Predictive Blends: Fundamental Indexing meets Markowitz

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Abstract

When constructing a portfolio of stocks, do you turn a blind eye to the firms' future outlooks based on careful consideration of companies' fundamentals, or do you ignore the stocks' correlation structures which ensure the best diversification? The fundamental indexing (FI) and Markowitz mean-variance optimization (MVO) approaches are complementary but, until now, have been considered separately in the portfolio choice literature. Using data on S&P 500 constituents, we evaluate a novel portfolio construction technique that utilizes the benefits of both approaches. Relying on the idea of forecast averaging, we propose to blend the two previously mentioned techniques to provide investors with a clear *bi*nocular vision. The out-of-sample results of the blended portfolios attest to their superior performance when compared to common market benchmarks, and to portfolios constructed solely based on the FI or MVO methods. In pursuit of the optimal blend between the two distinct portfolio construction techniques, MVO and FI, we find that the ratio of market capitalization to GDP, being a leading indicator for an overpriced market, demonstrates remarkably advantageous properties.

Keywords: fundamental indexing, portfolio optimization, equities, forecast averaging, blended portfolio.
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1 Introduction

The following analogy will help motivate our argument. Metallurgy teaches us that blending different metals produces alloys with better properties than their pure constituents. Even if new additions represent a very small percentage of the new alloy, its properties can change dramatically. For instance, duralumin, contains less than 6% of additives to 94% aluminium, but these additives dramatically change the properties of otherwise soft aluminium to an aircraft-grade strong alloy. We show that in composing stock portfolios the same phenomenon exists: blending portfolio construction approaches results in "blended" portfolios that outperform the benchmarks that sole-approach portfolios do not beat.

In this paper, we propose an innovative portfolio blending technique, combining the efficient portfolio selection method of Markowitz [1952] that takes into account the covariance structure of portfolio holdings and the fundamental indexing (FI) approach that favours investments with sound economic, financial, and managerial features.

Markowitz [1952] distinguishes between two stages in the portfolio selection process. The first stage is about forming beliefs about future performance. In practice, this often translates into reliance on historical data in estimating future rates of returns and their correlations. The second stage relies on the beliefs formed in the first stage and involves selecting a portfolio. Focusing only on the second stage, Markowitz [1952] introduces the mean-variance optimization (MVO) method for portfolio selection recommending that the choice of appropriate expected return and variance-covariance matrix "...should combine statistical techniques and the judgment of practical men..." [Markowitz, 1952, p.91]. The conventional approach often ignores the need to develop appropriate beliefs. As Markowitz emphasizes, it is our responsibility to use "observation and experience" to develop "beliefs about the future performances" [Markowitz, 1952, p.77]. While predicting future performance of stocks may be a daunting task, there is strong evidence that fundamental analysis may have some merit (Arnott et al. [2005], Walkshäusl and Lobe [2010], Basu and Forbes [2014]). As discussed in the forecast combination literature [Eklund and Karlsson, 2007, Smith and Wallis, 2009, etc.], we believe that fundamental analysis may improve the out-of-sample performance of MVO portfolios.

In practice, the MVO method relies on past returns to predict expected returns and estimate correlations. Past correlations predict future correlations much better that past returns predict future returns [Cuthbertson and Nitzsche, 2005, p.158]. Moreover, past returns fail to predict future returns in the long-run [Jorion, 1986, Poterba and Summers, 1988]. Given the volatile nature of these underlying processes, the MVO method likely produces superior out-of-sample results only for short-term investments. To mitigate this, frequent portfolio rebalancing based on the latest historical data is recommended for consistent superior results, but leads to high portfolio turnover and increased transaction costs. Transaction costs are of particular concern for funds with long-term performance objectives. Thus, in the industry, long-term investments are often based on "the judgment of practical men", rooted in fundamental analysis. In turn, fundamental analysis focuses on financial statements and the economic health of a company in an attempt to evaluate its long-term economic prospects, assessing its future growth, and investment potential.

Taken separately, both the classical MVO and the FI methods have their own limitations: the FI approach ignores the correlation structure of stocks' returns, while the classic MVO method is silent about the firms' fundamentals, which may well be the driving factors of the stocks' future performance. Berger et al. [2013] have also shown empirically that the MVO technique provides some diversification gains. Our blending technique combines the classical MVO method and the FI approach, by bridging the two stages of portfolio construction mentioned in Markowitz [1952]. Relying on 29 years of historical data we backtest and analyze out-of-sample performance of our proposed blending method and show that our blended portfolios are superior to conventional benchmarks as well as portfolios based on each method alone. Heteroskedasticity and autocorrelation (HAC) robust inference tests developed by Ledoit and Wolf [2008] show that our technique delivers statistically significantly higher Sharpe ratios than the (value weighted) S&P 500 and the Equally-Weighted S&P 500.

Currently the MVO and the FI literatures are isolated from each other.[1] Each of these literature streams considers stocks through a specific "oculus" described in the next two paragraphs. Up until now stocks have been considered separately through either one of these oculi.

In the first "oculus" considered, the MVO method, the expected returns and the variance-matrix are calculated based on in-sample information. Securities are sorted according to the MVO procedure, by maximizing the expected portfolio return while attaining a specific level of standard deviation. Since the introduction of the MVO by Markowitz [1952], a myriad of methods have been proposed in an attempt to refine this approach and offer superior out-of-sample performance. Among the most noticeable and practical extensions of the MVO method are those that control for outliers. Outliers often result in biased estimates of sample statistics translating in disproportionate portfolio holding weights. Several prominent robust techniques have been proposed to take this into account. For example, Ledoit and Wolf [2004] introduce a method that shrinks the sample covariance matrix to a well-conditioned parsimonious structure to reduce estimation errors that were shown to bias the classic MVO method. As an alternative to shrinkage methods, limiting portfolio holdings only to long positions, can produce similar results [Jagannathan and Ma, 2003]. However, Jagannathan and Ma [2003] note that such methods might lead to poor diversification, with only 20-25 stocks in the portfolio. Thus, to increase diversification and reduce the effect of measurement

[1]The FI approach was first proposed in Arnott et al. [2005] for US data; methodological improvement and empirical evidence can be found in Treynor [2005], Dopfel [2008]. Walkshäusl and Lobe [2010] and Basu and Forbes [2014] provide international evidence for the FI approach. Extensions and/or empirical evidence in favour of the MVO approach are too numerous to be listed here, however, for excellent surveys of the literature please refer to Markowitz et al. [2000] and Rubinstein [2002]. In a recent paper, Domowitz and Moghe [2018] consider a case where an exogenously pre-chosen "core" portfolio is complemented with other stocks based on the MVO method, without specifying how the "core" portfolio is constructed, and relying on expected returns of the individual components. To the best of our knowledge, no paper considers a portfolio construction strategy that combines the FI and MVO approaches. In our paper, we also propose the blending methodology based on economic conditions without relying on hard-to-predict expected returns of individual components.

errors, it is possible to set up an upper bound on weights (e.g., 5-10%)[2]. Since the MVO method suffers from the negative effects caused by measurement errors, outliers and *blindness* to firms' fundamentals (which are our second "oculus"), the performance of the classic MVO method, even with adjustments for outlier effects, often does not exceed market benchmarks such as equally- or capitalization-weighted portfolios in out-of-sample tests[3]. Hence, if the blended approach shows statistically significant results, they cannot be attributed to the MVO part of the technique alone.

We now shift our focus to the other "oculus", the FI approach, pioneered by Arnott et al. [2005]. In this approach, firms are ranked based on their fundamentals and securities are allocated proportionally to their overall fundamental scores. The fundamentals might include book value, free cash flow, revenue, sales, dividends, total employment, etc. In a recent paper, Asness et al. [2015] argue that FI indexing is, basically, systematic value investing. The FI approach significantly outperforms major benchmarks based on US market data [Arnott et al., 2005]. Walkshäusl and Lobe [2010] apply the FI approach to stocks from 50 countries and find that the FI approach outperforms capitalization-weighted portfolios in most countries. However, after applying the robust-to-fat-tails performance test proposed by Ledoit and Wolf [2008], the FI portfolios in only 6 countries and the global FI portfolio have statistically significant positive differences in Sharpe ratios. Our empirical results confirm that in the US, the FI portfolio outperforms the cap-weighted portfolio, but these results are not statistically significant[4]. Hence, if the blended approach shows statistically significant results in our US-based study, they cannot be attributed to the FI part of the technique alone.

