A blended portfolio construction technique using fundamentals and the Markowitz mean-variance methodology

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Abstract

When constructing a portfolio of stocks, do you turn a blind eye to the firms' future outlook based on careful consideration of companies' fundamentals, or do you focus on the stocks' correlation structures to ensure best diversification? The fundamental indexing (FI) and Markowitz mean-variance optimization (MVO) approaches are complementary but have until now been applied separately in the portfolio choice literature. This paper introduces a new portfolio selection technique that utilizes the benefits of both approaches. It relies on the idea of forecast averaging. In this paper, I propose to blend portfolios that provide investors with a clear **bi**nocular vision. I evaluate the proposed approach in out-of-sample tests and compare the blended portfolios to portfolios constructed solely based on the FI or MVO methods to attest to its superior performance.

Keywords: fundamental indexing, portfolio optimization, equities, forecast averaging, blended portfolio.
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1 Introduction and Literature Review

Markowitz [1952] identifies two stages in the portfolio selection process. The first stage is about forming beliefs about future performance. In practice, this often translates into reliance on historical data in estimating future rates of returns and their correlations. The second stage relies on the beliefs formed in the first stage and involves selecting a portfolio. Focusing only on the second stage, Markowitz [1952] introduces the mean-variance optimization (MVO) method for portfolio selection recommending that the choice of appropriate expected return and variance-covariance matrix "...should combine statistical techniques and the judgement of practical men..." [Markowitz, 1952, p.91]. The conventional approach often ignores the need to develop appropriate beliefs. As Markowitz emphasises, it is our responsibility to use "observation and experience" to develop "beliefs about the future performances" [Markowitz, 1952, p.77]. Although predicting future performance of stocks is a daunting task, the literature provides strong evidence that fundamental analysis may have some merit (Arnott et al. [2005], Walkshäusl and Lobe [2010], Basu and Forbes [2014]). Combined with evidence from forecast combination literature, we firmly believe that fundamental analysis may improve the out-of-sample performance of MVO portfolios.

In this paper we propose to use an innovative blended technique, combining the efficient portfolio selection method of Markowitz [1952] that takes into account the covariance structure of portfolio holdings and the fundamental indexing (FI) approach that identifies investments with sound economic, financial, and indirect managerial characteristics. In practice, the MVO method relies on past returns to predict future returns and correlations. Past correlations predict future correlations reasonably well, while past returns fail to predict future returns in a long run [Jorion, 1986, Poterba and Summers, 1988, Cuthbertson and Nitzsche, 2005, p.158]. Given the volatile nature of these underlying processes, the MVO method likely produces superior out-of-sample results for only short-term investments. However, frequent portfolio updates based on the latest historical data are recommended for consistent superior results, but lead to high portfolio turnover and increased transaction costs. Transaction costs buildup is of particular concern for long-term investments. Thus, in the industry, long-term investments are often based on "the judgement of practical men", rooted in fundamental analysis. It focuses on financial statements and the well-being of a company in an attempt to evaluate its long-term economic prospects, assessing its future growth and investment potential.

Taken separately, both the MVO and the FI have their own limitations: the FI approach ignores the correlation structure of stocks' returns, while the classic MVO method is silent about the firms' fundamentals, which may be the driving factors of the stocks' future performance. Our blending technique combines the classical method and the FI approach, by bridging the two stages of portfolio construction mentioned in Markowitz [1952]. As we will show, the resulting portfolios are superior in out-of-sample mean-variance space compared to portfolios based on each method alone, as well as conventional benchmarks.

The relevant literature has two streams: the MVO literature and the FI literature. Each of these streams consider stocks through a specific "oculus" described in the next two paragraphs. Up until now stocks have been considered through either of these oculus separately.

The first oculus we consider, the MVO method, was introduced by Markowitz [1952]. In this approach, the expected return and the variance-matrix are calculated based on in-sample information, and securities are allocated accordingly to the MVO procedure which maximizes the expected return given the variance. Since the introduction of the MVO, a myriad of methods have been proposed in an attempt to refine it and offer superior out-of-sample performance. Among the most noticeable and practical extension of the MVO method is that for outlier control. Outliers often result in biased estimates of sample statistics translating in disproportionate portfolio holding weights. Several prominent robust techniques have been proposed to take this into account. For example, Ledoit and Wolf [2004] introduce a method that shrinks the sample covariance matrix to a well-conditioned persimoneous structure to reduce estimation errors that were shown to bias the classic MVO method. As an alternative to the shrinkage method, limiting portfolio to long positions, can produce similar results [Jagannathan and Ma, 2003]. However, Jagannathan and Ma [2003] note that such method might lead to poor diversification, with only 20-25 stocks in the portfolio; thus, to increase diversification and reduce the effect of measurement errors, it is possible to set up an upper bound on weights (e.g., 5-10%)[1]. Since the MVO method suffers from the negative effects caused by measurement errors, outliers and ignorance of firms' fundamentals, the performance of the classic MVO method, even with adjustments for outlier effects, often does not exceed market benchmarks such as equally- or capitalisation-weighted portfolios in out-of-sample tests. Hence, if our technique shows statistically significant results, it cannot be attributed to the MVO part of the technique alone.

We shift our focus to the other "oculus", the FI approach introduced by Arnott et al. [2005]. In this approach, firms are ranked based on firms' fundamentals and securities are allocated proportionally to their overall fundamental scores. The fundamentals might include the book value, cash flow, revenue, sales, dividends, total employment, etc. Recent paper by Asness et al. [2015] argue that FI indexing is basically systematic value investing. The FI approach significantly outperforms major benchmarks based on US market data [Arnott et al., 2005]. Walkshäusl and Lobe [2010] apply the FI approach to stocks from 50 countries and find that the FI approach outperforms capitalization-weighted portfolios in most countries. However, after applying the robust-to-fat-tails performance test proposed by Ledoit and Wolf [2008], the FI portfolios in only 6 countries and the global FI portfolio have statistically significant positive differences in Sharpe ratios. The US FI portfolio outperformed the cap-weighted portfolio, but the result was not statistically significant. Hence, if our technique shows statistically significant results, it cannot be attributed to the FI part of the technique alone.

Given that the FI approach is relatively new, and is profoundly different from the MVO,

[1] Coincidentally, these weight recommendations are in accord with guidelines of many investment funds that try to avoid excessive dominance of a single security.

these two approaches have not yet been combined, even though each method offers distinctive benefits for portfolio choice problems. In fact, Hong and Wu [2016] show empirically that information on past returns and on the firms' fundamentals are complementary. They also show that in "good times", when volatility is low, past returns provide better information about future returns. However, fundamentals perform better in "bad times", when volatility on the market is high. In such periods, past returns are not that informative and investors are forced to rely on firms' fundamentals. Thus, a portfolio allocation strategy should rely more on past returns in times of low volatility and rely more on the firms' fundamentals in times of high volatility.

This paper fills an important gap in the literature by bridging the FI and the MVO approaches that are used widely, but separately, both by the academics and practitioners. By blending the two different approaches we take into account the firms' fundamentals and correlation structure of security returns. Out of all portfolios constructed with the MVO method, the most information about the correlation structure is used in the Global Minimum Variance (GMV) portfolio. More imortantly, construction of the GMV portfolio does not rely on estimates of expected returns, which makes it the portfolio of choice in blending with the FI portfolio. Firms' fundamentals help us detect and concentrate on 'healthy', undervalued stocks that are likely to grow in the long-run, while the assessment of the correlation structure allows us to construct a well-diversified portfolio.

The FI and the GMV portfolios are depicted in Figure 1. The technique that we propose blends these two portfolios into one. First, the FI portfolio is constructed based on firm' fundamentals using the FI approach. Second, the GMV portfolio is the minimum variance portfolio of the mean-variance portfolio frontier. To see whether it is worth blending, we try 101 blend combinations (in one percent increments) of these two portfolios, which generate the new, blended GMV/FI mean-variance frontier (GMV/FI frontier). On the new GMV/FI portfolio frontier, we select a portfolio tangent to the capital market line. This tangency portfolio is the final outcome of our blending of the GMV/FI techniques. This technique has several desirable features.

First, the two initial portfolios are formed using profoundly different methods, that should result in better performance of the combined model. Since we are concerned with out-of-sample performance of our portfolios in mean-variance space, our proposed blended approach parallel methods proposed in the forecast combination literature. Models with combined forecasts have been shown to outperform individual forecasts [Bates and Granger, 1969, Ericsson, 2017]. For an excellent survey of the literature, please see Hamilton [1994].

