Dynamic Competition in Electricity Markets: Hydroelectric and Thermal Generation

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Abstract

We study competition between hydro and thermal electricity generators that face uncertainty over demand and water flows where the hydro generator is constrained by water flows and the thermal generator by capacity. We compute the Feedback equilibrium for the infinite horizon game and show that there can be strategic withholding of water by the hydro generator. When water inflow is relatively low, however, the hydro generator may use more water than efficient as it faces an inefficiently low shadow price of water in this case. The inefficiency of the market outcome is tempered by the capacity constraints: for a large range of possible thermal production capacities and water flow levels, welfare loss under the duopoly market structure is much less than would occur in the absence of water and capacity constraints.

Keywords: Electricity markets; Hydroelectricity; Imperfect Competition

JEL classification: L13, L94, C63, C73

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1 Introduction

A common feature of many electricity markets is the co-existence of a variety of generation technologies, such as hydro, nuclear and thermal (coal, oil, gas) generation. Of these technologies, one special characteristic of hydroelectric power generation is that it is constrained by the dynamics of the availability of water. Generating now implies less water availability in the next period, and hence less generation in the future. In some jurisdictions, hydroelectric power generation is the dominant source of electricity. It accounts for 80% of generation in New Zealand, 97% in Brazil, 90% in Quebec, and 98% in Norway (Crampes and Moreaux[4]). In other jurisdictions, such as Ontario and the Western United States, it is a significant source of electricity, but not as dominant. It is not uncommon to observe a large hydro competing with thermal generators. For example, in Honduras, large state-owned hydro generation facilities coexist with privately owned thermal generators.[1] Colombia has a similar structure, a large hydro operator with 64% of the installed capacity coexists with a thermal production sector with the rest of the capacity.[2] Consequently, an interesting situation from the point of view of the dynamics of competition is when a hydroelectric generator coexists with a thermal generator. Hydroelectric generation can be characterized by low marginal cost when operating, but subject to the availability of water to drive the turbines. In contrast, thermal generation units have more flexibility in the sense that their inputs (gas, coal, etc.) are not subject to the same constraints as water in a reservoir, however the marginal cost of generation is higher as generators need to purchase the fuel inputs.

Another common feature of restructured electricity markets is price volatility. One reason for the relatively high volatility of electricity prices is the inability to store electricity at a scale that would enable speculation to smooth prices. However, the ability to store water behind a hydro dam does allow for some degree of price smoothing. A hydro operator may benefit from withholding water in periods with low prices in order to have more available for use in periods with high prices.[3] In a perfectly competitive market, it is likely that the hydro operators would choose their water release in an efficient way. However, in most jurisdictions, hydroelectric generators tend to be rather large producers, in which case there is no guarantee that water will be released efficiently in an unregulated environment. We investigate this issue by a dynamic game between a hydro generator and a thermal generator.

[1] Installed generation capacities are approximately two-thirds hydro and one-third thermal in Honduras (see ENEE at www.enee.hn).

[2] See www.creg.gov.co.

[3] A recent paper by Crampes and Moreaux [5] examines the use of hydro reservoirs to speculate across daily peak and off-peak price fluctuations.

Comparing the equilibrium of this game to the efficient outcome allows us to discuss the potential for inefficient water use in an imperfectly competitive market for electricity.

In light of the fact that hydro producers have relatively large market shares in many jurisdictions has led some authors to examine the issue of the use of market power by hydro producers. The papers most relevant to our work are Bushnell [2] and Crampes and Moreaux [4] as they examine a Cournot setting in which the hydro producer behaves strategically.[4] Bushnell [2] examines a Cournot oligopoly with fringe producers in which each producer controls both hydro and thermal generation facilities. Both hydro and thermal units face capacity constraints and the producers must decide how to allocate the available water over a number of periods. He solves the model with parameters calibrated to the western United States electricity market and finds that the dynamic allocation of water under imperfectly competitive conditions is not the efficient one. In particular, firms tend to allocate more water to off-peak periods than is efficient. Crampes and Moreaux [4] model a Cournot duopoly in which a hydro producer uses a fixed stock of water over two periods while facing competition from a thermal producer. They find that hydro production is tilted towards the second period in the closed-loop equilibrium relative to the open-loop, hence there is strategic withholding of water in the first period by the hydro producer. Our model differs from these two in a couple of ways. In both Bushnell [2] and Crampes and Moreaux [4], a fixed stock of water is allocated across a finite number of periods. In contrast, we examine an infinite horizon setting with inflows, so our model has a longer-term focus than theirs. Secondly, we allow for stochastic demand and water flows which, in combination with the infinite horizon, allows us to examine the implications of market power and water storage on the distribution of electricity prices.

