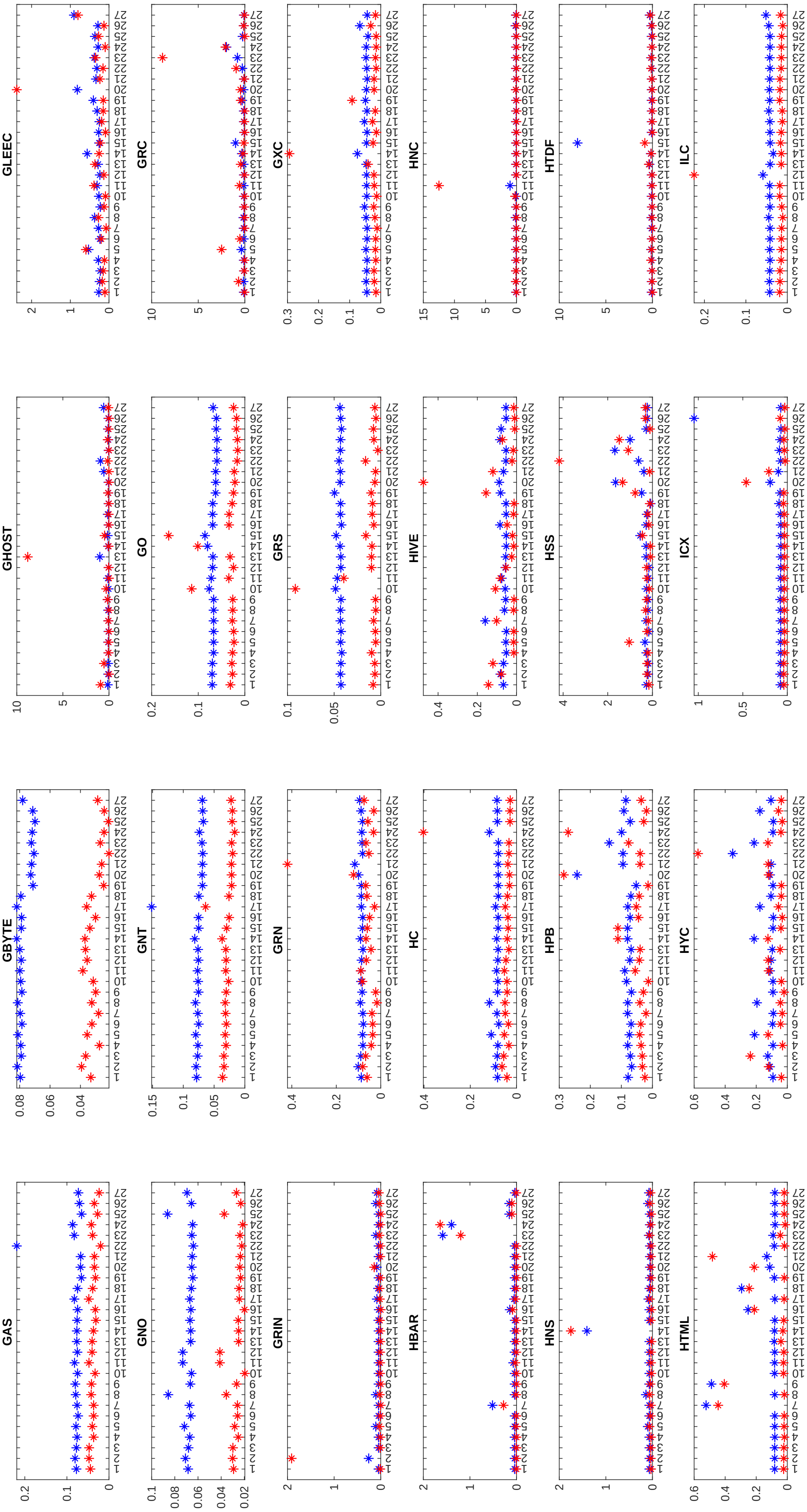
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