Second, since portfolios constructed based on the classic MVO approach (e.g. GMV) and the FI approach (e.g. FI) are most likely not perfectly correlated, the mean-variance optimal frontier based on these two portfolios will not result in a straight line. This "second-stage" mean variance optimization offers further refinement combining the weights of the GMV and the FI portfolios proportionally as in Figure 1. Since the FI portfolio brings additional relevant

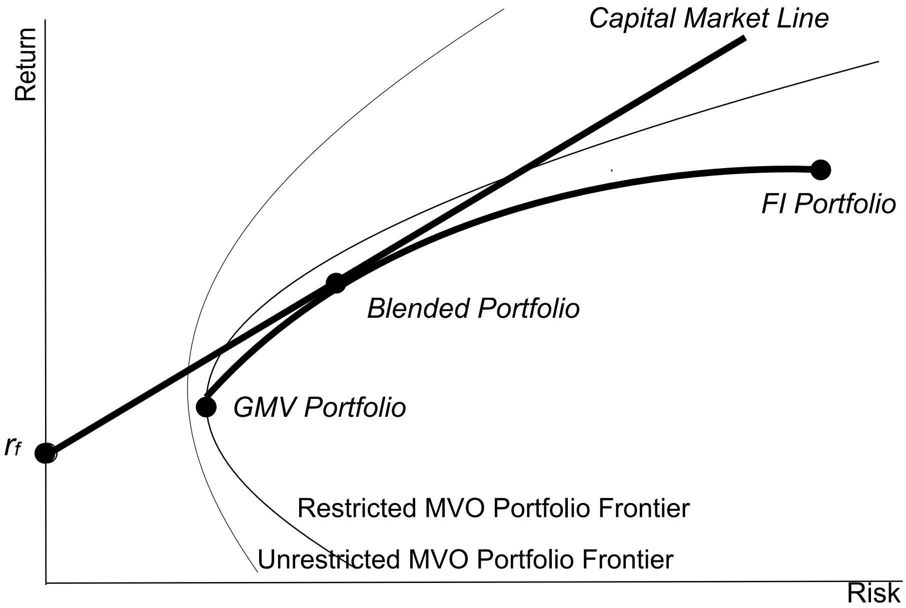

Figure 1: Bridging MVO and FI approaches.

information which was not included in the estimated mean-variance frontier, the new blended portfolio may generate a frontier that outperform the MVO efficient frontier in out-of-sample tests.

Third, construction of the GMV and FI portfolios does not depend on individual stocks' returns, which, as we mentioned earlier is the major source of error. In the second stage we calculate the expected return of the GMV and FI portfolios as a whole. In this case, the individual errors are pooled, which result in much lower estimation error of the portfolio's expected returns.

To test our technique we use a dataset of all common stocks listed on the NYSE, NASDAQ and AMEX for the last 26 years.

In out-of-sample tests the proposed blended portfolio outperforms the following benchmarks: $1/n$ strategy, S&P 500, market, GMV and FI portfolios based on each fundamental separately. It is not a surprise that our approach outperforms standard market capitalization approach, which suffers from allocating too much weight on overpriced stocks. In their provocative paper Arnott et al. [2015] argued that using stale market capitalization weights of surviving stocks from 20 years ago would beat the portfolio constracted with the current market capitalization weights in the out-of-sample tests.

The rest of the paper is organised as follows. We discuss the data preparation steps in Section 2. We introduce the methodology in Section 3 and discuss our empirical findings in Section 4. Finally, Section 5 concludes.

2 Data Description

We consider all common stocks listed on the NYSE, NASDAQ and AMEX from January 1990 to January 2016. To avoid survivorship bias we include delisted stocks in our analysis (see Brown et al., 1992). We obtain daily price indices, market values, price to book values and return indices (RI), which is price index plus dividend disbursements. We collect annual data on revenues, dividends per share, dividend yields and payout ratios. All data are from the Thomson Reuters Datastream.

To test our approach in an out-of-sample setting we construct 20 data samples using five year rolling windows with annual increments (1990-1995,1991-1996, etc) beginning on July 1 of every year to ensure public availability of fundamental data for previous calendar years. Each sample has six years of data; the first five years (1990-1994, 1991-1995, etc.) of each sample are used to analyse the stocks' in-sample information and to construct a portfolio. The last year of each sample (1995,1996, etc.) is used to test portfolios' out-of-sample performance. For each sample we select stocks with data available in the previous five years (2,039 stocks in an average sample, see Table 1).

To refine the data we apply several filters. If the return index of a stock changes more than 50% in one day, it means that either the stock is likely to be mispriced or there is some kind of data imperfection. We do not wish to consider such stocks in our portfolio, and we exclude them from the list of candidates for our portfolio. This filter is robust to market crashes, because historically the largest or most memorable daily percentage losses of Dow Jones Industrial Average were much smaller (22.61% on the October 19th, 1987 crash; 12.82% on October 28th, 1932, the biggest one day market crush during the Great Depression; and 7.87% on October 15th, 2008, the the ubiquitous economic weakness in all 12 Federal Reserve districts). Of course, we can apply this filter only for the in-sample period of the subset. Similarly, we only keep stocks that have positive book values and nonnegative dividends per share, dividend yields and distributed payout ratios. This filter allows us to avoid companies which distribute dividends in the periods with no profits. Furthermore, we exclude stocks priced more than 50 times higher than their book value per share. This filter still preserves most growth stocks, but eliminates stocks that are likely to be highly overpriced. We exclude stocks with dividend yields of more than 50%, and payout ratios of more than 200%. Please note, that the payout ratio can be more than 100%, when the company distributes the net income made in previous periods. The effects of these filters are presented in Table 1. An average sample that we analyse includes 1,384 stocks. The first sample (1990-1994) includes 577 stocks, and the last sample (2010-2014) includes 2,277 stocks. Overall the data analysed includes 21,996 firm-years.

3 Methodology

3.1 Creation of additional variables.

Since the total return index (RI) reflects both the price of an asset and any dividend disbursements, we obtain daily stock returns as follows:

$$r_{i,t} = \frac{RI_{i,t} - RI_{i,t-1}}{RI_{i,t-1}} \tag{1}$$

Table 1: Data Availability and Filters

Filter	Freq	Filter Description	N stocks	% of Market Cap
No Filter			1,976	98.52
Return Index (RI)	d	daily change≤50%	1,753	97.39
Market Value (MV)	d	50%≤percentile	1,102	96.48
Price to Book Value (PTBV)	d	0<PTBV≤50	1,046	93.96
Dividend per share (DPS)	a	0≤DPS	1,046	93.96
Dividend Yield (DY)	a	0≤DY≤50	1,045	93.95
Payout Ratio (POUT)	a	0≤POUT≤200	846	79.71
Revenue (Rev)	a		846	79.71

This table provides an average number of stocks for 20 in-sample rolling windows that are used to construct portfolios. "Freq" describes the frequency of underlying data with "d" and "a" denoting daily and annual data. "N stocks" is the average number of stocks left in the sample after each of filters is applied. "% of Market Cap" is the average capitalisation share of the stocks left after each filter. The base for the "% of Market Cap" is total market valuation of the companies that have data on RI for the last five years (=100%). Note, that on average 98.52% of the total market value is captured by stocks that have data on every variable of interest for at least five proceeding years. Then, on average 97.39% of the total market value is captured by stocks that satisfy RI filter (daily change $\leq$ 50%), etc.

Note, that using the simple return formula is essential for accurate aggregation of assets in portfolios, whereas log returns are convenient for time aggregation but result in inaccurate estimates when aggregated across several securities.

The book value, one of the main variables of interest, is published on different dates, which does not eliminate the possibility of mistakenly assigning the outdated book value to a specific date. Instead, market value and price to book value are reported daily. Thus, to obtain the book values, we use the calculated book value of the company, which ensures the availability of the most recent book values on daily basis:

$$BV_{i,t} = \frac{MV_{i,t}}{PTBV_{i,t}} \tag{2}$$

3.2 *Construction of the Global Minimum Variance (GMV) portfolio*

The GMV portfolio carries the most information about the diversification structure. In general, it is obtained from the optimization problem:

$$\mathbf{w}^{GMV} = \arg\min_{w} \sigma_p^2 \qquad \text{s.t.} \qquad \begin{cases} w'e = 1 \\ \sigma_p^2 = w'\Omega w \end{cases} \tag{3}$$

where, Ω is the variance-covariance matrix of all stocks, and e is the column vector of ones. The individual asset weights in GMV portfolio, w_{GMV}, the portfolio expected return, μ_{GMV}, and variance, σ^2_{GMV}, are obtained using formulas outlined in Algorithm 2.

Also we calculate restricted portfolio, with no short sales and the maximum of 10%. The restricted GMV portfolio is obtained from the optimization problem (3) with the added constraint on weights as in

$$0 \leq w \leq 0.1 \tag{4}$$

We calculate this and other restricted portfolios using Matlab portfolio object.

3.3 *Construction of the Fundamental Indexing (FI) portfolio*

Previous literature [Arnott et al., 2005, Walkshäusl and Lobe, 2010] considers fundamental indexes based on a single metric or an average of a number of fundamental factors. A single metric fundamental index can be calculated as:

$$w_i^X = \frac{X_i}{\sum_{j=1}^n X_j}, \tag{5}$$

where X_i can be any of the considered fundamental, e.g., BV, Rev, POUT, etc. Alternatively, X_i can represent a simple average of a number of fundamental factors:

$$X_i^{Composite} = \frac{1}{3}(w_i^{BV} + w_i^{DIV} + w_i^{Rev}) \tag{6}$$

In the case, when the were no dividend payments, X_i becomes:

$$X_i^{Composite,\ no\ dividends} = \frac{1}{2}(w_i^{BV} + w_i^{Rev}) \tag{7}$$

Weights in the FI portfolio are proportional to the value of the fundamental index for each stock. So, in general the weights are:

$$w_i^{FI} = \frac{FI_i}{\sum_{j=1}^n FI_j} \tag{8}$$

We test several methodologies of the FI construction, which are not necessary a simple average. Thus, in general fundamental index of each stock is i is:

$$FI_i = \sum_k F_{i,k}\phi_k, \tag{9}$$

where F_k is the value of the normalized fundamental characteristic k of the stock i (for example, $F_{MV} = w_{MV}$), ϕ_k is the weight of fundamental k in the FI scoring.