We model ongoing quantity competition between a hydro and a thermal generator using a stochastic, dynamic game over an infinite time horizon. The hydro generator is constrained by water availability and the thermal generator is constrained by its capacity. We solve the model using collocation techniques in order to compute approximations to the value function. To our knowledge, this is the first application of these techniques in this area. We demonstrate that the hydro producer engages in strategic withholding of water by comparing the feedback and open-loop equilibria of the game. In

[4]Scott and Read [12] and Barroso et.al. [1] also examine the behaviour of imperfectly competitive hydroelectric producers, but do not consider the dynamic aspects of the strategic behaviour of hydro producers. Garcia et.al. [7] examine a strategic pricing game between two hydro producers who have a capacity constraint on their reservoirs, demonstrating that the Bertrand paradox of marginal cost pricing is mitigated as firms incorporate the opportunity cost of using water today rather than in a future period.

addition, we compute the efficient solution to examine the departure from efficiency caused by market power in this setting. Our simulations show that, conditional on thermal capacity and water inflow not being too large, the outcome can be close to efficient. This result is interesting in light of the empirical work of Kauppi and Liski [11] who find only small welfare losses for the Nordic power market even though they uncover evidence of market power for hydro producers.

We turn next to a description of the basic aspects of the model for both the non-cooperative game between the hydro and thermal producers as well as the efficient outcome through the solution to a social planner's problem. In the third section we present our results via simulations of our numerical solution to both the game and the planner's problem. We do this for fixed thermal capacity and then allow capacity to vary in order to demonstrate how our model could be used to examine the incentives that the thermal player may have for investing in capacity.

2 The model

The behavior of consumers of electricity in any period $t = 0, 1, 2, ..., \infty$ is summarized by the following inverse demand function:

$$P_t = \alpha_t - \beta(h_t + q_t), \qquad \beta > 0. \tag{1}$$

The demand intercept, α_t, is stochastic and normally distributed, i.e., $\alpha_t \sim N(\mu, \sigma_\alpha^2)$, with a variance small enough relative to μ so that the probability of non-positive demand can be ignored.

There are two types of technologies used in the industry: a hydroelectric generator owns generation units that use water held behind dams to spin the electric generators and a thermal electric generator owns thermal units that burn fossil fuel. Thermal generation costs are quadratic, $C(q) = c_1 q + (c_2/2)q^2$, up to a capacity constraint, K. This results in a linear marginal cost up to capacity which is a commonly used functional form for modeling thermal generation marginal cost.[5]

Assuming that the hydro producer does not have to pay for the water it uses, it has a zero marginal cost of production. The hydro producer's electricity generation, h_t, is determined by a one-to-one relation with the amount of water it releases from its reservoir.[6] Its output is constrained

[5]Green and Newbery [9] and Wolfram [14] used a similar form for marginal cost in their empirical analyses of the British electricity market.

[6]In order to keep the model relatively simple, we are ignoring issues such as water pressure which can be an important consideration that links reservoir volume to electricity output.

by the amount of water available for release, W_t. The transition equation governing the level of water in the reservoir is

$$W_{t+1} = (1-\gamma)(W_t - h_t) + \omega_t, \tag{2}$$

where W_t is the level of the reservoir at the beginning of period t, γ is a parameter that determines the rate of evaporation/leakage in the reservoir over an interval of time, and ω_t is the rate of inflow into the reservoir over an interval of time. The rate of inflow is stochastic and distributed $\omega_t \sim N(\mu_\omega, \sigma_\omega^2)$, and is observed after period t decisions are made. There is no fixed capacity for the reservoir, although evaporation limits the accumulation of water.

Producers choose their outputs simultaneously in each period and both producers discount future payoffs with the common discount factor, $\delta \in (0,1)$. We next describe the game played by the duopoly, after which we describe the efficient solution. Following that, we analyse the differences in the two market structures by way of numerical solutions.

2.1 Duopoly

Each producer is assumed to maximize the discounted present value of profits, where each discounts the future using the common discount factor $\delta \in (0,1)$. We focus on the case in which producers use Feedback strategies, which are functions of the current state (W_t, α_t) only. Denote the strategies of the two producers by $s^H(\alpha_t, W_t)$ and $s^T(\alpha_t, W_t)$. We assume that both producers observe W_t and α_t before making decisions in period t. The Feedback equilibrium is a Nash equilibrium in Feedback strategies.

