Can earnings manipulation create value?*

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Abstract. Existing literature usually considers earnings manipulation to be a negative social phenomenon. We argue that earnings manipulation can be a part of the equilibrium relationships between firms’ insiders and outsiders. We consider an optimal contract between an entrepreneur and an investor where the entrepreneur is subject to a double moral hazard problem (one being the choice of production effort and the other being intertemporal substitution, which consists of transferring cash flows between periods). Investment and production effort may be below socially optimal levels because the entrepreneur cannot entirely capture the results of his effort. The opportunity to manipulate earnings protects the entrepreneur against the risk of a low payoff when the results of production are low. Ex-ante, this provides an incentive for the entrepreneur to increase his level of effort and invest efficiently.

Key words: earnings manipulation, intertemporal substitution, design of securities, property rights, moral hazard

JEL classification codes: G32, D92, D82
1 Introduction.

The recent wave of corporate scandals (Worldcom, Enron, Nortel etc.) has raised heated debates regarding the manipulation of earnings by firms’ insiders. Existing literature usually considers earnings manipulation (hereafter EM) to be a negative social phenomenon and suggests measures for its elimination. In the present paper, we argue that earnings manipulation can be a part of the equilibrium relationships between firms’ insiders and outsiders.

In contrast to earnings being misreported, which in most cases represents accounting fraud, we consider EM to be a transfer of funds between periods. This transfer does not create any social value (in contrast to productive effort). Some typical examples include delaying the approval of important decisions, inefficient investments, borrowing in order to manipulate financial results, inefficient discount policy etc. EM is well documented in empirical literature. For instance, Degeorge, Patel, and Zeckhauser (1999) discovered discontinuities in the distribution of corporate earnings at some specific values (thresholds). The number of reports with earnings just below the threshold is much lower than those just above the threshold. This suggests that insiders are involved in earnings manipulation around the threshold level. Burgstahler and Dichev (1997) show that 30-44% of firms with small pre-managed losses manage earnings to create a positive profit. Recently, Yu, Du, and Sun (2004) examined earnings management by Chinese firms and found earnings manipulation around two thresholds.

Degeorge, Patel, and Zeckhauser (1999) also present a theoretical model involving EM by a manager with a bonus-like contract. The authors show that the manager’s incentive to manipulate earnings depends on the values of the latent (pre-managed) earnings, the manager’s bonus, and the magnitude of the social loss from EM. The manager’s decision also relies on whether pre-

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1 For empirical evidence about earnings misreporting see Dechow, Sloan and Sweeney (1996) and Erickson, Hanlon and Maydew (2003). For theoretical papers see Cornelli and Yoscha (2003), Crocker and Slemrod (2005) and Johnsen and Talley (2005).

2 Other examples include the choice of inventory methods, allowance for bad debt, expensing of research and development, recognition of sales not yet shipped, estimation of pension liabilities, capitalization of leases and marketing expenses, and delay in maintenance expenditures (see Degeorge, Patel and Zeckhauser, 1999). Roychowdhury (2006) provides extensive evidence on earnings management through real activities manipulation.
dictions of future profits are certain or risky. In contrast, the model in the present paper contains a double-moral hazard problem (one being the choice of production effort and the other being the EM decision). We compare different contractual arrangements between an investor and an entrepreneur as well as their impact on the entrepreneur’s effort. This is important given that several recent papers analyze the links between financing structures and EM (see, for instance, Richardson, Tuna and Wu (2007), Hodgson and Stevenson (2000) and Jensen (2002)). Finally, we compare the model’s predictions with EM and without EM.

We analyze a model where a firm needs external financing. The firm’s value consists of current (first-period) earnings and the going concern value. In contrast to current earnings, it is costly to verify the going concern value of the firm and enforce payments contingent on it (for instance, since it is impossible to describe all states of nature in the future and all optimal actions, the firm’s owners may be able to divert all future earnings to their own pockets). The fact that it is impossible to write a complete contract contingent on the firm’s going concern value eliminates any opportunity to write a contract contingent on the firm’s total value (which would eliminate the problem of EM because EM cannot increase the firm’s total value). The financing contract includes cash payments and an allocation of rights on the firm’s going concern value - both being contingent on the magnitude of the firm’s current earnings. The contract may optimize the value of the parties’ cooperation because of the impact it has on the entrepreneur’s incentives to provide productive effort and engage in EM. For instance, if the going concern value represents a new firm and the party responsible for decision-making is the sole owner of this new firm, this party will be interested in shifting the value of the original business to the new firm (even if it is socially inefficient).

As mentioned above, we compare two situations. In the first, the entrepreneur chooses only a costly productive effort - assuming that the entrepreneur cannot be involved in EM. In the second, the entrepreneur is subject to a double-moral hazard problem which includes the choice of productive effort and the EM decision. It is shown that the entrepreneur’s productive effort may be higher in the second case. The following demonstrates the intuitions behind this result. Consider debt financing. If current earnings are below the

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3 For a similar approach see, for example, Hart, 1995.

4 See, among others, Kaplan and Stromberg (2004) for contingencies in financing contracts.
face value of debt, the firm is bankrupt and the entrepreneur gets nothing. If the amount of investment is relatively high, the debt face value should also be high. When the face value of debt is higher than the maximal value of current earnings, the entrepreneur receives nothing regardless of the effort provided. However, if he is able to transfer earnings between periods and the firm's going concern value is relatively high, the entrepreneur can increase current earnings by reducing the firm's going concern value. This allows the firm to avoid bankruptcy and make a positive profit. This in turn increases his ex-ante incentive to provide productive effort. This argument works even if the cost of intertemporal substitution is relatively high.