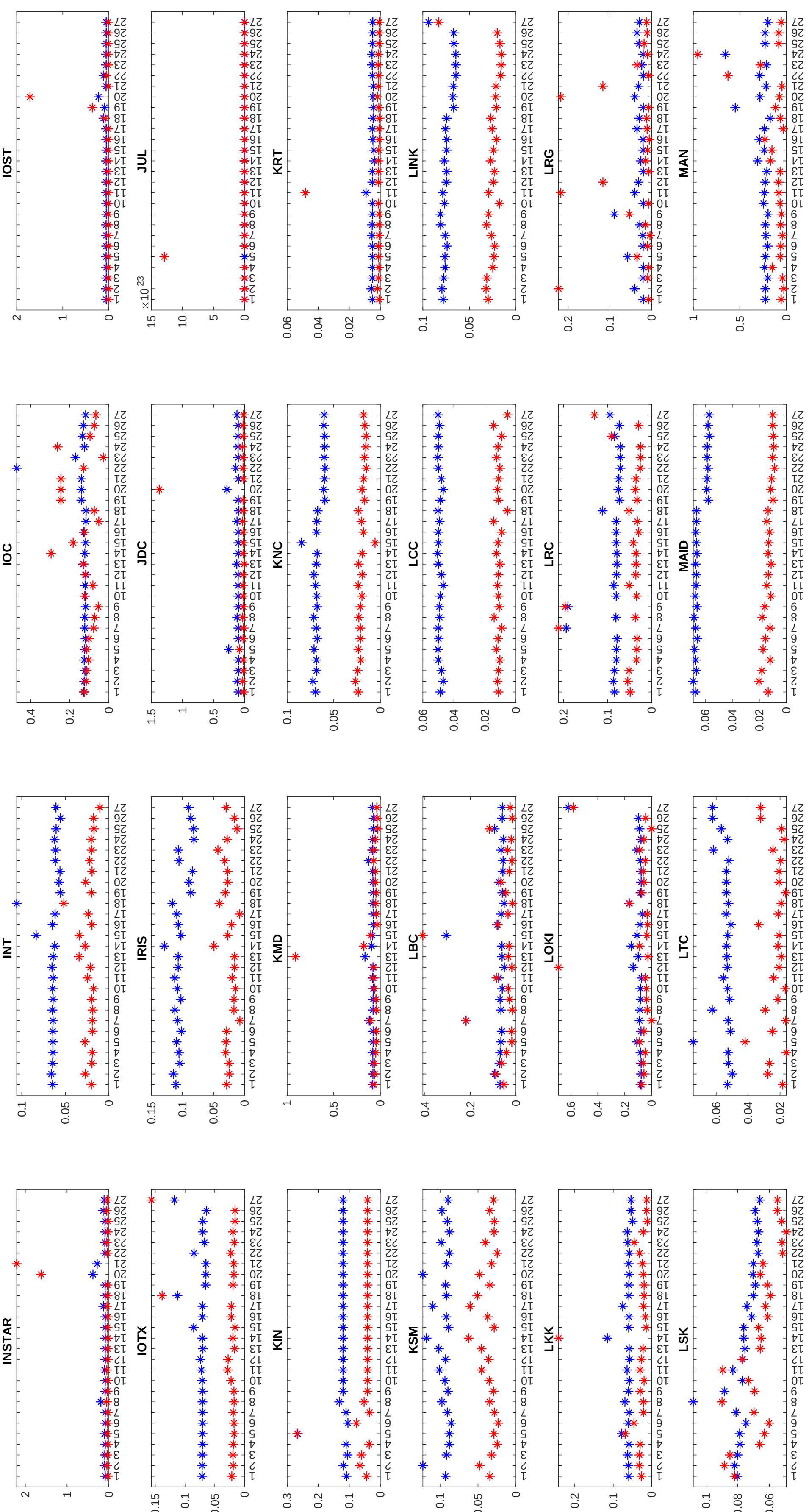
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