Project financing versus corporate financing under asymmetric information*

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Keywords: asymmetric information, non-recourse debt, project financing, asset securitization

JEL classification: C72, D82, G29, G32, O220

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Project financing versus corporate financing under asymmetric information

Abstract. In recent years financing through the creation of an independent project company or financing by non-recourse debt has become an important part of corporate decisions. Shah and Thakor (JET, 1987) argue that project financing can be optimal when asymmetric information exists between firm’s insiders and market participants. In contrast to that paper, we provide an asymmetric information argument for project financing without relying on corporate taxes, costly information production or an assumption that firms have the same mean of return. In addition, the model generates new predictions regarding asset securitization.

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1 Introduction.

Existing literature studies the effect of asymmetric information on many aspects of debt financing including debt maturity and seniority, collateral, liquidation rights, convertible debt, income bonds and sinking funds.\(^1\) Less is known about the effect of asymmetric information on firms’ incentive to issue non-recourse debt. The intention of this paper is to shed new light on this issue.

Project financing (non-recourse debt) differs from corporate financing in two ways: 1) the creditors do not have a claim on the profit from other projects if the project fails while corporate financing gives this right to the investors and; 2) it typically has priority on the cash flows from the project over any corporate claims. In recent years financing through the creation of an independent project company or financing by non-recourse debt has become an important part of corporate financing decisions. For example, Esty (2003, 2004) reports that total project-financed investments have grown from less than $10 billion per year in the late 1980s to more than $100 billion per year in 2001-2003. Within the United States, firms financed $68 billion in capital expenditures through project companies in 2001, approximately twice the amount raised in initial public offerings (IPOs) or invested by venture capital firms.

Existing literature suggests several explanations for project financing. Most of this literature is based on agency or moral hazard problems.\(^2\) This literature usually assumes that a firm’s insiders and investors have the same information at the beginning of the project. However, Flyvbjerg, Holm and Buhl (2002) found that the costs of most large infrastructure projects are underestimated. The authors argue that project initiators almost always provide misinformation. This suggest that there exists asymmetric information between insiders and outsiders. Similar conclusions can be found in Mao (1982), Merrow, McDonnell and Arguden (1988) and Miller and Lessard (2000). Also, the free cash flow and underinvestment problem arguments usually predict that project financing enhances performance. Some authors show that large projects often fail and argue that this is not consis-


tent with "pure" agency explanations (see, among others, Flyvbjerg et al. (2002) and Vilanova (2006)). Recent examples include EuroDisney and Eurotunnel which appeared to be structured to mitigate agency problems. Many other large projects have experienced financial distress (Iridium, Globalstar, Passific Crossing Cables etc.).

The present paper is based on asymmetric information between insiders and market participants. The extent to which the asymmetric information approach can be used to analyze project financing depends on many factors which are responsible for risk and uncertainty. These include technology transparency, availability of licensing documents and other contracts etc. In some industries, such as power generation, technological risks are relatively small (Chen, Kinsinger and Martin, 1989). However, large projects have many other aspects of risk including political risk, regulatory risk, country risk etc. It is noteworthy to mention, for instance, Calpine Corporation (power company) which was recently reorganized under Chapter 11 of the US Bankruptcy Code (this project is described in Esty, 2001). While a risky or uncertain environment does not represent per se a sufficient condition for asymmetric information, it definitely increases the probability of this situation and its potential extent. Flyvbjerg, et al (2002) suggests a number of policy applications including legislative and administrative control that can reduce asymmetric information problems. However, these options are not always available or efficient.

In Myers and Majluf’s (1984) pecking-order theory the underinvestment problem occurs because of asymmetric information about the value of both assets-in-place and the investment project. The authors provide an intuition that the underinvestment problem can be resolved by a spin-off project company. However, in this setting, a potential spin-off cannot resolve the adverse selection problem or the underpricing problem, which arises in the pooling equilibrium. In the present paper we focus not on the underinvestment problem (investments are always efficient in our model) but on the adverse selection problem, the resulting misvaluation of firms, and the existence of equilibria resolving or mitigating this problem. Esty (2004) noticed that Myers and Majluf’s (1984) insight cannot explain the nature of project financing or financing by non-recourse debt as compared, for instance, with standard senior debt which would have the same effect on the underinvestment problem.

\footnote{For additional evidence on project financing see, for example, Finnerty (1996).}
Shah and Thakor (1987) analyze optimal financing in the presence of corporate taxation. In their model projects have the same mean of return, the owners have private information about risk and investors may acquire (costly) information about the parameters of firms’ risks. If the benefits from information production are relatively high project financing is optimal because the cost of screening a separately incorporated project is low. Alternatively, project financing can result in higher leverage and provide greater tax benefits. This is because, under corporate financing, leverage is below the optimal level. In the absence of bankruptcy costs the first-best financing method is "pure" debt. However, firms reduce leverage in order to provide a credible signal about risk.

Note that in many cases projects are located in different states or countries, they are very long term, the corporate tax rates and their dynamics may differ across the projects, and the uncertainty surrounding the real tax advantage of project financing is large. For example, recently the Venezuelan government increased the corporate tax rate from 34 to 50%. In addition, it introduced a new oil tax. All this seriously affected the value of claims issued for the Petrozuata project (Esty, 1998). The present paper does not rely on tax considerations, transaction costs or an assumption that firms have the same mean of return. In contrast to Myers and Majluf (1984) and Shah and Thakor (1987), the firm has two projects available and must choose optimal financing for both of them. Projects’ performances are not necessarily positively correlated and the extent of asymmetry regarding different projects can be different. Among the reasons for this, note the following: projects can belong to different industries, projects can be managed by different management teams, projects can have different geographical locations including different countries, etc. We analyze the effect of different informational structures on firms’ financing decisions.

When high-profit firms have larger expected cash flows for both projects than low-profit firms a separating equilibrium does not exist: low-profit firms always mimic high-profit firms. However, when the extent of asymmetric information regarding firms’ total values is relatively small and that regarding the profile of performance across the projects is relatively large, this equilibrium may exist. The following explains the main ideas behind the

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4 This idea is related to literature with multidimensional signalling (see, for example, Chen (1997), Gertner, Gibbons and Sharfstein (1988), Grinblatt and Hwang (1989), Miglo (2007)).