Note that recent scandals have caused many authors to believe that linear contracts are the best contracts for managers (entrepreneurs) because they protect the firm against EM (see, for instance, Jensen, 2003). The main problem is that such contracts are not optimal if the entrepreneur is subject to moral hazard with regard to the choice of productive effort (Innes, 1990). However, Jensen (2003) argues that the benefits from non-linear contracts cannot be compared to the disadvantages of EM. This paper argues that non-linear contracts, including standard debt, can be better than linear contracts.

The rest of this paper is organized as follows: Section 2 describes the model; Section 3 explains optimal contracting without EM; Section 4 discusses optimal contracting when the entrepreneur is subject to a double moral hazard problem which includes EM. A comparison of the outcomes is presented in Section 5. Section 6 discusses the model’s implications with regard to empirical evidence and Section 7 presents the conclusions.

2 Model.

Consider a firm that has to make an investment \( b > 0 \). The firm's owner/entrepreneur (\( E \)) needs external financing from an outside investor (\( I \)). \( E \) and \( I \) are risk neutral. If the investment is made, the firm's performance depends on \( E \)'s effort \( e \in [0, 1] \). The cost of effort is \( e^2 \). The interim first-period cash flow \( r_0 \) equals 1 with probability \( e \) and 0 otherwise. The company's assets which remain at the end of first period may yield the revenue 2 in the second period.\(^5\) \( E \) may engage in EM. The firm’s final first-period profit is \( r = r_0 \)

\(^5\)For simplicity it is assumed that the firm’s going concern value does not depend on \( e \). The model can easily be generalized by allowing this. As far as we can see, no intuitions will be affected by this change. The specific value is chosen arbitrarily although it assures
$-a$, where $a$ is a profit correction arising from intertemporal substitution (EM takes place if $a \neq 0$). From an outsider’s perspective, $r$ is observable but $r_0$, $e$, and $a$ are unobservable. If $a \neq 0$, the firm’s going concern value at the end of first period is $v = 2 + a - c$, where $c$ is the cost of EM, $0 < c < 2$. EM is socially inefficient ($a^* = 0$, where $a^*$ denotes the socially optimal $a$).

To insure that earnings are non-negative in each period we assume

$$c - 2 \leq a \leq r_0$$

The first-best level of effort $e^*$ maximizes the firm’s expected value (which includes first and second-period earnings minus the cost of effort and investment). It is assumed for simplicity that the risk-free interest rate is zero and there is no discounting. Thus, the expected value can be written as $E[r + v - e^2 - b] = e + 2 - e^2 - b$. Obviously, $e^* = 1/2$. We assume that the project’s net present value is positive, i. e.

$$E_{e=1/2,a=0}[r + v - e^2] = 9/4 < b$$

A complete contract contingent on the firm’s going concern value is impossible to write. This stems from the idea that it is much more difficult to describe (ex-ante) all scenarios for long-term investments compared to short-term ones. Therefore, $E$ is not able to offer $I$ a complete contract contingent on the firm’s total value. As we discuss in Section 5, if this were possible, the problem of EM would not exist. Thus, we assume that $E$ can only offer a complete contract contingent on first-period earnings $r$, and that $E$ (the party in control) can capture the firm’s going concern value (similar to Hart, 1988). $E$ remains in control when the firm does not default. This leads to the following security design in the model which depends on the first-period sharing rule and the contingencies for shifting control in the second period.\footnote{In Section 5 we discuss different security designs.}

**Equity financing** (denote this strategy by $s$). In this case, $I$ gets a fraction $k$ of the firm’s earnings in the first period, $0 < k \leq 1$. $V_E = (1 - k)r + v$ and $V_I = kr$, where $V_E$ and $V_I$ denote the payoffs of $E$ and $I$ respectively.

**Debt** (denote this strategy by $d$). The firm issues debt with face value $D$ which matures at $t = 1$. If $r < D$ (default), $I$ gets the first-period earnings that the going-concern value of the firm is large enough compared to current earnings.

\footnote{The cost of EM includes mostly the time $E$ spends on creating the "technology" for EM (like creating a special purpose vehicle (firm) to hide losses in the case of Enron). This is not necessarily linked to the magnitude of EM. The model can be generalized by allowing different cost functions.}
and the firm’s going concern value. $E$ gets nothing. If $r \geq D$, $E$’s first-period earnings are $r - D$. He also obtains the firm’s going concern value. Therefore, $V_E = r - D + v$ and $V_I = D$.

The game is as follows:
1. Securities are issued and sold for an amount $b$. The investment is made.
2. $E$ chooses $e$.
3. $r_0$ is realized. $E$ observes $r_0$ and chooses $a$.
4. $r$ and $v$ become known. The parties get their payoffs according to the securities issued.

When choosing which securities to issue, $E$ maximizes the expected value of his net earnings (payoff on the securities minus the cost of effort). On the one hand, the contracts should provide $E$ with the optimal incentive to choose $e$ and $a$. On the other hand, the expected value of $I$’s payoff must cover the investment cost, $b$, in order for $I$ to accept the contract.