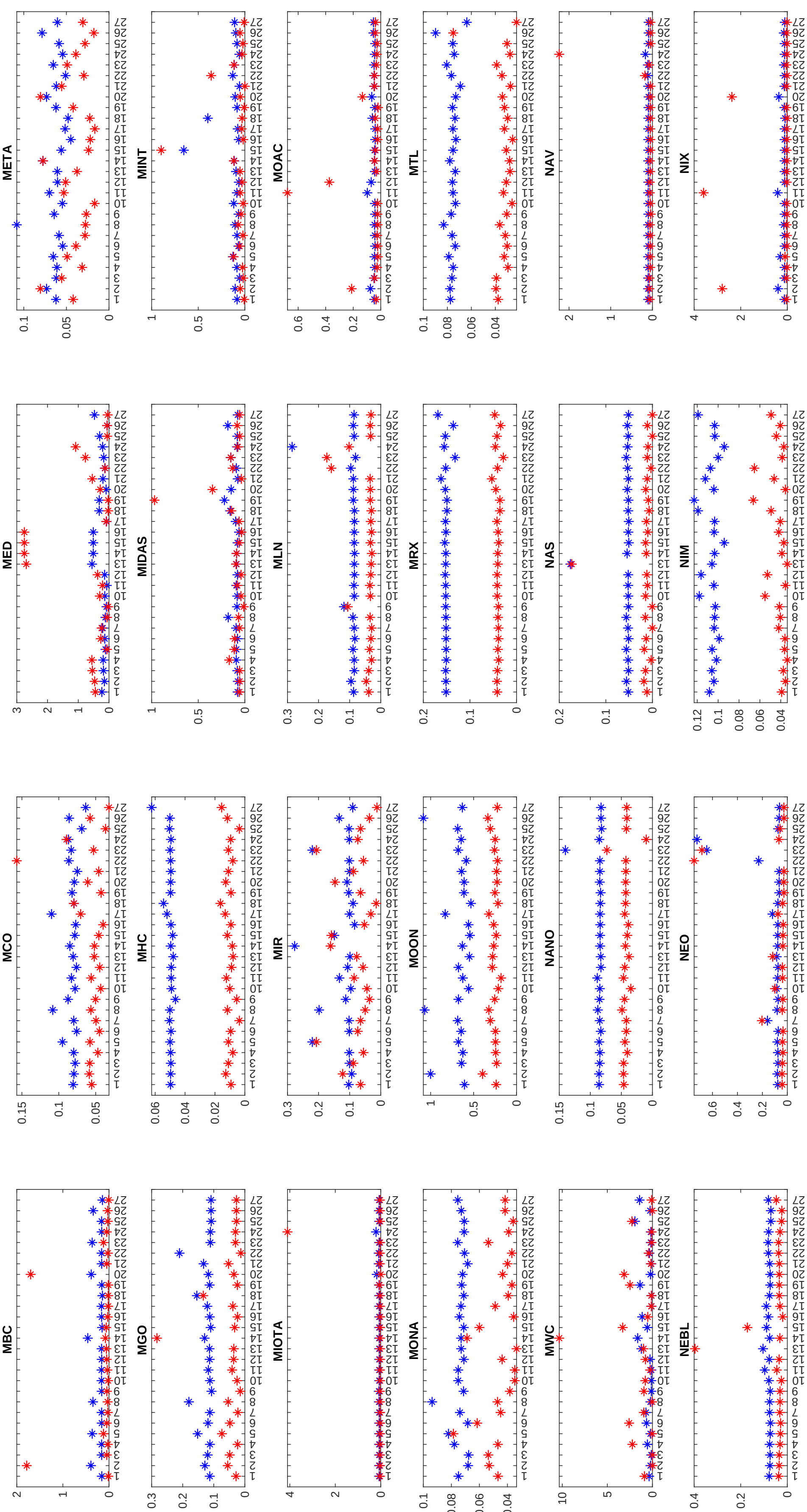
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