Imperfect Competition with Competitive Speculation

Sébastien Mitraille* Henry Thille†

March 4, 2011

Abstract

We examine how speculative storage affects the equilibrium outcomes for a market with imperfectly competitive production. Using a two-period model of quantity competition with random demand, we find that the ability of speculators to move the commodity across periods dramatically changes the predictions of the model. The usual Cournot result holds for a subset of model parameters, however, a variety of other equilibria are possible. We demonstrate that in addition to the expected price-smoothing equilibrium, equilibria exist in which i) speculators squeeze consumers out of the first period market; ii) nothing is produced in the first period; and iii) producers deter speculation by inducing a price higher than the Cournot level. This variety of possible equilibrium outcomes implies that the welfare effects of speculation are ambiguous.

JEL Classification: L13, D43
Keywords: Inventories, Cournot Oligopoly, Speculation.

*Toulouse Business School, s.mitraille@esc-toulouse.fr
†University of Guelph, hthille@uoguelph.ca

1 Introduction

The Cournot model of competition and the competitive model of speculative storage both have a long history in economics. Despite the attention given to each topic individually, there is not much known about how markets perform when there is both imperfect competition and the possibility of competitive storage. In an environment in which storage by speculators is possible the net demand faced by producers is affected by speculative activity. On the one hand, speculators increase demand for the product when price is low as they buy the good to store for future sale. On the other hand, speculators increase supply of the product when price is high and they wish to sell from their inventories. Hence speculators are competitors to producers when selling and competitors to consumers when buying. The net effect of speculators' actions is to change the effective demand faced by firms. Consequently, when production of the good is imperfectly competitive the net effect of speculative activity on the behaviour of firms is not obvious: demand is increased when speculators purchase the commodity, but then reduced in a subsequent period in which they sell. Do producers have an incentive to encourage or to discourage speculative purchases? How is this incentive affected by the number of producers active? What is the implication for welfare and the distribution of prices?

In order to answer these questions, we analyse a model in which an oligopoly produces a good in each of two periods with demand subject to stochastic shifts. In addition to producers and consumers there are competitive agents who are able to store the good between periods. We find that along with the two expected equilibria (one being the repetition of the Cournot outcome and the other being the standard smoothing equilibrium in which both consumers and speculators active in both periods), there are three additional ones. First, there is an equilibrium in which output is lower and price higher than in the Cournot equilibrium in the absence of storage. Price is maintained at a high level in order to deter speculators from purchasing in the first period.[1] Second, for relatively low first period demand, there is an equilibrium in which speculators buy the entire first period output, resulting in zero first period consumption. This occurs when speculators value the good in the first period much more highly than consumers do essentially outbidding them for the current production.[2] Finally, also

[1]This type of equilibrium was shown to be possible under a monopoly producer in Mitraille and Thille (2009) who called it a Limit equilibrium.

[2]This equilibrium is related to the "entitlement failure" used by Sen (1981) to explain how famines can occur even when production is not particularly low and storehouses

when first period demand is relatively low, an equilibrium exists in which zero output is produced in the first period. In this case firms would rather make no sales at all than sell to speculators who will be competing with them in the second period. We demonstrate the potential existence of all five of these equilibria analytically in our main theorem and then illustrate their existence with a couple of numerical examples.

An important driving force behind these results is the possibility that future demand might be entirely met by speculative stocks, which represents a "collapse" of the market from a producer's point of view. In this case producers make no sales in the second period. The probability that a collapse occurs affects their first period production choice since varying first period output has no effect on second period profit if firms make no sales in the second period. Therefore, the effect of first period production on future profit matter only to the extent that future demand is sufficiently large. The higher the probability of the collapse of the second period market, the higher the marginal profitability of selling to speculators in the first period, since there is a higher probability that those units sold will not diminish second period profit.[3] This effect of increased speculative purchases leading to an increased incentive to sell in the first period provides a counterweight to the strategic substitutability that is usually associated with quantity competition. Increased sales by rival firms can lead to an increased incentive to increase own sales to the extent that rival's sales are purchased by speculators, which adds a degree of strategic complementarity to producers' best responses. This effect is absent when demand is deterministic since the probability of the collapse of the second period market is zero when second period demand is not random.

The effects of speculative storage when production is perfectly competitive is fairly well understood, with important contributions made by Newbery and Stiglitz (1981), Newbery (1984), Williams and Wright (1991), Deaton and Laroque (1992, 1996), and McLaren (1999). The focus in these papers is on the effects of storage on the distribution of prices caused by the movement of production across periods due to random production (harvest) shocks. As aggregate inventories cannot be negative, speculators smooth prices across periods only when positive inventories exist. Unexpectedly

contain grain. In this situation, speculators' willingness to pay for the available supply exceeds that of consumers. This case is also relevant to the policy debate around anti-hoarding laws (see Wright and Williams (1984).

[3]Of course speculators must wish to purchase these units, which is less likely the higher the probability of collapse as that state represents a very low return for speculators. Here we are focusing only on how producers incentives are affected.

large prices result in stock-outs which leads to a breakdown of the price smoothing role of speculative storage. These occasional stock-outs lead to a skewed distribution of price. Market power has been considered by examining imperfect competition in the storage function (Newbery (1984), Williams and Wright (1991), and McLaren (1999)), but production itself remains perfectly competitive in these papers, hence there is no scope for strategic considerations on the part of producers. Another common element of these papers is the focus on agricultural commodities, for which the assumption of perfectly competitive production is justifiable. However, for other commodities, such as metals, the assumption of perfectly competitive production is harder to justify.

Newbery (1984) briefly touches on the problem of a monopolist producer facing competitive storage, simply noting that speculative storage results in a kinked demand curve for the monopolist and consequently discontinuous marginal revenue. Mitraille and Thille (2009) examine this problem in more detail using a model with capacity constrained speculators. They show that both the level and variance of prices are affected by speculation under monopolistic production. In particular, the presence of speculators may cause the monopolist to set a price higher than the monopoly one in order to eliminate competition from speculative sales in future periods, a result similar to that of limit pricing. In this paper we allow for an arbitrary number of firms and impose no constraint on speculative capacity. Our results are consistent with Mitraille and Thille (2009), however we demonstrate that additional equilibria exist in our more general model due to strategic interactions.

A relatively recent related literature has developed in which consumers are able to store a good across periods in a model of an imperfectly competitive market. Theoretical contributions include Anton and Das Varma (2005) and Guo and Villas-Boas (2007) who develop models of imperfectly competitive firms facing deterministic, but not necessarily constant, demand over two periods. The focus of these papers is on whether or not equilibrium involves consumers storing between periods. Storage by consumers results in a kinked demand curve in a similar way to what was pointed out in Newbery (1984), and so, whether or not storage occurs in equilibrium depends on the level of equilibrium production relative to the kink. The strategic consideration involves a firm's incentive to "steal" second period consumers by lowering first period price or increasing output which results in equilibria in which consumers store even if demand is identical in the two periods. In our model, firms have a similar strategic incentive: by increasing output in the first period speculative purchases can be induced which are then sold on the second period market, effectively supplying second period demand with first period production. We find a larger set of equilibria than is considered in the consumer storage literature. In addition, we demonstrate that demand uncertainty has interesting effects on the nature of the equilibrium.

After a description of the model we carefully discuss the implications of speculation for firms' residual demand in section 3 after which we examine the implication for marginal revenue and best-response functions in section 4. Section 5 contains the main theorem and examines the implications for welfare and the distribution of prices.

2 The Model

Consider the market for a homogeneous product consisting of producers, consumers, and speculators. We wish to focus on the effects of speculation alone, so we assume that all agents are risk-neutral. There are n producers in Cournot competition over two periods, indexed by $t \in \{1, 2\}$. Let $\delta \in (0, 1]$ be the agents' common discount factor. In each period t, firm $i \in \{1, ..., n\}$ chooses the quantity it wants to produce, $q_t^i \in \mathbb{R}^+$. All firms face the same quadratic cost function of $\frac{c}{2}{q_t^i}^2$. Convex production costs are not crucial to our results, however, as we discuss below, speculators' behavior can cause problems for the concavity of producer revenue. By allowing for sufficient convexity in the cost function we are able to simplify the exposition. We assume that firms are unable to store their production between the two periods. Clearly it would be more realistic to allow firms to also store, however that would add a substantial degree of complexity to the analysis potentially clouding the effects of independent speculators.[4] We denote the aggregate quantity produced in period t by Q_t, and the aggregate quantity produced by all firms but i by Q_t^{-i}, where $Q_t = \sum_{i=1}^n q_t^i$ and $Q_t^{-i} = \sum_{j=1, j \neq i}^n q_t^j$. Producer i's payoff, Π^i, is

$$\Pi^i = p_1 q_1^i - \frac{c}{2}{q_1^i}^2 + \delta E_1 \left(p_2 q_2^i - \frac{c}{2}{q_2^i}^2 \right). \tag{1}$$

In order for speculation to be potentially profitable, we require some price variation over time. We ensure this by having demand vary randomly across periods.[5] This assumption also has the implication that storage is

[4] See Arvan (1985) and Thille (2006) for analyses of producer storage under imperfect competition.

[5] Since we have assumed quadratic production costs, having the source of variation be in demand means that for a sufficiently large number of producers speculation will not occur as price will tend to equality in the two periods.

potentially socially valuable in this model as it allows producers to smooth their convex production costs across periods. In each period t, consumers' aggregate demand for the product is given by,

$$D_t(p_t) = \max\{a_t - p_t, 0\} \tag{2}$$

where a_t is independently and identically distributed over the support $[0, A]$. We denote the continuous cumulative distribution function for a_t as $F(a)$, with $f(a)$ the associated density function. Random changes in a_t may be interpreted as random shocks affecting the distribution of income in the population of consumers from period to period, modifying in turn the willingness to pay for the product sold by firms and stored by speculators. The distribution of the intercept of market demand is common knowledge to both producers and speculators who learn the current state of demand, a_t, prior to any production or storage decisions in a period. Information is consequently symmetric between producers and speculators.

We model speculation in the same way as is done in the competitive speculative storage literature (Newbery (1984), Deaton and Laroque (1992, 1996)). In each period a large number of risk-neutral, price-taking agents exist who have access to a storage technology and face no borrowing constraints. The storage technology allows speculators to store the commodity at a unit cost of w between periods. The expected return to storing a unit of the commodity from the first to the second period is $\delta(E_1[p_2] - w) - p_1$. The aggregate behavior of these speculators ensures that $p_1 \geq \delta(E_1[p_2] - w)$ with a stock-out occurring if the inequality is strict. We use X_t to denote aggregate speculative sales in period t (purchases have $X_t < 0$) and H_t to denote beginning of period stocks. We assume that $H_1 = 0$ and so $H_2 = -X_1$ describes the inventory dynamics.[6] We also assume that inventories unsold at the end of the second period can be disposed at no cost. To ensure that speculation is at least potentially profitable, we assume that consumers expected valuation exceeds the unit cost of storage, $E[a] \geq w$.