3 Optimal contracting without earnings manipulation.

Consider an optimal contract when $E$ does not manipulate earnings under any circumstance. This may be the case when the government puts in place a well developed system of corporate control which makes it highly probable that EM will be discovered. If the penalties for manipulating earnings are very high, $E$ cannot justify taking the risk. $E$’s problem can be written as follows (problem P1).

$$\max_{s,d} EV_E \text{ subject to }$$

$$e = \arg \max_e W_E$$

$$0 \leq e \leq 1$$

$$EV_I \geq b$$

where $W_E$ ($E$’s expected profit) equals $E$’s expected payoff minus the cost of effort: $W_E = EV_E - e^2$.

To solve P1 we will decompose it into two sub-problems. We first consider each financing strategy separately and will summarize the results in Proposition 1.
Lemma 1. 1) if $b > 1/8$, $s$ is not feasible; 2) If $b \leq 1/8$ and $s$ is chosen,

$$k = \frac{1 - \sqrt{1 - 8b}}{2}$$  \hspace{1cm} (3)

Proof. If $s$ is chosen, $W_E = E[(1 - k)r + 2 - e^2] = (1 - k)e + 2 - e^2$. Hence the optimal level of effort is

$$e = (1 - k)/2$$  \hspace{1cm} (4)

This is below the first-best level of effort: $E$ gets only a fraction of the firm’s profit but absorbs all the costs. $I$’s expected payoff is

$$EV_I = E[kr] = ke = k(1 - k)/2$$  \hspace{1cm} (5)

The optimal $k$ maximizes $W_E$ under the condition that $EV_I$ is not less than $b$. From (4) we get:

$$W_E = (1 - k)^2/4 + 2$$  \hspace{1cm} (6)

From (5), $I$’s payoff is maximized when $k = 1/2$ which implies that the maximal $EV_I$ is $1/8$. Thus, strategy $s$ is feasible only if $b \leq 1/8$. Since $E$’s expected profit is decreasing in $k$ (from (6)), the optimal $k$ can be found by equalizing (5) and $b$ which produces (3). End proof.

Intuitively, if $b$ is too large, the fraction of equity that must be given to $I$ is large enough to prevent $E$ from providing an effort level which will generate enough income to compensate $I$.

Now consider $d$.

Lemma 2. 1) If $b > 1$, $d$ is not feasible; 2) if $b \leq 1$ and $d$ is chosen

$$D = b$$  \hspace{1cm} (7)

Proof. If $d$ is chosen, $V_E = r_0 - D + 2$, if $r_0 = 1$ and $V_E = 0$ otherwise. Thus, $W_E = e(3 - D) - e^2$. The maximand of this expression is $e’ = \frac{3-D}{2}$. However, since $D \leq 1$ (otherwise $E$ gets nothing) we have $e’ > 1$ which implies $e = 1$. $I$’s payoff is $D$. Therefore, $D = b$ is optimal. This only works if $1 \geq b$. If $1 < b$ and $D \leq 1$, $I$’s payoff is not sufficient to cover the initial investment. If $D > 1$, $E$ provides no effort since he gets a payoff of zero and thus $I$ gets nothing. End proof.

An explanation for Lemma 2 is as follows. If $b$ is larger than the maximal first-period earnings, setting the debt face value below that maximal level
of earnings is not sufficient to ensure that the investor is repaid at least $b$. If debt face value is higher than the maximal first-period earnings, $E$ has nothing to gain and does not provide any effort.

**Proposition 1.** 1) If $b \leq 1/8$, $s$ is the optimal strategy. 2) If $1/8 < b \leq 1$, $d$ is optimal; 3) if $1 < b$, the project will not be undertaken.

**Proof.** From Lemma 1, if $s$ is chosen, $b = 8$ and

$$W_E = \frac{17 + \sqrt{1 - 8b} - 4b}{8} \quad (8)$$

If $d$ is chosen, $b \leq 1$ and

$$W_E = 2 - b \quad (9)$$

Proposition 1 follows from comparing (8) and (9) for different values of $b$. 

*End proof.*

The project will be undertaken if and only if $b < 1$. Thus, there is less inefficiency under small values of $b$ than under high values of $b$. Given that $E$’s portion of total profit increases as $b$ decreases, $E$ will provide a greater effort when $b$ is lower. Also, note that Innes (1990) analyzes a similar environment (where an entrepreneur’s effort is costly and EM is not allowed) with only one period (in terms of our model this means $v = 0$) and demonstrates that debt is the best financing strategy.

## 4 Optimal contracting with earnings manipulation.

Now suppose that $E$ can manipulate earnings. $E$’s problem (P2) can be written as follows:

$$\max_{s,d} W_E \text{ subject to}$$

$$a = \arg \max_a V_E$$

$$e = \arg \max_e W_E$$

$$EV_i \geq b$$

$$0 \leq e \leq 1$$

$$c - 2 \leq a \leq r_0$$

As in the previous section, we begin by considering each financing strategy separately.
Lemma 3. 1) s is feasible if and only if \( b \leq 1/8 \) and
\[
\frac{1 - \sqrt{1 - 8b}}{2} < c
\] (10)

2) if s is chosen, k is determined by (3).