Given the hydro producer's strategy, $s^H(\alpha_t, W_t)$, the problem for the thermal producer is then

$$\max_{\{q_t\}} E \sum_{t=0}^{\infty} \delta^t \left[(\alpha_t - \beta(s^H(\alpha_t, W_t) + q_t))q_t - c_1 q_t - (c_2/2){q_t}^2 \right] \tag{3}$$

subject to

$$0 \leq q_t \leq K.$$

The thermal producer's problem is simplified by the fact that the thermal producer does not influence the future state through its actions. Since its production decision does not affect its continuation payoff, thermal production is governed by its "static" best response function for an interior solution. Incorporating the capacity and non-negativity constraints, we have

$$s^T(\alpha_t, W_t) = \max \left[0, \min \left[\frac{\alpha_t - c_1 - \beta s^H(\alpha_t, W_t)}{2\beta + c_2}, K \right] \right] \tag{4}$$

Given the thermal producer's strategy, $s^T(\alpha_t, W_t)$, the problem faced by the hydro producer is

$$\max_{\{h_t\}} E \sum_{t=0}^{\infty} \delta^t \left[(\alpha_t - \beta(h_t + s^T(\alpha_t, W_t)))h_t \right] \tag{5}$$

subject to

$$0 \leq h_t \leq W_t$$

and

$$W_{t+1} = (1-\gamma)(W_t - h_t) + \omega_t,$$

The hydro producer's best response to the thermal producer's strategy, s^T, is determined by the solution to a dynamic optimization problem. The Bellman equation for the hydro producer's problem is

$$V(\alpha_t, W_t) = \max_{h_t \in [0, W_t]} \left\{ (\alpha_t - \beta(h_t + s^T(\alpha_t, W_t)))h_t + \delta E_t V(\alpha_{t+1}, W_{t+1}) \right\} \tag{6}$$

subject to (2). The solution to this problem yields $s^H(\alpha_t, W_t)$.

Define $\psi(h_t)$ as the derivative of the objective in the maximization problem in (6) with respect to h_t, i.e.,

$$\psi(h_t) = \alpha_t - 2\beta h_t - \beta s^T(\alpha_t, W_t) - \delta(1-\gamma)E_t V_W(\alpha_{t+1}, (1-\gamma)(W_t - h_t) + \omega_t). \tag{7}$$

Let b_{0t} and b_{Wt} be the Lagrange multipliers on the non-negativity and water availability constraints for the maximization problem in (6). The necessary conditions for optimal hydro output are then

$$\psi(h_t) + b_{0t} - b_{Wt} = 0 \tag{8}$$

$$b_{Wt}(W_t - h_t) = 0, \quad b_{Wt} \geq 0, \quad (W_t - h_t) \geq 0 \tag{9}$$

and

$$b_{0t} h_t = 0, \quad b_{0t} \geq 0, \quad h_t \geq 0. \tag{10}$$

We can illustrate the strategic effect by expanding the $E_t V_W(\alpha_{t+1}, W_{t+1})$ term in (7). Evaluating (6) at $t+1$ and differentiating with respect to W_{t+1} yields (when the derivative exists)

$$\begin{aligned} E_t V_W(\alpha_{t+1}, W_{t+1}) = E_t \Big[& (\psi(h_{t+1}) + b_{0t+1} - b_{Wt+1}) s^H_W(\alpha_{t+1}, W_{t+1}) \\ & - \beta s^T_W(\alpha_{t+1}, W_{t+1}) s^H(\alpha_{t+1}, W_{t+1}) \\ & + b_{Wt+1} + \delta(1-\gamma) E_{t+1} V_W(\alpha_{t+2}, W_{t+2}) \Big] \end{aligned} \tag{11}$$

Using (8) this simplifies to

$$E_t V_W(\alpha_{t+1}, W_{t+1}) = E_t \Big[-\beta s_W^T(\alpha_{t+1}, W_{t+1}) s^H(\alpha_{t+1}, W_{t+1}) \\ + b_{Wt+1} + \delta(1-\gamma) E_{t+1} V_W(\alpha_{t+2}, W_{t+2}) \Big] \quad (12)$$

Applying the same process for $V_W(\alpha_{T+2}, W_{t+2})$, $V_W(\alpha_{T+3}, W_{t+3})$, ... yields

$$E_t V_W(\alpha_{t+1}, W_{t+1}) = E_t \Big[-\beta \sum_{i=0}^{\infty} \delta^i (1-\gamma)^i s^H(\alpha_{t+1+i}, W_{t+1+i}) s_W^T(\alpha_{t+1+i}, W_{t+1+i}) \\ + \sum_{i=0}^{\infty} \delta^i (1-\gamma)^i b_{Wt+1+i} \Big]. \quad (13)$$

The strategic effect works through the influence of hydro output on future thermal output via future water availability. Since we expect $s_W^T(\alpha, W) \leq 0$, the hydro producer produces less output in the Feedback equilibrium relative to the Open Loop equilibrium. This will result in more water available in future periods and hence lower thermal output in those future periods.