5 In Miglo and Zenkevich (2006) and Miglo (2007) the order of different types of firms
separating equilibrium. First, it is well known that in a separating equilibrium each financing strategy is chosen by the worst possible type of firm for that strategy (from the investor’s viewpoint). Otherwise the firm will be mimicked by other firms which will benefit from the overvaluation of issued securities. The value of corporate claims depends on the firm’s total value and not on the profile of performance across the projects and the value of non-recourse debt relies on the expected performance of the project. If a firm with a high overall value issues corporate debt it will be mimicked because of the high value of this claim. However, if this firm issues non-recourse debt to finance the project with lower expected performance than that of other types of firms, and if the amount of investment in this project is sufficiently large, a separating equilibrium may exist. The adverse selection effect in valuing non-recourse debt may be larger than that of corporate claims issued for other projects. This may prevent firms with low overall values from mimicking. We also analyze the pooling equilibrium. We show that pooling with corporate debt minimizes mispricing if the asymmetry of information is uniformly distributed across the projects. However, if one of the projects contains less asymmetry than another the pooling equilibrium which minimizes mispricing may be issuing non-recourse debt for this project.

The model’s results are consistent with some important phenomena surrounding non-recourse debt such as the high leverage in project financing and the high risk of projects financed by non-recourse debt. In addition, the paper generates some new predictions regarding the link between the structure of asymmetric information between firms’ insiders and market participants and the choice between corporate debt and non-recourse debt. For example, we argue that: financing by non-recourse debt is more probable when the extent of asymmetric information regarding firms’ total values is small enough and that regarding performance profiles across the projects is large enough; the quality of firms issuing at least one claim without recourse is higher than that of firms issuing only corporate claims; when the asymmetry regarding firms values is large then issuing corporate claims is more probable if the asymmetry is uniformly distributed across the projects, and non-recourse debt must be issued if the asymmetry is not uniformly distributed. We discuss different strategies for testing these predictions. We also discuss the opportunities to apply the results to debtor-in-pocession claims and asset-backed securities.

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The rest of the paper is organized as follows. The next section provides a model description. Sections 3 and 4 analyze separating and pooling equilibria respectively. Section 5 discusses the model implications. The conclusion is presented in Section 6.

2 Model.

Consider a firm with two investment projects available indexed by \( i = 1, 2 \). In project \( i \) an amount \( k_i \) must be invested. Each project can be either successful or unsuccessful. There are two types of firms: for type \( g \) firms the probability of success for project \( i \) equals \( \theta_{gi} \) and for firm \( b \) it is \( \theta_{bi} \). The cash flow of type \( x = g, b \) from project \( i \) is denoted by \( c_{xi} \). In the case of success \( c_{xi} = 1 \), otherwise \( c_{xi} = 0 \). Total expected cash flow for type \( x \) over both projects is then \( v_x = \theta_{x1} + \theta_{x2} \). We assume the \( \theta \)'s are restricted to the interval \((k, 1]\), which implies that each project has a positive net present value. Let \( \rho^{0\beta}_x \) be the probability that \( c_{x1} = \alpha \) and \( c_{x2} = \beta \) and \( \rho^1_x \) be the probability that \( c_{x1} + c_{x2} = \gamma \). We have: \( \rho^{11}_x \equiv \rho^2_x = \theta_{x1}\theta_{x2}, \rho^{10}_x = \theta_{x1}(1-\theta_{x2}), \rho^{01}_x = (1-\theta_{x1})\theta_{x2}, \rho^1_x = \rho^{10}_x + \rho^{01}_x \) and \( \rho^{00}_x \equiv \rho^0_x = (1-\theta_{x1})(1-\theta_{x2}) \). We assume that the total cash flow of type \( g \) first-order dominates that of type \( b \):

\[
\rho^2_g > \rho^2_b \quad (1)
\]

\[
\rho^0_g < \rho^0_b \quad (2)
\]

(1) and (2) obviously imply that the total value of firm \( g \) is higher than firm \( b \):

\[
v_g > v_b \quad (3)
\]

The firm’s profit is observable and verifiable. There exists universal risk-neutrality and perfect competition among investors. This implies zero market profit and risk-neutral valuation for any security issued. The firm’s type is revealed to the entrepreneur in period 0 while financing and investments take place in period 1. The firm’s initial capital structure is 100% equity (which all belongs to the entrepreneur). Throughout this article, we use the concept of Perfect-Bayesian equilibria. We also use minimal mispricing criterion to refine the equilibrium when multiple pooling equilibria exist. The usage of this criterion in a game without repetition where the informed party moves first is quite common in existing literature.\(^7\)

\(^7\)See, for instance, Myers and Majluf (1984) or Nachman and Noe (1994).
2.1 Financing strategies

For each project firm \( x = g, b \) may issue debt with recourse (denote this strategy by \( d \)) and debt without recourse (\( n \)).

Strategy \( d \). The firm raises standard corporate debt (with recourse) totaling \( k_1 + k_2 \). The face value of debt is denoted by \( F_{dd} \). If \( c_{x1} = c_{x2} = 1 \), the creditors are paid in full. If \( c_{x1} = 0 \) and \( c_{x2} = 1 \) or when \( c_{x1} = 1 \) and \( c_{x2} = 0 \), two situations are possible. If \( F_{dd} < 1 \) the creditors are paid in full. Otherwise they get \( 1 \).^8

Strategy \( dn \). For the first project the firm issues debt with recourse with face value \( F_{dn}^1 \) and for the second project the firm issues non-recourse debt with face value \( F_{dn}^2 \). If \( c_{x1} = 1 \) the creditors are paid in full. If \( c_{x1} = 0 \) and \( c_{x2} = 1 \) two situations are possible. If \( 1 - F_{dn}^2 \geq F_{dn}^1 \) the creditors of project 1 are paid in full. Otherwise they get \( 1 - F_{dn}^2 \). The second project creditors are paid in full if \( c_{x2} = 1 \) and get nothing otherwise. The firm can also use strategy \( nd \). This is similar to \( dn \) except that in this case non-recourse debt will be used for the first project.

Strategy \( nn \). For the first project the firm issues non-recourse debt with face value \( F_{nn}^1 \) and for the second project the firm issues non-recourse debt with face value \( F_{nn}^2 \). If \( c_{xi} = 1 \) the creditors of project \( i \) are paid in full. Otherwise they get nothing.