Proof. Consider strategy s. Given the intermediate profit \( r_0 \) and action \( a \), E’s payoff is:
\[
(1 - k)r_0 + 2, \text{ if } a = 0 \tag{11}
\]
\[
(1 - k)(r_0 - a) + 2 + a - c, \text{ if } a \neq 0 \tag{12}
\]

Let \( \Delta \) be the difference between (11) and (12). We have \( \Delta = c - ka \). If \( r_0 = 0 \), then, from (1), \( a \leq 0 \). Thus \( \Delta > 0 \) and \( a = 0 \) is optimal. If \( r_0 = 1 \), then, from (1), \( a \leq 1 \). (12) is maximized when \( a = r_0 \) and it equals \( 3 - c \). Also, (11) equals \( 3 - k \). Thus, if \( k < c \), \( a = 0 \) is optimal. If \( k > c \), the optimal \( a = r_0 \) (when the cost of EM is relatively low, \( E \) will increase the firm’s going concern value).

If strategy s is chosen, I’s payoff is \( kr \). If \( k > c \), then it follows from the above paragraph that I’s payoff is 0 (this cannot be an equilibrium outcome). If
\[
k < c \tag{13}
\]
\( E \) does not manipulate earnings regardless of \( r_0 \). Thus, \( W_E = e(3 - k) + (1 - e)2 - e^2 \) (i.e. with probability \( e \), \( r_0 = 1 \) and \( E \) gets \( (1 - k)r_0 + 2 = 3 - k \) and with probability \( 1 - e \), \( r_0 = 0 \) and \( E \) gets 2). \( W_E \) is maximized when \( e = (1 - k)/2 \). Analogously to Lemma 1, we find that this only works if \( b \leq 1/8 \) and the optimal \( k \) is given by (3). From (3) and (13), this contract only works if the condition (10) holds. End proof.

An explanation of Lemma 3 is as follows. When strategy s is chosen and current earnings are low, \( E \) does not manipulate earnings since he receives the firm’s total going concern value and only a part of the firm’s current earnings. If current earnings are high, \( E \) can engage in EM (pumping up the firm’s going concern value) if the cost of EM is relatively low. The latter case can be detrimental for \( I \) since he does not have control over the firm’s going concern value under strategy s. The condition (10) requires that the cost of EM is relatively high. Thus, if s is chosen, \( E \) always chooses \( a = 0 \). The

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We assume that if \( E \) is indifferent between \( a = 0 \) and \( a = r_0 \), he chooses \( a = 0 \). This happens if \( k = c \).
payoffs are the same as those in the case without intertemporal substitution except for the condition (10).

**Lemma 4.** Consider strategy $d$. Let $c \leq 1$. 1) If $2 - c/2 < b$, $D = K$, where

$$K = \frac{5 - c - \sqrt{17 + c^2 - 2c - 8b}}{2} \quad (14)$$

2) If $2 - c/2 > b > 2 - c$, $D = 2 - c$. 3) If $2 - c > b$, $D = b$. Now, let $c > 1$. 1) If $b > 2$, $d$ is not feasible; 2) If $2 > b > 1 + c/2$, $D = K$; 3) if $1 + c/2 > b$, $D = 1$.

**Proof.** See Appendix.

The main result of Lemma 4 is that, in contrast to Lemma 2, debt financing is possible even when $b$ is relatively high. Without EM, a high $b$ would lead to a high debt face value which destroys $E$’s incentive to provide productive effort. With the possibility of EM, $E$ can make profit even if the debt face value is large and current earnings are low.

To illustrate the proof of Lemma 4, consider the case $c < 1$ and $b > 2 - c$.

Given the intermediate profit $r_0$ and action $a$, $E$’s payoff is:

- $0$, if $r_0 - a < D$ \quad (15)
- $r_0 - D + 2$, if $r_0 - a \geq D$ and $a = 0$ \quad (16)
- $r_0 - D + 2 - c$, if $r_0 - a \geq D$ and $a \neq 0$ \quad (17)

This means that if the firm defaults on its debt, $E$ gets nothing. If the firm does not default, $E$ gets the firm’s first-period profit plus the firm’s going-concern value minus the cost of EM. If $r_0 \geq D$, $a = 0$ is optimal. The same holds if $r_0 < D$ and $2 + r_0 - D - c < 0$. Otherwise, the optimal $a$ satisfies $a \leq r_0 - D$ and $2 + a - c \geq 0$. If these conditions are satisfied, $E$’s earnings remain the same regardless of $a$. Thus, for simplicity, we will assume $a = r_0 - D$. Finally, we have:

$$\begin{cases} 
  a = 0 & \text{if either } r_0 \geq D \text{ or } r_0 < D \text{ and } 2 + r_0 - D - c < 0 \\
  a = r_0 - D, & \text{if } r_0 < D \text{ and } 2 + r_0 - D - c \geq 0 
\end{cases} \quad (18)$$

If interim earnings are greater than the debt face value, the optimal strategy for $E$ is not to manipulate earnings. The same holds if bankruptcy is unavoidable (debt is too large and current earnings are too low). Otherwise, the optimal intertemporal substitution action is one that makes the firm’s first-period earnings just enough to cover the debt.
Three different situations are possible depending on the magnitude of $D$. Consider

\begin{align*}
2 - D - c & \geq 0 \\
1 & < D
\end{align*} \quad (19) \quad (20)

It will be shown that this case is never possible. The debt face value is less than $2 - c$ and less than the amount of investment $b$ (recall that by assumption $b > 2 - c$) which makes this case counterintuitive. It needs to be proven formally however, since $I$ can get a large portion of the firm’s going concern value if the firm defaults in the first period. By (18)-(20), $a = r_0 - D$, $\forall r_0$. This means that $E$ will manipulate earnings regardless of $r_0$ (the condition (20) implies that even if the firm performs well, the interim earnings are below the debt face value, and (19) ensures that the going-concern value is high enough to allow an increase first-period earnings to repay debt even if $r_0 = 0$). Thus, $I$’s payoff is $D$ regardless of $r_0$. From $I$’s budget constraint and (19) we have: $b \leq D \leq 2 - c$. This leads to a contradiction because $b > 2 - c$.