In order to describe the Feedback equilibrium strategies we need to find the value function for the hydro producer, which we do using numerical approximation techniques. Rather than approximate the value function itself, we solve the problem by approximating $E_t V(\alpha_{t+1}, W_{t+1})$, which has the benefit of allowing us to approximate a function of one state variable only (W_{t+1}) since the future demand shock is integrated out.[7]

2.1.1 Numerical algorithm: duopoly

We approximate the hydro producer's expected value function using the collocation method.[8] In particular,

$$E_t V(\alpha_{t+1}, W_{t+1}) \approx \sum_{i=1}^{n} d_i \phi_i(W_{t+1}) \equiv \widetilde{V}(W_{t+1}) \quad (14)$$

where the ϕ_i are known basis functions. Collocation proceeds by determining the d_i, $i = 1, ...n$, in order for the approximation to hold exactly at n collocation nodes, $W_+^1, W_+^2, ..., W_+^n$. The algorithm we use to solve the problem is described as follows:

[7] This is a consequence of the assumption that the demand states are i.i.d. If we were to allow any serial correlation in this process, the expected value function would be a function of two state variables as well.

[8] See Judd [10], Chapter 11.

0. Choose a starting approximation of $\widetilde{V}^0(W_{t+1})$, i.e., starting values $d_i^0,\ i = 1, 2, ..., n$.

1. Given the current approximation, $\widetilde{V}^k(W_{t+1})$ compute the value function at the collocation nodes, $W_+^1, W_+^2, ..., W_+^n$. In order to do this, we determine the Nash Equilibrium quantities for each producer. At every node i, conditional on the demand state, α_+:

   a) Use a root-finding algorithm to solve $\psi(h) = 0$ in which the thermal producer's strategy, (4), has s^H replaced with h (i.e. we replace the thermal producers strategy in (7) with its best-response to h). If a root does not exist on $(0, W_+^i)$, determine whether $h = 0$ or $h = W_+^i$ is appropriate.

   b) Given the value found for h, compute q from (4).

   Use these quantities to compute $V^k(\alpha_+, W_+^i)$. This step yields the value in the next period as a function of the demand state for each W_+^i.

2. Integrate the new value function numerically over demand states to update $\widetilde{V}(W_t^{k+1})$, i.e. find new values $d_i^1,\ i = 1, ..., n$.

3. If convergence achieved, stop. Else, return to step 1.

2.1.2 Constraint thresholds

Although $E_t V(\alpha_{t+1}, W_{t+1})$ is likely to be a smooth function of W_{t+1}, the value function itself will exhibit kinks at the thresholds where a constraint begins to bind. Finding expressions for these thresholds can aid the numerical integration step of the approximation algorithm. We will not concern ourselves with the non-negativity constraints as they are not binding in any of the situations that we examine, so the constraints of interest are $q_t \le K$ and $h_t \le W_t$. From the necessary conditions for the optimization problems of the two generators we can derive the following:

- If $q_t^* < K$ then $h_t^* = W_t$ if

$$\alpha_t \ge \frac{\beta(3\beta + 2c_2)W_t - \beta c_1 + (2\beta + c_2)\delta(1-\gamma)\widetilde{V}_W(\omega_t)}{\beta + c_2} \equiv H1 \quad (15)$$

- If $q_t^* = K$ then $h_t^* = W_t$ if

$$\alpha_t \ge 2\beta W_t + \beta K + \delta(1-\gamma)\widetilde{V}_W(\omega_t) \equiv H2 \quad (16)$$

- If $h_t^* = W_t$ then $q_t^* = K$ if

$$\alpha_t \geq c_1 + (2\beta + c_2)K + \beta W_t \equiv T1 \tag{17}$$

- If $h_t^* < W_t$ then $q_t^* = K$ if

$$\alpha_t \geq c_1 + (2\beta + c_2)K + \beta h_t^* \equiv T2 \tag{18}$$

Given an approximation $\widetilde{V}$ (and hence $\widetilde{V}_W$), the values $H1$, $H2$, and $T1$ are straightforward to compute and can be used to aid the numerical integration step of the algorithm. However, since h_t^* depends on α_t when $h_t^* < W_t$, $T2$ is relatively difficult to compute. Consequently, we use $H1$ and $T1$ to aid in the numerical integration of the value function, but not $H2$ and $T2$.