While we have chosen standard debt with recourse to model corporate financing, the model’s results can also be interpreted in terms of other forms of corporate financing (equity for instance). The first project under strategy \( dn \) can be seen as the parent company (financed totally by equity) and the second project is one financed by non-recourse debt. All major features of project financing and corporate financing (in terms of payoffs under different scenarios) remain the same as in the model. Strategy \( dd \) can be seen as one project being financed by non-recourse debt and the parent company is financed by non-secure corporate debt (such as the case when the parent company defaults but the project company is successful, the creditors of the

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^8Since both projects are financed by standard corporate debt there is no need to distinguish between creditors whose funds are used for financing project 1 or project 2. The payoff to the creditors only depends on the firm’s total performance and not on the performance of a particular project. This can be interpreted as all creditors having claims with the same seniority. An alternative way of modelling this strategy is to assume that the projects are financed by different creditors which have different priority. We omitted this possibility for brevity given that the seniority issue is not the focus of the paper and it does not affect the main results.
parent company have no recourse to profits from the second project).

3 Signalling with different kinds of debt.

Let $V_{st}^j$ be the expected payoff to the entrepreneur of type $s$ if the strategy $j, j \in \{dd, dn, nd, nn\}$ is played and the type is perceived by the market as type $t, s, t \in \{g, b\}$. A separating equilibrium is a situation where type $g$ plays strategy $j_1$, type $b$ plays strategy $j_2$ and no type has an incentive to mimic the other type:

$$V_{gb}^{j_2} \leq V_{gg}^{j_1}$$

$$V_{bg}^{j_1} \leq V_{bb}^{j_2}$$

Thus, it is clear that the analysis of the $V_{st}^j$ function is crucial. The value of $V_{st}^j$ depends on the performance of type $s$ and the prices of issued securities. Denote the prices of securities issued by type $x$ under symmetric information with a subscript $x$. For instance, $F_{x_1}^{nn}$ denotes the face value of non-recourse debt for the first project when strategy $nn$ is played. Consider strategy $dd$. We have: if $1 < F_{t_1}^{dd}$, then

$$V_{st}^{dd} = \rho_s^2(2 - F_{t_1}^{dd})$$

and otherwise

$$V_{st}^{dd} = \rho_s^2(2 - F_{t_1}^{dd}) + \rho_s^1(1 - F_{t_1}^{dd})$$

Let us turn to strategy $dn$. If $1 \leq F_{t_1}^{dn} + F_{t_2}^{dn}$ and $1 \leq F_{t_1}^{dn}$ then

$$V_{st}^{dn} = \rho_s^2(2 - F_{t_1}^{dn} - F_{t_2}^{dn})$$

If $1 \leq F_{t_1}^{dn} + F_{t_2}^{dn}$ and $1 > F_{t_1}^{dn}$ then

$$V_{st}^{dn} = \rho_s^2(2 - F_{t_1}^{dn} - F_{t_2}^{dn}) + \rho_s^1(1 - F_{t_1}^{dn})$$

and if $1 > F_{t_1}^{dn} + F_{t_2}^{dn}$

$$V_{st}^{dn} = \rho_s^2(2 - F_{t_1}^{dn} - F_{t_2}^{dn}) + \rho_s^1(1 - F_{t_1}^{dn}) + \rho_s^0(1 - F_{t_1}^{dn} - F_{t_2}^{dn})$$

For strategy $nn$ we get the following.

$$V_{st}^{nn} = \theta_{s_1}(1 - F_{t_1}^{nn}) + \theta_{s_2}(1 - F_{t_2}^{nn})$$
The following lemma determines the prices of issued securities under symmetric information that are necessary for the analysis of the $V_{st}^j$ function.

**Lemma 1.** If information is symmetric then, for type $x = g, b$:

\[
F_{dd}^x = \begin{cases} 
\frac{k_1 + k_2 - \rho_x^2}{1 - \rho_x^2}, & k_1 + k_2 \geq 1 - \rho_x^0 \\
\frac{k_1 + k_2}{1 - \rho_x^0}, & k_1 + k_2 < 1 - \rho_x^0
\end{cases} \tag{12}
\]

\[
F_{dn}^x = \begin{cases} 
\frac{k_1}{1 - \rho_x^0}, & \frac{k_1}{1 - \rho_x^0} + \frac{k_2}{\theta_x^0} \leq 1 \\
\frac{k_1}{\theta_x^0} + \frac{k_2}{\theta_x^0}, & \frac{k_1}{1 - \rho_x^0} + \frac{k_2}{\theta_x^0} > 1
\end{cases} \tag{13}
\]

\[
F_{x2}^d \equiv F_{x2}^n = k_1 / \theta_{x1} \tag{14}
\]

\[
F_{x2}^n = k_2 / \theta_{x2} \tag{15}
\]

(Proofs of all lemmas and propositions are collected in the Appendix. Also note that $F_{x2}^{nd}$ and $F_{x2}^{nd}$ are omitted for brevity - their formulas mirror those for $F_{x2}^{dn}$ and $F_{x2}^{dn}$ respectively by substituting the parameters of the second project with those of the first one and vice versa.)

As one can see from Lemma 1 the values of different securities depend in different ways on the firm’s expected performance in each period. Since each type performs differently in each project the value of securities issued by different types are different. To avoid mimicking, firms will issue securities which have a lower value for investors than if they were issued by the other type. In this sense the following remarks about Lemma 1 are useful.

From (14) and (15) the non-recourse debt face value is positively linked to the amount of financing and negatively related to the expected performance of the project for which the debt is issued. If corporate debt is used for both projects, the payoff to the debtholders depends only on the total earnings $c_1 + c_2$ and thus the value of debt depends only on the probabilities attributed to the firm’s total earnings (equ. (12)). If strategy $dn$ or $nd$ is used the value of debt with recourse relies on the cross-probabilities of default in both projects or that of success in both projects.

It follows from Lemma 1 and the definition of $V_{st}^j$ that $V_{ss}^j = \theta_{k1} + \theta_{k2} - k_1 - k_2, j \in \{dd, dn, nd, nn\}, s \in \{b, g\}$. The right side shows the expected payoff of type $s$ under symmetric information: it equals the total expected cash flow minus the costs of investment which is not surprising in this Modigliani-Miller environment. This can be proven by substituting the prices of securities under symmetric information into the expressions for $V_{st}^j$. 

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Lemma 2. If \( \theta_{g1} \geq \theta_{b1} \) and \( \theta_{g2} \geq \theta_{b2} \) a separating equilibrium does not exist.

It follows from Lemma 2 and Corollary 1 that if one type has higher performance in both periods than the other, a separating equilibrium does not exist: the type with higher value will always be mimicked. Thus, we consider the case where type \( b \) has higher profitability in one of the projects. Without loss of generality we assume:

\[
\theta_{g1} > \theta_{b1}; \theta_{g2} < \theta_{b2}
\]  \hfill (16)

Proposition 1. A separating equilibrium where \( g \) plays \( dd \) does not exist.