The timing of moves within a period is simply the standard Cournot timing with demand adjusted for net speculative supply: producers set outputs and then price adjusts to clear the market with demand now equal to the sum of consumer demand and speculators' net demand. We search for the subgame-perfect Nash equilibrium to this game with rational expectations on the part of speculators. We begin our analysis in the next section by examining the implications of speculation for producers' inverse demand.

[6]We seek to analyze the effects of the *entry* of speculators. Allowing positive initial stocks would introduce an exogenous source of supply, clouding the comparisons with the standard Cournot model.

3 Effects of Speculation on Net Demand

In this section we derive the inverse demand function faced by firms after the behaviour of speculators is taken into account. Since speculators' behaviour depends on the expected price in the second period, we start by deriving the second period equilibrium for a given level of speculative inventories. We then examine the effect of speculation on the oligopolist's first period inverse demand.

Since there is no salvage value to inventories held at the end of the second period, speculators will sell the quantity they have stored as long as $p_2 > 0$. In this case aggregate speculative sales are equal to $X_2 = H_2$. If on the other hand the aggregate quantity stored by speculators is such that the market price is equal to zero once H_2 is sold, then the aggregate speculative sales cannot exceed the maximal quantity consumers are ready to buy given the aggregate production of firms, $a_2 - Q_2$. In that case, aggregate speculative sales are simply $X_2 = a_2 - Q_2$ and the price is zero. Consequently, the inverse demand faced by producers in the second period is

$$P_2(H_2, Q_2) = \max\{a_2 - H_2 - Q_2, 0\}. \tag{3}$$

Given the residual inverse demand $P_2(H_2, Q_2)$ it faces, each producer chooses the quantity q_2^i to maximize its second period profit, $\pi_2^i = P_2(H_2, Q_2)q_2^i - \frac{c}{2}{q_2^i}^2$, with respect to the quantity produced q_2^i. For the ease of exposition we introduce the following notation:

Definition 1 *Let*

(a) $\beta = (1 + c)/(n + 1 + c) \in (0, 1)$.

(b) $\gamma = (2 + c)/2(n + 1 + c)^2 \in (0, 1)$.

For a given c, β reflects the impact of competition on the market price and γ reflects the impact of competition on each firm's profit. The larger the number of producers the closer β and γ are to zero. Using Definition 1, the unique and symmetric pure strategy Nash equilibrium for the second period subgame is given by the following functions of inventories and demand[7] (H_2, a_2):

$$q_2^*(H_2, a_2) = \frac{1}{n}(1-\beta)\max[a_2 - H_2, 0], \quad Q_2^*(H_2, a_2) = (1-\beta)\max[a_2 - H_2, 0], \tag{4}$$

[7] We will often suppress the arguments of these functions and simply refer to them as q_2^*, p_2^*, Q_2^* and π_2^* as convenient.

$$p_2^*(H_2, a_2) = \beta \max[a_2 - H_2, 0], \quad (5)$$

$$X_2^*(H_2, a_2) = \min[H_2, a_2], \quad (6)$$

$$\pi_2^*(H_2, a_2) = \gamma(\max[a_2 - H_2, 0])^2. \quad (7)$$

It is important to note that an increase in speculative inventories H_2 reduces each firm second period marginal profit by a factor which decreases with the number of producers. That is, the larger the number of producers, the less sensitive producers become to the competition of speculators in second period.

Turning now to the first period, we note that the non-negativity constraint on aggregate speculative inventories implies that speculators aggregate behaviour in the first period satisfies the complementarity condition

$$-X_1 \geq 0, \quad p_1 - \delta(E_1[p_2^*] + w) \geq 0, \quad -X_1(p_1 - \delta(E_1[p_2^*] + w)) = 0. \quad (8)$$

Either no inventories are carried and the return to storage is negative, or inventories are carried and the return to storage is zero. We will use $X_1^*(Q_1)$ to denote the equilibrium storage undertaken when producers sell Q_1 in aggregate. The market clearing price, $P_1(Q_1)$, must be such that, given Q_1, the total of consumer and speculative purchases satisfy

$$D_1(P_1(Q_1)) - X_1^*(Q_1) = Q_1. \quad (9)$$

There are three possible situations that we must concern ourselves with depending on who purchases first period output. First, speculators may buy nothing in the first period and producers sell only to consumers. Second, both speculators and consumers may purchase the good in the first period. We will refer to this case as the "smoothing" outcome, as it corresponds to the standard case of speculators smoothing prices across time. Third, consumers may buy nothing in the first period and producers sell only to speculators.

Consider first the case in which speculators do not purchase. Consumers will buy the entire industry supply if their marginal willingness to pay for it exceeds speculators expected return of storing, $p_1 = a_1 - Q_1 \geq \delta(\beta E[a] - w)$, as $E_1[p_2^*] = \beta E[a]$ when no inventories are carried into the second period. If this were not the case inventories could profitably be carried forward. Hence, a unique threshold output exists, denoted Q_L, such that $p_1 = \delta(\beta E[a] - w)$ given by

$$Q_L = a_1 - \delta(\beta E[a] - w). \quad (10)$$

Q_L is the level of industry output that just renders speculators indifferent between storing and not storing. If $0 \leq Q_1 < Q_L$, speculators do not find profitable to store the product, as consumers have a higher marginal willingness to pay for Q_1. If $Q_1 \geq Q_L$, speculators may store the product and consumers purchase it.

If Q_L is negative, that is if $a_1 < \delta(\beta E[a] - w)$, then speculators can never be excluded from the market. However in this case consumers may be excluded if consumers willingness to pay is low relative to the expected return speculators may obtain by storing. In this case speculators may purchase the good at prices that exclude consumers in first period. An important effect to keep in mind here is that the lower is a_1, the higher is speculators' demand (if a low a_1 indeed corresponds to a low equilibrium p_1). Hence, speculator and consumer demands are negatively correlated, introducing the possibility that speculators squeeze consumers out of the market.

We next turn to the possibility that consumers purchase nothing in the first period and all production is bought by speculators. Consumers will not purchase if $p_1 \geq a_1$ and in order for speculators to purchase $X_1 = -Q_1$ it must be the case that $p_1 = \delta(E[p_2^*(Q_1, a_2)] - w)$, as otherwise the market would not clear. We can define a threshold output, $\widehat{Q}$, such that speculators purchase the entire first period output for $Q_1 < \widehat{Q}$. This threshold is defined by the output at which consumer demand is just extinguished, $p_1 = a_1$, and speculators are willing to store the first period output, $p_1 = \delta(E[p_2^*(Q_1, a_2)] - w))$. Consequently, this threshold is defined by

$$\delta\left(\beta \int_{\widehat{Q}}^{A} (a - \widehat{Q}) dF(a) - w\right) - a_1 = 0. \tag{11}$$

If Q_1 exceeds $\widehat{Q}$ both speculators and consumers purchase the product in period one.

Finally, when both consumers and speculators purchase the commodity in the first period the quantity stored is determined implicitly by the relationship between first period and expected second period prices. We define the position of speculators when smoothing occurs at non-zero[8] p_1 to be $\widetilde{X}(Q_1)$, which is the solution in X to

$$a_1 - X - Q_1 = \delta\left(\beta \int_{-X}^{A} (a + X) dF(a) - w\right). \tag{12}$$

We summarize these results for speculators' behavior with the following lemma:

[8] There is also a possibility that speculators buy at a zero first period price, however, we do not discuss this case as it will not occur in equilibrium.

Lemma 1 *There exist unique Q_L, $\widehat{Q}$, and $\widetilde{X}(Q_1)$ solving (10), (11) and (12) with the properties*

(i)

$$\frac{d\widetilde{X}}{dQ_1} = -\frac{1}{1+\delta\ \beta(1-F(-\widetilde{X}(Q_1)))} < 0$$

(ii)

$$\widehat{Q} = 0 \Leftrightarrow Q_L = 0 \quad and \quad sign(\widehat{Q}) = -sign(Q_L)$$

Proof. See appendix.‖

Part *(i)* of Lemma 1 establishes that speculative sales are monotonically decreasing in first period output (as $X_1 \leq 0$ this means that purchases are increasing). Part *(ii)* of the lemma establishes that one and only one of Q_L and $\widehat{Q}$ is positive.

In summary, aggregate speculative sales in the first period are given by

$$X_1^*(Q_1) = \begin{cases} 0 & \text{if } 0 \leq Q_1 \leq Q_L \\ -Q_1 & \text{if } 0 \leq Q_1 \leq \widehat{Q} \\ \widetilde{X}(Q_1) & \text{if } \max[Q_L, \widehat{Q}] < Q_1. \end{cases} \tag{13}$$

The inverse demand that is faced by producers in period one is then

$$P_1(Q_1) = \begin{cases} a_1 - Q_1 & \text{if } 0 \leq Q_1 \leq Q_L \\ \delta\beta \int_{Q_1}^{A} (a - Q_1) dF(a) - \delta w & \text{if } 0 \leq Q_1 \leq \widehat{Q} \\ a_1 - \widetilde{X}(Q_1) - Q_1 & \text{if } \max[Q_L, \widehat{Q}] < Q_1. \end{cases} \tag{14}$$

We can see in (14) that producers only will care about second period demand to the extent that they are able to make sales in the second period: the expectation in the second line of (14) is only over values of a_2 that exceed Q_1 (which is the inventories carried in this case), and in (12) where the expectation is only for values of a_2 larger than $-X$.

The effects of uncertainty

In this subsection we elaborate on the effects of uncertain future demand on first period residual demand since randomness in demand can have significant effects on the behavior of imperfectly competitive producers. This is due to price being more dependent on demand under an oligopolistic market structure than under perfect competition. Therefore demand uncertainty matters more to an oligopoly than it does to a perfectly competitive industry.

The benchmark case without uncertainty[9] has $a_2 = E[a]$ with probability one. From (10) we see that the threshold at which speculators become active, Q_L, does not depend on any other characteristic of the distribution of a_2 other than $E[a]$, so $Q_L^o = Q_L$. However, the threshold at which speculators purchase the entire first period output (defined by $\delta p_2^*(\widehat{Q}^o, E[a]) - \delta w = a_1$) becomes

$$\widehat{Q}^o = -Q_L/(\delta\beta). \tag{15}$$

Comparing $\widehat{Q}^o$ with $\widehat{Q}$ we see that uncertainty increases the range of first period output over which consumers are squeezed out by speculators:

Lemma 2 $\widehat{Q}^o < \widehat{Q}$.

Proof. See appendix.‖

When $Q_L > 0$, the position of speculators when future demand is certain is determined by a simplified[10] version of (12):

$$a_1 - X - Q_1 = \delta\beta E[a] + \delta\beta X - \delta w \tag{16}$$

giving

$$X_1^o(Q_1) = \widetilde{X}^o(Q_1) = \frac{Q_L - Q_1}{1 + \delta\beta}. \tag{17}$$

Using the lemma above, we can prove:

Proposition 1 *Compared to the case of certain future demand, demand uncertainty leads to more inventories being carried between periods and causes inventories to be more sensitive to first period production.*

Proof. See appendix.‖

[9]We will denote variables in this case with a o superscript.