2.2 Efficient solution

We wish to compare the outcome under duopoly to what is efficient. To this end, we solve the problem faced by a social planner choosing thermal and hydro generation with the objective of maximizing the expected present value of the stream of consumer surplus less generation costs:

$$\max_{\{h_t, q_t\}} \sum_{t=0}^{\infty} \delta^t \left(\alpha_t (h_t + q_t) - \frac{\beta}{2}(h_t + q_t)^2 - c_1 q_t - \frac{c_2}{2} {q_t}^2 \right) \tag{19}$$

subject to

$$\begin{aligned} W_{t+1} = (1-\gamma)(W_t - h_t) + \omega_t, \\ 0 \leq h_t \leq W_t, \\ 0 \leq q_t \leq K, \end{aligned}$$

and

$$\alpha_{t+1} \sim N(\mu, \sigma_\alpha^2).$$

The planner's value function then satisfies the Bellman equation:

$$V^P(\alpha_t, W_t) = \max_{h_t, q_t} \left\{ \alpha_t (h_t + q_t) - \frac{\beta}{2}(h_t + q_t)^2 - c_1 q_t - \frac{c_2}{2} {q_t}^2 + \delta E_t V^P(\alpha_{t+1}, W_{t+1}) \right\} \tag{20}$$

subject to the above constraints.

The necessary conditions for the maximization problem are

$$\alpha_t - \beta(h_t + q_t) - \delta(1-\gamma) E_t V_W^P(\alpha_{t+1}, W_{t+1}) - b_t^W + b_t^0 = 0 \tag{21}$$

and

$$\alpha_t - \beta(h_t + q_t) - c_1 - c_2 q_t - a_t^K + a_t^0 = 0 \tag{22}$$

where b_t^W and b_t^0 are the Lagrange multipliers on hydro production's capacity and non-negativity constraints and a_t^K and a_t^0 are the multipliers on thermal production's capacity and non-negativity constraints. Equations (21) and (22) imply

$$a_K - a_0 + c_1 + c_2 q_t = \delta(1-\gamma)E_t V_W^P(\alpha_{t+1}, W_{t+1}) + b_W - b_0 \tag{23}$$

which for an interior solution simplifies to

$$\delta(1-\gamma)E_t V_W^P(\alpha_{t+1}, W_{t+1}) = c_1 + c_2 q_t, \tag{24}$$

the marginal value of retained water is equalized with the marginal cost of thermal production.

The numerical algorithm used to solve the planner's problem is similar to that described for the duopoly, using collocation to approximate the planner's expected value function.

3 Results

We analyse the model by computing solutions numerically for particular parameter values. We focus on demand uncertainty and water inflow uncertainty separately, beginning with the former. We will use thermal cost function parameters of $c_1 = 10$ and $c_2 = 0.025$. These are chosen to be roughly consistent with the ratio of these two parameters that was used in Green and Newbery [9]. The demand parameters are chosen so that there is a relatively small demand elasticity when used to compute the unconstrained (Cournot) solution. Setting $\beta = 20$ and $\mu_\alpha = 200$ gives a demand elasticity of 0.54 at the Cournot solution.[9] Finally we choose $\delta = 0.9$, and $\gamma = 0.3$.

A useful benchmark to keep in mind is what the equilibrium of an unconstrained situation would be. If neither producer were ever constrained (i.e., K and ω sufficiently large) the model would be a simple repeated Cournot game with random demand and firms having asymmetric costs. For these parameters, the hydro producer would produce approximately 3.5 units and the thermal producer 3 units on average resulting in an average price of 70.

We also present results for the Open Loop equilibrium for comparison. For the Open Loop solution we use the S-adapted open loop concept in which producers are able to respond to the realization of the demand state (α_t) but the hydro producer does not strategically adjust future water levels (essentially, the $E_t V_W$ term in (7) is absent), so the strategies are functions of the demand shocks only ($s^H(\alpha_t)$ and $s^T(\alpha_t)$). This results in the Open Loop solution being the Cournot equilibrium for the particular realizations of the random demand and inflow variables.

[9]This is near the upper end of the range of demand elasticities examined by Green and Newbery [9]

3.1 Demand uncertainty

To analyse the model with demand uncertainty only, we set $\sigma_\omega = 0$ and choose the standard deviation for α_t to be 10% of the mean, so we have $\sigma_\alpha = 20$. We present results for alternative values of μ_ω and a large capacity for the thermal producer ($K = 6.0$, which is twice the average level of thermal output in the unconstrained game). After presenting results for this level of capacity we will demonstrate the effects of varying the thermal capacity on the equilibrium outcome of the game.