Proposition 1 is based on Lemma 1 and (1) and (2). An explanation for this result is as follows. Since the corporate debt represents a monotone claim on the firm’s total cash flow and since the total cash flow of type \( g \) first-order dominates that of type \( b \), a separating equilibrium is impossible (Brennan and Kraus, 1987).

From (16) type \( g \) appears to have a "lemon" advantage with regard to the second project: lower profits in this project mean that this type of firm can capitalize on the adverse selection problem. On the other hand, in the first project, the "lemon" advantage belongs to type \( b \). Generally speaking, for \( g \) to separate from \( b \), \( g \) must issue claims with a value which depends heavily on the second-project expected performance where \( g \) is weak. When asymmetric information regarding firms’ total values is large, a separating equilibrium does not exist: the type with a low value mimics that with a high value. When asymmetric information regarding firms’ total values is relatively low and that regarding the earnings profiles is relatively high, a separation may exist. In this case the types can be separated by issuing claims on the cash flow without cross pledging (at least for one project).

Proposition 2. A separating equilibrium where \( g \) plays \( nd \) does not exist.

Proposition 2 is based on Lemma 1. An explanation for this result is as follows. Since the price of corporate debt depends on the value of the firm, and not just second-project-performance, \( g \) cannot benefit from its “lemon” advantage in the second project. Moreover, \( g \) will lose in the first project because of \( b \)’s “lemon” advantage.

Proposition 3. A separating equilibrium where \( g \) plays \( nn \) exists if and only if \( \frac{k_1(1 - \frac{\theta_{b1}}{\theta_{g1}})}{k_2(\frac{\theta_{b2}}{\theta_{g2}} - 1)} \).

Consider the interpretation of Proposition 3. From (16) \( \theta_{g2} < \theta_{b2} \) and \( \theta_{g1} > \theta_{b1} \). Thus the condition \( k_1(1 - \frac{\theta_{b1}}{\theta_{g1}}) \leq k_2(\frac{\theta_{b2}}{\theta_{g2}} - 1) \) holds if: 1) \( \theta_{b2} \) is
sufficiently greater than \( \theta_{g2} \) and \( k_2 \) is sufficiently large and/or 2) when \( \theta_{b1} \) is sufficiently close to \( \theta_{g1} \) and \( k_1 \) is sufficiently small. It assures that an adverse selection problem concerning the second project (large amount of investments and uncertainty about the project’s performance) where type \( g \) has "lemon" advantage is more important than that concerning the first project. This makes mimicking type \( g \) unattractive to type \( b \).

Now consider the separating equilibrium where \( g \) plays \( d_n \). The intuition here is similar to that of Proposition 2.

**Proposition 4.** A separating equilibrium where \( g \) plays \( d_n \) exists if one of the following holds: 1) \( \frac{k_1}{1-\rho_g^0} + \frac{k_2}{\theta_{g2}} \leq 1 \) and \( k_1(1 - \frac{1-\rho_g^0}{1-\rho_g}) \leq k_2(\frac{\theta_{g2}}{\theta_{g2}} - 1) \); 2) \( \frac{k_1}{1-\rho_g^0} + \frac{k_2}{\theta_{g2}} > 1 \) and \( k_1(1 - \frac{\theta_{g1}}{\theta_{b1}}) \leq k_2(\frac{\theta_{g1}(\theta_{g2}e^{0} + \theta_{b2})}{\theta_{g1}e^{0}} - 1) + \frac{\theta_{g2} - \theta_{g1} - \theta_{b1}(\theta_{g2}e^{0} + \theta_{b2})}{\theta_{g1}e^{0}} \).

The interpretation of condition \( k_1(1 - \frac{1-\rho_g^0}{1-\rho_g}) \leq k_2(\frac{\theta_{g2}}{\theta_{g2}} - 1) \) is as follows. If \( \frac{k_1}{1-\rho_g^0} + \frac{k_2}{\theta_{g2}} \leq 1 \) then \( 1 > F_{d_n} + F_{d_2} \) (see the proof of Lemma 1). Thus \( 1 - \rho_g^0 \) shows the probability of solvency for type \( g \). Also from (16) \( \theta_{g2} < \theta_{b2} \) and from (2) \( 1 - \rho_g^0 > 1 - \rho_b^0 \). Thus the condition \( 2 < \frac{\theta_{b2}}{\theta_{g2}} + \frac{1-\rho_g^0}{1-\rho_g^0} \) holds if \( \theta_{b2} \) is sufficiently greater than \( \theta_{g2} \) and/or when \( 1 - \rho_b^0 \) is sufficiently close to \( 1 - \rho_g^0 \) and/or \( k_1 \) is sufficiently small. This means that a separating equilibrium exists when the asymmetry regarding the firm’s credit rating (probability of default in both projects) is sufficiently small while the asymmetry regarding the project (assets), for which the firm issues non-recourse debt, is large and/or when the amount of investment in project 2 (financed by a non-recourse debt) is sufficiently greater than that in project 1. The spirit of second part of Proposition 4 is similar. The asymmetry regarding the first project and the amount of investment in that project must be sufficiently low in order for a separating equilibrium to exist. This is because type \( b \) has "lemon" advantage in the first project.

Additional insights can be discovered if one consider the case \( k_1 = k_2 = k \). In this case the extent of asymmetric information regarding different projects can be easily compared using \( \theta \). Let \( r_x = \theta_{x1}/\theta_{x2} \). This ratio can be used to compare the profiles of project profitability. From (16) we have

\[
r_g > r_b
\]

(17)

The greater the difference between the firms’ \( r_x \), the greater the difference between firms’ profitability profiles across the projects. The firm’s perfor-
mance can be described by a pair \((v_x, r_x)\). The probabilities of success in each project are then:

\[
\theta_{x1} = \frac{v_x r_x}{1 + r_x} ; \theta_{x2} = \frac{v_x}{1 + r_x}
\]  

(18)

For our purposes it is suitable to present a set of exogenous parameters describing the model as \((v_g, r_g, v_b, r_b, k)\).

**Corollary 1.** If \(r_g = r_b\) a separating equilibrium does not exist.

Corollary 1 follows immediately from Lemma 2. It means that if firms have the same profiles of performance across the projects and differ only in their total values a separating equilibrium does not exist.