For PTBV we construct two types of fundamental characteristics: one is similar to 5:

$$F_{PTBVai} = \frac{PTBV_i^{-1}}{\sum_{j=1}^n PTBV_j^{-1}} \tag{10}$$

And the second one based on the ranking. We rank all firms in the ascending PTBV order. That is firm with the highest PTBV will be ranked as number 1. Then,

$$F_{PTBVbi} = \frac{rank(PTBV_{it})}{\sum_{j=1}^n rank(PTBV_{jt})} \tag{11}$$

Firms with the lowest price of their balance sheet equity (value stocks) have greater F_{PTBVi}.

We test multiple approaches in FI construction by changing ϕ_k in 9. However, for each FI we apply the same consequent steps. Thus, regardless of the specific construction technique the portfolio constructed in this subsection is referred as FI portfolio for the rest of the section.

The expected return and variance of FI portfolio are calculated as described in the algorithm 2.

3.4 *Construction of the Blended GMV/FI (B) portfolio*

We define the Blended GMV/FI portfolio (denoted with a subscript B) as the tangency portfolio between the capital market line originating at r_f and the efficient frontier based on the two risky assets - the GMV and FI portfolios. But before developing best blended portfolio strategies, it is worth checking whether there are some blending strategies that could outperform benchmarks. To do so, we take 101 combinations of GMV and FI portfolios: (0% FI, 100% GMV), (1% FI, 99% GMV), ... , (100%FI, 0% GMV). Then we compare these portfolios to benchmarks in out-of-sample.

If there are some blended portfolios combinations, that are superior to benchmarks, then it is worth discussing how to find the best of them beforehand. We attempt to provide some guidelines, of finding the best blended portfolios out-of-sample.

There are two ways to calculate weights of the blended portfolio. First, is to calculate the weights of GMV and FI portfolio in the blended portfolio:

$$\begin{pmatrix} v_{GMV} \\ v_{FI} \end{pmatrix} = \underset{v_{GMV},v_{FI}}{\arg max} \frac{r_p - r_f}{\sigma_p} \tag{12}$$

subject to:

$$\begin{cases} v_{GMV} + v_{FI} = 1 \\ r_p = [\mu_{GMV}\ \mu_{FI}]'[v_{GMV}\ v_{FI}] \\ \sigma_p^2 = (\sigma_{GMV} v_{GMV})^2 + (\sigma_{FI} v_{FI})^2 + 2 v_{GMV} v_{FI} cov_{GMV,FI} \end{cases}$$

where μ_{GMV} and μ_{FI} are expected returns of the GMV and FI portfolios, and σ_{GMV} and σ_{FI} is the standard deviation of these portfolios and $cov_{GMV,FI}$ is the covariance between the return of the GMV and FI portfolios. The solution for the mentioned maximization problem in matrix notation is:

$$\begin{pmatrix} v_{GMV} \\ v_{FI} \end{pmatrix} = \frac{\begin{pmatrix} \sigma_{GMV}^2 & Cov_{GMV,FI} \\ Cov_{GMV,FI} & \sigma_{FI}^2 \end{pmatrix}^{-1} \begin{pmatrix} \mu_{GMV} - r_f \\ \mu_{FI} - r_f \end{pmatrix}}{[1\ 1] \begin{pmatrix} \sigma_{GMV}^2 & Cov_{GMV,FI} \\ Cov_{GMV,FI} & \sigma_{FI}^2 \end{pmatrix}^{-1} \begin{pmatrix} \mu_{GMV} - r_f \\ \mu_{FI} - r_f \end{pmatrix}}, \tag{13}$$

or alternatively Bodie et al. [2014, p.217]:

$$v_{GMV} = \frac{[\mu_{GMV} - r_f]\sigma_{FI}^2 - [\mu_{FI} - r_f]Cov_{GMV,FI}}{[\mu_{GMV} - r_f]\sigma_{FI}^2 + [\mu_{FI} - r_f]\sigma_{GMV}^2 - [\mu_{GMV} - r_f + \mu_{FI} - r_f]Cov_{GMV,FI}} \tag{14}$$

$$v_{FI} = 1 - v_{GMV} \tag{15}$$

The equation 14 can be simplified given that the covariance of the GMV portfolio with any other portfolio is equal to the variance of the GMV portfolio (please, see the proof in A.6):

$$Cov_{GMV,FI} = \sigma^2_{GMV}. \tag{16}$$

Thus,

$$v_{GMV} = \frac{[\mu_{GMV} - r_f]\sigma^2_{FI} - [\mu_{FI} - r_f]\sigma^2_{GMV}}{[\mu_{GMV} - r_f]\sigma^2_{FI} - [\mu_{GMV} - r_f]\sigma^2_{GMV}} = \frac{[\mu_{GMV} - r_f]\sigma^2_{FI} - [\mu_{FI} - r_f]\sigma^2_{GMV}}{[\mu_{GMV} - r_f][\sigma^2_{FI} - \sigma^2_{GMV}]} \tag{17}$$

Calculated weights of the GMV and FI portfolios in the BGMV/FI portfolio allow us to find the final BGMV/FI portfolio composition in terms of stocks:

$$w_B = w_{GMV} v_{GMV} + w_{FI} v_{FI} \tag{18}$$

Alternatively, we can apply a closed form solution for the individual stocks' weights in the BGMV/FI portfolio as a function of initial data. In other words, the two steps procedure can be reduced to one step (please see the proof in the appendix B.):

$$w_B = \frac{[(\mu' - r_f e')(\Omega^{-1} e FI' \Omega - FIe')FI\mathbf{I} + FI\mu'(\mathbf{I}e' FI - FIe')]\Omega^{-1} e}{e'(\Omega^{-1} e FI' \Omega - FIe')FI(\mu' - r_f e')\Omega^{-1} e}, \tag{19}$$

where μ is a column vector of individual stocks' column vector, r_f is a scalar representing the risk-free rate, e is a column vector of ones, FI is a column vector of individual stocks' fundamental indexes; all vectors have the same dimensions equal to the number of stocks in the consideration; Ω is the variance-covariance matrix of individual stocks under the consideration, with the number of raws and columns identical to the length of vectors in the equation.

In this section we analysed stocks in-sample and constructed the GMV, FI and BGMV/FI portfolios. The next section provides the results on how the GMV, FI and the Blended portfolios perform out-of-sample.

4 Results

In this section we discuss two sets of results. First, we analyse portfolios constructed without constraints on holdings weights (unconstrained portfolios). Second, we analyse portfolios when no short sales constraint is implemented with a maximum holdings weights at most 10% of the portfolio at the time of construction (constrained portfolios).

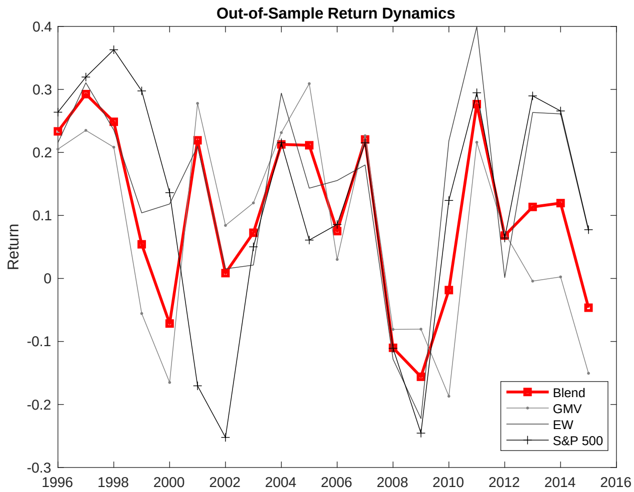

Figure 2: OUT-OF-SAMPLE RETURN DYNAMICS. On July 1 of every year from 1995 to 2014, we use the preceding five years of data to estimate portfolio weights for the GMV, FI and Blended portfolios. Using these weights we calculate one year out-of-sample returns for each of these portfolios plus the S&P500 and the Equally Weighted (EW) portfolios. All returns are annualised.

4.1 Unconstrained Portfolios

In Figure 2 we plot out-of-sample returns for the unconstrained portfolios. None of the portfolios exhibits consistent superior returns. The returns of blended portfolios, even in out-of-sample, are never the highest or the lowest among the considered alternatives.

However, Figure 3 shows that out-of-sample standard deviation of blended portfolios is always lower than that of S&P500 and Equally Weighted (EW) portfolios. Surprisingly, the out-of-sample risk of the blended portfolio is lower than the risk of GMV portfolio in years 2000, 2006, 2010 and 2011. This suggests that the economic footprint of companies captured by the FI and Blended portfolios conveys more useful information than the GMV portfolio in risk prevention before or after crises times (the Dot-com bubble and the Great Recession periods).

Consecutively, Figure 4 shows that the Sharpe ratio of the Blended portfolio is never the lowest except for one period compared to three other benchmarks. It is a desirable property for the fund managers evaluated annually - they are not likely to “lose their job for consecutive underperformance” if they employ the blended portfolio technique. In contrast, the S&P500 and GMV portfolios can underperform consecutively for several years. The only benchmark that does not underperform consecutively is EW strategy.