Table 1 displays statistics for variables of interest under equilibrium strategies. They are created by generating 100 simulations of the model over 1,000 periods each.[10] The values in Table 1 are averages over the 100 runs. To compute the equilibrium strategies we use Chebyshev polynomials for the ϕ_i functions and n varies by example and market structure.[11] We report n as well as an estimate of the maximum approximation residual for each case in the last two rows of Table 1.

3.1.1 High inflow

The high inflow case represents a benchmark in which neither of the constraints (water or capacity) are binding for the duopoly. For this scenario, we choose an inflow of water that is double mean hydro production in the unconstrained game discussed above ($\mu_\omega = 7.0$). In this case the approximation residual is very small (of the order 10^{-9}) with $n = 2$, which is expected given that the value function is quadratic if the constraints do not bind.

Not surprisingly, the duopoly equilibrium outcomes are what occur in the repeated Cournot game. Neither producer operates at capacity, so we just have an interior solution that replicates the Cournot outcome. This is not efficient, since the planner would like to use more of the low cost technology, having the hydro producer at capacity in all periods. This scenario results in the largest welfare loss of the three examined. The duopoly price is seven times the efficient level and there is substantial under-utilization of water, the water level being more than double the efficient level on average. This results in a shadow price of water that is zero for the duopoly hydro producer but significant (approximately one-third of the price level) for the planner.

The comparison between the duopoly and efficient outcomes in this case represents a measure of the effect of "static" market power alone. The open loop solution is the same as the Feedback one as there are no strategic effects when the constraint on water availability does not bind.

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[10] An initial run of 100 periods precedes the 1,000 period sample to minimize any effects of starting values.

[11] The computations are done with C++ and make use of routines for Chebyshev approximation, numerical integration, and root finding from the Gnu Scientific Library. [6]

	Low inflow ($\mu_\omega = 1.75$)		Medium inflow ($\mu_\omega = 3.5$)		High Inflow ($\mu_\omega = 7.0$)	
	Duopoly	Efficient	Duopoly	Efficient	Duopoly	Efficient
Quantities:						
$E(h)$	1.74	1.72	3.26	3.46	3.50	7.00
$E(q)$	3.87	5.99	3.11	5.67	3.00	2.49
$\%(h = W)$	81.20	78.36	1.04	46.85	0.00	100.00
b_W	9.99	17.33	0.04	6.67	0.00	3.80
$\%(q = K)$	0.00	100.00	0.00	55.84	0.00	0.02
a_K	0.00	35.51	0.00	7.16	0.00	0.00
Price:						
$E(p)$	87.58	45.66	72.36	17.30	70.00	10.06
st.dev.(p)	9.58	17.09	7.08	11.04	6.66	0.05
skew.(p)	0.18	0.64	0.09	1.79	0.00	4.37
Water:						
$E(W)$	1.77	1.83	4.05	3.59	15.17	7.00
st.dev.(W)	0.05	0.19	0.26	0.08	0.32	0.00
min(W)	1.75	1.75	3.50	3.50	13.81	7.00
max(W)	2.37	3.77	5.27	3.69	16.51	7.00
Payoffs:						
$E\Pi^H$	1533.72	807.69	2389.12	617.44	2479.76	705.68
$E\Pi^T$	3061.57	2146.08	1973.80	449.80	1825.34	0.95
E(Welfare)	7787.57	8918.52	8485.45	9444.03	8583.69	9848.04
Open Loop:						
$E(h)$	1.77		3.48		3.50	
$E(q)$	3.86		3.01		3.00	
Approx.Res.	10^{-7}	10^{-5}	10^{-4}	10^{-5}	10^{-9}	10^{-9}
n	10	10	13	4	2	2

Table 1: Simulated Descriptive Statistics

3.1.2 Low inflow

For a low water inflow scenario, we choose $\mu_\omega = 1.75$, which is half of the average level of hydro production in the unconstrained version of the game (the high inflow case above). In this example, we expect hydro production to be frequently constrained by water availability.

For this low water inflow case, the hydro producer exhausts the available water 81% of the time which is actually more frequent than is efficient (78%) and we see that the average hydro output is actually higher than efficient. The reason for this is that the planner places a substantially higher value on water: the shadow price of water (b_W) is 17.3 for the planner vs. 9.99 for the hydro duopolist. This results in the planner wishing to maintain a higher average reservoir level ($E(W)$ of 1.83 vs. 1.77) for which it needs to produce less hydro electricity on average. As thermal production is always at capacity, higher levels of available water are desirable to smooth out the more substantial price volatility (the standard deviation of price for the planner is 17.09 vs. 9.58 for the duopoly).