Out of all portfolios constructed with the MVO method, the richest information about the correlation structure is contained in the Global Minimum Variance (GMV) portfolio[5], which is based solely on the variance-covariance matrix and achieves the highest level of diversification. More importantly, construction of the GMV portfolio does not rely on often noisy estimates of individual expected returns, which makes it the portfolio of choice in blending with the FI portfolio. Firms' fundamentals help us detect and concentrate on 'healthy' stocks that are likely to grow in the long-run, while the assessment of the correlation structure allows us to construct well-diversified portfolios.

Before we discus the "how" in our next section, one question remains: In what proportion do we combine the GMV and FI portfolios? Given that the FI approach is relatively new, and is profoundly different from the MVO method, these two approaches have not yet been combined, even though each method offers distinctive benefits for portfolio choice problems.

[2]Coincidentally, these weight recommendations are in accord with guidelines of many investment funds that try to avoid excessive dominance of a single security.

[3]The p-value for the tangency MVO portfolio vs the Equally-Weighted S&P 500 is 0.543; the p-value for the GMV portfolio against the Equally-Weighted S&P 500 is 0.098. We show p-values of all portfolios against the benchmarks in Table 2.

[4]The p-value for the difference in Sharpe ratios of FI portfolio vs the Equally-Weighted S&P 500 is 0.235, which is not statistically significant at conventional levels.

[5]In the GMV portfolio we will find mostly low volatility companies. As Walkshäusl [2013] shows, high quality firms exhibit lower volatility than low quality firms, hence we expect the GMV portfolio to include a larger number of high quality firms than S&P 500.

In fact, Hong and Wu [2016] show empirically that information on past returns and on the firms' fundamentals are complementary. They show that in "good times", when volatility is low, past returns provide better information about future returns. However, fundamentals perform better in "bad times", when volatility in the market is high. In such periods, past returns are not that informative and investors are forced to rely on firms' fundamentals. Thus, a portfolio allocation strategy should rely more on past returns (the GMV portfolio) in times of low volatility and rely more on the firms' fundamentals (the FI portfolio) in times of high volatility. It is a daunting task to predict "good" and "bad" times. We, however, use a metric often mentioned by Warren Buffett as a lead indicator of a stock market "bubble" - the market capitalization to nominal GDP ratio.[6] This approach is in the same spirit as Shiller's cyclically adjusted price-to-earnings (CAPE) ratio [Campbell and Shiller, 1988], where earnings per share are averaged over a long period. When this ratio indicates overpricing, and the likelihood of "bad times" is higher, we tilt the blend of our portfolio closer to the FI and away from the GMV portfolio. We discuss this in more detail in the methodology section.

The rest of the paper is organized as follows. We introduce the method of blended portfolios in Section 2. We summarize our data and empirical findings in Sections 3 and 4, respectively. Finally, Section 5 concludes.

2 Methodology

The FI and the GMV portfolios are depicted in Figure 1, which illustrates our proposed technique of blending these two portfolios into one. First, the FI portfolio is constructed based on firms' fundamentals using the FI approach. Second, the GMV portfolio is identified on the mean-variance portfolio frontier. We construct 101 blended combinations (in one percent increments) of these two portfolios, which generate the new, blended GMV/FI mean-variance frontier (in red). On the blended GMV/FI portfolio frontier, we select a portfolio depending on prediction of stock market correction (captured by the Buffett Indicator Index, which is discussed in more detail in Subsection 2.3). This Predictive Blended (PB) portfolio is the final outcome of our blended GMV/FI technique. It is the performance of this portfolio that we compare to our benchmarks, the S&P 500 index and the S&P 500 Equally-Weighted index. Next, we describe several desirable features of our proposed technique.

First, the two initial portfolios are formed using profoundly different methods, that should result in better performance of the combined model. Since we are concerned with out-of-sample performance of our portfolios in mean-variance space, our blended approach is inspired by methods proposed in the forecast combination literature. Models with combined forecasts have been shown to outperform individual forecasts [Bates and Granger, 1969, Ericsson, 2017].[7]

Second, since portfolios constructed based on the classic MVO (e.g., GMV) and FI approaches (e.g., Arnott FI), are most likely not perfectly correlated, the mean-variance optimal

[6] We use nominal GDP since we employ nominal market capitalization.
[7] For an excellent survey of the literature, see Hamilton [1994].

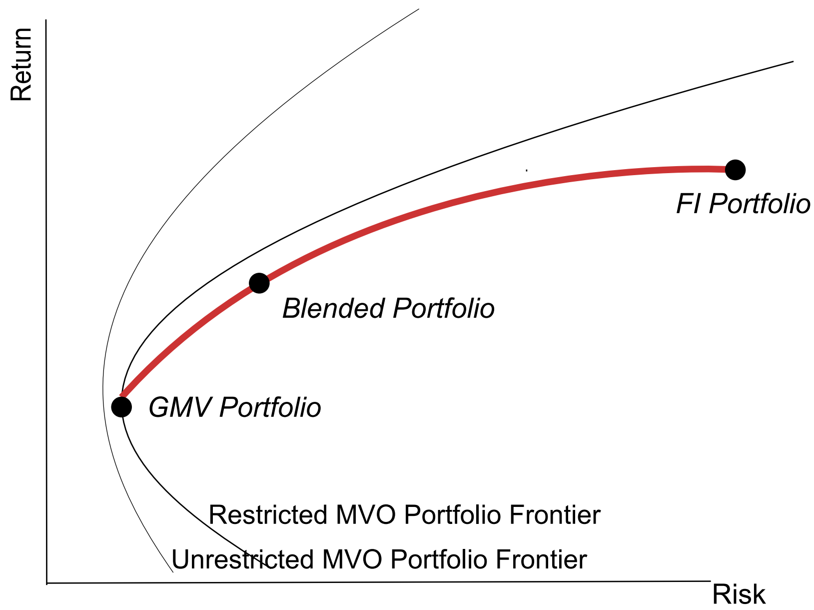

Figure 1: Bridging MVO and FI approaches. The figure illustrates hypothetical unrestricted and restricted minimum variance sets (MVS) based on Markowitz mean–variance optimization, incorporating short-sale and no short-sale constraints, respectively. The FI portfolios are constructed with long positions only, thus appearing in the interior of the restricted MVS. Typically, construction of the GMV and FI portfolios result in conceptually different asset allocation which allows for nontrivial correlation, and results in the MVS being located between these two portfolios, as depicted by the bold red curve.

frontier (red curve in Figure 1) will not result in a straight line. This "second-stage" (blended GMV/FI) mean-variance frontier offers further refinement combining the weights of the GMV and the FI portfolios proportionally as in Figure 1. Since the FI portfolio brings additional forward-looking information which was not included in the estimated mean-variance frontier, the new blended portfolio may generate a frontier that outperforms the MVO efficient frontier in out-of-sample tests.

Third, construction of the GMV and FI portfolios does not depend on individual stocks' expected returns, which, as we mentioned earlier, is a major source of error in portfolio optimization problems. Blending the GMV and FI together also does not depend on their expected returns. Instead, we employ the Buffett Indicator Index discussed below.

2.1 Construction of the Global Minimum Variance (GMV) portfolio

The GMV portfolio carries the most information about the diversification structure. In general, it is obtained from the optimization problem:

$$w^{GMV} = \arg\min_{w} w'\Omega w \qquad \text{s.t.} \qquad w'e = 1, \tag{1}$$

where, Ω is the $N \times N$ variance-covariance matrix of stocks' returns, N is the number of assets, e is the $N \times 1$ column vector of ones, and w is the $N \times 1$ vector of weights, w^{GMV} is a vector of individual asset weights in the GMV portfolio.