[10]The future price in this case is $p_2^* = \beta \max[E[a] + X, 0]$, but we ignore the possibility of a zero second period price as speculators would not wish to carry stocks if that were the case.

4 Effects of Speculation on Producer Behaviour

Using (13) and (14) we now describe how the addition of speculative storage affects the payoffs of producers. We will let $\Pi^i(q_1^i, Q_1^{-i})$ denote the total profit to firm i when it produces q_1^i and the other firms produce Q_1^{-i} in aggregate while incorporating the equilibrium of the second period game:

$$\Pi^i(q_1^i, Q_1^{-i}) = P_1(Q_1)q_1^i - \frac{c}{2}{q_1^i}^2 + \delta E_1[\pi_2^*] \tag{18}$$

with $Q_1 = q_1^i + Q_1^{-i}$. As speculators position $X_1^*(Q_1)$ and inverse demand $P_1(Q_1)$ are continuous, individual profit also continuous. The marginal profit of a firm choosing a level of output q_1^i in period one, where it exists, is equal to

$$\Pi_q^i(q_1^i, Q_1^{-i}) = P_1'(Q_1)q_1^i + P_1(Q_1) - c\ q_1^i + \delta\ \frac{\partial E_1[\pi_2^*]}{\partial Q_1} \tag{19}$$

where first period output affects expected second period profit because $H_2 = -X_1^*(Q_1)$, and $P_1'(Q_1)$ denotes the derivative of first period inverse demand with respect to the industry output Q_1. Profit is continuous but only piecewise differentiable, as noticed from the expression of inverse demands in period one. The effects of first period output choice on expected second period profit is summarized in Lemma 3.

Lemma 3 *A producer's expected second period profit is decreasing in aggregate first period production, Q_1, when speculators purchase in the first period:*

$$\frac{\partial E_1[\pi_2^*]}{\partial Q_1} = 2\gamma\ \frac{dX_1}{dQ_1} \int_{-X_1(Q_1)}^{A} (a + X_1(Q_1))dF(a) \leq 0 \tag{20}$$

where dX_1/dQ_1 is equal to -1 when speculators buy the entire production in first period, or to $d\widetilde{X}/dQ_1 \in (-1, 0)$ when both consumers and speculators buy in first period.

Proof. See appendix.‖

We are now in a position to describe the marginal profit to a producer's choice of first period output. When $a_1 \leq \delta(\beta E[a] - w)$ speculators are always active and consumers are excluded from the market when total output is lower than $\widehat{Q}$. The marginal profit of firm i for an output level $q_1^i \leq \widehat{Q} - Q_1^{-i}$ is equal to

$$\Pi_q^S(q_1^i, Q_1^{-i}) \equiv \delta(\beta - 2\gamma) \int_{Q_1}^{A} (a - Q_1)dF(a) - \delta w - \delta\beta(1 - F(Q_1))q_1^i - cq_1^i, \tag{21}$$

where we use an S superscript to denote case in which speculators purchase the entire first period output. If $q_1^i \geq \widehat{Q} - Q_1^{-i}$ marginal profit is equal to

$$\Pi_q^{CS}(q_1^i, Q_1^{-i}) \equiv a_1 - \widetilde{X}(Q_1) - Q_1 - \left(1 + \frac{d\widetilde{X}}{dQ_1}\right) q_1^i - cq_1^i \\ - \frac{2\gamma(a_1 - \widetilde{X}(Q_1) - Q_1 + \delta w)}{\beta(1 + \delta\beta(1 - F(-\widetilde{X}(Q_1)))}, \quad (22)$$

where we use a CS superscript to denote the case in which both consumers and speculators purchase in the first period.

When $a_1 \geq \delta(\beta E[a] - w)$ consumers are always active and speculators are excluded from the first period market when total output is lower than Q_L. The marginal profit of firm i for an output level $q_1^i \leq Q_L - Q_1^{-i}$ is equal to

$$\Pi_q^C(q_1^i, Q_1^{-i}) \equiv a_1 - 2q_1^i - Q_1^{-i} - cq_1^i, \quad (23)$$

where a C superscript denotes the case in which consumers purchase the entire first period. Finally, if $q_1^i \geq Q_L - Q_1^{-i}$ then the marginal profit is again given by equation (22). To summarize,

$$\Pi_q^i(q_1^i, Q_1^{-i}) = \begin{cases} \Pi_q^o(q_1^i, Q_1^{-i}) & \text{if } 0 \leq Q_1 \leq Q_L \\ \Pi_q^S(q_1^i, Q_1^{-i}) & \text{if } 0 \leq Q_1 \leq \widehat{Q} \\ \Pi_q^{CS}(q_1^i, Q_1^{-i}) & \text{otherwise.} \end{cases} \quad (24)$$

The behaviour of a producer's marginal profit across the kinks in the inverse demand curve is important for our analysis of the equilibria of this game. We establish the following result.

Lemma 4 *The marginal profit of producer i, $\Pi_q^i(q_1^i, Q_1^{-i})$,*

(i) jumps up at the output level such that consumers start to buy the product, $q_1^i = \widehat{Q} - Q_1^{-i}$, whenever this output level exists and is positive;

(ii) jumps up or down at the output level such that speculators start to buy the product, $q_1^i = Q_L - Q_1^{-i}$, whenever this output level exists and is positive.

Proof. See appendix.‖

The consequences of this lemma are the following. First, as is well known, the possibility of upward jumping marginal profit may generate multiple local solutions to the firm's optimization problem. Second, the possibility of an downward jump at $q_1^i = Q_L - Q_1^{-i}$ suggests that an equilibrium may exist in which all producers choose output levels such that the industry output is equal to Q_L. In this case speculators do not buy the product and the first period price is equal to $\delta(\beta E[a] - w)$. As Mitraille and Thille (2009) showed for a monopoly, this limit outcome is a possibility in an oligopoly as well.

We now analyze the shape of the best-response functions in the regions where speculators buy and store the product in order to clearly see the strategic effects of speculators' behavior. Applying the implicit function theorem to the first order condition $\Pi_q^i(q_1^i, Q_1^{-i}) = 0$ (where the derivative exists) gives the slope of the reaction function

$$dq_1^i/dQ_1^{-i} = -\Pi_{qQ}^i(q_1^i, Q_1^{-i})/\Pi_{qq}^i(q_1^i, Q_1^{-i}), \tag{25}$$

where $\Pi_{qq}^i(q_1^i, Q_1^{-i})$ indicates the derivative of the marginal profit of firm i with respect to its own output, and $\Pi_{qQ}^i(q_1^i, Q_1^{-i})$ indicates the derivative of firm i marginal profit with respect to competitors output Q_1^{-i}. This slope differs depending whether speculators buy the entire production Q_1 or not, and may be positive. If positive, producers output choices are strategic complements which is an unexpected feature in a game of Cournot competition. We first establish the lemma and then discuss this issue.

Lemma 5 *The slope of the best response of each producer to competitors output when speculators buy in period one are given by:*

(i) when only speculators buy the product,

$$\frac{dq_1^S}{dQ_1^{-i}} = -\frac{\Pi_{qQ}^S}{\Pi_{qQ}^S - \delta\beta(1 - F(Q_1)) - c},$$

where

$$\Pi_{qQ}^S = \delta(2\gamma - \beta)(1 - F(Q_1)) + \delta\beta f(Q_1) q_1^i$$

(ii) when consumers and speculators buy the product is

$$\frac{dq_1^{CS}}{dQ_1^{-i}} = -\frac{\Pi_{qQ}^{CS}}{\Pi_{qQ}^{CS} - c - \left(1 + \frac{d\widetilde{X}}{dQ_1}\right)},$$

where

$$\Pi_{qQ}^{CS} = \left(1 + \frac{d\widetilde{X}}{dQ_1}\right)\left[-1 + \frac{2\gamma}{\beta(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))}\right] + \frac{\delta\beta f(-\widetilde{X}(Q_1))}{(1+\delta\beta(1-F(-\widetilde{X}(Q_1)))^3}\left[q_1^i - 2\frac{\gamma}{\beta}(P_1(Q_1) + \delta w)\right]$$

Proof. See appendix.∥

As the first terms in Π^S_{qQ} and Π^{CS}_{qQ} are negative, the sign of these expressions will depend on the second terms in each of them. Consider first Π^S_{qQ}: clearly the second term is positive, and if it is large then this may very well force the marginal profit of each individual firm to increase with the output of its competitors, in contrast to the standard Cournot model. The sign of Π^S_{qQ} depends on i) the effect of an increase in output on the likelihood of facing a zero price in second period, $f(Q_1)$, ii) the size of q^i_1, and iii) the number of competitors through the effect of n on β. On the other hand Π^S_{qq} is equal to Π^S_{qQ} plus a negative term containing the slope of the marginal cost c. It c is large enough, this may be negative for some distributions $F()$ and model parameters. Consequently the slope of the reaction function may be positive for some level of output Q^{-i}_1, and negative for others, depending on the sign of Π^S_{qQ}. From the discussion above, this may be more likely for a small number of competitors and a density function, $f()$, large for specific values of a. The economic intuition is the following: when increasing their output, a firm's competitors increase the likelihood of getting a zero price in second period. In that case a firm is better off selling today at a positive price (proportional to the expectation of the second period price) rather than waiting, and will follow its competitors by increasing its output. Searching to sell at a positive price results in local strategic complementarity.

Similar logic can be applied to case (*ii*) of the lemma. However an upward sloping best-response function is less likely to occur in this case as the second term of Π^{CS}_{qQ} contains an additional negative term.

The effects of uncertainty

Uncertain future demand has a non-trivial effect on best-response functions and marginal profit due to the possibility that the sale of inventories in the second period might drive price to zero. This effect is seen in Lemma 5 by the terms involving $1 - F(Q_1)$ or $1 - F(\widetilde{X}(Q_1))$ which measure the probability that demand will be high enough for speculators to be unable to drive price to zero.

In the absence of future demand uncertainty:

$$\frac{\partial \pi^*_2}{\partial Q_1} = 2\gamma \frac{dX_1}{dQ_1} \left(E[a] + X_1(Q_1)\right) \tag{26}$$

Comparing this with expected second period marginal profit we have

Proposition 2 *Expected second period marginal profit of an oligopolist is smaller under demand uncertainty compared to the certainty case when speculators store and consumers buy the product ($Q_1 > Q_L$ or $Q_1 > \widehat{Q}$), or when only speculators store with and without uncertainty ($Q_1 \leq \widehat{Q}^o$).*

Proof. See appendix.‖

Proposition 3 *Future demand uncertainty increases the marginal profit of an oligopolist compared to the no uncertainty case when speculators store and consumers do not buy the product ($Q_1 \leq \widehat{Q}^o$).*

Proof. See appendix.‖

The consequence of this proposition is that whenever a consumer squeeze equilibrium exists, the individual output of each producer is larger with future demand uncertainty than without. As seen in Lemma 4, this may also result in strategic complementarity.