Comparing the Sharpe ratios of different blended portfolios to the Sharpe ratio of the EW portfolio for every period shows mixed results, as it is seen on the Figure 5 (left).

However, Figure 5 (right) shows that the average Sharpe ratio of any Blended portfolio over the years is higher than the average Sharpe ratio of the S&P500 portfolio. Moreover, most Blended portfolios have higher average Sharpe ratios than the Equally Weighted port-

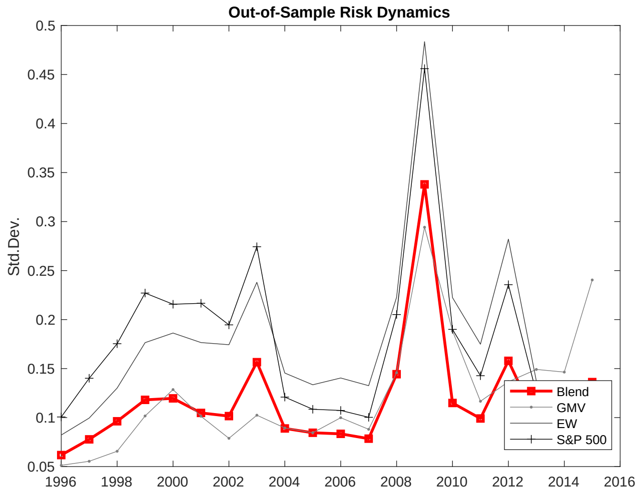

Figure 3: Out-of-sample risk dynamics. On July 1 of every year from 1995 to 2014, we use the preceding five years of data to estimate the portfolio weights for the GMV, FI and Blended portfolios. Using these weights we obtain one year of daily returns out-of-sample and calculate standard deviation of these returns for each of the portfolios plus the S&P500 and the Equally Weighted (EW) portfolios. Standard deviations are annualised.

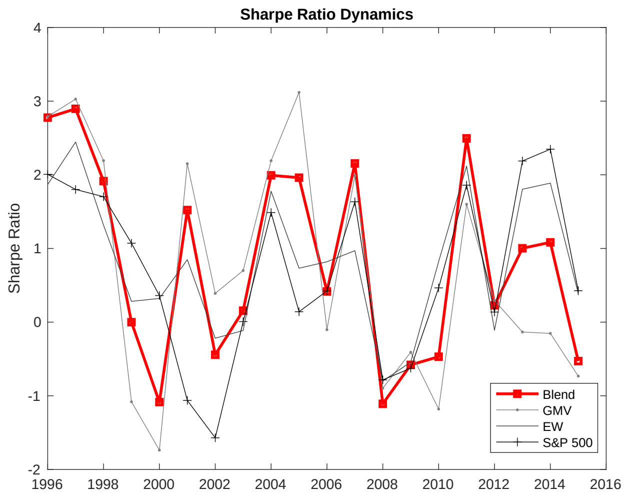

Figure 4: Out-of-sample Sharpe ratio dynamics for restricted portfolios. We use the preceding five years of data to estimate portfolio weights; weights are restricted: $0 \leq w \leq 10\%$. Using these weights we calculate the one year out-of-sample Sharpe ratio. Out-of-sample Sharpe ratios for years beginning July 1, 1995 to June 30, 2015 are plotted in the figure.

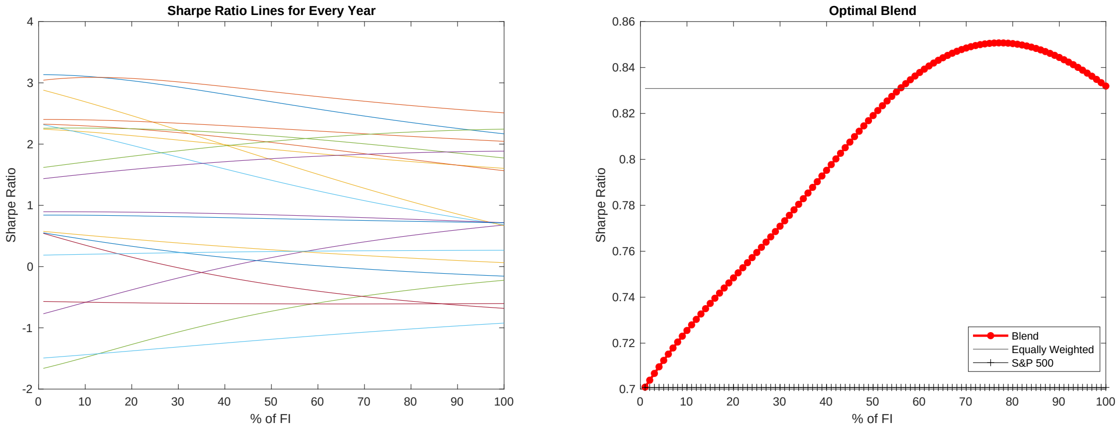

Figure 5: Out-of-sample Sharpe ratio for a given year for unrestricted portfolios (left); average out-of-sample Sharpe ratio line compared to Equally Weighted and S&P500 portfolio benchmark (right). **Each line** represents different combinations of GMV and FI portfolios in the Blended Portfolio **for a given year**. Lines are plotted for years beginning July 1, 1995 to June 30, 2015. On the right figure we plot an average out-of-sample Sharpe ratio line across 20 years versus the average Equally Weighted and the S&P500 benchmarks across 20 years.
Note:

folio, with a peak approximately 12.5% higher than the Sharpe ratio of the Equally Weighted portfolio (.92 vs .83). This suggests, that employing the Blended portfolio approach is a superior investment strategy if there are no restrictions on weights.

4.2 Constrained Portfolios

Constrained portfolios do not include short positions and the maximal weight of any stock is limited to 10% of the portfolio. The blended portfolio in this subsection is chosen to be 50/50 of the GMV and FI portfolios. In our later research we will investigate methods of better blending. We plot the out-of-sample returns for constrained portfolios in Figure 6. None of the portfolios exhibits superior returns on a constant basis. Predictably, the out-of-sample returns of the blended portfolio are never highest or lowest compared to other benchmarks.

However, Figure 7 shows that the out-of-sample risk of the blended portfolio is always lower than the risk of the S&P500 and the Equally Weighted (EW) portfolios. The GMV portfolio shows the lowest risk as expected. However, for the year 2007, and 2014 the GMV and Blended portfolios have the same risk. Given that the restricted Blended portfolio is constructed as 50/50 of the restricted GMV and FI portfolios, this equality suggests that in these years the economic footprint of companies captured by the FI and Blended portfolios provide enough information for diversification up to the level of the GMV portfolio.

Consequently, we can see in the Figure 8 that the Sharpe ratio of the Blended portfolio is never the lowest compared to the three other benchmarks.

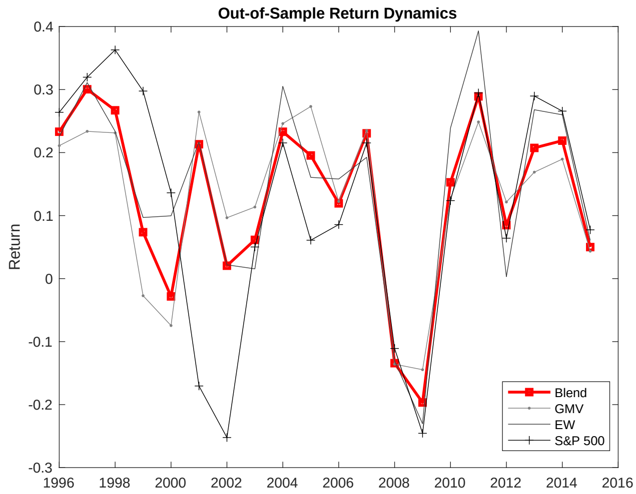

Figure 6: Out-of-sample return dynamics for restricted portfolios. We use proceeding five years of data to estimate portfolio weights; weights are restricted: 0≤w≤10%. Using these weights we calculate one year out-of-sample returns. Out-of-sample returns for years beginning July 1, 1995 to June 30, 2015 are plotted in the figure.

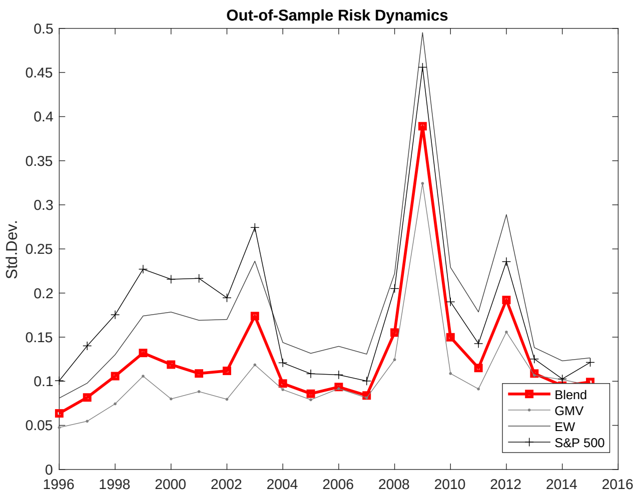

Figure 7: Out-of-sample risk dynamics for restricted portfolios. We use proceeding five years of data to estimate portfolio weights; weights are restricted: 0≤w≤10%. Using these weights we calculate one year out-of-sample risk. Out-of-sample standard deviation for years beginning July 1, 1995 to June 30, 2015 are plotted in the figure.