Note, that we calculate weight-restricted portfolios, with no short sales and a maximum weight of 10%. The restricted GMV portfolio is obtained by solving the optimization problem (1) with the added constraint of $0 \leq w \leq 0.1$.

2.2 *Construction of the Fundamental Indexing (FI) portfolio*

Previous literature [Arnott et al., 2005, Walkshäusl and Lobe, 2010] considers fundamental indexes based on a single metric or an average of a number of fundamental factors. A single metric fundamental index can be calculated as:[8]

$$FI_i^X = \frac{max\{0, X_i\}}{\sum_{j=1}^{n} max\{0, X_j\}}, \tag{2}$$

where X_i is a numeric value for the considered fundamentals for stock i, e.g., book value *(BV)*, dividends paid *(D)*, free cash flows *(FCF)*, revenues *(REV)*, among others.[9] We side with Arnott et al. [2005]'s composite approach in constructing our FI portfolios as follows:

$$FI_i^{COMP} = \begin{cases} \frac{1}{4}(FI_i^{BV} + FI_i^{D} + FI_i^{FCF} + FI_i^{REV}), & \text{in the presence of dividends for } i; \\ \frac{1}{3}(FI_i^{BV} + FI_i^{FCF} + FI_i^{REV}), & \text{otherwise.} \end{cases} \tag{3}$$

Then, the weights in the FI portfolio are normalized values of the fundamental index constructed above:

$$w_i^{FI} = \frac{FI_i}{\sum_{j=1}^{n} FI_j}. \tag{4}$$

Similarly to Arnott et al. [2005], we use book value for the preceding fiscal year, and trailing five-year averages of free cash flows, revenues and dividends. Combined with equation (2), equation (4) ensures no-short sales, full investment and under-weighting of stocks with non-positive fundamentals.

Arnott's portfolio consists of 1000 stocks; in Walkshäusl and Lobe [2010] portfolio sizes vary. To make sure that the performance of our blending method compared with the S&P 500 is not driven by mid- or small-cap stocks, we include only the top 500 stocks ranked by their market capitalization.[10]

Arnott et al. [2005] rebalance portfolios on January 1st. Since the fundamentals of the preceding fiscal year might be unavailable by January 1st, we follow the Walkshäusl and Lobe [2010] methodology to rebalance portfolios on July 1st, using the data for the preceding fiscal year.

[8] The use of $max()$ in Equation 2 ensures no short sales in the FI portfolios.

[9] Other fundamentals might include employment, income, sales [see Arnott et al., 2005, Basu and Forbes, 2014]. However, evidence on outperformance of these alternative FI portfolios relative to the originally proposed baseline, FI by Arnott et al. [2005], is mixed.

[10] Although the list of top-500 stocks by market capitalization is not identical to the list of the S&P 500, it mimics it closely.

2.3 *Construction of Predictive Blended Portfolios*

We define our blended portfolios as the portfolios based on the two risky assets - the GMV and FI portfolios. We consider 101 combinations of GMV and FI portfolios: (0% FI & 100% GMV), (1% FI &99% GMV), ... , (100% FI & 0% GMV).

Our in-sample results suggest that the optimal blend depends on whether or not financial markets are in turmoil. To avoid look-ahead bias but incorporate this feature, as a proxy for a looming crisis, we use a metric often mentioned by Warren Buffett: Total Market Capitalization divided by GDP. Buffett and Loomis [2001, p.93] argue that this is "the best single measure of where valuations stand at any given moment". We will refer to this ratio as the Buffett Indicator (BI):

$$BI_t = \frac{\text{Wilshire 5000}_t}{\sum_{\tau=t-4}^{t} GDP_\tau/5}, \tag{5}$$

where, the Wilshire 5000 is a market capitalization-weighted index of the market value of all stocks actively traded in the US (the actual number of stocks in the index may vary), and GDP is annualized US nominal GDP in the last five years. Similar to recent literature we favour GDP over GNP.[11] Nominal GDP is chosen because the Wilshire 5000 is also nominal. The Wilshire 5000 is highly correlated with the S&P 500 but more commonly used in the literature for calculating Market Capitalization-to-GDP ratio.

To adjust BI for cycles, in the spirit of Campbell and Shiller [1988], we test the BI ratio, taking ten-, five-, and one- year US GDP. The time horizon for the GDP average in BI calculations does not play a crucial role, producing similar results. Thus, we take the average GDP over a time span of five years.

We propose to use the Buffett Indicator Index:[12]

$$BII_t = \frac{BI_t - \min\{BI_\tau\}_{\tau=t-4}^{t}}{\max\{BI_\tau\}_{\tau=t-4}^{t} - \min\{BI_\tau\}_{\tau=t-4}^{t}} * 100\% \tag{6}$$

We propose[13] to choose the optimal blend proportionally to BII:

$$w_t^{PB} = BII_t w_t^{FI} + (1 - BII_t)\, w_t^{GMV}, \tag{7}$$

[14]

[11] The appropriateness of GDP vs GNP in equation (5) is contentious. Some implementations with GDP can be found in the World Bank and World Federation of Exchanges databases as well as among the Corporate Finance Institute (CFI)'s resources at `https://corporatefinanceinstitute.com/resources/knowledge/valuation/market-cap-to-gdp-buffett-indicator/`.

[12] Note, this formula is similar to the Dimension Index (attainment levels) in the Human Development Index [Sen, 1994, p.8]

[13] We focus on the linear relation between BII and the optimal blending proportion. In our future research we will consider alternatives for $\alpha = f(BII)$, e.g., sigmoid functions for $f()$ as a smoothing alternative.

[14] However, in this paper we round the exact value of BII to the nearest percentage point to improve calculation speed, obtaining: $w_t^{PB} = \alpha *_t w_t^{FI} + (1 - \alpha *_t)\, w_t^{GMV}$, where $\alpha = round(BII)$ is the proportion of the FI portfolio in a blended portfolio strategy. We focus on the linear relation between BII and the optimal blending proportion. In our future research we will consider alternatives for $\alpha = f(BII)$, e.g., sigmoid functions for $f()$ as a smoothing alternative.

When the market is likely to be undervalued, and the likelihood of steady growth increases, it is prudent to invest in a well-diversified portfolio, best captured by the GMV portfolio. If the current BI is at its lowest point ($BII = 0\%$), we suggest that an investor should invest fully in the GMV portfolio.

When the market is likely to be overvalued, and the likelihood of a market crash increases, it is prudent to invest based on the economic footprint of companies, which is best captured by the FI portfolio. If the current BI is at its highest point ($BII = 100\%$), we suggest that an investor should invest fully in the FI portfolio.

When the market is neither undervalued nor overvalued, the likelihood of a crash or expected boom are unclear. This situation is somewhere between the two extremes, expected crash or expected boom. Thus, a blended portfolio constructed from the GMV and FI should be proportional to how close to either extremes the market happens to be.

For example, on July 3rd, 2017[15] the BI metric was 141%; in the preceding five years the minimum BI was 109%, the maximum BI was 141%, thus according to equation 6, the Buffett Indicator Index is equal to 100%. In such a case, we argue that the PB portfolio should be the 100% FI portfolio.

In this section, we analyzed stocks in-sample and constructed the GMV, FI and PB portfolios out-of-sample. Before we perform the empirical investigation of our technique in Section 4, we describe our data and data preparation procedures in the following section.

3 Data Description and Preparation

3.1 Data Description

Our investable universe consists of the S&P 500 constituents listed on the NYSE, NASDAQ and AMEX from January 1990 to August 2018. To avoid survivorship bias we include delisted stocks in our analysis (see Brown et al., 1992). We obtain daily market values (MV) and return indices (RI), which are price index plus dividend disbursements. We collect annual data on book values (BV), dividends (Div), free cash flows (FCF) and revenues (Rev). We also consider the Wilshire 5000[16] (daily) and nominal GDP (annual) data from 1971 to 2018 to construct the Buffett Indicator. These data are sourced from Thomson Reuters Datastream.