Consider the interpretation of Proposition 3. Two ideas underline the analysis below. First when the difference between firms’ total values is large enough a separating equilibrium does not exist because the type with a low total value will mimic the high value type. Secondly, a large difference in the firms’ rates of earnings growth contributes to the existence of a separating equilibrium by making it possible for \(g\) to design debt claims which will not be mimicked by \(b\). To see this let us rewrite the condition in Proposition 3 as follows:

\[
\frac{v_b(1 + r_g)(r_g + r_b)}{v_g(1 + r_b)r_g} \geq 2
\]  

(19)

**Corollary 2.** A separating equilibrium exists if and only if the following holds: 1) \(v_g\) is sufficiently small (other parameters being equal); 2) \(v_b\) is sufficiently large; 3) \(r_g\) is sufficiently large; 4) \(r_b\) is sufficiently small.

Corollary 2 follows directly from (19) by analyzing the partial derivatives of left side.

Figure 1 illustrates Propositions 3 and 4 and Corollary 1 and 2. Here \(v_g = 1.5, r_g = 1.6, \theta_{g1} = 0.96, \theta_{g2} = 0.64\) and \(k_1 = k_2 = 0.2\). The figure shows the values of \(r_b\) and \(v_b\) for which separating equilibriums may exist. In the space below both thick lines \((A)\) both separating equilibria (one where \(g\) plays \(nn\) and one where \(g\) plays \(dn\)) exist. In \(B\) only the separating equilibrium where \(g\) plays \(dn\) exists. Note that for any value of \(v_b\) a separating equilibrium exists if \(r_b\) is low enough and for any \(r_b\) a separating equilibrium exists if \(v_b\) is high enough. In other words a separating equilibrium exists if asymmetric information about rate of earnings growth is more important than that concerning the firms’ total values. Also note that a separating equilibrium does not exist when \(r_b = r_g = 1.5\) for any value of \(v_b\) as was discussed previously.
4 Pooling equilibria.

Let $V^j_{s\mu}$ be the expected payoff to the entrepreneur of type $s$ if strategy $j, j \in \{dd, dn, nd, nn\}$ is played and the market perceives the type as $g$ with probability $\mu$ and respectively as type $b$ with probability $1 - \mu$. A pooling equilibrium is a situation where both types play strategy $j_1$, off-equilibrium beliefs about observing strategy $j_2$ are that the type is $g$ with probability $\mu(j_2)$ and for each type $i \in b, g$

$$V^j_{i\mu(j_2)} \leq V^j_{i\mu}$$ (20)

Consider the $V^j_{s\mu}$ function. Denote the prices of securities when the proportion of type $g$ firms is $\mu$ with a subscript $\mu$. For instance, $F_{\mu1}^{nn}$ denotes the face value of non-recourse debt for the first project when the equilibrium is pooling with $nn$. Note that the case when $\mu = 1$ corresponds to the symmetric information prices for type $g$ from the previous section and $\mu = 0$ corresponds to the symmetric information prices for type $b$. Consider strategy $dd$. If $1 < F_{\mu}^{dd}$, then

$$V^dd_{s\mu} = \rho_s^2(2 - F_{\mu}^{dd})$$ (21)

and otherwise

$$V^dd_{s\mu} = \rho_s^2(2 - F_{\mu}^{dd}) + \rho_s^1(1 - F_{\mu}^{dd})$$ (22)

Let us turn to strategy $dn$. If $1 < F_{\mu1}^{dn} + F_{\mu2}^{dn}$ and $1 < F_{\mu1}^{dn}$ then

$$V^{dn}_{s\mu} = \rho_s^2(2 - F_{\mu1}^{dn} - F_{\mu2}^{dn})$$ (23)
If $1 < F_{\mu_1}^{dn} + F_{\mu_2}^{dn}$ and $1 > F_{\mu_1}^{dn}$ then

$$V_{\mu_1}^{dn} = \rho_s^2 (2 - F_{\mu_1}^{dn} - F_{\mu_2}^{dn}) + \rho_s^{10} (1 - F_{\mu_1}^{dn})$$

and If $1 > F_{\mu_1}^{dn} + F_{\mu_2}^{dn}$

$$V_{\mu_1}^{dn} = \rho_s^2 (2 - F_{\mu_1}^{dn} - F_{\mu_2}^{dn}) + \rho_s^{10} (1 - F_{\mu_1}^{dn}) + \rho_s^{01} (1 - F_{\mu_1}^{dn} - F_{\mu_2}^{dn})$$

For strategy $nn$ we get the following,

$$V_{nn}^{dn} = \theta_{s1} (1 - F_{\mu_1}^{nn}) + \theta_{s2} (1 - F_{\mu_2}^{nn})$$

The following lemma determines the prices of issued securities that are necessary for the analysis of the $V_{\mu}^{dn}$ function. The bar will mean the average value of the parameter. For example, $\bar{\theta}_1 = \mu \theta_{g1} + (1 - \mu) \theta_{b1}, \bar{\rho} = \mu \bar{\rho}_g + (1 - \mu) \bar{\rho}_b$.

**Lemma 4.**

$$F_{\mu}^{dd} = \begin{cases} \frac{k_1 + k_2 - \rho \bar{\rho}}{\rho \bar{\rho}}, & k_1 + k_2 \geq 1 - \bar{\rho} \\ \frac{k_1 + k_2}{1 - \rho \bar{\rho}}, & k_1 + k_2 < 1 - \bar{\rho} \end{cases}$$

$$F_{\mu_1}^{dn} = \begin{cases} \frac{k_1 + k_2}{1 - \rho \bar{\rho}}, & \frac{k_1 + k_2}{\bar{\theta}_1} + \frac{k_2}{\bar{\theta}_2} < 1 \\ \frac{k_1 + k_2}{1 - \rho \bar{\rho}} - \frac{k_1 + k_2}{\bar{\theta}_1} + \frac{k_2}{\bar{\theta}_2} \geq 1 \end{cases}$$

$$F_{\mu_2}^{dn} = F_{\mu_2}^{nn} = k_2 / \bar{\theta}_2$$

$$F_{\mu_1}^{nn} = k_1 / \bar{\theta}_1$$

As one can see, Lemma 1 and Lemma 4 have a lot in common.

From (29) and (30) the face value of non-recourse debt is positively linked to the amount of financing and negatively related to the expected performance of the project. The value of debt with recourse also relies on the cross-probabilities of default in both projects or that of success in both projects.