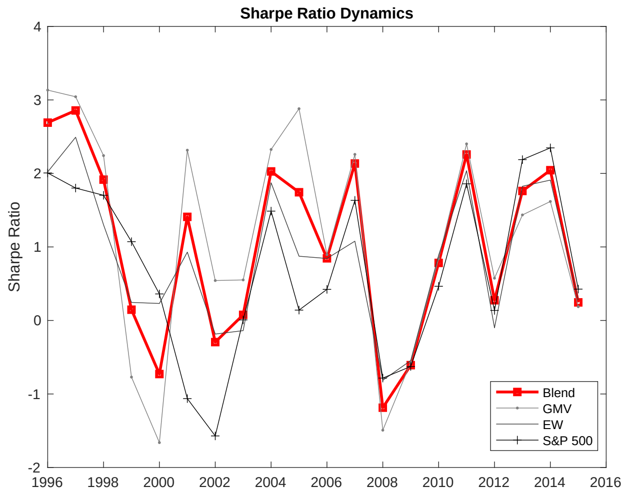

Figure 8: OUT-OF-SAMPLE SHARPE RATIO DYNAMICS FOR RESTRICTED PORTFOLIOS. We use proceeding five years of data to estimate portfolio weights; weights are restricted: $0 \leq w \leq 10\%$. Using these weights we calculate one year out-of-sample Sharpe ratio. Out-of-sample Sharpe ratios for years beginning July 1, 1995 to June 30, 2015 are plotted in the figure.

As in the unconstrained case, the Sharpe ratios of different constrained blended portfolios exhibit mixed results as it is seen in the Figure 9 (left).

However, the average Sharpe ratio of any of the Blended portfolio over the years is higher than the average Sharpe ratio of the S&P500 portfolio as it is shown in the Figure 9 (right). For the constrained case the GMV portfolio has the highest average Sharpe ratio, which is the driving factor of the high average Sharpe ratio of the Blended portfolio. Even the FI portfolio has an average Sharpe ratio that is on the level of the Equally Weighted portfolio. This suggests, that the blended approach is a superior investment strategy compared to the Equally Weighted and the S&P500 portfolios for the constrained case, two very difficult benchmarks to beat.

For the unconstrained case the Blended portfolio outperforms the GMV portfolio and FI portfolios. For the constrained case the Blended portfolio underperforms the GMV portfolio. In both constrained and unconstrained cases the GMV and Blended portfolios outperform the hard to beat Equally Weighted Portfolio and S&P500 benchmarks. Note, that to focus only on the blended technique we used the FI portfolio technique similar to the classic Arnott methodology. In our continuing research we will investigate different fundamentals and methodologies for constructing FI portfolios.

5 Conclusion

In this paper we propose a technique of portfolio construction that combines the benefits of Mean Variance Optimization (MVO) and of Fundamental Indexing (FI). Given that the FI approach is relatively new, and is profoundly different from the MVO, these two approaches

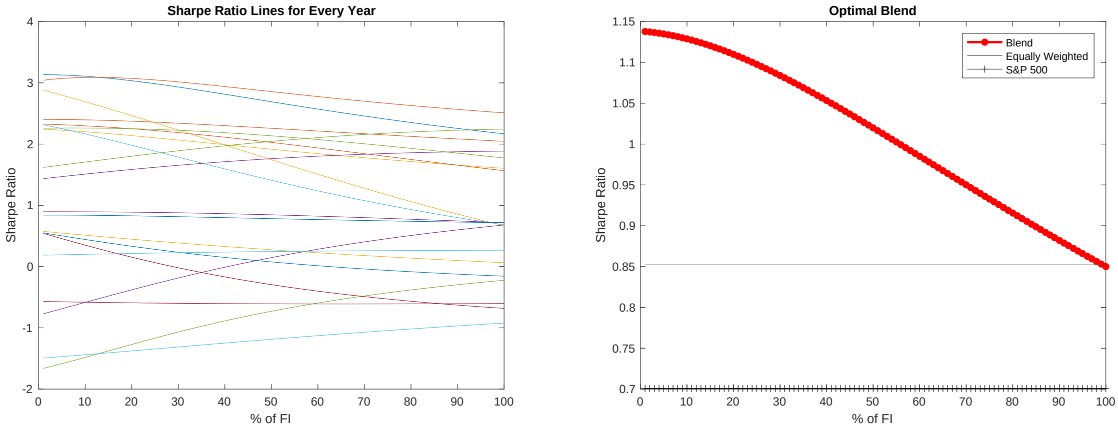

Figure 9: Out-of-sample Sharpe ratio for a given year for restricted portfolios (left); average out-of-sample Sharpe ratio line compared to Equally Weighted and S&P500 portfolio benchmark (right). **Each line** represents different combinations of GMV and FI portfolios in the Blended Portfolio **for a given year**. Lines are plotted for years beginning July 1, 1995 to June 30, 2015. On the right figure we plot an average out-of-sample Sharpe ratio line across 20 years versus average Equally Weighted and S&P500 benchmarks across 20 years.

have not yet been combined, even though each method offers distinctive benefits for portfolio choice problems. Our paper fills this gap in the literature, while our results attest to a superior performance of the Blended portfolio compared to two hard to beat benchmarks - the Equally Weighted portfolio and the S&P 500.

Applying the MVO method proposed by Markowitz [1952] we find the portfolio that contains the most information about the variance-covariance structure of stock returns - the Global Minimum Variance portfolio (GMV). Applying the FI method proposed by Arnott et al. [2005], we construct a portfolio of stocks that have a large economic footprint. Blending these two portfolios generates a portfolio that has better diversification than the FI portfolio and better return than the GMV portfolio. We derive a closed form solution for the optimal Blended portfolio weights when they are not restricted.

We test out-of-sample performance of various combinations of the Blended portfolio using 26 years worth of data from the NYSE, NASDAQ and AMEX. Specifically, we test two cases: when the Blended portfolio has weights that are not restricted and the case when short sales are not allowed and maximum weights are limited to 10%. In both cases, the average Sharpe ratio of most combinations of the Blended portfolio are higher than the benchmarks (Equally Weighted and S&P 500 portfolios).

For the unrestricted case, the optimal Blended portfolio has a higher average Sharpe ratio compared to the GMV, FI, Equally Weighted and S&P 500 portfolios. For the more practical restricted case the Blended portfolio outperformed the FI, Equally Weighted and S&P 500 portfolios, but underperformed the GMV portfolio. Given this evidence we can state that the Blended portfolio is a superior portfolio strategy.

To keep the focus of the paper on the blending technique, we employ the classic FI introduced by Arnott et al. [2005]. Our future research will be concentrated on finding the FI portfolio technique that could improve the Blended portfolio performance even further.

References

Robert Arnott, Noah Beck, and Vitali Kalesnik. Rip van winkle indexing. *The Journal of Portfolio Management*, 41(4):50–67, 2015.

Robert D. Arnott, Jason Hsu, and Philip Moore. Fundamental indexation. *Financial Analysts Journal*, 61(2):83–99, 2005.

Clifford Asness, Andrea Frazzini, Ronen Israel, and Tobias Moskowitz. Fact, fiction, and value investing. *The Journal of Portfolio Management*, 42(1):34–52, 2015.

Anup K Basu and Brigette Forbes. Does fundamental indexation lead to better risk-adjusted returns? new evidence from australian securities exchange. *Accounting & Finance*, 54(3): 699–728, 2014.

John M Bates and Clive WJ Granger. The combination of forecasts. *Journal of the Operational Research Society*, 20(4):451–468, 1969.

Zvi Bodie, Alex Kane, and Alan J Marcus. *Investments*. McGraw-Hill Irwin, revised 1973 edition edition, 2014.

Stephen J Brown, William Goetzmann, Roger G Ibbotson, and Stephen A Ross. Survivorship bias in performance studies. *Review of Financial Studies*, 5(4):553–580, 1992.

M. Carhart. On persistence in mutual fund performance. *Journal of Finance*, 52:57–82, 1997.

Keith Cuthbertson and Dirk Nitzsche. *Quantitative financial economics: stocks, bonds and foreign exchange*. John Wiley & Sons, 2005.

Neil R Ericsson. Economic forecasting in theory and practice: An interview with david f. hendry. *International Journal of Forecasting*, 2017.

James Douglas Hamilton. *Time series analysis*, volume 2. Princeton university press Princeton, 1994.

KiHoon Hong and Eliza Wu. The roles of past returns and firm fundamentals in driving US stock price movements. *International Review of Financial Analysis*, 43(C):62–75, 2016.

Ravi Jagannathan and Tongshu Ma. Risk reduction in large portfolios: Why imposing the wrong constraints helps. *The Journal of Finance*, 58(4):1651–1684, 2003.

Philippe Jorion. Bayes-stein estimation for portfolio analysis. *Journal of Financial and Quantitative Analysis*, 21(3):279–292, 1986.

Oliver Ledoit and Michael Wolf. Robust performance hypothesis testing with the sharpe ratio. *Journal of Empirical Finance*, 15(5):850–859, 2008.

Olivier Ledoit and Michael Wolf. Honey, I shrunk the sample covariance matrix. *The Journal of Portfolio Management*, 30(4):110–119, 2004.

Harry Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, 1952.

James M Poterba and Lawrence H Summers. Mean reversion in stock prices: Evidence and implications. *Journal of financial economics*, 22(1):27–59, 1988.