5 Analysis of Equilibrium

We now turn to an analysis of the equilibria of this game, outlining regions of the parameter space in which the various equilibria occur. It is important to keep in mind that the concavity of the firms' profit functions is not granted: indeed when speculators are active, the linkage of the period one price to the period two price leads to the period one price being convex in Q_1. This raises the possibility that a firm's payoff function is not concave. The results we present in this section are conditional on the assumption that the equilibrium period one price is not so convex that it renders payoffs convex. A sufficient condition for this to be the case is that the density of the demand intercept, $f(a)$, cannot be too large for any a. We illustrate below that this is indeed the case when demand follows a uniform distribution. Given this assumption, we first present results for a general $F(a)$ and then follow that with a computed example with a uniform distribution for a.

5.1 Equilibrium

We first describe the various equilibria that may occur and then state the main theorem. First, the Cournot equilibrium, which we label C, obtains for the obvious case in which speculators entry is blockaded: if the model parameters are such that the second period expected price without storage, $\beta E[a]$, is lower than the unit cost of storage, w, then speculators never find it profitable to store regardless of the level of demand in first period. In this situation the unique symmetric Nash equilibrium is the Cournot outcome, with individual producer output of

$$q_C = \frac{1}{n}(1-\beta)a_1. \tag{27}$$

Note that as n increases, β falls, so for $w > 0$ there is a threshold number of producers above which the Cournot equilibrium is unique outcome for any value of a_1.

If $\beta E[a] > w$, then speculators might be active. However, the Cournot outcome can still be an equilibrium if i) speculators do not wish to purchase at the Cournot price and, ii) a firm has no incentive to increase output to a level that induces purchases by speculators when all other firms are producing q_C. This will be the case if $\Pi_q^o(q_1^i, Q_1^{-i}) < 0$ at the threshold where speculators enter and no individual producer can earn higher profits by increasing production to sell to speculators.

It is also possible that firms' choose outputs below the Cournot level in order to ensure that speculators do not purchase anything in the first period. We say that speculation is *deterred* in this case. Let $(q_1^{1*}, ..., q_1^{n*})$ denote the n-tuple of individual output such that this occurs, then speculation is deterred if

$$\sum_{i=1}^{n} q_1^{i*} = a_1 - \delta(\beta E[a] - w). \tag{28}$$

In this case the market price in first period is equal to $\delta(\beta E[a] - w)$, which is the minimum price consistent with zero storage. This will happen if $\Pi_q^o(q_1^i, Q_1^{-i}) > 0$ and $\Pi_q^{CS}(q_1^i, Q_1^{-i}) < 0$ at the equilibrium individual outputs produced by all firms. We will use an L subscript to denote this equilibrium.

The outcome involves smoothing if speculators are active and link prices between the two periods. This can happen with and without consumers active in the first period market. We use the label CS to denote an equilibrium in which both speculators and consumers purchase the good in the first period. Individual firm output, q_{CS}, is the solution of

$$\Pi_q^{CS}(q_{CS}, (n-1)q_{CS}) = 0, \tag{29}$$

which exists only if $\Pi_q^{CS}(q_1^i, Q_1^{-i}) > 0$ at the threshold at which both consumers and speculators wish to buy. Depending on the model parameters this threshold is either Q_L or $\widehat{Q}$. In addition, no individual producer has an incentive to reduce production in order to drive the price up, causing speculative purchases to go to zero.

We denote the potential equilibrium in which consumers are excluded with an S label. In this equilibrium firms produce q_S, which is the solution to

$$\Pi_q^S(q_S, (n-1)q_S) = 0. \quad (30)$$

A solution exists if $\Pi_q^S(q_1^i, Q_1^{-i}) < 0$ at the threshold at which consumers enter, $\widehat{Q} - (n-1)q_S$. In addition, this equilibrium requires that no individual firm has an incentive to increase production in order to induce sales to consumers.

Finally, we allow for the possibility that first period output is zero and denote this equilibrium with a NP subscript. Firms may find it preferable to produce nothing than to sell to speculators. This will occur if $\Pi_q^S(q_1^i, Q_1^{-i}) <$ 0 negative when production is zero. Since marginal profit jumps upward at $\widehat{Q}$, the NP equilibrium requires $\Pi_q^{CS}(\widehat{Q}, 0) < 0$, which ensures that marginal profit is negative for all levels of output.

The following theorem establishes that each one of these potential equilibria is indeed possible.

Theorem 1 *If marginal profit is well behaved, in the sense that (29) and (30) have solutions, then there exist parameter values for which any of the equilibria C, L, CS, S, and NP obtain.*

Proof: See appendix.‖

In the proof of Theorem 1 we demonstrate existence of the different equilibria without fully characterizing when each equilibria occurs. However, we can say something about the circumstances under which the various equilibria obtain, in doing so we will focus our attention on values for the first period demand intercept, a_1, relative to the other parameters under which each equilibrium occurs.

The condition $\beta E[a] - w \leq 0$ can be interpreted as a condition on the intensity of competition: as n increases, β falls, so for $w > 0$ there is a threshold number of producers above which the Cournot equilibrium is unique. Consequently if competition on the second period market is intense enough to induce the expected marginal return of speculation to be lower than the marginal cost of storage, then speculative storage is never profitable and the unique equilibrium is the Cournot outcome in both periods. Speculators entry is blockaded in that case.

The static Cournot equilibrium occurs when first period demand is high enough, while the limit equilibrium Q_L occurs for intermediate-high values of first period demand as the proof of Theorem 1 shows. In the limit equilibrium, first period price is precisely that at which speculators would obtain zero profits upon entry. The benefit to a firm from increasing output in order to make sales to speculators is insufficient to offset the loss it incurs due to the addition of speculative competition in the second period. Interestingly, this equilibrium occurs only in the case where the static Cournot price in first period is low enough to allow speculation to be profitable, $\beta a_1 \leq \delta(\beta E[a] - w)$: to deter speculation, oligopolists choose to reduce their output compared to static Cournot competition to force the price to remain at a level higher than the expected second period price net of the cost of storage.

The equilibrium in which consumers purchase and speculators store the product, (CS), occurs for intermediate-low values of first period demand, that is for a_1 around the threshold $\delta(\beta E[a] - w)$. Most interestingly, it can occur for values of a_1 such that the static Cournot price in first period, βa_1, would be larger than the threshold $\delta(\beta E[a] - w)$, that is $a_1 \geq \delta E[a] - \delta w/\beta$. In that case although the Cournot equilibrium would be protected from speculation, oligopolists increase their production to force the first period price down compared to the second period expected price in order to sell to speculators.

Note that multiple equilibria of the C and the CS type are possible. Indeed it suffices that the marginal profit is upward jumping around Q_L and that no profitable individual deviations from q_C to a larger output or from q_{CS} to a smaller output are simultaneously possible.

The equilibrium with exclusion of consumers from the first period market, S, occurs when the intercept of first period demand, a_1, is low. Indeed as competitive speculators are ready to pay an amount based on the expected second period price for the oligopolists production, consumers may be outbid on the first period market and therefore excluded when their willingness to pay driven by a_1 is low.

Finally the equilibrium in which no production takes place, NP, occurs if consumers first period demand is low and the expected second period demand is also low. In this case payoffs gained from selling to speculators in the first period are more than offset by the loss of payoffs due to the increased speculative supply in the second period. Since consumer demand is also low, producers prefer to produce nothing.

5.2 Implications for the distribution of price and welfare

We now address a couple of questions pertaining to the equilibrium outcomes. First, what is the effect of speculation on the distribution of price in an imperfectly competitive market? Second, what is the effect of speculation on consumers surplus and social welfare compared to the situation where speculators are absent?

5.2.1 Price distribution

We wish to examine the implications of this analysis for the ex-ante first and second moments of first period price and for the correlation between first and second period price. By "ex-ante", we are thinking about the distribution of first period price prior to knowledge of a_1. Our two period model is admittedly not ideal for examining the effects of speculation on the distribution of price as the analysis is conditional on zero initial stocks, and so cannot account for longer term inventory dynamics. However, it does provide some insight into the type of effects that speculation can have on oligopoly price distributions.

Whether p_1^* is above or below the Cournot price depends on which equilibrium obtains. If the equilibrium is of type L, S, or NP, p_1^* exceeds the Cournot price, while if it is of type CS, p_1^* is lower than the Cournot price. In terms of variance, when p_1^* is higher than the static Cournot price, equilibria of type S, NP or L, p_1^* is independent of a_1 and therefore does not contribute to the ex-ante variance. When the p_1^* is strictly lower than the static Cournot price (the CS equilibrium), computing the variance also leads to a smaller contribution to the ex-ante variance than in the Cournot equilibrium. We summarize this in the following corollary

Proposition 4 *Compared to static Cournot competition, the presence of speculators:*

(i) can increase or decrease the ex-ante mean of the price p_1^* depending on the distribution of the demand intercept, $F()$,

(ii) unambiguously decreases the ex-ante variance of the price p_1^*.

The relationship between p_1^* and p_2^* is also dependent on which equilibrium type obtains. When either CS or S is the equilibrium, prices are tied together due to speculation, $p_1^* = \delta(E[p_2^*] - w)$. As we demonstrate below with an example, the likelihood of the region in which prices are tied together between periods can be larger when competition is imperfect than when it is perfect or when there is a monopoly. This suggests that speculators may be more active under imperfectly competitive production than under perfectly competitive production. This finding is interesting in relation to Deaton and Laroque (1992), (1996), who amongst other results identify more price autocorrelation in their data than what their theoretical model predicts. Our results suggest that one possible explanation, realistic for many commodity markets, lies in the fact that competition is imperfect.