To test our approach we construct 23 trailing sub-samples of six years each: five years are used for estimation (July 1, 1990 - June 30, 1995; July 1, 1991 - June 30, 1996 etc.) with the remaining one year for out-of-sample performance (July 1, 1995 - June 30, 1996; July 1, 1996 - June 30, 1997, etc.). Portfolios are rebalanced on July 1 (or the next available trading day) of every year to ensure availability of fundamental data from previous calendar years. In each in-sample sub-period we select 500 stocks with the highest market values on the date of portfolio construction; these are closely related to our main benchmark, S&P 500.[17] Please

[15]Since scheduled rebalancing day July 1st, 2017 was a Saturday, the actual rebalancing day was the first following trading day, Monday July 3rd, 2017

[16]The Wilshire 5000 is a market capitalization index.

[17]We find a high degree of concordance between the market values and free float market capitalization resulting in minimal changes in composition of our universe of 500 stocks.

Table 1: Descriptive Statistics for the period from July 1, 1990 to June 30, 2018. All values are in billions of USD.

\$bn	Mean	StDev	5%	50%	95%	Skew	Kurt
Market Value (MV)	10.75	30.86	0.08	2.48	44.12	8.44	115.21
Book Value (BV)	4.08	13.16	0.03	1.00	15.65	10.37	152.15
Total Dividends (Div)	0.21	0.79	0.00	0.02	0.88	10.92	236.08
Free Cash Flows (FCF)	0.99	3.84	-0.01	0.21	3.99	9.34	256.78
Revenue (Rev)	7.40	21.06	0.06	1.77	30.06	9.51	145.07

see Table 1 for descriptive statistics of the data for stocks that are included at least once in our sample (1095 stocks, for the period of 28 years).[18]

3.2 *Data Preparation*

Since the total return index (RI) reflects both the price of an asset and any dividend disbursements, we obtain daily stock returns as follows:

$$r_{i,t} = \frac{RI_{i,t} - RI_{i,t-1}}{RI_{i,t-1}} \tag{8}$$

Note, that using the simple return formula is essential for accurate aggregation of assets in portfolios, whereas log returns are convenient for time aggregation but result in inaccurate estimates when aggregated across several securities.

Our next section discusses the results of out-of-sample tests on the proposed blended portfolios comparing their performance to common market benchmarks, namely the S&P 500 Index, the Equally-Weighted portfolio comprised of the S&P 500 constituents, the GMV and Arnott's FI portfolios.

4 Results

We analyze portfolios when a "no short-sales" constraint is implemented with maximum holding weights of at most 10% of the portfolio at the time of construction. Table 2 shows the following central findings of our paper.

The first, and the most important result of this study is that over the period 1995-2018 in out-of-sample tests, the Predictive Blended (PB) portfolio, based on the Buffett Indicator discussed in Section 2.3, outperforms in terms of Sharpe ratio scores the Markowitz Tangency, GMV, Arnott FI portfolios and any fixed blend of the GMV and FI portfolios. In Table 2 refer to the second column and the first row: the Sharpe ratio of the PB portfolio is 0.610; this is the highest in the out-of-sample calculations. The PB portfolio is the only portfolio that has a statistically significant outperformance compared to the Equally-Weighted S&P 500 portfolio. Given that all these methods use the same universe of stocks (the S&P 500 constituents lists), the only source of better performance is likely to be a superior methodological approach.

[18] We do not require normality for the distribution of returns, as we use the Ledoit and Wolf [2008] test to calculate heteroskedasticity and autocorrelation-consistent p-values for statistical significance tests of portfolios' Sharpe ratios.

Second, even if the Predictive Blended approach is not applied, blending the GMV and FI portfolios in fixed proportions (for example 25%FI + 75%GMV) produces results stronger (Sharpe ratio is 0.558) than those of the S&P 500 (0.346), Equally-Weighted S&P 500 (0.511), or the FI portfolio (0.453).[19] The difference in Sharpe ratios between the fixed blend (25% FI, 75% GMV) and the Equally-Weighted S&P 500 portfolio is statistically significant at the 90% level[20], a result that is only outmatched by the Predictive Blended portfolio (see the right hand column in Table 2). This confirms the point we made earlier in Section 2 that blending portfolios produce better results than the pure Markowitz MVO (GMV) or Arnott's FI approaches.

Third, even if the Predictive Blended portfolio is based on consistently flawed forecasts the result would not be much different from the capitalization-weighted S&P 500: Table 3 shows that even when we consistently choose the worst blend, the since-inception Sharpe ratio is 0.288, compared to 0.346 for the S&P 500. In contrast, in the equally unrealistic case, when our forecasts are consistently right (the best blend), the Sharpe ratio is 0.791, compared to 0.346 for the S&P 500.

Fourth, the Predictive Blended portfolio produces a higher since-inception return (12.61%) than the GMV (10.54%) and FI (11.97%) portfolios taken separately. The S&P 500, GMV and FI portfolios have the lower returns since inception: 10.26%, 10.54%, and 11.97% respectively; in contrast, returns on the PB (12.61%), Equally-Weighted S&P 500 (13.31%), and Tangency (14.07%) portfolios are higher. The Predictive Blended portfolio provides somewhat lower returns than the Tangency and the Equally-Weighted S&P 500 portfolios, but with the benefit of much lower volatility.

Fifth, the PB portfolio is less volatile (σ =14.29%) than the Tangency (σ =17.44%), FI (σ =17.83%), S&P 500 (σ =18.37%), and Equally-Weighted S&P 500 (σ =18.43%) over the period 1995 - 2018. This property makes the PB portfolio the portfolio of choice for investors with high aversion to volatility, but who still would like to earn returns higher than those of the GMV portfolio (with the lowest volatility of σ =11.53%).

Interestingly, in out-of-sample (1995-2018) tests the Sharpe ratio of the GMV portfolio (0.576) is close to that of the Tangency portfolio (0.583). This may be due to the fact that in out-of-sample-tests the Tangency portfolio moves further inside the Minimum Variance Set than the GMV portfolio. This illustrates the point we made earlier that the GMV portfolio does not suffer as much from estimation errors of its inputs: the covariance structure needed for both of them is more robust than the hard-to-predict expected returns needed only for the Tangency portfolio.

Table 3 shows that Predictive Blend is the best strategy over the long-term, even though, there is a possibility that some other strategy might be better in specific years (see Table 4 for year-by-year performance). In fact, the PB strategy has the highest Sharpe ratios since

[19] However, the GMV portfolio outperforms fixed blends, having a Sharpe ratio of 0.576.

[20] Interestingly, we noticed that the fixed blend (25% FI, 75% GMV) has higher statistical significance and a lower Sharpe ratio than those of the GMV and Tangency portfolios.

Table 2: OUT-OF-SAMPLE SHARPE RATIO ANALYSIS. This table outlines the results of significance tests for the difference in Sharpe ratios (Sharpe ratios are highlighted in bold) of various portfolios (in rows) against the two benchmarks (in columns 2-3-4-5) for the period from July 1, 1995 to June 30, 2018. We apply the methodology in Ledoit and Wolf [2008] to calculate heteroskedasticity and autocorrelation-consistent (HAC) *p*-values for the difference in Sharpe ratios of two portfolios. (*), (**), and (***) represent respectively the 90%, 95%, and 99% significance levels. The out-of-sample Sharpe ratios of our constructed portfolios are, generally, higher than those of the two benchmarks considered. The bottom two rows contain the best and the worst blends of FI and GMV portfolios under unrealistic perfect foresight scenarios, representing the most liberal and conservative thresholds.