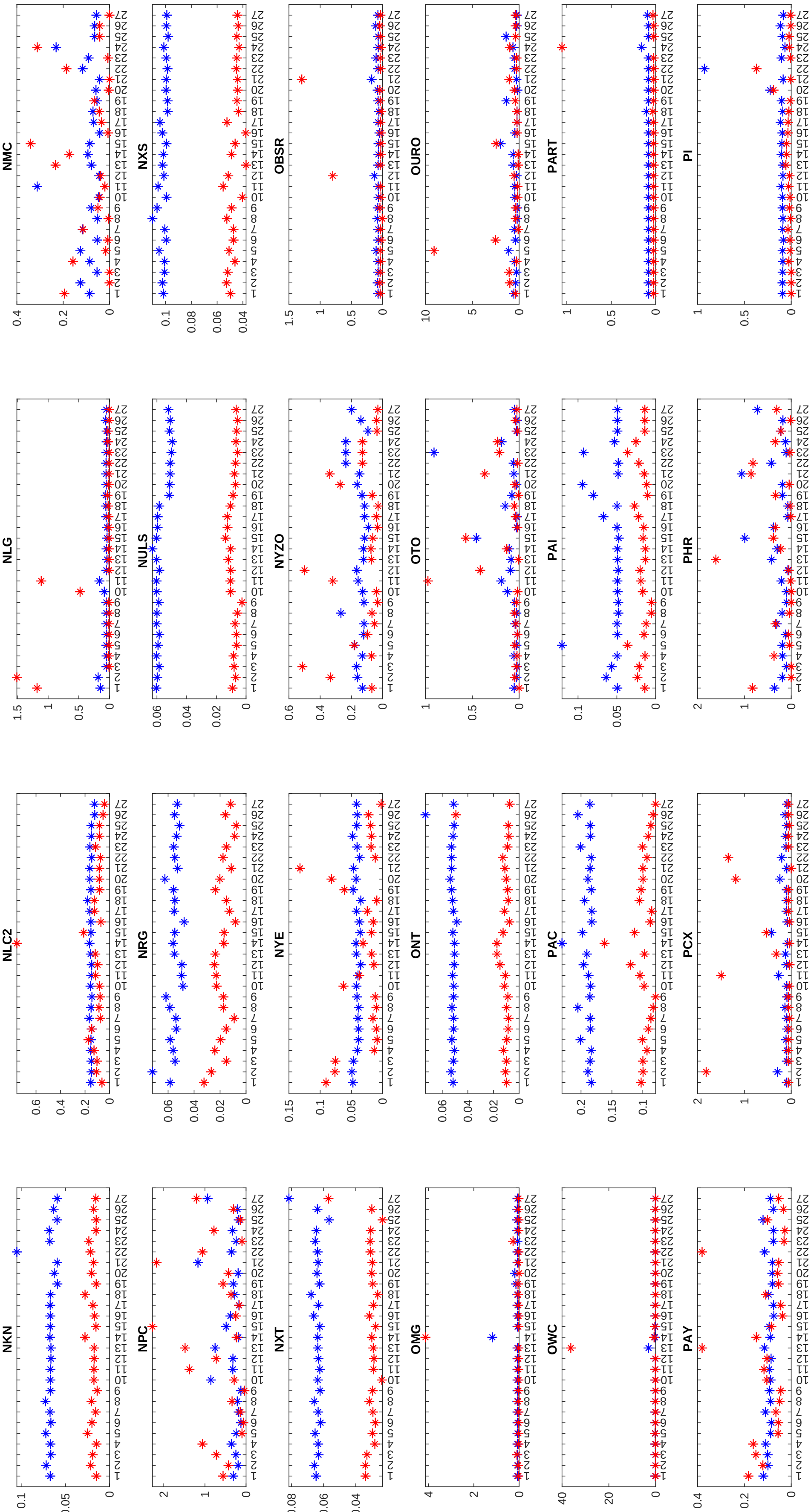
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