Finally, the effect of the number of firms on how speculation affects the equilibrium is of interest. Consider a situation with consumers absent from the market in period one, $a_1 = 0$. This situation, even if it occurs with zero probability under our assumption on demand, allows us to clarify the importance of speculators for producers to allow them compete for future market share. In this case the inverse demand in period one comes from speculators only,

$$P_1(Q_1) = \delta(E_1[p_2^*(-Q_1, a_2)] - w) \tag{31}$$

and the effect of an increase of first period production on the second period expected profit is equal to $\delta \frac{\partial}{\partial Q_1} E_1[\pi_2^i(Q_1)] = -2\delta \ \frac{\gamma}{\beta} \ E_1[p_2^*(-Q_1, a_2)]$ so marginal total profit is

$$\delta E_1[p_2^*(-Q_1, a_2)] - \delta w + \delta \frac{\partial}{\partial Q_1} E_1[p_2^*(-Q_1, a_2)] q_1^i - c \, q_1^i - 2\delta \ \frac{\gamma}{\beta} \ E_1[p_2^*(-Q_1, a_2)].$$

To characterize the incentive of producers to sell output to speculators, consider the case of an aggregate output of the industry equal to 0. In this case the marginal profit of producer i is equal to

$$\delta E_1[p_2^*(0, a_2)] - \delta w - 2\delta \ \frac{\gamma}{\beta} \ E_1[p_2^*(0, a_2)] = \delta \left((\beta - 2\ \gamma) \ E[a] - w \right) \tag{32}$$

Note that

$$\beta - 2\gamma = \frac{n(1+c) + c^2 + c - 1}{(n+1+c)^2} \geq 0, \tag{33}$$

with equality when there is only one firm and $c = 0$. Speculators are therefore of no value to a monopolist with zero cost of production with zero consumer demand: the marginal profit of a monopolist facing no demand in first period is non-positive when its cost of production is nil. But this is not the case in oligopoly! For $n > 1$, even if $c = 0$, the difference $\beta - 2\gamma$ is strictly positive. Speculators are valuable to oligopolists even with zero consumer demand and zero cost of production. The main reason for this is that they allow oligopolists to compete in advance for next period sales by selling to speculators in the first period. Moreover this effect is non-monotonic in n.[11]

5.2.2 Welfare

We now turn to an analysis of the welfare implications of speculation in a Cournot market. To compute discounted social welfare, we need to aggregate the discounted sum of consumers surplus, producers profits and speculators payoffs across the two periods, taking the expectation of second period payoffs with respect to the demand intercept a_2. Note that, although speculators payoffs do contribute to second period welfare if the realized market price exceeds their marginal cost of storage, they do not contribute to discounted social welfare when aggregating across the two periods. Indeed given their equilibrium position in period one, X_1^*, speculators payoffs are equal to $(p_1^* - \delta E(p_2^*) + \delta w)X_1^*$. Either a stock-out occurs, $X_1^* = 0$, or speculators store and resell in second period, but in that case $p_1^* = \delta E(p_2^*) - \delta w$. In both cases their discounted expected payoff is zero. Consequently, we can ignore speculative profits in our welfare analysis.

In the cases where speculators are inactive and do not store in the first period (C, L and NP), consumers surplus and producers profits are unaffected by speculation in the second period. In first period, either the quantity produced is equal to the Cournot outcome (C), or it is strictly lower (L, and NP). We deduce immediately:

Proposition 5 *When speculation is possible but no stocks are carried into the second period, the present value of consumer surplus and producer profits are no higher than and can be lower than the values they obtain under Cournot competition in the absence of speculation.*

The presence of speculators, even if they are inactive, can be detrimental to social welfare.

In the cases where speculators are active in period one (outcomes CS and S), consumer surplus increases in the second period. Indeed the total quantity sold on the market is equal to the sum of speculators position and producers output, $-X_1^* + Q_2^*(-X_1^*, a_2)$, which always exceeds the static Cournot output.[12] Consequently, second period consumer surplus with active speculators exceeds that without active speculators. Producer profits

[11] For $c = 0$, $\beta - 2\gamma = (n-1)/(n+1)^2$ reaches a maximum for $n = 3$ and stays above $1/9$ (its value when $n = 2$) for n between 2 and 5. It decreases to 0 when n goes to infinity.

[12] From (4), producers reduce their output by a factor $1 - \beta$ when speculators increase their first period inventories by 1 unit, and this factor is strictly lower than 1 as long as the number of producers is finite (see Definition 1).

are clearly reduced by the release of speculative inventories on the market in the second period relative to profits in the absence of speculators. Consequently, speculation can result in either an increase or a decrease in expected second period total surplus.

We can now turn our attention to the first period. In the case where consumers are excluded by speculators on the market (S), consumers surplus is clearly reduced compared to Cournot competition. The loss in consumer surplus is precisely the consumer surplus available under the Cournot equilibrium. On the other hand producer profits are larger than their Cournot static counterpart: the equilibrium quantity is larger and the price is higher. Again which effect dominates, the gain in producers profits or the loss in consumers surplus, depends on particular values of the model parameters.

When consumers can purchase the product and speculators store in first period (CS), consumers surplus increases due to speculation since the market price is lower than the static Cournot price. Consumers purchase a larger quantity at a lower price and consequently enjoy a larger surplus. Producer profit can increase or decrease compared to static Cournot competition. To summarize, we have established:

Proposition 6 *When speculators carry stocks the net effect on welfare is ambiguous:*

(i) in the CS equilibrium, consumer surplus increases in both periods, while producer profits can increase or decrease in period one and decrease in period two,

(ii) in the S equilibrium, consumers surplus decreases in period one and increases in period two, and producers profits increase in period one and decrease in period two.

In both cases, depending on the distribution $F()$ and the model parameters, the discounted social welfare computed across the two periods may increase or decrease compared to Cournot competition.

5.3 Example

We now use a numerical example to illustrate when the various equilibria described in Theorem 1 occur. For this example we assume that a_2 is uniformly distributed over $[0, 20]$, and choose $w = 0.3$, $\delta = 0.95$, and $c = 0.2$. These parameters are chosen largely since they result in a fairly clear depiction of the possibility that any of the five equilibria are possible. We solve the model for a range of values for n and a_1, in particular, for $n = 1, 2, ..., 40$ and for 500 equally spaced values for $a_1 \in (0, 20]$. It is straightforward to show that with uniform demand and these parameter values a firm's marginal profit is decreasing everywhere apart from possibly at the thresholds, so finding the zeros of the marginal profit functions is relatively straightforward.

The solution is computed by first determining whether a particular type of equilibrium is possible by checking for zeros of (21), (22), and (23) while imposing that all firms produce the same output. Second, we ensure that the resulting output lies in the correct interval for the candidate equilibrium, i.e., relative to Q_L or $\widehat{Q}$. Finally, when there is a possibility of an upward jump in marginal revenue we check whether we in fact have an equilibrium by checking that there are no profitable deviations. With a uniform distribution for a_2, $\widehat{Q}$ and $\widetilde{X}(Q_1)$ can be found analytically and the only numerical solution required is to find output when smoothing occurs, i.e. for the CS and S equilibria.

We illustrate the nature of the equilibrium for each (n, a_1) pair in Figure 1, color coded to denote the type of equilibrium. The Cournot equilibrium (C), colored in blue and pink, is the most common equilibrium. It is the unique equilibrium (blue) for "large" first period demand or large numbers of firms (39 or more). The smoothing equilibrium (CS), colored in red, pink and yellow, is the next most common, occurring for values of a_1 that are relatively low (but not very low) when there are fewer than 38 firms. The red color illustrates the situations in which CS is the unique equilibrium. There are a substantial number of (n, a_1) pairs for which we have multiple equilibria of either C and CS (colored in pink). The consumer exclusion equilibrium (S), colored in orange and yellow, occurs for a fairly substantial range of parameters when the market is not very competitive. It is the unique equilibrium in the orange section. There is also a relatively small number of cases in which either S or CS can be the equilibrium (yellow). The (NP) equilibrium, colored in green, does occur in this example, but only for very low a_1 when there are 37 or 38 firms in the market and is rather hard to see. The limit equilibrium (L), colored in purple, occurs also when there are 37 or 38 firms.

To see the effects of storage costs and to demonstrate that the NP equilibrium need not be as rare as we see in Figure 1, in Figure 2 we plot the equilibria that obtain with a value of $w = 1.1$ and all other parameters the same as in Figure 1. Not surprisingly, equilibria with storage occur much less frequently. L and NP equilibria now occur for the monopoly.[13]

[13]This potential for the region where the NP equilibrium occurs to be non-monotonic

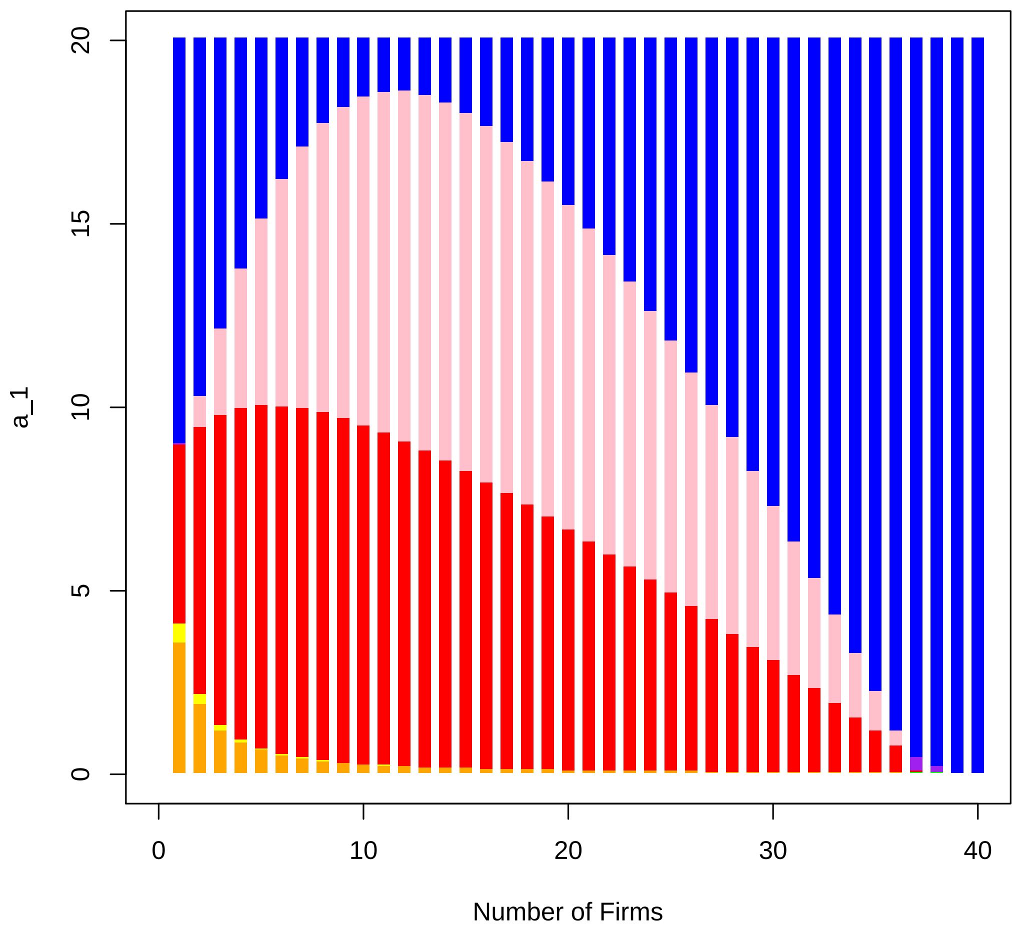

Figure 1: Depiction of alternative equilibria: Blue (C), Red (CS), Pink (C or CS), Orange (S), Yellow (S or CS), Green (NP), and Purple (L).