Now consider the case $1 \geq D$ (and $2 - D - c > 0$ because $c < 1$). This case is also counterintuitive because $D < b$. Here, the firm is solvent if $r_0 = 1$, and $E$ can increase first-period earnings to avoid bankruptcy if $r_0 = 0$. In both cases, $I$’s payoff is $D$. This does not work because $b > 2 - c > D$ (recall that $c < 1$).

Finally, consider

\begin{equation}
2 - c - D < 0 \quad (21)
\end{equation}

By (18) and (21) we have, $a = r_0 - D$ if $r_0 = 1$, and $a = 0$ if $r_0 = 0$. The choice of $e$ maximizes

\begin{equation}
e(3 - D - c) - e^2 \quad (22)
\end{equation}

This means that, with probability $e$, $E$ gets current earnings of 0 and the firm’s going concern value 2 reduced by the amount of EM $(D - 1)$ and the cost of EM. The maximand of (22) is $e'' = (3 - D - c)/2$. Thus,

\begin{align*}
e & = \quad e'' \text{ if } 3 - D - c \geq 0 \\
e & = \quad 0 \text{ if } 3 - D - c < 0
\end{align*} \quad (23)

(Note that $e'' < 1$ because $2 > 3 - D - c$). $I$’s expected payoff is

\begin{equation}
EV_I = (3 - D - c)D/2 + (-1 + D + c) \quad (24)
\end{equation}
This means that debtholders receive $D$ (when $r_0 = 1$) with probability $e''$ and they receive the firm’s going concern value $2$ with probability $1 - e'' = (-1 + D + c)/2$ (when $r_0 = 0$). From (22), $W_E = (3 - D - c)^2/4$ which decreases in $D$. Thus, the optimal $D$ is the minimal one which makes (24) equal to at least $b$ under conditions (21) and (23). Solving this optimization problem we get the following (note that (14) denotes the minimal value of $D$, which makes (24) equal to $b$). If $2 - c/2 \leq b$, $D = K$. If $2 - c/2 > b$, $D = 2 - c$.

An interpretation of Lemma 4 is as follows. If $b$ is relatively low ($b < 2 - c$) and the cost of EM is relatively low ($c < 1$), debt is risk-free ($D = b$). The face value of debt is low and $E$ is able to manipulate earnings to attain the threshold to avoid bankruptcy. If $b$ is relatively large ($b > 2$) and the cost of EM is relatively high ($c > 1$), debt is not feasible (EM is not possible). Otherwise, $E$ delivers some reasonable level of effort which implies some positive probability of default making debt risky. Lemmas 3 and 4 lead to the following proposition.

**Proposition 2.** If $b \leq 1/8$, $s$ is optimal if $c > 1 - \sqrt{1 - b^2}/2$ and $d$ is optimal if $c \leq 1 - \sqrt{1 - b^2}/2$. If $1/8 < b \leq 2$, $d$ is optimal. If $b > 2$, $d$ is optimal if $c \leq 1$, and the project will not be undertaken if $c > 1$.

**Proof.** Consider $b \leq 1/8$. Suppose $c > 1$. If $s$ is chosen, $E$’s payoff is $17 + \sqrt{1 - 8b - 4b}/8$ by (8) and Lemma 3. If $d$ is chosen, $E$’s payoff is $(2 - c)^2/4$ (see the proof of Lemma 4). The former is not less than 33/16 (this value is attained when $b = 1/8$) and the latter is not greater than 1/2 (this value is attained when $c = 0$). Thus, $s$ is optimal. Consider $1 - \sqrt{1 - b^2}/2 < c \leq 1$. If $s$ is chosen, $E$’s payoff is $17 + \sqrt{1 - 8b - 4b}/8$. If $d$ is chosen, $E$’s payoff is $9/4 - b - c/2 + c^2/4$. Again, the payoff from $s$ is higher. To see this, note that the payoff from $d$ decreases in $c$. When $c = 1 - \sqrt{1 - b^2}/2$, the payoff from $s$ is still larger. Thus, it is also larger under other values of $c$. Consider $1 - \sqrt{1 - b^2}/2 > c$. By Lemma 3, $s$ is not feasible. $d$ is feasible and thus is optimal.

Consider $1/8 < b \leq 2$. $s$ is not feasible. $d$ is feasible and thus is optimal.

Consider $2 < b$. $s$ is not feasible. If $c > 1$, no contract is feasible. If $c \leq 1$, $d$ is feasible and thus is optimal. **End proof.**

Proposition 2 is intuitive. First, if $b$ is large, $s$ is not feasible - as discussed in the case without EM. Thus, debt is the optimal financing choice if the cost of EM is low. For other values of $b$, we have the following. A low $c$ is detrimental to $s$ because it creates opportunities for $E$ to engage in EM, thereby shifting the firm’s value away from $I$’s pockets. $d$ is almost
always accompanied by EM, so reducing the cost of EM is beneficial for debt financing.