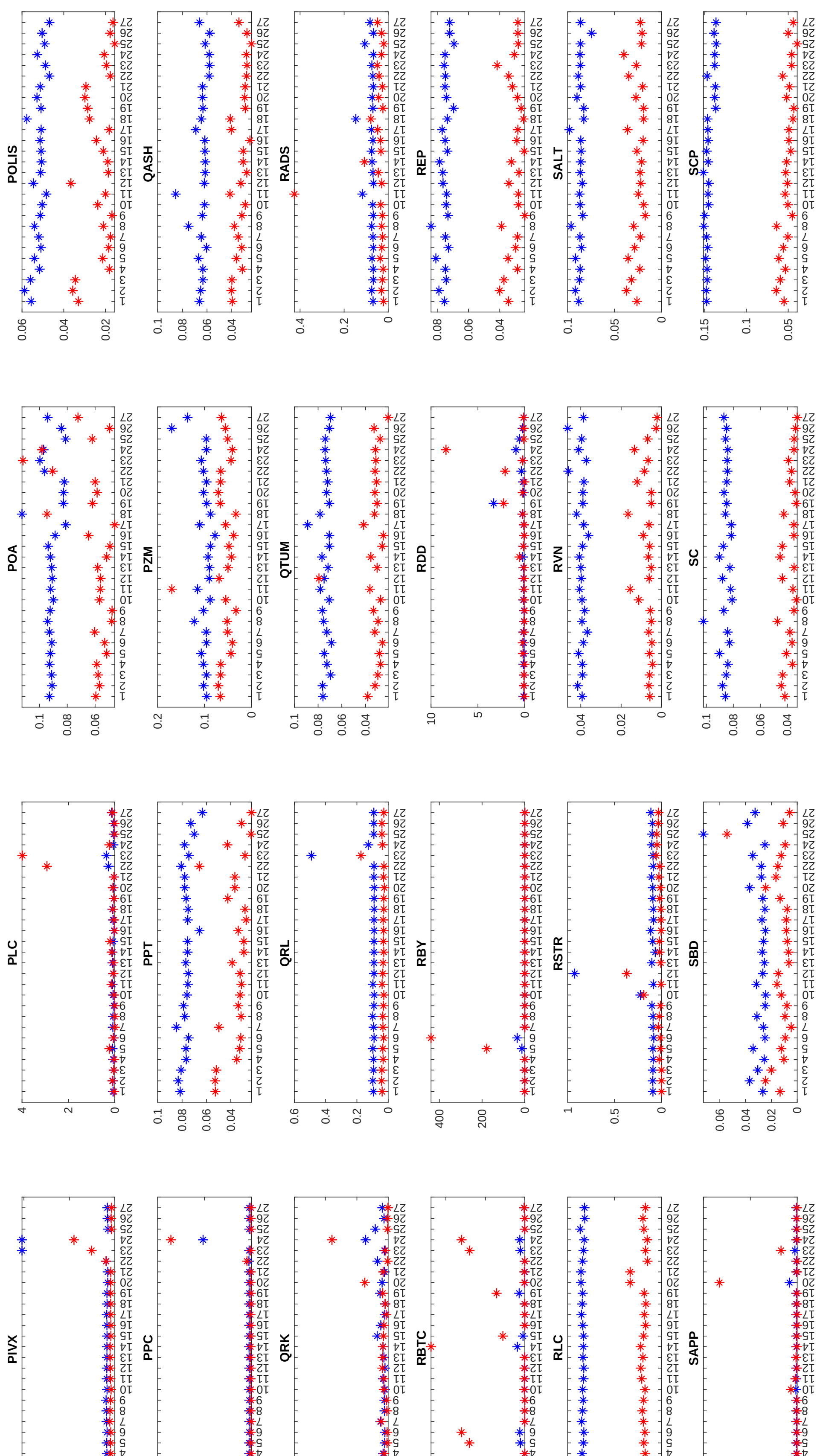
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