	Portfolios:		S&P 500		Eq.-Weighted S&P 500	
		Sharpe ratio	**0.346**		**0.511**	
			p-values		*p*-value	
			HAC	HAC (pre-whitened)	HAC	HAC (pre-whitened)
Out-of-sample	PB	**0.610**	0.001***	0.000***	0.040**	0.041**
	Tangency	**0.583**	0.044**	0.021**	0.431	0.438
	GMV	**0.576**	0.021**	0.013**	0.131	0.132
	25%FI+75%GMV	**0.558**	0.011**	0.004***	0.097*	0.099*
	50%FI+50%GMV	**0.525**	0.011**	0.002***	0.171	0.175
	75%FI+25%GMV	**0.488**	0.031**	0.003***	0.767	0.770
	FI	**0.453**	0.131	0.025**	0.229	0.235
	Best Blend	**0.791**	0.000	0.000	0.001	0.001
	Worst Blend	**0.288**	0.684	0.620	0.003	0.004

inception in 9 out of the 23 years we considered (see Table 3). Even though the year-by-year Table 4 shows that the GMV portfolio outperforms other portfolios in 11 out of 23 years taken separately, it is not a reliable strategy in the long term. For example, during the Asian and the Long Term Capital Management crises in 1998-1999, the GMV portfolio was the only portfolio in our set to show negative returns (the GMV return was -2.11%, while the S&P 500 return was +22.45%, and FI +17.37%) and consequently, Sharpe ratios (for the GMV it was -0.659, while for the S&P 500 the Sharpe ratio was 0.796, and for the FI it was 0.686). The following year, in 1999 to 2000, we face a similar situation: returns were -1.03% for the GMV vs +8.58% for the S&P 500, and +0.79% for the FI portfolio.[21]

Table 5 presents year-by-year returns, standard deviations and Sharpe ratios for the six portfolios we study: Global Minimum Variance (GMV), Arnott Fundamental Index (FI), Predictive Blend (PB), Tangency based on the restricted MVO frontier, the S&P 500 index, and Equally-Weighted S&P 500. We note in the bottom row (1995-2018) that the PB portfolio outperforms all other portfolios in terms of Sharpe ratios, and the difference in performance is statistically significant at the 99% level compared with the S&P 500, 95% level compared with the Equally-Weighted S&P 500 as our main benchmarks. Taking year-by-year changes in risk-adjusted return performance, the PB underperformed GMV or other portfolios in certain years, but over the long term the PB proved to be the most successful. Even though the cumulative over-performance of the PB over the GMV portfolio is not statistically significant, cumulative returns of the PB dominate those of the GMV portfolio in 9 out of 23 years, not a

[21] Refer to Table ??.

Table 3: Out-of-sample Sharpe ratios for portfolios first built in 1995, and ending in various years, assuming annual rebalancing on July 1 of each year. The bottom row represents the number of years a portfolio had the highest Sharpe ratio among the benchmarks considered.

Period		Out-of-sample Sharpe ratios						Perfect foresight	
Start	End	PB	Tangency	GMV	FI	S&P 500	Eq.Weighted	Best Blend	Worst Blend
1995	1996	1.914	2.847	**3.132**	1.893	1.718	2.575	3.132	1.893
1995	1997	1.976	2.430	**2.606**	1.944	1.771	2.155	2.600	1.944
1995	1998	1.729	2.096	**2.453**	1.719	1.499	1.724	2.471	1.719
1995	1999	1.346	1.281	1.240	**1.351**	1.225	1.222	1.405	1.171
1995	2000	0.941	0.904	0.737	0.949	**0.952**	0.904	0.882	0.833
1995	2001	0.893	0.695	0.818	0.877	0.589	**0.877**	0.914	0.767
1995	2002	**0.743**	0.416	0.716	0.619	0.286	0.679	0.824	0.482
1995	2003	**0.688**	0.376	0.625	0.501	0.273	0.574	0.747	0.377
1995	2004	**0.770**	0.461	0.751	0.565	0.312	0.661	0.836	0.460
1995	2005	0.817	0.518	**0.849**	0.564	0.313	0.670	0.908	0.468
1995	2006	0.803	0.505	**0.839**	0.557	0.317	0.669	0.893	0.468
1995	2007	0.848	0.522	**0.865**	0.612	0.371	0.707	0.948	0.499
1995	2008	0.646	0.484	**0.672**	0.465	0.255	0.552	0.780	0.370
1995	2009	**0.448**	0.285	0.428	0.266	0.116	0.332	0.548	0.180
1995	2010	**0.485**	0.336	0.478	0.302	0.152	0.379	0.587	0.225
1995	2011	0.551	0.437	**0.555**	0.370	0.226	0.455	0.651	0.300
1995	2012	0.529	0.468	**0.567**	0.351	0.226	0.422	0.657	0.285
1995	2013	0.576	0.510	**0.602**	0.401	0.264	0.468	0.715	0.315
1995	2014	**0.619**	0.577	0.611	0.439	0.308	0.511	0.761	0.327
1995	2015	**0.605**	0.593	0.576	0.433	0.311	0.502	0.740	0.310
1995	2016	0.585	0.566	**0.610**	0.418	0.305	0.483	0.764	0.300
1995	2017	**0.607**	0.558	0.595	0.445	0.331	0.504	0.793	0.295
1995	2018	**0.610**	0.583	0.576	0.453	0.346	0.511	0.791	0.288
No. of superior years		9	0	11	1	1	1		

Table 4: One-year out-of-sample Sharpe ratios for portfolios established in various periods, starting on July 1 of each year. The bottom row represents the number of years a portfolio had the highest Sharpe ratio among the benchmarks considered.

Period		Out-of-sample						Perfect foresight	
Start	End	PB	Tangency	GMV	FI	S&P 500	Eq.Weighted	Best Blend	Worst Blend
1995	1996	1.914	2.847	**3.132**	1.893	1.718	2.575	3.132	1.899
1996	1997	2.051	2.137	**2.175**	2.019	1.848	1.908	2.263	2.019
1997	1998	1.467	1.689	**2.300**	1.462	1.184	1.248	2.300	1.462
1998	1999	0.617	0.161	-0.659	0.686	**0.796**	0.463	0.686	-0.659
1999	2000	-0.322	-0.037	-0.672	-0.298	**0.125**	0.065	-0.298	-0.672
2000	2001	0.631	0.059	**1.221**	0.514	-0.906	0.758	1.221	0.514
2001	2002	-0.460	-1.412	**0.064**	-0.763	-1.438	-0.325	0.064	-0.764
2002	2003	**0.318**	0.166	**0.318**	0.095	0.223	0.230	0.318	0.095
2003	2004	**1.652**	1.474	1.611	1.312	0.860	1.539	1.660	1.312
2004	2005	1.507	1.332	**1.761**	0.602	0.372	0.819	1.761	0.602
2005	2006	0.659	0.354	**0.727**	0.500	0.440	0.685	0.727	0.500
2006	2007	1.622	0.744	1.249	**1.674**	1.480	1.362	1.674	1.249
2007	2008	-0.912	**0.265**	-0.864	-0.907	-0.918	-0.822	-0.864	-0.920
2008	2009	-0.342	-0.623	**-0.273**	-0.439	-0.530	-0.411	-0.273	-0.439
2009	2010	1.100	1.023	**1.260**	0.775	0.708	0.953	1.260	0.775
2010	2011	2.000	**2.391**	2.097	1.720	1.864	1.901	2.097	1.720
2011	2012	0.279	**0.887**	0.750	0.117	0.227	0.014	0.750	0.117
2012	2013	1.724	1.421	1.378	**1.765**	1.404	1.762	1.765	1.378
2013	2014	1.857	2.031	0.851	1.883	1.929	**2.040**	1.883	0.851
2014	2015	0.289	**1.077**	-0.233	0.289	0.451	0.294	0.289	-0.233
2015	2016	0.218	-0.021	**1.246**	0.093	0.163	0.053	1.246	0.098
2016	2017	1.567	0.370	0.161	1.752	**1.766**	1.569	1.752	0.161
2017	2018	0.717	**1.388**	0.057	0.748	0.886	0.842	0.748	0.057
No. of superior years		2	5	11	2	3	1		

bad property for practitioners who target higher returns than those of the GMV portfolio.

5 Conclusion

In this paper, we propose a new portfolio construction technique that combines the benefits of Mean-Variance Optimization (MVO) and Fundamental Indexing (FI). Given that the FI approach is relatively new, and is profoundly different from the MVO, these two approaches have not yet been combined, even though each method offers distinctive benefits for portfolio choice problems. Our paper fills this gap in the literature. Our results attest to the superior performance of the proposed Predictive Blended (PB) portfolio compared to two hard-to-beat benchmarks, the S&P 500 and the Equally-Weighted S&P 500.