The low water inflow reduces hydro producer payoffs and welfare relative to the high-inflow scenario. The thermal producer is better off, as it produces more output at a higher price relative to the high inflow case.

3.1.3 Medium inflow

In order to relax the constraint on hydro production somewhat, we now set $\omega = 3.5$ which is the hydro producer's average level of output in the unconstrained game. This guarantees that there is enough water in any period for the hydro producer to produce the same output as it would in the Open Loop game. Hence, this case allows the sharpest view on the extent to which the hydro producer will strategically withhold water.

Now the hydro producer chooses output that drains its reservoir only 1% of the time, whereas the planner would have it do so 46% of the time. This reduction of output by the hydro producer, along with the thermal producer's inefficiently low output now results in a larger price gap between the duopoly and efficient outcomes. The duopoly price falls compared to the low inflow case, but not by as much as is efficient. Furthermore, average hydro production in this case is 3.26, lower than what it would produce on average in the unconstrained case (3.5). Even though the thermal producer produces more (3.11 vs. 3.0), average price is higher (72.36 vs. 70.0). Even though the water inflow is sufficient to generate the Open Loop output for the hydro producer, it chooses to produce a lower quantity than this in equilibrium. The difference between the Open Loop output of 3.48 and the equilibrium output of 3.26 represents the strategic effect.

Again, in this scenario, the average water level exceeds the efficient level (4.05 vs. 3.59) with a substantially lower value put on water by the hydro duopolist than is efficient (b_W of 0.04 vs. 6.67).

As this is the case with the largest approximation residuals for the duopoly, we plot the residual function in the Appendix. The residuals display the oscillatory nature expected of Chebyshev approximation.

3.1.4 Price volatility

From the solutions presented in Table 1, we see that price volatility, as measured by the standard deviation in price, is lower under the duopoly than is efficient when the water inflow is relatively low. However, the reverse occurs in the high inflow case. One force at work is the under-use of water by the hydro producer. When it is efficient to be using as much hydro generation as possible, water is not used for price smoothing in a significant way in the efficient solution. When constrained by water availability, there is simply no role for using water to smooth demand fluctuations. In contrast, under the duopoly market structure, the hydro producer is less often constrained by water availability and so reacts more to demand fluctuations resulting in smoother prices.

The effect of the water-availability constraint on price volatility works in the opposite direction of market power. As shown in Thille [13], in a Cournot model with uncertainty, duopolists will not adjust output as much as is efficient in response to demand shocks. This effect dominates the effect of the constraint on price volatility in the high inflow case, resulting in the duopoly producing higher price variability than is efficient in that case.

3.1.5 Effects of Thermal Capacity

In order to analyse how thermal generation capacity, K, affects the equilibrium outcome under the two market structures, we examine the medium inflow case of section 3.1.3 allowing for different levels of K. We solve the model for 20 different capacities ranging between zero and five units. For each solution, we simulate the model as above and plot some of the resulting statistics in Figures 1 and 2.

The top row of Figure 1 plots the average outputs of each producer by market structure. At low levels of thermal capacity, the thermal producer is essentially always operating at capacity, which is efficient. At large levels of capacity the thermal producer reduces output below capacity more frequently, resulting in inefficiently low output at higher capacities. In contrast, the hydro producer's average output is below capacity for any level K, although the size of the difference to the efficient level is not large.