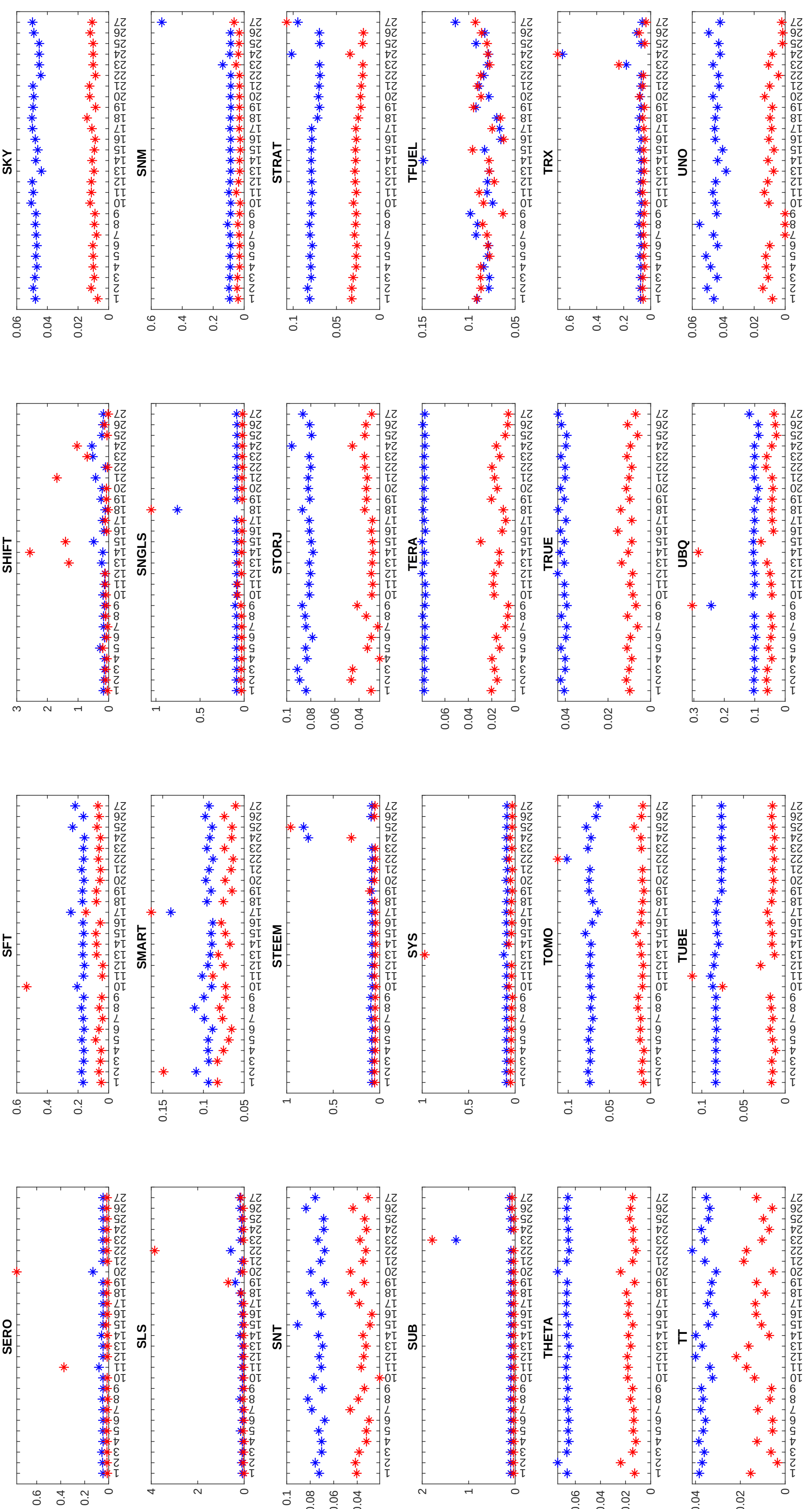
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