Applying the MVO method proposed by Markowitz (1952), we find the portfolio that contains the most information about the variance-covariance structure of stock returns - the Global Minimum Variance portfolio (GMV). Applying the FI method proposed by Arnott et al. [2005], we construct a portfolio from stocks that are in sound financial health. Blending these two portfolios generates a portfolio that has better diversification than the FI portfolio and better risk-adjusted return characteristics than the GMV portfolio. Although, ad-hoc static fixed-proportion blends provide promising results compared to the benchmarks, we find that the dynamic Predictive Blended portfolio is remarkably superior.

We test the out-of-sample performance of the predictive and fixed blends (for example 25% FI and 75% GMV) using 29 years worth of data from S&P 500 companies. The suggested PB approach is the only portfolio that provides statistically significant superior (over the S&P 500 and Equally-Weighted S&P 500 benchmarks) Sharpe ratios in out-of-sample tests. The FI, GMV or classic Markowitz Tangency portfolios taken separately do not have statistically significant Sharpe ratios over the hard-to-beat Equally-Weighted S&P 500 benchmark.

The second major result of our paper is that almost any fixed blend between the GMV and FI portfolios performs better than the S&P 500 (but not necessarily better than the Equally-Weighted benchmark).

Our future research will focus on finding improved FI techniques that would enhance our predictive blended portfolios even further. In particular, within-industry analysis of the FI portfolios could enable portfolio managers to fine-tune prediction metrics and optimal blends during industry-specific crises vs market-wide turmoils. In addition, given the limited number of studies on FI strategies for non-US markets, a comparative study assessing predictive blended portfolios in global markets is worth pursuing.

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Appendix

5.1 *Additional tables*

Table 8: Annualized Out-of-Sample Portfolio Performance Statistics Portfolio of year-by-year portfolios. First three portfolios are blended portfolios constructed in fixed proportions; the Best and the Worst Blends are constructed given unrealistic perfect foresight scenarios, and PB(alt) is an alternative modification of the Predictive Blended portfolio (where, GDP from a preceding year is used, instead of using an average over 5 years GDP). We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks.

July 1, *yyyy*		25%FI+75%GMV			50%FI+50%GMV			75%FI+25%GMV			Best Blend			Worst Blend			PB (alt)		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	21.62	5.29	2.95	21.96	6.21	2.57	22.34	7.44	2.19	21.30	4.88	3.13	22.74	8.83	1.89	22.71	8.72	1.91
1996	1997	21.75	6.79	2.26	24.64	8.31	2.20	27.50	10.04	2.10	21.40	6.63	2.26	30.33	11.86	2.02	29.09	11.05	2.05
1997	1998	23.54	8.68	2.06	24.74	10.50	1.82	26.00	12.58	1.62	22.38	7.27	2.30	27.31	14.81	1.46	27.26	14.71	1.47
1998	1999	2.94	11.68	-0.17	7.86	13.42	0.22	12.67	15.62	0.49	17.37	18.06	0.69	-2.11	10.75	-0.66	15.50	17.06	0.62
1999	2000	-0.76	11.06	-0.61	-0.36	12.63	-0.50	0.16	14.85	-0.39	0.79	17.45	-0.30	-1.03	10.45	-0.67	0.61	16.70	-0.32
2000	2001	14.60	9.02	1.03	13.91	10.44	0.82	13.29	12.30	0.65	15.36	8.24	1.22	12.73	14.43	0.51	13.21	12.54	0.63
2001	2002	2.13	8.98	-0.30	-1.10	11.02	-0.54	-4.36	13.54	-0.68	5.35	7.71	0.06	-7.64	16.36	-0.76	0.06	10.22	-0.47
2002	2003	7.48	15.99	0.23	6.92	18.95	0.16	6.54	22.36	0.12	8.20	13.68	0.32	6.35	26.12	0.10	8.20	13.68	0.32
2003	2004	20.82	10.03	1.66	20.41	10.05	1.61	20.06	10.70	1.48	20.80	10.02	1.66	19.77	11.88	1.31	20.99	10.18	1.65
2004	2005	17.52	8.75	1.53	15.15	8.92	1.23	12.74	9.43	0.91	19.87	8.93	1.76	10.31	10.26	0.60	17.43	8.75	1.52
2005	2006	10.30	8.54	0.68	10.02	8.87	0.62	9.75	9.38	0.56	10.60	8.41	0.73	9.50	10.03	0.50	10.21	8.63	0.66
2006	2007	15.47	7.60	1.42	17.13	8.06	1.55	18.77	8.68	1.63	20.41	9.41	1.67	13.80	7.33	1.25	18.38	8.52	1.61
2007	2008	-8.82	14.24	-0.90	-10.30	15.61	-0.92	-11.75	17.20	-0.92	-7.32	13.15	-0.86	-10.94	16.28	-0.92	-12.73	18.40	-0.91
2008	2009	-8.53	33.87	-0.35	-11.66	37.42	-0.40	-14.59	41.66	-0.43	-5.25	30.98	-0.27	-17.26	46.59	-0.44	-8.27	33.61	-0.34
2009	2010	17.75	13.06	1.09	18.32	15.56	0.95	18.98	18.24	0.85	17.29	10.98	1.26	19.72	20.98	0.77	17.71	12.88	1.11
2010	2011	23.77	10.20	2.04	25.21	11.50	1.93	26.68	13.00	1.82	22.36	9.23	2.10	28.17	14.62	1.72	24.17	10.54	2.01
2011	2012	10.35	15.71	0.53	8.44	18.09	0.35	6.58	20.78	0.22	12.33	13.73	0.75	4.81	23.74	0.12	7.69	19.13	0.30
2012	2013	17.11	9.75	1.57	19.34	10.38	1.69	21.56	11.32	1.75	23.77	12.46	1.77	14.87	9.50	1.38	19.88	10.58	1.71
2013	2014	13.05	8.67	1.20	15.85	8.81	1.50	18.61	9.24	1.72	21.31	9.90	1.88	10.19	8.85	0.85	20.66	9.73	1.85
2014	2015	1.34	9.75	-0.09	2.73	10.14	0.05	4.13	10.75	0.18	5.54	11.54	0.29	-0.04	9.61	-0.23	5.54	11.54	0.29
2015	2016	14.30	13.14	0.94	10.80	14.02	0.63	7.22	15.26	0.34	17.72	12.63	1.25	3.56	16.83	0.09	5.33	16.04	0.21
2016	2017	7.05	7.28	0.68	10.65	7.29	1.18	14.16	7.88	1.53	17.60	8.86	1.75	3.36	7.92	0.16	14.72	8.01	1.58
2017	2018	5.11	9.03	0.29	7.19	9.76	0.48	9.24	10.66	0.63	11.28	11.68	0.75	3.03	8.55	0.06	10.87	11.47	0.73

Table 5: Annualized Out-of-Sample Portfolio Performance Statistics beginning in 1995 and ending in various years. All portfolios are based on S&P 500 stocks. Portfolios are rebalanced every year on July 1. GMV is the Global Minimum Variance portfolio, FI is Arnott's FI, PB is Predictive Blended portfolio (which blends the GMV and FI), Tangency portfolio is a portfolio tangent to the MVO frontier, S&P 500 is market capitalization-weighted portfolio, while Equally-Weighted is a portfolio, where each stock has equal presence. We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks. Figures in bold represent the lowest portfolio risk, the highest portfolio return and Sharpe ratio among the portfolio strategies and benchmarks for a given period under consideration. It is not surprising that the GMV portfolio consistently results in the lowest risk, even in out-of-sample. However, the average portfolio return and Sharpe ratio suggest that the PB portfolio exhibits superior out-of-sample performance.