From Lemma 1 any claim issued by type $g$ has a higher value than those issued by type $b$. Therefore, in a pooling equilibrium type $g$ is underpriced. Thus, we will look for a pooling equilibrium which minimizes mispricing for type $g$. Let us consider the case $k_1 + k_2 < 1 - \bar{\rho}, \frac{k_1 + k_2}{\bar{\theta}_1} + \frac{k_1}{\bar{\theta}_1} < 1$ and
$\frac{k_1+k_2}{1-\theta^*} + \frac{k_2}{\theta_2} < 1$. Other cases are omitted for brevity. They do not change the main results (the proof is available upon request).  

**Proposition 5.** Pooling with $dd$ minimizes mispricing if and only if  
\[ \frac{\theta_{g1}}{\theta_{b1}} < \frac{\theta_{g2} (1-\theta_{b2})}{\theta_{b2} (1-\theta_{g2})} \quad \text{and} \quad \frac{\theta_{g2}}{\theta_{b2}} < \frac{\theta_{g1} (1-\theta_{b1})}{\theta_{b1} (1-\theta_{g1})}. \]

An explanation for this result is as follows. Since the price of corporate debt depends on the firm’s overall performance, and not just the performance in one project (as in the price of non-recourse debt), it does not make sense to issue non-recourse debt if asymmetry is uniform: corporate debt will better. This is the case when the values of $\theta_{g1}/\theta_{b1}$ and $\theta_{g2}/\theta_{b2}$ are close. However, if the amount of asymmetry in one project is smaller than that regarding the second project, issuing non-recourse debt for this project is beneficial. In the extreme case, when private information is one-dimensional, the equilibrium is pooling where the project with known profitability is financed by non-recourse debt and the second project is financed by corporate debt: by pledging earnings from the project with known profitability the firm minimizes adverse selection problems in financing the second project.

![Figure 2. Pooling equilibria with corporate debt/non-recourse debt.](image)

Figure 2 illustrates Proposition 5. Here $\mu = 0.5$, $\theta_{b2} = 0.5$, $\theta_{g2} = 0.64$ and $k_1 = k_2 = 0.2$. The figure shows the values of $\theta_{b1}$ and $\theta_{g1}$ for which a pooling equilibrium with corporate debt minimizes mispricing: this is the

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9 Also we have verified that off-equilibrium beliefs survive Cho and Kreps’ (1987) intuitive criterion.
space between thick lines \((B)\). In this space the extent of asymmetry concerning the first project \((\theta_{y1}/\theta_{b1})\) is sufficiently close to that in the second project \((\theta_{y2}/\theta_{b2})\). In spaces \(A\) and \(C\) a pooling equilibrium where at least one project is financed by non-recourse debt minimizes mispricing.

5 Implications

The analysis in this paper reveals some insights about how different structures of asymmetric information (regarding firms’ earnings potentials) between insiders and market participants affects firms’ incentives to issue non-recourse debt versus standard debt with recourse.

(i) If the extent of asymmetric information regarding firms’ total values is small enough (compared to the extent of asymmetric information regarding performance profiles across the projects) a separating equilibrium may exist. Financing both projects with standard debt with recourse will never allow the good type to signal its quality. However, a separating equilibrium may exist where the good type issues non-recourse debt (for at least one project). Proposition 4 describes separating equilibria with the most frequently observed financial structure involving project financing: one project (parent company) is financed by corporate claims and one project is financed by non-recourse debt. As follows from Proposition 4, the existence of this equilibrium is probable when the amount of investment financed by non-recourse debt is sufficiently large with regard to corporate investment. The results of Proposition 3 are similar. Also, as follows from Propositions 3 and 4, the uncertainty regarding the performance of projects financed by non-recourse debt is greater than that of projects financed by corporate debt. While a complete test of the separating equilibria and implications of these equilibria must be based on identifying firms with low asymmetry regarding total firm value and high asymmetry regarding performance profiles across the projects there exists some evidence consistent with the spirit of the above predictions. Brealey, Cooper and Habib (1996), Esty (2003, 2004), McGuinty (1981), and Nevitt (1979) argue that non-recourse debt is typically used for financing large, capital-intensive, projects and that the leverage ratio of project companies is typically larger than that of parent companies. Also, this is consistent with the evidence that project financing is usually used for financing risky projects (see for instance: Esty (2002, 2004), Flybjerg et al. (2003), McGuinty (1981), Merrow et al. (1988), Miller and Lezard (2000), and Nevitt
(119).

(ii) Since, in a separating equilibrium, non-recourse debt is issued by the good type (while the financing policy of the bad type is irrelevant) this equilibrium implies that the quality of firms issuing at least one claim without recourse is higher than that of firms issuing only corporate claims. The same prediction can be found in John and John (1991). There also exists literature that analyzes debtor-in-possession financing (DIP) which has a lot in common with project financing. For example, Dahiya, John, Puri and Ramirez (2003) show that firms which emerge from bankruptcy and which use DIP financing have higher quality than firms which do not use DIP financing.

(iii) Several empirical predictions which have not been tested in existing literature follow from Propositions 3, 4 and 5. The model predicts that: 1) if the extent of asymmetric information regarding firms’ total values is high enough, a separating equilibrium does not exist, and 2) if the asymmetry is uniformly distributed across the projects, the pooling equilibrium which minimizes mispricing for the good type is one where both types issue corporate debt for both projects. However, if the asymmetry is not uniformly distributed, the equilibrium which minimizes mispricing is one where both types issue non-recourse debt for at least one project. This is implied by Proposition 5. If, on the other hand, the degree of asymmetric information is uniform across the projects then there is no need for project financing.

The existence of separating equilibria and the issuance of non-recourse debt should more frequently be observed when asymmetric information regarding the profile of earnings across the projects is larger than that regarding the total cash flows (Corollary 2). Possible tests of this prediction will be based on the following. One can use the spread in analysts’ valuations of firms’ shares as a proxy for the extent of asymmetric information regarding the firms’ total values. Also, firms investing in different industries or in different countries can be seen as ones with a high degree of asymmetric information regarding the profile of earnings across the projects. The same holds for firms manipulating earnings since earnings management can often be seen as a redistribution of earnings (between periods and projects) rather than accounting fraud (small effect on the extent of asymmetric information regarding firms’ total values).

Note that Schipper and Smith (1983) found that 72 out of 93 firms in their sample of spin-offs involved parent companies and subsidiaries with different industry membership (cross-industry spin-offs). Strategy nn and dn can be interpreted as involving a spin-off because they contain financing by
non-recourse debt and the creation of an independent company respectively. Thus, Schipper and Smith’s (1983) results are consistent with the spirit of the present paper (points (i) and (iii)). In addition to point (ii) note Daley, Mehrotra and Sivakumar (1997). They found that value creation occurs in cross-industry spin-offs due to the parent company "taking out the trash" by separating poor performing units. This is consistent with our separating equilibria where a firm with high overall expected performance creates a stand alone company for at least one of the projects and this project has high risk.