The monopolist chooses zero production for values of a_1 within roughly the bottom 20% of the distribution. With higher storage costs, the NP equilibrium supplants the S equilibrium for the monopolist compared the the situation in Figure 1. The reason for this is that when selling to speculators $p_1 = \delta(\beta E[a] - w)$. For relatively small w, sales are made at a price not much discounted from the expected second period price and so the monopolist does not mind selling to speculators instead of consumers when first period consumer demand is low. However, for larger storage costs, first period price must be discounted more relative to expected second period price and sales to speculators become less desirable. For w large enough the monopolist would rather make no sales in the first period than sell to speculators at this low price.

6 Conclusion

We have demonstrated that there exists a rich set of equilibria when competitive storage is allowed in a standard Cournot oligopoly setting. In addition to the standard price-smoothing effects of storage, the equilibrium can result in the complete shutdown of production in the first period or in the complete absence of consumers from the first period market. Consequently, the impact of competitive speculation on consumer surplus and on social welfare is ambiguous. These results were derived for a relatively general distribution of second period demand shocks, so we expect the results to be fairly robust.

Although we were able to discuss some results for the equilibrium distribution of prices, the two-period model employed limits the generality of those results as they are sensitive to the starting and ending conditions. Extension to a multi-period model is required to obtain more details about the equilibrium price and welfare distributions. However, we expect the qualitative nature of the results to survive to a setting with a long time horizon: speculators' behavior will affect firms' residual demands in the same manner as it has done in this paper.

can be seen from (32) and (33) which form one of the conditions defining the NP region. It is quadratic in n and has two roots for the parameters underlying Figure 2.

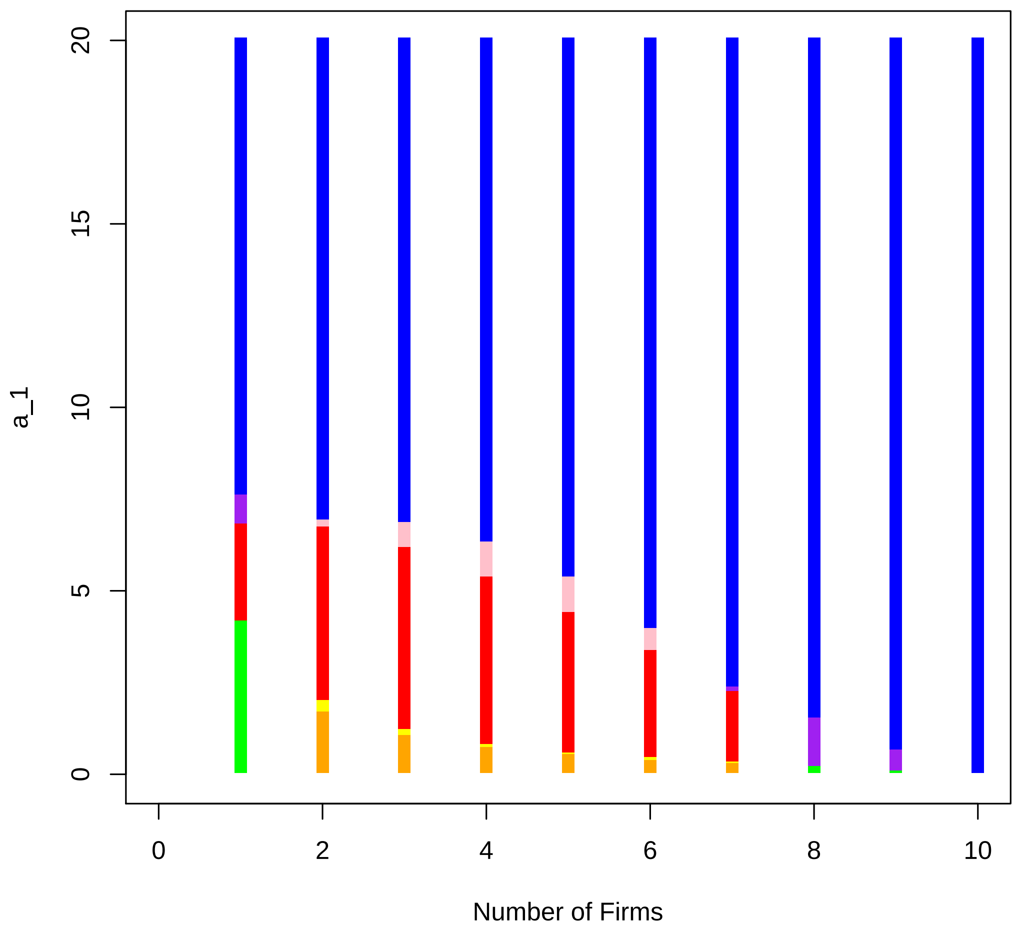

Figure 2: Depiction of alternative equilibria: Blue (C), Red (CS), Pink (C or CS), Orange (S), Yellow (S or CS), Green (NP), and Purple (L).

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Appendix

Proof of lemma 1

The uniqueness of Q_L is obvious from (10). The uniqueness of $\widehat{Q}$ is straightforward as the left-hand side of (11) is strictly decreasing in $\widehat{Q}$, equal to $\delta(\beta E[a] - w) - a_1 > 0$ for $\widehat{Q} = 0$ and equal to $-\delta w - a_1 < 0$ for $\widehat{Q} = A$.

To prove the uniqueness of $\widetilde{X}(Q_1)$, we first establish a bound to the aggregate quantity of inventories carried into the second period. Define $-X^0$ to be the quantity of inventories held that causes the expected second period revenue from speculation to be 0. Clearly, speculators would not carry more inventory than $-X^0$. To find X^0 we define

$$g_0(X) = \beta \int_{-X}^{A} (a + X)\ dF(a) - w, \tag{34}$$

for $X \in (-A, 0)$. We have $g_0'(X) = \beta(1 - F(-X)) > 0$, $g_0(0) = \beta E[a] - w > 0$, and $g_0(-A) = -w < 0$. Hence $g_0(X) = 0$ defines a unique X^0. We define another function,

$$g(X, Q_1) = a_1 - X - Q_1 - \delta \left(\beta \int_{-X}^{A} (a + X)\, dF(a) - w \right) \tag{35}$$

where the domain on which g is defined changes depending on which quantity amongst Q_L and $\widehat{Q}$ is positive. Equation (12) is equivalent to $g(X, Q_1) = 0$, which defines $\widetilde{X}(Q_1)$. The function g is continuous and differentiable in both X and Q, and decreasing in X given Q_1. Indeed:

$$\frac{\partial g}{\partial X}(X, Q_1) = -1 - \delta\beta(1 - F(-X)) < 0. \tag{36}$$

First consider the case in which speculators can be excluded from the market, $Q_L > 0$. In this case, $\widetilde{X} : [Q_L, \bar{Q}] \to [X^0, 0]$, where $\bar{Q} = a_1 - X^0$. As g is strictly decreasing in X, to prove that there exists a unique $\widetilde{X}(Q_1)$ it suffices to prove that $g(X^0, Q_1) \geq 0$ and $g(0, Q_1) \leq 0$. We have

$$\begin{aligned} g(X^0, Q_1) &= a_1 - X^0 - Q_1 - \delta\Big(\beta \textstyle\int_{-X^0}^{A} (a + X^0)\, dF(a) - w\Big) \\ &= a_1 - X^0 - Q_1 \end{aligned} \tag{37}$$

by definition of X^0. Since $Q_1 \leq \bar{Q} = a_1 - X^0$, clearly $g(X^0, Q_1) \geq 0$. Similarly,

$$\begin{aligned} g(0, Q_1) &= a_1 - Q_1 - \delta(\beta E[a] - w) \\ &= Q_L - Q_1 \end{aligned} \tag{38}$$

so we have $g(0, Q_1) \leq 0$ for $Q_1 \geq Q_L$. Hence, there is a unique value of $\widetilde{X}(Q_1)$ associated with each $Q_1 \in [Q_L, \bar{Q}]$.

Next consider the case in which consumers can be excluded from the market, $\widehat{Q} \geq 0$. In this case, $\widetilde{X} : [\widehat{Q}, \bar{Q}] \to [X^0, -\widehat{Q}]$. We now need to show $g(X^0, Q_1) \geq 0$ and $g(-\widehat{Q}, Q_1) \leq 0$ for a unique $\widetilde{X}(Q_1)$. The value for $g(X^0, Q_1) \geq 0$ is unchanged and

$$g(-\widehat{Q}, Q_1) = a_1 + \widehat{Q} - Q_1 - \delta\Big(\beta \int_{\widehat{Q}}^{A} (a - \widehat{Q})\, dF(a) - w\Big). \qquad (39)$$

Using $g(-\widehat{Q}, \widehat{Q}) = 0$ by the definition of $\widehat{Q}$ and the fact that $\frac{\partial g}{\partial Q_1}(X, Q_1) = -1 < 0$, clearly $g(-\widehat{Q}, Q_1) < 0$ for $Q_1 \in [\widehat{Q}, \bar{Q}]$. Hence, there is a unique value of $\widetilde{X}(Q_1)$ associated with each $Q_1 > [\widehat{Q}, \bar{Q}]$ when $\widehat{Q} > 0$.

Applying the Implicit Function Theorem to $g(X, Q_1) = 0$ gives immediately part (i) of the lemma.

Finally, to prove part (ii) of the Lemma, note that $Q_L = 0$ is equivalent to $a_1 = \delta(\beta E[a] - w)$ by (10). Replacing this value of a_1 in (11) gives

$$\delta\beta \int_{\widehat{Q}}^{A} (a - \widehat{Q}) dF(a) - \delta w - \delta(\beta E[a] - w) = 0$$

or

$$\int_{\widehat{Q}}^{A} (a - \widehat{Q}) dF(a) = E[a] = \int_{0}^{A} a dF(a),$$

hence, $\widehat{Q} = 0$. From it is clear that Q_L is strictly increasing with a_1, while it is straightforward to show that $\widehat{Q}$ is strictly decreasing with a_1. As we have just shown, $Q_L = \widehat{Q} = 0$ when $a_1 = \delta(\beta E[a] - w)$. Consequently $sign(\widehat{Q}) = -sign(Q_L)$ for $a_1 \neq \delta(\beta E[a] - w)$.