July 1, *yyyy*		PB			Tangency			GMV			FI			S&P 500			Equally-Weighted		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	22.71	8.72	1.91	27.18	7.43	2.85	21.30	4.88	3.13	22.74	8.83	1.89	22.94	9.85	1.72	24.44	7.15	2.58
1995	1997	25.96	10.00	1.98	26.93	8.53	2.43	20.06	5.32	2.61	26.54	10.46	1.94	27.12	11.81	1.77	24.99	8.71	2.16
1995	1998	26.39	11.78	1.73	26.56	9.80	2.10	20.84	6.04	2.45	26.80	12.09	1.72	26.73	13.81	1.50	24.07	10.47	1.72
1995	1999	23.67	13.31	1.35	21.91	12.61	1.28	15.09	7.53	1.24	24.44	13.83	1.35	25.66	16.24	1.23	21.38	12.79	1.22
1995	2000	19.04	14.06	0.94	18.57	14.12	0.90	11.86	8.21	0.74	19.69	14.64	0.95	22.23	17.25	0.95	18.53	14.08	0.90
1995	2001	18.07	13.82	0.89	16.58	15.63	0.69	12.44	8.21	0.82	18.53	14.61	0.88	16.28	17.92	0.59	18.30	14.35	0.88
1995	2002	15.52	13.37	0.74	12.03	15.46	0.42	11.43	8.14	0.72	14.81	14.88	0.62	10.77	18.08	0.29	15.61	14.74	0.68
1995	2003	14.61	13.41	0.69	11.41	16.07	0.38	11.02	9.02	0.63	13.75	16.71	0.50	10.64	19.29	0.27	14.88	16.57	0.57
1995	2004	15.32	13.09	0.77	12.42	15.59	0.46	12.17	9.22	0.75	14.42	16.24	0.56	11.05	18.61	0.31	15.99	16.25	0.66
1995	2005	15.52	12.72	0.82	12.99	15.15	0.52	12.94	9.19	0.85	14.01	15.74	0.56	10.75	17.96	0.31	15.76	15.84	0.67
1995	2006	15.04	12.40	0.80	12.60	14.90	0.51	12.73	9.12	0.84	13.60	15.31	0.56	10.59	17.41	0.32	15.46	15.51	0.67
1995	2007	15.34	12.14	0.85	12.72	14.71	0.52	12.82	8.99	0.87	14.16	14.91	0.61	11.31	16.91	0.37	15.77	15.17	0.71
1995	2008	13.18	12.73	0.65	12.55	15.68	0.48	11.27	9.38	0.67	12.06	15.27	0.46	9.34	17.17	0.25	13.59	15.63	0.55
1995	2009	11.65	15.21	0.45	10.11	18.49	0.28	10.09	12.26	0.43	9.96	19.29	0.27	7.21	20.40	0.12	11.43	19.83	0.33
1995	2010	12.05	15.07	0.48	10.99	18.55	0.34	10.57	12.18	0.48	10.62	19.40	0.30	7.82	20.26	0.15	12.35	20.05	0.38
1995	2011	12.82	14.83	0.55	12.65	18.33	0.44	11.31	12.02	0.55	11.71	19.14	0.37	9.14	19.93	0.23	13.65	19.81	0.45
1995	2012	12.50	15.14	0.53	13.17	18.56	0.47	11.37	12.13	0.57	11.31	19.44	0.35	9.03	20.12	0.23	12.98	20.16	0.42
1995	2013	12.94	14.94	0.58	13.71	18.38	0.51	11.56	12.00	0.60	12.00	19.12	0.40	9.56	19.76	0.26	13.62	19.83	0.47
1995	2014	13.35	14.71	0.62	14.76	18.23	0.58	11.49	11.85	0.61	12.49	18.75	0.44	10.21	19.37	0.31	14.19	19.45	0.51
1995	2015	12.96	14.57	0.61	14.81	17.99	0.59	10.91	11.75	0.58	12.14	18.46	0.43	10.08	19.06	0.31	13.76	19.14	0.50
1995	2016	12.60	14.64	0.58	14.18	17.93	0.57	11.24	11.79	0.61	11.73	18.38	0.42	9.82	18.96	0.30	13.24	19.03	0.48
1995	2017	12.69	14.40	0.61	13.79	17.62	0.56	10.88	11.65	0.59	12.00	18.06	0.45	10.11	18.60	0.33	13.38	18.70	0.50
1995	2018	12.61	14.29	0.61	14.07	17.44	0.58	10.54	11.53	0.58	11.97	17.83	0.45	10.26	18.37	0.35	13.31	18.43	0.51

Table 6: Annualized Out-of-Sample Portfolio Performance Statistics of year-by-year portfolios. GMV is the Global Minimum Variance portfolio, FI is Arnott's FI, PB is Predictive Blended portfolio (which blends the GMV and FI), Tangency portfolio is a portfolio tangent to the MVO frontier, S&P 500 is market capitalization-weighted portfolio, while Equally-Weighted is a portfolio, where each stock has equal presence. We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks. Figures in bold represent the lowest portfolio risk, the highest portfolio return and Sharpe ratio among the portfolio strategies and benchmarks for a given period under consideration. It is not surprising that GMV portfolio consistently results in the lowest risk, even in out-of-sample. The evidence of out-of-sample superior performance of PB portfolios is less apparent from year-by-year results compared to results in Tables 5, suggesting that this strategy is more appropriate for long-term investments. Figures in bold represent the lowest portfolio risk, the highest portfolio return and Sharpe ratio among the portfolio strategies and benchmarks for a given period under consideration.

July 1, *yyyy*		PB			Tangency			GMV			FI			S&P 500			Equally-Weighted		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	22.71	8.72	1.91	27.18	7.43	2.85	21.30	4.88	3.13	22.74	8.83	1.89	22.94	9.85	1.72	24.44	7.15	2.58
1996	1997	29.20	11.12	2.05	26.68	9.50	2.14	18.83	5.72	2.17	30.33	11.86	2.02	31.29	13.48	1.85	25.52	10.03	1.91
1997	1998	27.26	14.71	1.47	25.82	11.94	1.69	22.38	7.27	2.30	27.31	14.81	1.46	25.94	17.12	1.18	22.25	13.29	1.25
1998	1999	15.50	17.06	0.62	7.97	18.62	0.16	-2.11	10.75	-0.66	17.37	18.06	0.69	22.45	21.97	0.80	13.32	18.02	0.46
1999	2000	0.61	16.70	-0.32	5.28	18.95	-0.04	-1.03	10.45	-0.67	0.79	17.45	-0.30	8.58	20.76	0.12	7.19	18.32	0.07
2000	2001	13.21	12.54	0.63	6.58	21.65	0.06	15.36	8.24	1.22	12.73	14.43	0.51	-13.57	20.84	-0.91	17.14	15.62	0.76
2001	2002	0.19	10.13	-0.46	-15.36	14.31	-1.41	5.35	7.71	0.06	-7.64	16.36	-0.76	-22.38	18.93	-1.44	-0.62	16.86	-0.32
2002	2003	8.20	13.68	0.32	7.13	19.78	0.17	8.20	13.68	0.32	6.35	26.12	0.10	9.72	26.23	0.22	9.83	25.98	0.23
2003	2004	20.99	10.18	1.65	20.46	11.04	1.47	21.30	10.63	1.61	19.77	11.88	1.31	14.32	11.81	0.86	24.83	13.42	1.54
2004	2005	17.33	8.75	1.51	18.08	10.47	1.33	19.87	8.93	1.76	10.31	10.26	0.60	8.04	10.49	0.37	13.64	11.60	0.82
2005	2006	10.18	8.65	0.66	8.74	12.03	0.35	10.60	8.41	0.73	9.50	10.03	0.50	9.03	10.33	0.44	12.51	11.71	0.69
2006	2007	18.64	8.62	1.62	13.97	12.54	0.74	13.80	7.33	1.25	20.41	9.41	1.67	19.27	9.88	1.48	19.20	10.68	1.36
2007	2008	-12.67	18.32	-0.91	10.51	24.45	0.26	-7.32	13.15	-0.86	-13.18	18.99	-0.91	-14.34	20.03	-0.92	-12.62	20.26	-0.82
2008	2009	-8.27	33.61	-0.34	-21.59	39.82	-0.62	-5.25	30.98	-0.27	-17.26	46.59	-0.44	-20.43	44.61	-0.53	-16.59	48.24	-0.41
2009	2010	17.73	12.97	1.10	23.27	19.36	1.02	17.29	10.98	1.26	19.72	20.98	0.77	16.34	18.19	0.71	25.18	22.79	0.95
2010	2011	24.29	10.64	2.00	37.57	14.46	2.39	22.36	9.23	2.10	28.17	14.62	1.72	28.98	13.93	1.86	33.13	15.84	1.90
2011	2012	7.46	19.45	0.28	21.50	21.94	0.89	12.33	13.73	0.75	4.81	23.74	0.12	7.24	22.98	0.23	2.39	25.10	0.01
2012	2013	20.41	10.80	1.72	22.90	14.86	1.42	14.87	9.50	1.38	23.77	12.46	1.77	18.50	11.90	1.40	24.42	12.85	1.76
2013	2014	20.77	9.75	1.86	33.69	15.27	2.03	10.19	8.85	0.85	21.31	9.90	1.88	21.98	10.01	1.93	24.42	10.67	2.04
2014	2015	5.54	11.54	0.29	15.83	12.65	1.08	-0.04	9.61	-0.23	5.54	11.54	0.29	7.55	11.87	0.45	5.61	11.61	0.29
2015	2016	5.48	15.97	0.22	1.64	16.66	-0.02	17.72	12.63	1.25	3.56	16.83	0.09	4.73	16.83	0.16	2.88	16.77	0.05
2016	2017	14.58	7.98	1.57	5.45	9.10	0.37	3.36	7.92	0.16	17.60	8.86	1.75	16.25	8.02	1.77	16.32	9.08	1.57
2017	2018	10.71	11.39	0.72	20.23	12.74	1.39	3.03	8.55	0.06	11.28	11.68	0.75	13.36	12.21	0.89	11.91	11.12	0.84