The results of the paper can also be applied to asset-backed securities (ABS). Suppose that the firm can issue ABS to finance the first project. If the project fails then the creditors (or the holders of ABS) do not have any legal rights of recourse to the assets of the firm. In addition, there is a bankruptcy remoteness condition. If the parent company fails it cannot use the assets of the project company. Therefore, formally this debt is analogous to the case of non-recourse debt issued for both projects in the model. ABS are now used by many corporations as a financing method. The standard explanation in existing literature is that these securities exist primarily for regulatory reasons (for instance, banks were trying to avoid minimal capital requirements). However, recent empirical literature (Calomaris and Mason, 2004) argues that securitization seems to be motivated more by reasons related to efficient contracting.

6 Conclusion

This paper has analyzed the choice between project financing (with non-recourse debt) and corporate financing in situations where corporate insiders have private information about the qualities of their firms’ investment projects. The paper explains how asymmetric information can affect firms’ financing policies. The model’s results are consistent with some important phenomena surrounding non-recourse debt such as the high leverage in project financing and the high risk of projects financed by non-recourse debt. Also, the model predicts that; financing by non-recourse debt is more probable when the extent of asymmetric information regarding firms’ total values is small enough and that regarding performance profiles across the projects is large enough; the quality of firms issuing at least one claim without recourse is higher than that of firms issuing only corporate claims; when
the asymmetry regarding firms values is large then issuing corporate claims is more probable if the asymmetry is uniformly distributed across the projects, and non-recourse debt must be issued if the asymmetry is not uniformly distributed.

Appendix

Proof of Lemma 1. Consider strategy $nn$ played by type $x, x \in g, b$. For project $i$ we have the following equation:

$$k_i = E_{c_{xi}} \min\{c_{xi}, F_{xi}^{nn}\} \tag{31}$$

which produces:

$$F_{xi}^{nn} = k_i / \theta_{xi} \tag{32}$$

Now consider strategy $dn$. The following equations determine the prices of issued securities:

1) market valuation of debt for financing the first project:

$$k_1 = E_{c_{x1}} [\min\{d_{x1} + \max\{c_{x2} - F_{x2}^{dn}, 0\}, F_{x1}^{dn}\}] \tag{33}$$

Equation (33) takes into account that if the cash flow from the first-project is not sufficient to pay short-term debt, the creditors have the right to seize the earnings from the second project.

2) market valuation of debt for financing the second project:

$$k_2 = E_{c_{x2}} \min\{c_{x2}, F_{x2}^{dn}\} \tag{34}$$

which implies:

$$F_{x2}^{dn} = k_2 / \theta_{x2} \tag{35}$$

Consider the case

$$1 > F_{x1}^{dn} + F_{x2}^{dn} \tag{36}$$

Equation (33) can be written as

$$k_1 = (\theta_{x1} \theta_{x2} + \theta_{x1}(1 - \theta_{x2}) + \theta_{x2}(1 - \theta_{x1})) F_{x1}^{dn} \tag{37}$$

Now consider the case

$$F_{x1}^{dn} < 1 \leq F_{x1}^{dn} + F_{x2}^{dn} \tag{38}$$
Equation (33) can be written as

\[ k_1 = \theta_{x1}F_{x1}^{dn} + \theta_{x2}(1 - \theta_{x1})(1 - F_{x2}^{dn}) \]  

(39)

From (37) and (39) we get

\[ F_{x1}^{dn} = \frac{k_1 - (\theta_{x2} - k_2)(1 - \theta_{x1})}{\theta_{x1}} \]

Finally consider the case

\[ F_{x1}^{dn} \geq 1 \]  

(40)

Equation (33) can be written as

\[ k_1 = \theta_{x1}\theta_{x2}F_{x1}^{dn} + \theta_{x2}(1 - \theta_{x1})(1 - F_{x2}^{dn}) + \theta_{x1}(1 - \theta_{x2}) \]  

(41)

From (37) and (41) we have:

\[ F_{x1}^{dn} = \frac{k_1 - (\theta_{x2} - k_2)(1 - \theta_{x1}) - \theta_{x1}(1 - \theta_{x2})}{\theta_{x1}\theta_{x2}} \]

Consider \( F_{x1}^{dn} - 1 \). It equals \( \frac{k_1 - (\theta_{x2} - k_2)(1 - \theta_{x1}) - \theta_{x1}}{\theta_{x1}\theta_{x2}} < 0 \). This contradicts (40). Therefore the case \( F_{x1}^{dn} \geq 1 \) is never possible.

Strategy \( dd \). We have:

1) value of debt:

\[ k_1 + k_2 = E_{C_{x1},C_{x2}} \min\{C_{x1} + C_{x2}, F_{x}^{dd}\} \]  

(42)

First consider the case

\[ 1 > F_{x}^{dd} \]  

(43)

Equation (42) can be written as

\[ k_1 + k_2 = (\theta_{x1}\theta_{x2} + \theta_{x1}(1 - \theta_{x2}) + \theta_{x2}(1 - \theta_{x1}))F_{x}^{dd} \]  

(44)

Now consider the case

\[ 1 \leq F_{x}^{dd} \]  

(45)

Equation (42) can be written as

\[ k_1 + k_2 = \theta_{x1}\theta_{x2}F_{x}^{dd} + \theta_{x1}(1 - \theta_{x2}) + \theta_{x2}(1 - \theta_{x1}) \]  

(46)
Proof of Lemma 2. If \( \theta_{g1} \geq \theta_{b1} \) and \( \theta_{g2} \geq \theta_{b2} \) then by Lemma 1, any claim issued by \( g \) has a higher value than that of type \( b \). This means that \( b \) will always mimic \( g \) (if they play a different strategy) and a separating equilibrium does not exist. \textit{End proof.}

Proof of Proposition 1. Suppose the opposite is true and such an equilibrium exists. Consider the case \( k_1 + k_2 \geq 1 - \rho^0_g \). If \( b \) mimics \( g \) then its payoff equals \( V^d b = \theta_{b1} \theta_{b2} (2 - F^d g) = \frac{\theta_{b1} \theta_{b2} (\theta_{g1} + \theta_{g2} - k_1 - k_2)}{\theta_{b1} \theta_{b2}} \). Thus, \( V^d b - (\theta_{b1} + \theta_{b2} - k_1 - k_2) = \theta_{b1} \theta_{b2} (\frac{\theta_{g1} + \theta_{g2} - k_1 - k_2}{\theta_{b1} \theta_{b2}} - \frac{\theta_{b1} + \theta_{b2} - k_1 - k_2}{\theta_{b1} \theta_{b2}}) > 0 \). The latter follows from (1) and (2). It is analogous for the case \( k_1 + k_2 < 1 - \rho^0_g \). \textit{End proof.}