Proof of lemma 2

Total differentiation of (11) yields $\frac{d\widehat{Q}}{da_1} = -\frac{1}{\delta\beta(1-F(\widehat{Q}))} < 0$, so we have $\frac{d\widehat{Q}}{da_1} < \frac{d\widehat{Q}^o}{da_1} = -\frac{1}{\delta\beta} < 0$. When $a_1 = \delta\beta E[a] - \delta w$, $\widehat{Q} = \widehat{Q}^o = 0$. For $a_1 < \delta\beta E[a] - \delta w$ (i.e. such that $Q_L < 0$), $\widehat{Q} > \widehat{Q}^o$ as $\widehat{Q}$ is increasing at a faster rate as a_1 declines. Consequently $\widehat{Q} > \widehat{Q}^o$ for any a_1 such that $Q_L < 0$.‖

Proof of Proposition 1

Comparing i) of Lemma 1 with (17) we have that

$$\frac{d\widetilde{X}}{dQ_1} = -\frac{1}{1 + \delta\beta(1 - F(\widetilde{X}(Q_1)))} < \frac{d\widetilde{X}^o}{dQ_1}. \qquad (40)$$

We can then distinguish 4 cases:

- When $Q_1 \leq \widehat{Q}^o$, $X_1^o(Q_1) = X_1(Q_1) = -Q_1$, and both have the same sensitivity to an increase of output.
- When $\widehat{Q}^o \leq Q_1 \leq \widehat{Q}$ and $Q_L < 0$, then $X_1(Q_1) = -Q_1 < \widetilde{X}^o(Q_1) < 0$. Speculators buy less than first period output without uncertainty while they still buy Q_1 with demand uncertainty.
- When $Q_1 > \widehat{Q}$ and $Q_L < 0$, since $\widetilde{X}(Q_1)$ is negative, decreasing and steeper than $\widetilde{X}^o(Q_1)$, and since $\widetilde{X}^o(\widehat{Q}) > -\widehat{Q} = \widetilde{X}(\widehat{Q})$, we have $-Q_1 < \widetilde{X}(Q_1) < \widetilde{X}^o(Q_1) < 0$.
- When $Q_L > 0$, $\widetilde{X}(Q_L) = \widetilde{X}^o(Q_L) = 0$. For $Q_1 > Q_L$, since $\widetilde{X}(Q_1)$ is steeper than $\widetilde{X}^o(Q_1)$, we have $\widetilde{X}(Q_1) < \widetilde{X}^o(Q_1) < 0$, with equality at $Q_1 = Q_L$.

We can conclude that $X_1(Q_1)$ is always larger in absolute value under future demand uncertainty than when demand is certain.$\|$

Proof of lemma 3

The expected second period profit can be expanded as

$$E_1[\pi_2^*] = \gamma \left(\int_{-X_1^*(Q_1)}^{A} a^2 dF(a) + 2X_1^*(Q_1) \int_{-X_1^*(Q_1)}^{A} a dF(a) + (X_1^*(Q_1))^2 \int_{-X_1^*(Q_1)}^{A} dF(a) \right).$$

Its derivative with respect to Q_1 is equal to:

$$\begin{aligned} \frac{\partial E_1[\pi_2^*]}{\partial Q_1} &= \left(\gamma(-X_1^*(Q_1))^2 + 2\gamma X_1^*(Q_1)(-X_1^*(Q_1)) + \gamma(X_1^*(Q_1))^2\right)\frac{dX_1^*}{dQ_1} f(-X_1^*(Q_1)) \\ &+2\gamma\frac{dX_1^*}{dQ_1}\int_{-X_1^*(Q_1)}^{A} a dF(a) + 2\gamma\frac{dX_1^*}{dQ_1} X_1^*(Q_1)\int_{-X_1^*(Q_1)}^{A} dF(a), \end{aligned}$$

which simplifies to

$$\begin{aligned} \frac{\partial E_1[\pi_2^*]}{\partial Q_1} &= 2\gamma\frac{dX_1^*}{dQ_1}\left(\int_{-X_1^*(Q_1)}^{A} a dF(a) + 2\gamma X_1^*(Q_1)\int_{-X_1^*(Q_1)}^{A} dF(a)\right) \\ &= 2\gamma\frac{dX_1^*}{dQ_1} \int_{-X_1^*(Q_1)}^{A}(a + X_1^*(Q_1)) dF(a) \\ &= 2\frac{\gamma}{\beta}\frac{dX_1^*}{dQ_1} \frac{P_1(Q_1)+\delta w}{\delta} \\ &\leq 0 \end{aligned}$$

since dX_1^*/dQ_1 is equal to -1 when speculators buy the entire production in first period, or to $d\widetilde{X}/dQ_1 \in (-1, 0)$ when both consumers and speculators buy in first period.

Proof of lemma 4

(i) For $0 < q_1^i < \widehat{Q} - Q_1^{-i}$, $p_1 > a_1$ and consumer demand is zero, while if $q_1^i > \widehat{Q} - Q_1^{-i}$, $p_1 < a_1$ and both consumer and speculative demand is positive. Consequently, the individual firm's first period inverse demand flattens as the threshold $\widehat{Q} - Q_1^{-i}$ is crossed, which results in the firm's profit being less sensitive to increases in output for q_1^i larger than $\widehat{Q} - Q_1^{-i}$ than for smaller levels of output. Hence, marginal profit must take an upward jump at $q_1^i = \widehat{Q} - Q_1^{-i}$.

(ii) In this case the discontinuity in marginal profit occurs at $q_1^i = Q_L - Q_1^{-i}$. Only consumers buy the product when the output of firm i is lower than this value, and consequently the marginal profit at the left of the discontinuity is

$$\Pi_q^o(Q_L - Q_1^{-i}, Q_1^{-i}) = a_1 - (2+c)Q_L + (1+c)Q_1^{-i}. \tag{41}$$

Since $\widetilde{X}(Q_L) = 0$, marginal profit at the right of the discontinuity is

$$\Pi_q^{CS}(Q_L - Q_1^{-i}, Q_1^{-i}) = a_1 - Q_L - \left(1 + c - \frac{1}{1+\delta\beta}\right)(Q_L - Q_1^{-i}) - \frac{2\gamma(a_1 - Q_L + \delta w)}{\beta(1+\delta\beta)}, \tag{42}$$

and the difference across the discontinuity is equal to

$$\Pi_q^{CS}(Q_L - Q_1^{-i}, Q_1^{-i}) - \Pi_q^o(Q_L - Q_1^{-i}, Q_1^{-i}) = \frac{Q_L - Q_1^{-i} - 2\ \delta\gamma\ E[a]}{1+\delta\beta}. \tag{43}$$

Even if Q_1^{-i} is such that $Q_L - Q_1^{-i} > 0$, this difference may be larger or smaller than $2\ \delta\gamma\ E[a]$ and consequently the marginal profit may upward or downward jumping at $Q_L - Q_1^i$.

Proof of lemma 5

Denoting $P_1''(Q_1)$ the second derivative of first period inverse demand, the general expression of the derivative $\Pi_{qq}^i(q_1^i, Q_1^{-i})$ is given by:

$$\begin{aligned}\Pi_{qq}^i(q_1^i, Q_1^{-i}) &= P_1''(Q_1)q_1^i + 2P_1'(Q_1) - c \\ &+2\delta\gamma\left(\tfrac{d^2X_1^*}{d(Q_1)^2}\int_{-X_1^*(Q_1)}^{A}(a + X_1^*(Q_1))dF(a) + \left(\tfrac{dX_1^*}{dQ_1}\right)^2(1 - F(-X_1^*(Q_1)))\right)\end{aligned} \tag{44}$$

Similarly the the general expression of the derivative $\Pi^i_{qQ}(q^i_1, Q^{-i}_1)$ is given by:

$$\Pi^i_{qQ}(q^i_1, Q^{-i}_1) = P''_1(Q_1)q^i_1 + P'_1(Q_1)$$
$$+\delta 2\gamma \left(\frac{d^2 X^*_1}{d(Q_1)^2} \int_{-X^*_1(Q_1)}^{A} (a + X^*_1(Q_1))dF(a) + \left(\frac{dX^*_1}{dQ_1}\right)^2 (1 - F(-X^*_1(Q_1))) \right) \tag{45}$$

Consider first case (i) in which $X^*_1(Q_1) = -Q_1$. The first period price is decreasing and convex in Q_1:

$$P'_1(Q_1) = -\delta\beta(1 - F(Q_1)) \leq 0 \text{ and } P''_1(Q_1) = \delta\beta f(Q_1) \geq 0. \tag{46}$$

and (44) simplifies to

$$\Pi^S_{qq}(q^i_1, Q^{-i}_1) = 2\delta(\gamma - \beta)(1 - F(Q_1)) - c + \delta\beta f(Q_1)q^i_1, \tag{47}$$

and similarly

$$\Pi^S_{qQ}(q^i_1, Q^{-i}_1) = \delta(2\gamma - \beta)(1 - F(Q_1)) + \delta\beta f(Q_1)q^i_1. \tag{48}$$

Second, consider case (ii) in which $X^*_1(Q_1) = \widetilde{X}(Q_1)$. The derivatives $P'(Q_1)$ and $P''(Q_1)$ are equal to:

$$P'_1(Q_1) = -1 - \frac{d\widetilde{X}}{dQ_1} \leq 0 \text{ and } P''_1(Q_1) = -\frac{d^2\widetilde{X}}{d(Q_1)^2} \tag{49}$$

From lemma 1,

$$\frac{d^2\widetilde{X}}{d(Q_1)^2} = \frac{\delta\beta\frac{d\widetilde{X}}{dQ_1} f(-\widetilde{X}(Q_1))}{[1 + \delta\beta(1 - F(-\widetilde{X}(Q_1)))]^2} = -\frac{\delta\beta f(-\widetilde{X}(Q_1))}{[1 + \delta\beta(1 - F(-\widetilde{X}(Q_1)))]^3} \tag{50}$$

which is negative. We have

$$2\gamma\frac{d^2\widetilde{X}}{d(Q_1)^2}\int_{-\widetilde{X}(Q_1)}^{A}(a + \widetilde{X}(Q_1))dF(a) + 2\gamma\left(\frac{d\widetilde{X}}{dQ_1}\right)^2(1 - F(-\widetilde{X}(Q_1)))$$
$$= 2\gamma\frac{d^2\widetilde{X}}{d(Q_1)^2}\frac{a_1 - \widetilde{X}(Q_1) - Q_1 + \delta w}{\delta\beta} + 2\gamma\left(\frac{d\widetilde{X}}{dQ_1}\right)^2\frac{(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))\left(1+\frac{d\widetilde{X}}{dQ_1}\right)}{\delta\beta} \tag{51}$$

and consequently (44) reduces to

$$\Pi^{CS}_{qq}(q^i_1, Q^{-i}_1) = 2\left(1 + \frac{d\widetilde{X}}{dQ_1}\right)\left[-1 + \frac{\gamma}{\beta(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))}\right] - c$$
$$+\frac{\delta\beta f(-\widetilde{X}(Q_1))}{(1+\delta\beta(1-F(-\widetilde{X}(Q_1)))^3}\left[q^i_1 - 2\frac{\gamma}{\beta}(P_1(Q_1) + \delta w)\right] \tag{52}$$

Similarly we have:

$$\begin{aligned}\Pi^{CS}_{qQ}(q^i_1, Q^{-i}_1) = &\left(1 + \tfrac{d\widetilde{X}}{dQ_1}\right)\left[-1 + \tfrac{2\gamma}{\beta(1+\delta\beta(1-F(-\widetilde{X}(Q_1))))}\right] \\ &+ \tfrac{\delta\beta f(-\widetilde{X}(Q_1))}{(1+\delta\beta(1-F(-\widetilde{X}(Q_1)))^3}\left[q^i_1 - 2\tfrac{\gamma}{\beta}(P_1(Q_1)+\delta w)\right].\end{aligned} \tag{53}$$

Proof of Proposition 2

Consider the different cases:

- If $Q_1 < \widehat{Q}^o$, $X^o_1(Q_1) = X_1(Q_1) = -Q_1$, so $\frac{\partial E\pi^*_2}{\partial Q_1} < \frac{\partial \pi^*_2}{\partial Q_1}$ if

  $$-2\gamma \int_{Q_1}^{A} (a - Q_1) dF(a) < -2\gamma \left(E[a] - Q_1\right). \tag{54}$$

  Re-arranging yields

  $$\int_0^{Q_1} (Q_1 - a) dF(a) > 0 \tag{55}$$

  which is true since $Q_1 - a \geq 0$ in this situation.