Corollary 1 considers the effect of changes in $b$ on the optimal choice of contract. It is shown that when $c$ is relatively small, firms with a high $b$ issue debt while firms with the same $c$ but a low $b$ issue equity. If $b$ is relatively small, $E$ will finance the project by issuing stock. The firm’s going concern value will fully cover the investor’s investment. The entrepreneur will keep 100% of current period earnings which will mitigate the moral hazard problem. If $b$ is large, then financing in this way may not be feasible. Therefore, debt becomes optimal.

**Corollary 1.** If $\frac{1-\sqrt{1-c^2}}{2} \geq c$, $d$ is optimal. If $\frac{1-\sqrt{1-c^2}}{2} < c \leq 1$, $s$ is optimal if $b \leq 1/8$, and $d$ is optimal if $b > 1/8$. If $c > 1$, $s$ is optimal if $b \leq 1/8$, $d$ is optimal if $1/8 < b < 2$, and no contract is feasible if $b > 2$.

**Proof.** Follows directly from Proposition 2.

**Corollary 2.** Earnings manipulation can appear in equilibrium. Earnings manipulation is more probable as $c$ decreases and $b$ increases.

**Proof.** From Proposition 2, if, for instance, $c < 1$ and $b > 2 - c$, the equilibrium outcome is financing by debt and, if $r_0 = 1$, the firm will manipulate earnings. Also, from Proposition 2, for a given $b$, debt financing is optimal when $c$ is relatively low. In most cases, debt financing, in contrast to equity financing, will be accompanied by earnings manipulation (see the proof of Lemma 4). From Corollary 1, the same holds for high values of $b$. End proof.

## 5 Can earnings manipulation enhance a firm’s value?

We now compare firms that are involved in EM (Section 4) with those that are not (Section 3). If the amount of investment is large ($b > 1$), a firm that does not manipulate earnings will not undertake projects with positive value. In contrast, a firm that manipulates earnings will undertake the same projects. If the amount of investment is low and EM is not possible, financing with equity is optimal. If a firm can manipulate earnings, equity may still be optimal. However, for equity to be optimal, the cost of EM must be high - otherwise the entrepreneur will "convert" current earnings into inefficient long-term projects making the issuance of equity unfeasible (ex-ante). In the latter case, debt becomes optimal. This will usually be accompanied by EM:
the entrepreneur will try to achieve the threshold to avoid bankruptcy. It follows that there is a trade-off in social efficiency between the benefits from EM improving the entrepreneur’s effort and the costs of EM.

**Proposition 3.** If $1 < b \leq 2$, firms that manipulate earnings have a higher value than firms that do not. Otherwise, firms that manipulate earnings have a higher value if and only if the cost of manipulation is low.

**Proof.** Let $V_{EM}$ denote the value of firms that can manipulate earnings and let $V_N$ denote the value of firms that cannot manipulate earnings. As follows from Proposition 1, if $b > 1$ and earnings manipulation is not allowed, the firm does not invest and thus $V_N = 0$. According to Proposition 2, if $1 < b \leq 2$ or if $b > 2$ and $c < 1$, firms that can engage in EM will use debt financing and invest in the project. The value of these firms will be positive. Consider $1/8 < b \leq 1$. According to Proposition 1, $V_N = 2 - b$. If $c = 1$, $V_{EM} = 9/4 - b - c/2 + c^2/4$ (from the proof of Proposition 2). This expression decreases in $c$ when $c \leq 1$. The minimal value, $2 - b$, is attained when $c = 1$. Therefore, the value of firms that can engage in EM is greater than or equal to the value of firms that are not involved in EM. If $c > 1$, $V_{EM} = (2 - c)^2/4$. This is less than $2 - b$. Therefore, firms that do not manipulate earnings have a higher value. If $1/8 \geq b$, $V_N = \frac{17 + \sqrt{1 - 8b - 4b^2}}{8}$. If $c > \frac{1 - \sqrt{1 - 8b}}{2}$, firms that manipulate earnings have the same value as firms that do not. If $c \leq \frac{1 - \sqrt{1 - 8b}}{2}$, $V_{EM} = 9/4 - b - c/2 + c^2/4$. Consider $\Delta = V_{EM} - V_N$. This expression decreases in $c$. When $c = 0$, $\Delta > 0$. When $c = \frac{1 - \sqrt{1 - 8b}}{2}$, $\Delta < 0$. The proposition follows from the continuity of $\Delta$ in $c$. *End proof.*

6 Model discussion.

1. Suppose that it is possible to write an enforceable contract contingent on the firm’s total value. Then, for any contract found in section 4, there exists an alternative contract contingent on the firm’s total value that will provide $E$ with a higher payoff. To illustrate this, consider $c < 1$ and $b < 2 - c$. If a firm can engage in EM, the optimal contract is analogous to the one described in proposition 2. $E$’s effort is $e = 1/2$ and the parties expected payoffs are:

$$W_E = 9/4 - b - c$$  \hspace{1cm} (25)

and $EV_f = b$. $D = b$ is optimal. $E$ manipulates earnings regardless of $r_0$. When $r_0 = 0$ he receives $2 - b - c$ and when $r_0 = 1$ he receives $3 - b - c$. 