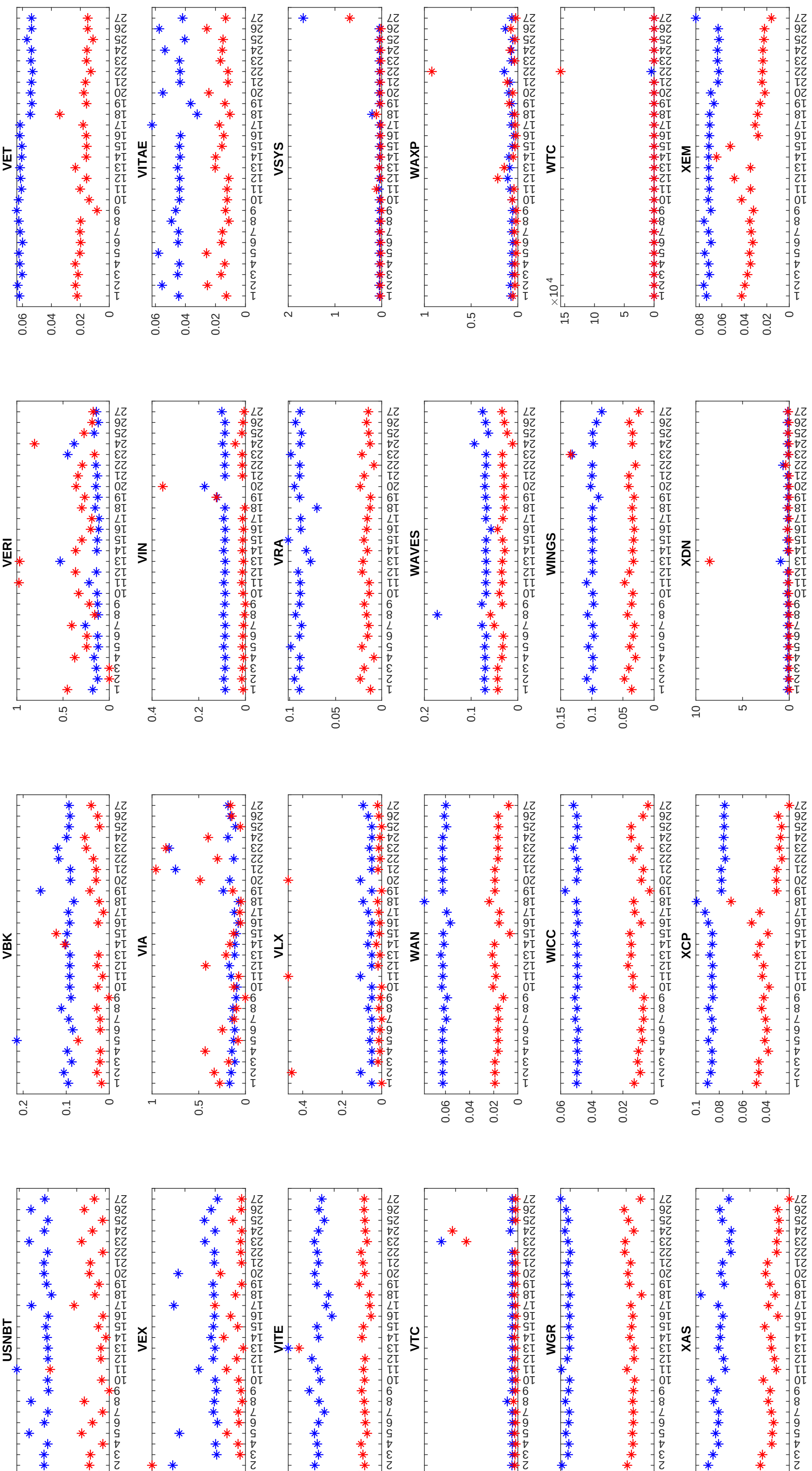
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