- If $Q_1 > \widehat{Q}$ or if $Q_L > 0$, then $X^o_1(Q_1) = \widetilde{X}^o(Q_1)$ and $X_1(Q_1) = \widetilde{X}(Q_1)$. In this case the second period price must be equal to $p_1 + \delta w$, and therefore

  $$\frac{\partial E\pi^*_2}{\partial Q_1} = 2\gamma \frac{d\widetilde{X}}{dQ_1} \int_{-\widetilde{X}}^{A} (a + \widetilde{X}) dF(a) = -2 \frac{\gamma\left(a_1 - \widetilde{X}(Q_1) - Q_1 + \delta w\right)}{\beta\left(1 + \delta\beta(1 - F(-\widetilde{X}(Q_1)))\right)} \tag{56}$$

  and

  $$\frac{\partial \pi^*_2}{\partial Q_1} = 2\gamma \frac{d\widetilde{X}^o}{dQ_1}\left(E[a] + \widetilde{X}^o(Q_1)\right) = -2 \frac{\gamma\left(a_1 - \widetilde{X}^o(Q_1) - Q_1 + \delta w\right)}{\beta\left(1 + \delta\beta\right)} \tag{57}$$

  and we have $\frac{\partial E\pi^*_2}{\partial Q_1} < \frac{\partial \pi^*_2}{\partial Q_1}$ if

  $$(1 + \delta\beta)\left(\widetilde{X}^o(Q_1) - \widetilde{X}(Q_1)\right) + \delta\beta\widetilde{F}\left(a_1 - \widetilde{X}^o(Q_1) - Q_1 + \delta w\right) > 0, \tag{58}$$

  where $\widetilde{X} < \widetilde{X}^o < 0$ implies that the first term of this last equation is positive, and where the second term, equal to $p_1 + \delta w$, is also positive. Consequently the inequality holds.

We have therefore established the result.‖

Proof of Proposition 3

When $Q_1 < \widehat{Q}^o$ the marginal profit under certainty is equal to:

$$\Pi_q^{So}(q_1^i, Q_1^{-i}) = \delta(\beta - 2\gamma)\left(E[a] - Q_1\right) - \delta w - \delta\beta q_1^i - cq_1^i \tag{59}$$

where $-\delta\beta q_1^i < -\delta\beta(1 - F(Q_1))q_1^i < 0$ and $E[a] - Q_1 < \int_{Q_1}^{A}(a - Q_1)dF(a)$ as we have seen. Consequently since $\beta - 2\gamma \geq 0$ we have directly that $\Pi_q^S(q_1^i, Q_1^{-i}) > \Pi_q^{So}(q_1^i, Q_1^{-i})$ in the range of output $Q_1 < \widehat{Q}^o$. ‖

Proof of theorem 1

We prove the theorem by demonstrating for each candidate equilibrium that a non-empty set of parameters exists for which the candidate is indeed an equilibrium.

C **equilibrium** The Cournot equilibrium obtains trivially when $\beta E[a] - w \leq 0$. In this case Q_L exceeds a_1 and $\widehat{Q} < 0$ so that no profitable speculation can occur over the range of productions $[0, a_1]$. For the case in which speculation is not blockaded, we use the necessary condition that $\Pi_q^o(Q_L - (n-1)q_C, (n-1)q_C) \leq 0$ must hold in a Cournot equilibrium. This condition is $a_1 - (2+c)Q_L + (1+c)(n-1)q_C \leq 0$, which upon substituting for Q_L and q_C we get

$$a_1 \geq \delta E[a] - \delta w/\beta. \tag{60}$$

In addition to this condition, a deviation to a higher level of output than q_C must not be profitable. A sufficient condition for this is that $\Pi_q^{CS}(Q_L - (n-1)q_C, (n-1)q_C) \leq 0$, from which we derive

$$a_1 \geq \frac{\left[((2+c)(1+\delta\beta) - 1)(\delta\beta E[a] - \delta w) - 2\delta\gamma E[a]\right] n(n+1+c)}{((1+c)(1+\delta\beta) - 1)(n^2+1+c)}. \tag{61}$$

Since the right-hand sides of both (60) and (61) are finite, there exists parameters and distribution function for a_2 that satisfies both (60) and (61) and a Cournot equilibrium exists.

L **equilibrium.** In order to demonstrate the existence of an L equilibrium, it is sufficient to examine the symmetric case ($q_i = Q_L/n \; \forall i$) as it is easy to show that the symmetric equilibrium exists if any asymmetric ones do. An L equilibrium exists if $\Pi_q^o(Q_L/n, (n-1)Q_L/n) \geq 0$, $\Pi_q^{CS}(Q_L/n, (n-1)Q_L/n) \leq 0$ and $\beta E[a] - w \geq 0$. The first condition reduces to

$$a_1 \leq \delta E[a] - \delta w/\beta \tag{62}$$

which is the complement to (60). The second condition reduces to

$$a_1 \geq \left[\frac{n(1+\delta\beta)}{\delta\beta + c(1+\delta\beta)} + 1\right] \delta(\beta E[a] - w) - \frac{2n\delta\gamma E[a]}{\delta\beta + c(1+\delta\beta)} \tag{63}$$

Conditions (62) and (63) can both hold if

$$w \geq \frac{(1+c)(n-1)}{(n+1+c)^2} E[a]. \tag{64}$$

The condition $\beta E[a] - w \geq 0$ requires

$$w \leq \frac{1+c}{n+1+c} E[a], \tag{65}$$

so an L equilibrium exists as these two constraints on w can simultaneously hold.

CS **equilibrium.** Consider $a_1 \leq \delta(\beta E[a] - w)$, in which case marginal profit jumps upward when total output equals $\widehat{Q}$, so q_{CS} is an equilibrium if $\Pi_q^S(\widehat{Q} - (n-1)q_{CS}, (n-1)q_{CS}) \geq 0$ or

$$a_1 - [\delta\beta(1 - F(\widehat{Q})) + c] \, (\widehat{Q} - (n-1)q_{CS}) - \frac{2\gamma(a_1 + \delta w)}{\beta} \geq 0. \tag{66}$$

For $a_1 = 0$ this condition cannot hold, while for $a_1 = \delta(\beta E[a] - w)$, $\widehat{Q} = 0$ and (66) reduces to $(\delta\beta + c)(n-1)q_{CS} + \delta(\beta - 2\gamma)E[a] - \delta w \geq 0$. As $\beta - 2\gamma \geq 0$, this condition will hold for w small enough, which is enough to insure that the region of parameters in which the CS equilibrium exists is not empty.

S **equilibrium.** When $a_1 \leq \delta(\beta E[a] - w)$ so that $\widehat{Q} > 0$, by Lemma 4 we know that marginal profit is upward jumping at $q_1^i = \widehat{Q} - Q_1^{-i}$. Consequently an S equilibrium will exist if $\Pi_q^S(0, (n-1)q_S) \geq 0$ and $\Pi_q^{CS}(\widehat{Q} - (n-1)q_S, (n-1)q_S) \leq 0$ (which is sufficient for $\Pi_q^S(\widehat{Q} - (n-1)q_S, (n-1)q_S) \leq 0$ as marginal profit jumps up at this point).

Using the definition of q_S and the fact that $\int_Q^A (a - Q)dF(a)$ decreases with Q, we have

$$\begin{aligned}\Pi_q^S(0,(n-1)q_S) &= \delta(\beta-2\gamma)\int_{(n-1)q_S}^{A}(a-(n-1)q_S)dF(a)-\delta w \\ &\geq \delta(\beta-2\gamma)\int_{nq_S}^{A}(a-nq_S)dF(a)-\delta w \\ &= \delta\beta(1-F(nq_S))q_S+cq_S \\ &> 0 \qquad (67)\end{aligned}$$

for $q_S > 0$. For q_S to be positive, it must be the case that $\Pi_q^S(0,0) > 0$, that is $\delta(\beta-2\gamma)E[a]-\delta w > 0$. As $\beta > 2\gamma$ this condition can be satisfied.

$\Pi_q^{CS}(\widehat{Q}-(n-1)q_S,(n-1)q_S) \leq 0$ if

$$a_1 - \left[\frac{\delta\beta(1-F(\widehat{Q}))}{1+\delta\beta(1-F(\widehat{Q}))}+c\right](\widehat{Q}-(n-1)q_S) - \frac{2\gamma(a_1+\delta w)}{\beta[1+\delta\beta(1-F(\widehat{Q}))]} \leq 0, \qquad (68)$$

which is clearly satisfied as a_1 tends to 0.

NP **equilibrium.** An NP equilibrium requires $\widehat{Q} > 0$ or, $a_1 \leq \delta(\beta E[a] - w)$. In addition, marginal profit must be negative for any output, so we require $\Pi_q^{CS}(\widehat{Q},0) \leq 0$ and $\Pi_q^S(0,0) \leq 0$. We have

$$\Pi_q^{CS}(\widehat{Q},0) = a_1 - \left(\frac{\delta\beta(1-F(\widehat{Q}))}{1+\delta\beta(1-F(\widehat{Q}))}\right)\widehat{Q} - c\widehat{Q} - \frac{2\gamma(a_1+\delta w)}{\beta(1+\delta\beta(1-F(\widehat{Q})))} \qquad (69)$$

so $\Pi_q^{CS}(\widehat{Q},0) \leq 0$ for a_1 small enough as the last three terms of this expression are negative.

$\Pi_q^S(0,0) = \delta((\beta-2\gamma)E[a]-w)$, so $\Pi_q^S(0,0) < 0$ if $\beta-2\gamma < w/E[a]$. At the same time, speculation must not be blockaded, so we require $w/E[a] < \beta$. As this condition does not depend on a_1, we have that NP can be an equilibrium for a_1 small and

$$\beta - 2\gamma < \frac{w}{E[a]} < \beta,$$

a condition which is feasible for some model parameters.