Christian Walkshäusl and Sebastian Lobe. Fundamental indexing around the world. *Review of Financial Economics*, 19(3):117–127, 2010.

Appendix

A. Notation

Variable	Description
PI	price index
RI	total return index (includes change in price and dividends)
r_{it}	simple return for stock i in day t (based on RI)
r_i;$\bar{r}$	expected return for stock i; vector of r_i's
r_f	risk-free rate
R_{it}	gross return for the stock i in day t
FI_i	fundamental index of stock i
w_i^{FI}	weight of stock i in a portfolio obtained using FI approach
$\overline{w^{MVO}}$	stocks' weights vector obtained using MVO method
Ω	expected variance covariance matrix of stocks
DPS	dividends per share
DY	dividends yield
MV	market value, capitalization
POUT	(dividend per share)/(earnings per share) for the last financial period
PTBV	(current price per share)/(the latest book value per share)
Rev	revenue for the last financial period

B. Proofs

Weights of the GMV and FI portfolios in the blended portfolio are:

$$v = \begin{pmatrix} v_{GMV} \\ v_{FI} \end{pmatrix} = \frac{\Omega^{-1}_{GMV,FI}(\mu_{GMVFI} - r_f e)}{e'\Omega^{-1}_{GMV,FI}(\mu_{GMVFI} - r_f e)} \tag{A.1}$$

Note, that the inversion of variance-covariance matrix between two assets, the GMV and FI yields:

$$\Omega^{-1}_{GMV,FI} = \begin{pmatrix} \sigma^2_{GMV} & \sigma_{GMV,FI} \\ \sigma_{GMV,FI} & \sigma^2_{FI} \end{pmatrix}^{-1} = \frac{\begin{pmatrix} \sigma^2_{FI} & -\sigma_{GMV,FI} \\ -\sigma_{GMV,FI} & \sigma^2_{GMV} \end{pmatrix}}{\sigma^2_{GMV}\sigma^2_{FI} - \sigma^2_{GMVFI}\sigma^2_{GMVFI}} \tag{A.2}$$

Thus, we rewrite A.1:

$$v = \frac{\begin{pmatrix} \sigma^2_{FI} & -\sigma_{GMV,FI} \\ -\sigma_{GMV,FI} & \sigma^2_{GMV} \end{pmatrix} \frac{\mu_{GMVFI} - r_f e}{\sigma^2_{GMV}\sigma^2_{FI} - \sigma^2_{GMVFI}\sigma^2_{GMVFI}}}{e' \begin{pmatrix} \sigma^2_{FI} & -\sigma_{GMV,FI} \\ -\sigma_{GMV,FI} & \sigma^2_{GMV} \end{pmatrix} \frac{\mu_{GMVFI} - r_f e}{\sigma^2_{GMV}\sigma^2_{FI} - \sigma^2_{GMVFI}\sigma^2_{GMVFI}}} \tag{A.3}$$

Cancelling the scalar, non-zero determinant of the variance covariance matrix:

$$v = \frac{\begin{pmatrix} \sigma^2_{FI} & -\sigma_{GMV,FI} \\ -\sigma_{GMV,FI} & \sigma^2_{GMV} \end{pmatrix} (\mu_{GMVFI} - r_f e)}{e' \begin{pmatrix} \sigma^2_{FI} & -\sigma_{GMV,FI} \\ -\sigma_{GMV,FI} & \sigma^2_{GMV} \end{pmatrix} (\mu_{GMVFI} - r_f e)} \tag{A.4}$$

Now, let us expand each element of the matrix in the equation:

$$\sigma^2_{FI} = w'_{FI} \Omega w_{FI} \tag{A.5}$$

Please note, for that the three remaining matrix elements have the similar form. Let us analyse it:

Proposition 1. *The covariance of the GMV portfolio with any other portfolio is equal to the variance of the GMV portfolio.*

Proof. Noting that we require that sum of weights in any portfolio X to be equal to one,

■

$$\sigma_{X,GMV} = w'_X \Omega w_{GMV} = w'_X \Omega \Omega^{-1} \frac{e}{e'\Omega^{-1}e} = w'_X e \frac{1}{e'\Omega^{-1}e} = \frac{1}{e'\Omega^{-1}e}. \tag{A.6}$$

Remark 1. Alternative proof (by contradiction): Suppose that the GMV portfolio had two distinct covariances with two other portfolios. This means that additional diversification using these 3 assets would result in a portfolio with a variance lower than that of the GMV. This would be contradictory to the fact that the GMP portfolio has the lowest possible variance for a given covariance matrix Ω. Therefore, the covariance of the GMV portfolio with any other asset or portfolio is constant and is equal to the variance of the GMV portfolio.

Applying this finding to the each remaining element of the matrix discussed:

$$\sigma^2_{GMV} = \sigma_{GMV,GMV} = \sigma_{GMV,FI} = \sigma_{X,GMV} = \frac{1}{e'\Omega^{-1}e} \tag{A.7}$$

Let us take the scalar from A.7 out of the matrix in A.4:

$$\begin{pmatrix} \sigma^2_{FI} & -\sigma_{GMVFI} \\ -\sigma_{GMVFI} & \sigma^2_{GMV} \end{pmatrix} = \begin{pmatrix} w'_{FI}\Omega w_{FI} & -\frac{1}{e'\Omega^{-1}e} \\ -\frac{1}{e'\Omega^{-1}e} & \frac{1}{e'\Omega^{-1}e} \end{pmatrix} = \frac{1}{e'\Omega^{-1}e} \begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} \tag{A.8}$$

Now, let us rewrite A.4 and cancel a positive scalar from the numerator and denominator:

$$v = \frac{\frac{1}{e'\Omega^{-1}e} \begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e)}{e' \frac{1}{e'\Omega^{-1}e} \begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e)} =$$

$$= \frac{\begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e)}{e' \begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e)} \tag{A.9}$$

Let us analyse the denominator of the equation A.9:

$$e' \begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e) = [1\,1] \begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e) =$$

$$= (w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1 \;\; 0)(\mu_{GMVFI} - r_f e) = (w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1 \;\; 0)(\begin{matrix} \mu_{GMV} - r_f \\ \mu_{FI} - r_f \end{matrix}) =$$

$$= (w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1)(\mu_{GMV} - r_f) = (w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1)(\mu_{GMV} - r_f) \tag{A.10}$$

Let us substitute A.10 into A.9

$$v = \frac{\begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} (\mu_{GMVFI} - r_f e)}{(w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1)(\mu_{GMV} - r_f)} =$$

$$= \frac{\begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \mu_{GMV} - r_f \\ \mu_{FI} - r_f \end{pmatrix}}{(w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1)(\mu_{GMV} - r_f)} =$$

$$= \frac{\begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e(\mu_{GMV} - r_f) - (\mu_{FI} - r_f) \\ -(\mu_{GMV} - r_f) + (\mu_{FI} - r_f) \end{pmatrix}}{(w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1)(\mu_{GMV} - r_f)} =$$

$$= \frac{\begin{pmatrix} w'_{FI}\Omega w_{FI} e'\Omega^{-1}e(\mu_{GMV} - r_f) - (\mu_{FI} - r_f) \\ -\mu_{GMV} + \mu_{FI} \end{pmatrix}}{(w'_{FI}\Omega w_{FI} e'\Omega^{-1}e - 1)(\mu_{GMV} - r_f)} =$$

$$= \frac{\begin{pmatrix} \frac{FI'}{e'FI}\Omega\frac{FI}{e'FI} e'\Omega^{-1}e(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f) - (\frac{\mu' FI}{e'FI} - r_f) \\ -\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} + \frac{\mu' FI}{e'FI} \end{pmatrix}}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI} e'\Omega^{-1}e - 1)(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f)} \tag{A.11}$$

Thus, combining weights of the individual stocks in the blended portfolio are:

$$w_B = w_{GMV} v_{GMV} + w_{FI} v_{FI} = \frac{\Omega^{-1}e}{e'\Omega^{-1}e} v_{GMV} + \frac{FI}{e'FI} v_{FI} \tag{A.12}$$

First, we will simplify each part and the we will bring them back together.

$$w_{GMV}v_{GMV} = \frac{\frac{\Omega^{-1}e}{e'\Omega^{-1}e}[\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f) - (\frac{\mu'FI}{e'FI} - r_f)]}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f)} =$$

$$= \frac{\Omega^{-1}e[\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f) - \frac{1}{e'\Omega^{-1}e}(\frac{\mu'FI}{e'FI} - r_f)]}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f)} \tag{A.13}$$

$$w_{FI}v_{FI} = \frac{\frac{FI}{e'FI}(-\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} + \frac{\mu'FI}{e'FI})}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f)} \tag{A.14}$$