Table 7: ANNUALIZED OUT-OF-SAMPLE PORTFOLIO PERFORMANCE STATISTICS beginning in 1995 - ending in different years to 2018. First three portfolios are blended portfolios constructed in fixed proportions; the Best and the Worst Blends are constructed given unrealistic perfect foresight scenarios, and PB(alt) is an alternative modification of the Predictive Blended portfolio (where, GDP from a preceding year is used, instead of using an average over 5 years GDP). We estimate annualized average portfolio returns, $\bar{r}$, portfolio standard deviations, $\hat{\sigma}$, and Sharpe ratios, S, for various portfolio strategies and benchmarks.

July 1, *yyyy*		25%FI+75%GMV			50%FI+50%GMV			75%FI+25%GMV			Best Blend			Worst Blend			PB (alt)		
Start	End	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S	$\bar{r}$(%)	$\hat{\sigma}$ (%)	S
1995	1996	21.62	5.29	2.95	21.96	6.21	2.57	22.34	7.44	2.19	21.30	4.88	3.13	22.74	8.83	1.89	22.71	8.72	1.91
1995	1997	21.68	6.09	2.54	23.31	7.34	2.33	24.92	8.84	2.12	21.35	5.82	2.60	26.54	10.46	1.94	25.90	9.96	1.98
1995	1998	22.30	7.06	2.31	23.78	8.53	2.08	25.28	10.24	1.88	21.70	6.34	2.47	26.80	12.09	1.72	26.36	11.76	1.73
1995	1999	17.46	8.47	1.38	19.80	9.99	1.41	22.13	11.82	1.38	20.61	10.57	1.40	19.56	11.79	1.17	23.64	13.29	1.35
1995	2000	13.80	9.06	0.88	15.75	10.58	0.94	17.72	12.50	0.95	16.63	12.28	0.88	15.43	11.55	0.83	19.02	14.05	0.94
1995	2001	13.93	9.05	0.91	15.45	10.56	0.92	16.98	12.47	0.90	16.42	11.70	0.91	14.98	12.07	0.77	18.05	13.81	0.89
1995	2002	12.25	9.05	0.74	13.09	10.63	0.70	13.94	12.63	0.66	14.84	11.22	0.82	11.76	12.78	0.48	15.49	13.37	0.74
1995	2003	11.66	10.18	0.62	12.32	11.99	0.58	13.01	14.22	0.54	14.01	11.56	0.75	11.08	15.11	0.38	14.58	13.41	0.69
1995	2004	12.68	10.17	0.73	13.22	11.79	0.68	13.80	13.87	0.62	14.77	11.40	0.84	12.05	14.79	0.46	15.29	13.09	0.77
1995	2005	13.16	10.03	0.80	13.41	11.54	0.72	13.69	13.50	0.63	15.28	11.18	0.91	11.88	14.40	0.47	15.51	12.72	0.82
1995	2006	12.90	9.91	0.79	13.10	11.32	0.71	13.34	13.17	0.63	14.86	10.96	0.89	11.66	14.06	0.47	15.02	12.40	0.80
1995	2007	13.12	9.74	0.83	13.44	11.09	0.76	13.79	12.86	0.68	15.32	10.84	0.95	11.84	13.63	0.50	15.30	12.13	0.85
1995	2008	11.43	10.16	0.64	11.61	11.51	0.58	11.82	13.25	0.52	13.57	11.04	0.78	10.08	13.85	0.37	13.15	12.73	0.64
1995	2009	10.00	13.34	0.39	9.95	14.94	0.34	9.93	16.95	0.30	12.23	13.48	0.55	8.13	18.26	0.18	11.62	15.21	0.45
1995	2010	10.52	13.32	0.43	10.51	14.98	0.38	10.54	17.04	0.34	12.57	13.33	0.59	8.90	18.46	0.23	12.02	15.07	0.48
1995	2011	11.35	13.15	0.51	11.43	14.79	0.46	11.55	16.82	0.41	13.18	13.11	0.65	10.11	18.24	0.30	12.78	14.82	0.55
1995	2012	11.29	13.31	0.51	11.25	15.00	0.45	11.25	17.08	0.40	13.13	13.15	0.66	9.80	18.61	0.29	12.48	15.11	0.53
1995	2013	11.61	13.14	0.55	11.70	14.79	0.50	11.83	16.81	0.45	13.72	13.11	0.72	10.08	18.23	0.31	12.89	14.90	0.57
1995	2014	11.69	12.95	0.57	11.92	14.53	0.53	12.18	16.50	0.48	14.12	12.97	0.76	10.08	17.86	0.33	13.30	14.67	0.62
1995	2015	11.17	12.81	0.55	11.46	14.35	0.51	11.78	16.26	0.47	13.69	12.90	0.74	9.58	17.54	0.31	12.91	14.53	0.60
1995	2016	11.32	12.82	0.57	11.43	14.33	0.52	11.56	16.21	0.46	13.88	12.89	0.76	9.29	17.50	0.30	12.55	14.61	0.58
1995	2017	11.13	12.62	0.57	11.39	14.09	0.53	11.68	15.93	0.49	14.05	12.73	0.79	9.02	17.18	0.29	12.65	14.37	0.61
1995	2018	10.86	12.49	0.56	11.21	13.93	0.53	11.57	15.74	0.49	13.93	12.69	0.79	8.76	16.90	0.29	12.57	14.26	0.61

B. *Notation*

Variable	Description
i and t	subscripts denoting stock i and period t
RI	Total Return Index (includes change in price and dividends)
r_{it}	Simple return (based on RI)
FI	Fundamental Index
w^{FI}	Vector of weights of the FI portfolio
w^{GMV}	Vector of weights of the GMV portfolio
w^{PB}	Vector of weights of the Predictive Blended portfolio
Ω	Expected variance-covariance matrix of stocks
BI	Buffett Indicator = Wilshire 5000 / nominal GDP
BII	Buffett Indicator Index = (BI - min(BI)/(max(BI) - min (BI))
BV	Book Value
Div	Dividends for the last year
FCF	Free Cash Flows
MV	Market Value, capitalization
Rev	Revenue for the last year