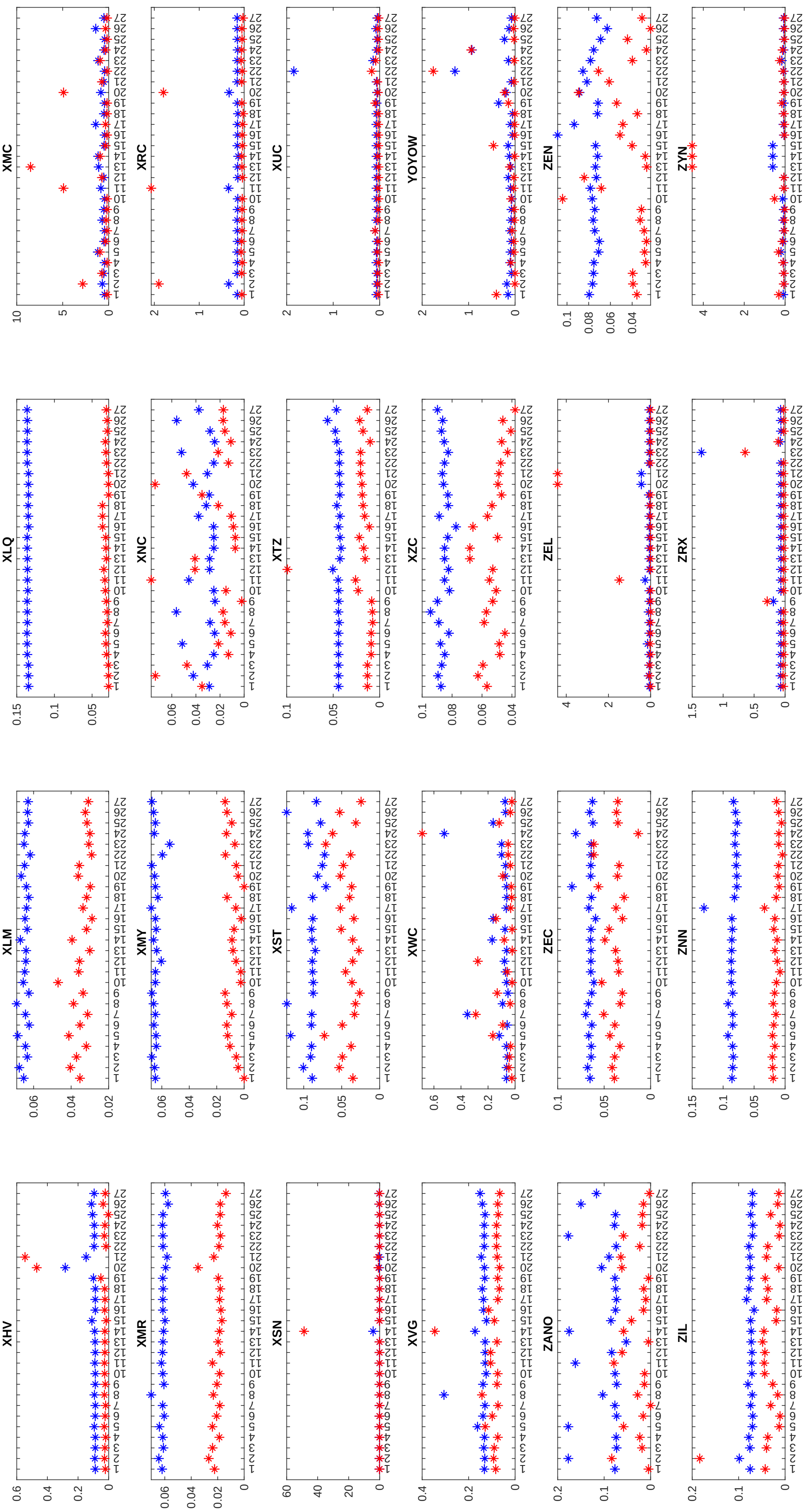
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