Proof of Proposition 2. Suppose the opposite is true and such an equilibrium exists. Consider the case \( k_1 + k_2 \geq 1 - \rho^0_g \). If \( b \) mimics \( g \) then its payoff equals \( V^d b_g = \theta_{b1} \theta_{b2} (2 - F^d g) = \frac{\theta_{b1} \theta_{b2} (\theta_{g1} + \theta_{g2} - k_1 - k_2)}{\theta_{b1} \theta_{b2}} \). Thus, \( V^d b_g - (\theta_{b1} + \theta_{b2} - k_1 - k_2) = \theta_{b1} \theta_{b2} (\frac{\theta_{g1} + \theta_{g2} - k_1 - k_2}{\theta_{b1} \theta_{b2}} - \frac{\theta_{b1} + \theta_{b2} - k_1 - k_2}{\theta_{b1} \theta_{b2}}) > 0 \). The latter follows from (1) and (2). It is analogous for the case \( k_1 + k_2 < 1 - \rho^0_g \). \textit{End proof.}

Proof of Proposition 3. A separating equilibrium exists if and only if \( \theta_{b1} (1 - F^d g) + \theta_{b2} (1 - F^d g) \leq \theta_{b1} + \theta_{b2} - k_1 - k_2 \). By Lemma 1, this can be rewritten as \( k_1 (1 - \frac{\theta_{b1}}{\theta_{g1}}) \leq k_2 (\frac{\theta_{g2}}{\theta_{g2}} - 1) \). \textit{End proof.}

Proof of Proposition 4. Condition (4) always holds. Consider the case \( \frac{k_1}{1 - \rho^0_g} + \frac{k_2}{\theta_{g2}} \leq 1 \). We have \( V^d b_g = \theta_{b1} \theta_{b2} (2 - F^d g) + \theta_{b1} (1 - \theta_{b2}) (1 - F^d g) \). Therefore, condition (5) can be rewritten as: \( F^d g (1 - \rho^0_b) + F^d g \theta_{b2} \geq k_1 + k_2 \)

Now consider the case \( \frac{k_1}{1 - \rho^0_g} + \frac{k_2}{\theta_{g2}} > 1 \) and \( \frac{k_1 - (\theta_{g2} - k_2)(1 - \theta_{g1})}{\theta_{g1}} \leq 1 \). \( V^d b_g = \theta_{b1} \theta_{b2} (2 - F^d g - F^d g) + \theta_{b1} (1 - \theta_{b2}) (1 - F^d g) \). Therefore, condition (5) can be rewritten as:

\[
\begin{align*}
  k_1 (1 - \frac{\theta_{b1}}{\theta_{g1}}) + k_2 (1 - \frac{\theta_{b1} \theta_{b2}}{\theta_{g2}} - \frac{\theta_{b1} (1 - \theta_{g1})}{\theta_{g1}}) &\leq \theta_{b2} - \theta_{b1} \theta_{b2} - \frac{\theta_{g2} \theta_{b1} (1 - \theta_{g1})}{\theta_{g1}} \\
\end{align*}
\]

\textit{End proof.}

Proof of Proposition 5. Compare \( V^{dd} g, V^{dd} g, V^{nd} g, \) and \( V^{nn} g \). Compare \( V^{dd} g \) and \( V^{dn} g \). We have: \( V^{dd} g = \theta_{g1} \theta_{g2} (2 - F^{dd} g) + (\theta_{g1} (1 - \theta_{g2}) + (1 - \theta_{g1}) \theta_{g2}) (1 - F^{dd} g) \).
and \( V_{g\mu}^{dn} = \theta_{g1}\theta_{g2}(2 - F_{\mu1}^{dn} - F_{\mu2}^{dn}) + \theta_{g1}(1 - \theta_{g2})(1 - F_{\mu1}^{dn}) + (1 - \theta_{g1})\theta_{g2}(1 - F_{\mu1}^{dn} - F_{\mu2}^{dn}) \). Using Lemma 5 we get: \( V_{g\mu}^{dd} - V_{g\mu}^{dn} = \frac{k_1(\theta_{g2}\theta_{b1}(1 - \theta_{b2}) - \theta_{g1}\theta_{b2}(1 - \theta_{b2}))}{\theta_{g2}(1 - \rho^2)} \). So \( V_{g\mu}^{dd} > V_{g\mu}^{dn} \) if and only if

\[ \frac{\theta_{g1}}{\theta_{b1}} < \frac{\theta_{g2}}{\theta_{b2}} \frac{(1 - \theta_{b2})}{(1 - \theta_{g2})} \]  

(47)

Comparing \( V_{g\mu}^{dd} \) and \( V_{g\mu}^{nd} \) we analogously get that \( V_{g\mu}^{dd} > V_{g\mu}^{nd} \) if and only if

\[ \frac{\theta_{g2}}{\theta_{b2}} < \frac{\theta_{g1}}{\theta_{b1}} \frac{(1 - \theta_{b1})}{(1 - \theta_{g1})} \]  

(48)

Now compare \( V_{g\mu}^{dd} \) and \( V_{g\mu}^{nn} \). \( V_{g\mu}^{nn} = \theta_{g1}(1 - F_{\mu1}^{nn}) + \theta_{g2}(1 - F_{\mu2}^{nn}) \). Using Lemma 5 we get: \( V_{g\mu}^{dd} - V_{g\mu}^{nn} = (1 - \mu)\left[ \frac{k_1\theta_{b1}(1 - \theta_{b1})}{\theta_{g2}(1 - \rho^2)} \left( \frac{\theta_{b2}(1 - \theta_{b2})}{\theta_{g1}(1 - \theta_{g1})} \right) - \frac{\theta_{g1}}{\theta_{g2}} \right] + \frac{k_2\theta_{b2}(1 - \theta_{b2})}{\theta_{g1}(1 - \rho^2)} \left( \frac{\theta_{b1}(1 - \theta_{b1})}{\theta_{g2}(1 - \theta_{g2})} \right) \]. So if conditions (47) and (48) hold, \( V_{g\mu}^{dd} > V_{g\mu}^{nn} \).

End proof.

References