Combining last two equations:

$$w_B = \frac{\frac{\Omega^{-1}e}{e'\Omega^{-1}e}[\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f) - (\frac{\mu'FI}{e'FI} - r_f)] - \frac{FI}{e'FI}(-\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} + \frac{\mu'FI}{e'FI})}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f)} =$$

$$= \frac{\frac{\Omega^{-1}e}{e'\Omega^{-1}e}[\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}(\mu'\Omega^{-1}e - r_f e'\Omega^{-1}e) - (\frac{\mu'FI}{e'FI} - r_f)] - \frac{FI}{e'FI}(-\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} + \frac{\mu'FI}{e'FI})}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\frac{\mu'\Omega^{-1}e}{e'\Omega^{-1}e} - r_f)} =$$

$$= \frac{\Omega^{-1}e[\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}(\mu'\Omega^{-1}e - r_f e'\Omega^{-1}e) - (\frac{\mu'FI}{e'FI} - r_f)] + \frac{FI}{e'FI}(\mu'\Omega^{-1}e - \frac{\mu'FI}{e'FI}e'\Omega^{-1}e)}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\mu'\Omega^{-1}e - r_f e'\Omega^{-1}e)} =$$

$$= \frac{[[\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}(\mu'\Omega^{-1}e - r_f e'\Omega^{-1}e) - (\frac{\mu'FI}{e'FI} - r_f)]\mathbf{I} + \frac{FI}{e'FI}(\mu' - \frac{\mu'FI}{e'FI}e')]\Omega^{-1}e}{(\frac{FI'}{e'FI}\Omega\frac{FI}{e'FI}e'\Omega^{-1}e - 1)(\mu' - r_f e')\Omega^{-1}e} =$$

$$= \frac{[[FI'\Omega FI(\mu'\Omega^{-1}e - r_f e'\Omega^{-1}e) - (\mu'FI - r_f e'FI)e'FI]\mathbf{I} + FI(\mu'e'FI - \mu'FIe')]\Omega^{-1}e}{(e'\Omega^{-1}eFI'\Omega FI - e'FIe'FI)(\mu' - r_f e')\Omega^{-1}e} =$$

$$= \frac{[[(\mu' - r_f e')\Omega^{-1}eFI'\Omega - (\mu' - r_f e')FIe']FI\mathbf{I} + FI\mu'(e'FI - FIe')]\Omega^{-1}e}{e'(\Omega^{-1}eFI'\Omega - FIe')FI(\mu' - r_f e')\Omega^{-1}e} =$$

$$= \frac{[(\mu' - r_f e')(\Omega^{-1}eFI'\Omega - FIe')FI\mathbf{I} + FI\mu'(\mathbf{I}e'FI - FIe')]\Omega^{-1}e}{e'(\Omega^{-1}eFI'\Omega - FIe')FI(\mu' - r_f e')\Omega^{-1}e} \tag{A.15}$$

C. Algorithms

Data cleaning and pre-processing procedures are detailed in Algorithm 1. Steps for blended portfolio construction are summarised in Algorithm 2. Performance evaluation procedures for our proposed approach are laid out in Algorithm 3.

Derivation

Descriptive Statistics

The data of out-of-sample values (the following year after the portfolio has been constructed), contain the same number of stocks as the data of in-sample values. The descriptive

Algorithm 1 Data Preparation

1. Raw data are from the Thomson Reuters Datastream.
   (a) Daily: PI, MV, PTBV and RI.
   (b) Annual: Rev, DPS, DY, and payout ratios (POUT).
2. Full sample filtering.
   (a) Bankruptcy filter. Upon the first occurrence of RI or PI =0, a NaN (Not-a-Number) is assigned to all subsequent PI or RI values.
3. Subsample filtering. Using a six-year rolling window with a one-year increments to reflect annual rebalancing of portfolios, we construct 26 subsamples (1990-1995,1991-1996, ..., 2010-2015). Within each subsample, the first 5 years of data us used as in-sample information to construct portfolios. The remaining one year in each subsample is used to test portfolios' out-of-sample performance. Within each sample we filter stocks based on the first five years of the data. Stocks falling outside the following ranges are removed from consideration in portfolio construction:
   (a) Does not have missing values;
   (b) -50% $\leq$Daily RI change $\leq$50%;
   (c) 0$\leq$PTBV$\leq$50;
   (d) 0$\leq$DPS;
   (e) 0$\leq$DY$\leq$50;
   (f) 0$\leq$POUT$\leq$0.
4. Creating the matrix of simple returns:
$$r_{(\tau_{sample}-1)\times n} = (RI_{2:\tau_{sample}\times 1:n} - RI_{1:(\tau_{sample}-1)\times 1:n}) \oslash RI_{1:(\tau_{sample}-1)\times 1:n} - e_{(\tau_{sample}-1)\times 1}e'_{1\times n}, \quad \text{(A.16)}$$
   where τ_{sample} is the length of the sample dates
5. The expected annualized return for the each stock is calculated as the geometric return average from the in-sample RI data:
$$\mu_{n\times 1.daily} = (RI'_{in-sample\ end} \oslash RI'_{in-sample\ beginning})^{\circ \frac{1}{NumberOfDays}} - e_{n\times 1}, \quad \text{(A.17)}$$
$$\mu_{n\times 1} = (e_{n\times 1} + \mu_{n\times 1.daily})^{\circ 260} - e_{n\times 1}, \quad \text{(A.18)}$$
   where $e_{n\times 1}$ is the vector of ones.
6. The in-sample variance-covariance matrix calculation is based on the daily returns data for the 5 in-sample years
$$\Omega_{n\times n} = [r'_{n\times(\tau_{in-sample}-1)} - \mu_{n\times 1,daily}e_{1\times(\tau_{in-sample}-1)}][r'_{n\times(\tau_{in-sample}-1)} - \mu_{n\times 1,daily}e_{1\times(\tau_{in-sample}-1)}]', \quad \text{(A.19)}$$
   where $\tau_{in-sample}$is the length of in-sample dates
7. The book value (BV) calculation:
$$BV_{\tau\times n} = MV_{\tau\times n} \oslash PTBV_{\tau\times n} \quad \text{(A.20)}$$

Algorithm 2 Blended portfolio construction.

1. The Global Minimum Variance (GMV) portfolio construction.
   (a) The weights are:
$$w_{GMV\ n\times1} = \frac{\Omega^{-1}_{n\times n} e_{n\times1}}{e'_{1\times n}\Omega^{-1}_{n\times n} e_{n\times1}} \tag{A.21}$$
   (b) The expected return is:
$$\mu_{GMV\ 1\times1} = \frac{\mu'_{1\times n}\Omega^{-1}_{n\times n} e_{n\times1}}{e'_{1\times n}\Omega^{-1}_{n\times n} e_{n\times1}} \tag{A.22}$$
   (c) The variance is:
$$\sigma^2_{GMV\ 1\times1} = \frac{1}{e'_{1\times n}\Omega^{-1}_{n\times n} e_{n\times1}} \tag{A.23}$$
2. The Fundamental Index (FI) portfolio construction
   (a) The weights are (we will test various variables for the FI base): Needs a full description of FI index construction in a separate bullet point(s)
$$w_{FI\ n\times1} = \frac{FI_{n\times1}}{e_{1\times n} FI_{n\times1}} \tag{A.24}$$
   (b) The expected return is:
$$\mu_{FI\ 1\times1} = \frac{\mu'_{1\times n} FI_{n\times1}}{e'_{1\times n} FI_{n\times1}} \tag{A.25}$$
   (c) The variance is:
$$\sigma^2_{FI\ 1\times1} = w'_{FI\ 1\times n}\Omega_{n\times n} w_{FI\ n\times1} \tag{A.26}$$
3. The blended GMV/FI (BGMV/FI) portfolio construction
   (a) The weights of the GMV and FI portfolios in the BGMV/FI portfolio (please, see the proof):
$$v_{GMV\ 1\times1} = \frac{[\mu_{GMV\ 1\times1} - r_{f\ 1\times1}]\sigma^2_{FI\ 1\times1} - [\mu_{FI\ 1\times1} - r_{f\ 1\times1}]\sigma^2_{GMV\ 1\times1}}{[\mu_{GMV\ 1\times1} - r_{f\ 1\times1}][\sigma^2_{FI\ 1\times1} - \sigma^2_{GMV\ 1\times1}]} \tag{A.27}$$
$$v_{FI\ 1\times1} = 1 - v_{GMV\ 1\times1} \tag{A.28}$$
   (b) Calculation of weights of individual stocks in the BGMV/FI portfolio (using equations A.21, A.27,A.24, and A.28.)
$$w_{BGMV/FI\ n\times1} = w_{GMV\ n\times1} v_{GMV\ 1\times1} + w_{FI\ n\times1} v_{FI\ 1\times1} \tag{A.29}$$
   (c) Calculation of the expected return and variance of the BGMV/FI portfolio.
      i. $\mu_{BGMV/FI\ 1\times1} = w'_{BGMV/FI\ 1\times n}\mu_{n\times1}$
      ii. $\sigma^2_{BGMV/FI\ 1\times1} = w'_{BGMV/FI\ 1\times n}\Omega_{n\times n} w_{BGMV/FI\ n\times1}$

Algorithm 3 Performance and Out-Of-Sample Tests

1. Out-of-sample performance for every sample (note: τ_{out} is the length of out-of-sample dates). For every portfolio X, $X \in \{FI, GMV, BGMV/FI\}$ we calculate:

$$r_{X\ \tau_{out}\times 1} = r_{\tau_{out}\times n} w_{X\ n\times 1} \tag{A.30}$$

$$\mu^{realised}_{X\ 1\times 1} = \prod_{i=1}^{\tau_{out}} (1 + r_{X\ i}) - 1 \tag{A.31}$$

$$\mu^{realised}_{X\ daily\ 1\times 1} = (1 + \mu^{realised}_{X\ 1\times 1})^{\frac{1}{\tau_{out}}} - 1 \tag{A.32}$$

$$\sigma^{2\ realised}_{X\ 1\times 1} = [r_{X\ \tau_{out}\times 1} - e_{\tau_{out}\times 1}\mu^{realised}_{X\ daily\ 1\times 1}]'[r_{X\ \tau_{out}\times 1} - e_{\tau_{out}\times 1}\mu^{realised}_{X\ daily\ 1\times 1}] \tag{A.33}$$

$$Sharpe_X = \frac{\mu^{realised}_{X\ 1\times 1} - r_{f\ 1\times 1}}{\sigma^{realised}_{X\ 1\times 1}} \tag{A.34}$$