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Now suppose the parties write a contract where $E$ gets $2 - b$ if the firm’s total value is 2 or less and $3 - b$ if the firm’s total value is greater than 2. The optimal effort maximizes $e(3 - b) + (1 - e)(2 - b) - e^2$. $e = 1/2$ is optimal. Also, $a = 0$ because any $a > 0$ will only reduce the firm’s total value. $E$’s expected payoff is $9/4 - b$ which is greater than (25). $EV_I = 1/2(3 - (3 - b)) + 1/2(2 - (2 - b)) = b$. Therefore, we have a better contract which does not involve EM.

2. Now suppose that the model does not contain productive effort. In this case, equity is the optimal financing contract since it eliminates the intertemporal substitution problem (this idea is developed in Jensen, 2003). Other securities will be useless. This scenario is not realistic since firms do not issue equity alone.

3. Suppose that the firm can issue convertible debt. This is similar to standard debt described in the model except that $I$ can purchase a fraction of the firm’s shares when it is solvent. However, since $E$ remains in control, he will cream-off the firm’s going concern value. Hence, the modelling is similar to standard debt.

4. Long-term debt is not considered (in the spirit of incomplete contract literature) because it cannot be enforced. Since the creditors do not have property rights on the remaining assets, the owners will capture the firm’s entire going-concern value.

5. One can make additional assumptions about the first and second period sharing rules based on a continuous earnings distribution function or different control shifting scenarios. These scenarios may yield some new results. For instance, one can assume that $E$ also has some private benefits from controlling the firm. However, the main idea that EM can improve productive effort will not be affected.

7 Implications.

1. We have shown that EM can be a part of the equilibrium relationship between firms’ insiders and outsiders. This holds even if the cost of EM is relatively high (as follows from Proposition 2). Investors accept some degree of EM because this increases the insiders’ incentive to provide a high level productive effort.

2. From Proposition 3, if the cost of EM is relatively low, EM can be socially efficient. EM can enhance a firm’s value when compared to the case
without EM. If the cost of EM is relatively high, the opportunity to engage in EM either does not affect firms’ values (when they do not use EM in equilibrium) or is detrimental to firms’ values (when firms engage in EM in equilibrium).

3. EM should more frequently be observed in industries characterized by incomplete contracts. If complete contracts can be written, the parties can write a contract contingent on the firm’s overall earnings which eliminates the possibility of EM. Thus, firms in industries which are characterized by a high degree of technological or market uncertainty (such as software, internet, biomedical etc.) are more likely to be engaged in EM.

4. As implied by Corollary 2, EM should more frequently be observed among less profitable firms (high $b$). This prediction is consistent with Burgstahler and Dichev (1997).

5. Firms which manipulate earnings issue more debt (Lemmas 3 and 4). This is consistent with Richardson, Tuna and Wu (2002) and Hodgson and Stevenson (2000) where firms which have excessive debt are more likely to be involved in EM.

6. It follows from Corollary 1 that firms with a higher $b$ (and lower profitability respectively) issue debt more often than firms with a lower $b$. This is consistent with a very important corporate finance phenomenon: the negative correlation between debt and profitability (see, among others, Titman and Wessels (1988), and Rajan and Zingales (2000)).

Since EM can be socially efficient, the question of its regulation depends on the industry and any parameters related to the firm’s projects. If the cost of EM is relatively low, putting in place an expensive public system of EM prevention cannot be efficient: entrepreneurs will invest less funds in socially efficient projects and will not provide high levels of productive effort. According to our analysis (proof of Proposition 3), such a system should target average-profit firms (when the cost of EM is relatively high) or high-profit firms (when the cost of EM is in the intermediate range).

8 Conclusion.

This paper analyzes a model where an entrepreneur needs external financing for a profitable investment project and his productive effort is not observable by outsiders. The security design should provide the entrepreneur with the optimal incentive to provide productive effort. We have a standard moral
hazard problem when the entrepreneur is not able to manipulate earnings. The equilibrium level of effort is below the socially optimal level and in some cases (if the amount of investment is relatively large), the entrepreneur will not invest in socially efficient projects. Following this, we analyze the case where the entrepreneur is also able to manipulate earnings. More specifically, the entrepreneur can transfer cash flow between periods. Our main finding is that the existence of EM can lead to increased output (including the entrepreneur’s effort and the amount of investment) and therefore improved social efficiency. It is also shown that EM should more frequently be observed among firms with low profitability, low costs of EM, and extensive debt financing. The main policy implication is that putting in place an expensive system to prevent EM may be socially inefficient.

Appendix

Proof of Lemma 5. Suppose \( c < 1 \). First, consider the choices of \( a \) and \( e \). Three situations are possible. Consider \( 2 - c \geq D > 1 \). By (16), in this case \( a = r_0 - D, \forall r_0 \). This implies

\[
EV_I = D. \tag{26}
\]

and

\[
WE = e(3 - D - c) + (1 - e)(2 - D - c) - e^2 \tag{27}
\]

Thus, \( e = 1/2 \) is optimal.