2. Out-of-sample performance across samples. For every portfolio X, $X \in \{FI, GMV, BGMV/FI\}$:

   (a) First, we construct the long vectors of out-of-sample returns, that include out-of-sample returns of multiple samples combined:

$$r^{across}_{X\ T\times 1} = [r^{sample1}_{X\ \tau_{out}\times 1}; r^{sample2}_{X\ \tau_{out}\times 1}; ...; r^{sampleLast}_{X\ \tau_{out}\times 1}], \tag{A.35}$$

   where $T_{out-of-sample} = \tau^{sample1}_{out} + \tau^{sample2}_{out} + ... + \tau^{sampleLast}_{out}$

   (b) The final (future) value of the one dollar invested using the strategy X with annual rebalancing:

$$FV^{realised}_{X\ 1\times 1} = \prod_{i=1}^{\tau_{out}} (1 + r^{across}_{X\ i}) \tag{A.36}$$

   (c) The geometric average annual return:

$$\mu^{realised}_{X\ average\ annual} = (FV^{realised}_{X\ 1\times 1})^{\frac{1}{number\ of\ samples}} - 1, \tag{A.37}$$

   (d) The geometric average daily return:

$$\mu^{realised}_{X\ all\ samples\ daily} = (FV^{realised}_{X\ 1\times 1})^{\frac{1}{T_{out-of-sample}}} - 1, \tag{A.38}$$

   (e) The variance of the all out-of-sample periods combined:

$$\sigma^{2\ realised}_{X\ 1\times 1} = [r^{across}_{X\ T\times 1} - e_{T\times 1}\mu^{realised}_{X\ all\ samples\ daily\ 1\times 1}]'[r^{across}_{X\ T\times 1} - e_{T\times 1}\mu^{realised}_{X\ all\ samples\ daily\ 1\times 1}] \tag{A.39}$$

   (f) The Sharpe Ratio of the all out-of-sample periods combined:

$$Sharpe^{overall}_X = \frac{FV^{realised}_{X\ 1\times 1} - \prod_{i=1}^{number\ of\ samples}(1 + r_{f\ i})_{1\times 1}}{\sigma^{realised}_{X\ 1\times 1}} \tag{A.40}$$

Table 2: Descriptive Statistics of In-Sample Data Used for Portfolio Selection

	Mean	StDev	Skew	Kurtosis	5%	50%	95%
r (%)	17.95	44.56	0.65	20.21	-3.87	0	4.16
MV ('000,000)	5,724	22,646	10.13	144.96	31.95	713	22,336
PTBV	2.64	2.58	4.98	45.39	0.74	1.93	6.74
DPS (%)	0.41	1.54	28.84	1,233.90	0.00	0.06	1.60
DY (%)	1.45	2.15	3.19	28.41	0.00	0.48	5.49
POUT (%)	24.19	33.19	1.81	6.69	0.00	9.73	94.05
Rev ('000)	3,998,115	15,916,774	14	292	23,212	578,237	16,046,250

In this table r is the annualised net return, MV is expressed in millions, and Rev is in thousands.

Table 3: Descriptive Statistics of Out-of-Sample Data

	Mean	StDev	Skew	Kurtosis	5%	50%	95%
r (%)	18.08	45.52	13.28	5813.77	-3.89	0.00	4.17
MV ('000,000)	6,005	23,441	10	136	32	748	23,500
PTBV	2.66	2.62	4.94	44.39	0.74	1.94	6.84
DPS (%)	0.46	1.67	27.09	1058.09	0.00	0.08	1.70
DY (%)	1.42	2.20	5.48	100.32	0.00	0.47	5.25
POUT (%)	656	67,568	116	13,925	0	6	100
Revenues ('000)	4,419,780	17,212,498	13	256	24,155	641,422	17,971,800

In this table r is the annualised net return, MV is expressed in millions, and Rev is in thousands.

statistics of these data are in Table 3. The descriptive statistics in both tables are almost identical in both periods. For example the annual return is only 0.13% higher for the out-of-sample period. Market Values are typically higher in the out-of-sample date, due to the market upward sloping trend. The only striking difference between in-sample and out-of-sample descriptive statistics is related to the payout ratio (POUT): the standard deviation and skewness of which are dramatically higher in the out-of-sample data (33.19 vs 67,568 and 1.81 vs 116 respectively). This phenomenon happens when dividends are not sufficiently reduced despite a significant drop in earnings per share.

D. Assessment of fundamentals in predicting returns

The four-factor model by Carhart [1997] is commonly used as an active management and mutual fund evaluation model. After controlling for the four factors (market risk, company size, company price-to-book ration, and momentum), we include our fundamental factors to check their potency to explain stock returns in-sample,

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + s_i SMB_t + h_i HML_t + m_i UMD_t + \sum_{j=1}^{k} \gamma_{i,j} F_{i,j} + \epsilon_{i,t}, \qquad \text{(A.41)}$$

where $r_{i,t}$ is the return on stock i on day t, $r_{m,t}$ and $r_{f,t}$ are the returns on the broad market index and a risk-free asset, and SMB, HML, and UMD are premia of the size factor, book-to-market factor, and the premium on winners minus losers respectively as in Fama-French (1993) and Carhart (1997).

The assessment of estimates of γ pinpoint the fundamentals worth considering in FI portfolio construction.

6 Mapping of Functions

Subscript dimensional notations:

T - number of daily observations in full sample

t - number of daily observations in subsample (in-sample, i.e., the length of the in-sample window for daily data)

A - number of annual observations in full sample

a - number of annual observations in subsample (in-sample)

f - number of daily observations in out-of-sample (i.e., the length of the out-of-sample window)

Inputs $\Longrightarrow$	**Function** $\Longrightarrow$	**Outputs**
$\underset{T\times N}{RI}$	**SimpleReturns**	$\underset{(T-1)\times N}{r}$
$\underset{a\times N}{DPS},\underset{a\times N}{DY},\underset{a\times N}{POUT},\underset{a\times N}{REV},\underset{t\times N}{MV},\underset{t\times N}{PTBV},\underset{t\times N}{RI},$ FilterCriteria structure, TimeIndexDaily=ti:(ti+t), TimeIndexAnnual=ai:(ai+a)	**FilteredStocks**	$\underset{N\times 1}{FilteredStocks}$
$\underset{T\times N}{RI},\underset{(t-1)\times N}{r},$ $\underset{N\times 1}{FilteredStocks},$ TimeIndexDaily=ti:(ti+t)	**RVarCovInSample**	$\underset{N\times 1}{\mu},\underset{N\times N}{\Omega}$
$\underset{N\times 1}{\mu},\underset{N\times N}{\Omega},$ $\underset{N\times 1}{FilteredStocks}$	**WeightsGMV**	$\underset{N\times 1}{w_{gmv}},\underset{1\times 1}{\mu_{gmv}},\underset{1\times 1}{\sigma^2_{gmv}}$
$\underset{a\times N}{DPS},\underset{a\times N}{DY},\underset{a\times N}{POUT},\underset{a\times N}{REV},\underset{t\times N}{MV},\underset{t\times N}{PTBV},\underset{t\times N}{RI},$ FundamentalCriteria structure, TimeIndexDaily=ti:(ti+t), TimeIndexAnnual=ai:(ai+a)	**FundamentalIndex**	$\underset{N\times 1}{FI}$
$\underset{N\times 1}{\mu},\underset{N\times N}{\Omega},\underset{N\times 1}{FI}$ $\underset{N\times 1}{FilteredStocks}$	**WeightsFI**	$\underset{N\times 1}{w_{fi}},\underset{1\times 1}{\mu_{fi}},\underset{1\times 1}{\sigma^2_{fi}}$
$\underset{N\times 1}{w_{gmv}},\underset{1\times 1}{\mu_{gmv}},\underset{1\times 1}{\sigma^2_{gmv}},$ $\underset{N\times 1}{w_{fi}},\underset{1\times 1}{\mu_{fi}},\underset{1\times 1}{\sigma^2_{fi}},$ $\underset{T\times 1}{r_f},\underset{N\times N}{\Omega},\underset{N\times 1}{\mu},$ TimeIndexDaily=ti:(ti+t), $\underset{N\times 1}{FilteredStocks}$	**WeightsB**	$\underset{N\times 1}{w_b},\underset{1\times 1}{\mu_b},\underset{1\times 1}{\sigma^2_b},\underset{1\times 1}{v_{gmv}},\underset{1\times 1}{v_{fi}}$

List of structured items in variable **FilterCriteria**:

FilterCriteria.r=[-0.5 0.5], FilterCriteria.PTBV=[0 50], FilterCriteria.DPS=[0 Inf], FilterCriteria.DY=[0 50],

FilterCriteria.POUT=[0 200],