Now consider \( 2 - D - c < 0 \). Again by (16), \( a = D - 1 \) if \( r_0 = 1 \) and \( a = 0 \) if \( r_0 = 0 \). The choice of \( e \) maximizes \( e(3 - D - c) - e^2 \). The maximand of (22) is \( e'' = (3 - D - c)/2 \). Thus,

\[
e = e'' \text{ if } 3 - D - c > 0 \tag{28}
\]

\[
e = 0 \text{ if } 3 - D - c \leq 0
\]

(Note that \( e'' < 1 \) because \( 2 > 3 - D - c \)). The case \( e = 0 \) is not interesting because \( E \)'s payoff is 0 and thus debt is never the optimal contract. If \( 3 - D - c > 0 \), \( I \)'s payoff is

\[
(3 - D - c)D/2 + (-1 + D + c) \tag{29}
\]

\[
WE = (3 - D - c)^2/4 \tag{30}
\]
If $1 \geq D$ (and $2 - D - c > 0$ because $c < 1$), by (16), $a = D$ if $r_0 = 0$ and $a = 0$ if $r_0 = 1$. The choice of $e$ maximizes $e(3 - D) + (1 - e)(2 - D - c) - e^2$. Thus, $e = (1 + c)/2$. Therefore,

$$EV_I = D. \quad (31)$$

Now we turn to the analysis of the choice of optimal contract. The case $b > 2 - c$ was described in the text.

Consider the case $1 < b < 2 - c$. For the case $2 - c \geq D > 1$, by (26), $D = b$ is optimal. From (27)

$$W_E = 9/4 - b - c \quad (32)$$

Consider the case

$$2 - D - c < 0 \quad (33)$$

Since $W_E$ decreases in $D$ by (30), the optimal $D$ is the minimal one that makes (29) at least equal to $b$. Taking into account (33) and (28) we get $D = 2 - c$. (Note that when $b < 2 - c$, $K < 2 - c$ which makes the constraint (33) binding). Therefore,

$$W_E = 1/4 \quad (34)$$

The case $1 \geq D$ does not work because $b > 1 \geq D = EV_I$. The latter inequality follows from (31).

Finally, for the case $2 - c > b > 1$, we have the following. There are two candidates for the optimal contract. One contract (with $D = b$) implies earnings manipulation regardless of $r_0$ and the other implies earnings manipulation when $r_0 = 1$ (with $D = 2 - c$). Comparing (32) and (34) shows that a higher output is produced when $D = b$.

Now consider $b < 1$. For the case $2 - c \geq D > 1$, $E$’s objective function is not binding. The optimal level of debt is $D = 1 + \varepsilon$, where $\varepsilon$ should be as small as possible, $\varepsilon > 0$ (it cannot be equal 0 since $D > 1$). $E$’s payoff is $5/4 - \varepsilon - c$. In the limit (when $\varepsilon \to 0$) it equals

$$5/4 - c \quad (35)$$

For the case $2 - D - c < 0$, $D = 2 - c$ is optimal. Therefore,

$$W_E = 1/4 \quad (36)$$
For the case $1 \geq D$, $D = b$ is optimal. Thus,

$$W_E = 9/4 - b - c/2 + c^2/4 \quad (37)$$

Comparing (35), (36), and (37) we find that if $b < 1$, $D = b$ is optimal. $W_E = 9/4 - b - c/2 + c^2/4$.

Now suppose $c > 1$. First, we analyze the choices of $a$ and $e$.

If $1 - D < 0$, by (16), $a = D - 1$ if $r_0 = 1$ and $a = 0$ if $r_0 = 0$. The choice of $e$ maximizes $e(3 - D - c) - e^2$. Thus, $e = (3 - D - c)/2$ if $3 - D - c \geq 0$ and 0 otherwise. The latter case is not interesting because $E$’s payoff is 0 and thus debt is never the optimal contract. In the former case, $I$’s payoff is

$$(3 - D - c)D/2 + (-1 + D + c) \quad (38)$$

$$W_E = (3 - D - c)^2/4 \quad (39)$$

If $2 - D - c > 0$, by (16), $a = D$ if $r_0 = 0$ and $a = 0$ if $r_0 = 1$. The choice of $e$ maximizes $e(3 - D) + (1 - e)(2 - D - c) - e^2$. Thus, $e = (1 + c)/2$. Therefore, $I$’s payoff is $D$.

If $1 > D$ and $2 - D - c < 0$ (no EM), the choice of $e$ maximizes $e(3 - D) - e^2$. Thus, $e = 1$ because $1 - D > 0$. Therefore, $I$’s payoff is $D$.

We now turn to the analysis of optimal contracts.

Consider $b > 2$. The case $1 - D < 0$ is not feasible because $EV_I < b$ for any $D > 1$. The case $2 - D - c > 0$ is not feasible either. This works only if $b < 2 - c$.

If $1 > D$ and $2 - D - c < 0$ (no EM), $I$’s payoff is $D$. $D = b$ is optimal. This only holds if $b < 1$.

Therefore, for the case $b > 2$, debt is not feasible.

Now consider $2 > b$.

If $1 - D < 0$, then if $0 < 2 + c - 2b$, $D = 1$ is optimal. This implies that $W_E = (2 - c)^2/4$. If $0 > 2 + c - 2b$, $D = K$ is optimal. This implies that $W_E = L$, where

$$L = \frac{(1 - c + \sqrt{17 + c^2 - 2c - 8b})^2}{16}$$

The case $2 - D - c > 0$ works only if $b < 2 - c$.

The case $1 > D$ and $2 - D - c < 0$ (no EM) works only if $b < 1$.

Therefore, for the case $2 > b$ we have the following. If $0 < 2 + c - 2b$, $D = 1$ is optimal. This implies that $W_E = (2 - c)^2/4$. If $0 > 2 + c - 2b$, $D = K$ is optimal. This implies that $W_E = L$. End proof.
References