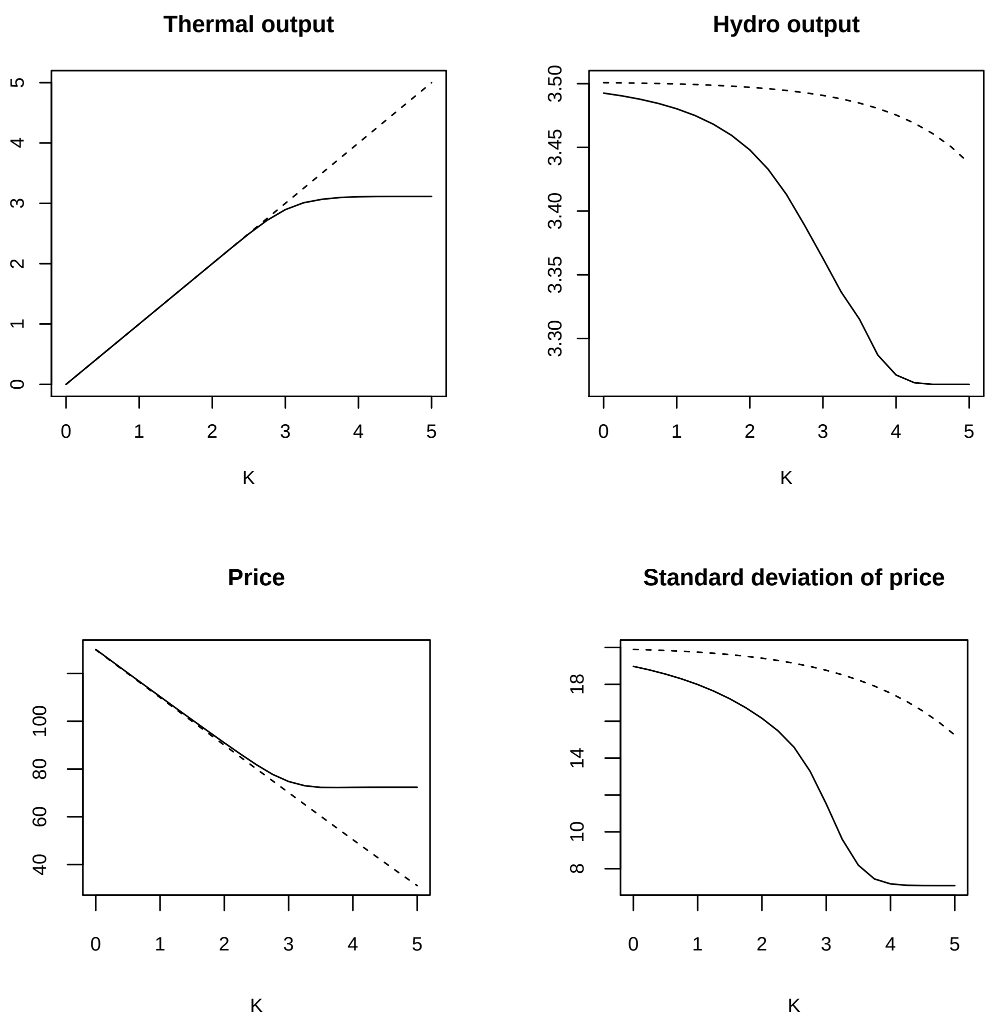

Figure 1: Average model values for alternative thermal capacities: Duopoly (solid line) and Efficient (dashed line)

The bottom left graph in Figure 1 demonstrates that price is very close to the efficient level until thermal capacity reaches approximately 2.5. After this point, price levels off and slowly falls to the Cournot price of 70.0, whereas the planner has price falling until thermal capacity is beyond 5.0. The implications for price volatility are demonstrated in the bottom right graph in Figure 1. The duopoly results in prices that are less volatile than is efficient. It is notable that the gap between price volatility is largest for intermediate values of K.

We plot payoffs and social welfare in Figure 2. Each payoff is very close to the efficient one for thermal capacities less than 2.5. From the above discussion we know that this is because the thermal constraint frequently binds under both market structures and the hydro producer does not reduce output greatly under duopoly.

An interesting question to now address is what thermal capacity would be chosen if the thermal producer could choose capacity in a previous period? Consider allowing the thermal producer to make a one time investment in capacity before time 0. The slope of the thermal producer's payoff (solid line) in Figure 2 measures the benefit to the producer of a marginal addition to capacity. The thermal producer would choose a level of capacity that results in a significant departure from efficiency only if the the marginal cost of capacity is relatively low. Notice that for the outcome to be significantly inefficient in this case would require a capacity investment that exceeds the "average" Cournot output of the thermal producer (3.0 in this case). Since the region of capacity levels for which the thermal producer's payoff is relatively steep coincides with the region where the equilibrium is near efficiency, we can suggest that *conditional on the level of capacity chosen*, the equilibrium in the dynamic duopoly game can be "close" to the efficient one if the marginal cost of capacity is not too low. Of course the planner may wish to choose a higher level of capacity, so an interesting extension of this game would be to examine optimal versus actual capacity choices.[12]

3.2 Water inflow uncertainty

An important characteristic of hydroelectric generation is that the flow of water into the system is often uncertain.[13] We now examine the case in which demand is certain, but water inflows contain a random component: $\sigma_\alpha^2 = 0$ and $\sigma_\omega^2 > 0$. A simplifying feature of this case is that the dimension of the

[12] The efficiency of thermal investment decisions is examined for a two-period version of this game in Genc and Thille [8]. They find that equilibrium investment may be higher or lower than efficient.

[13] See Bye et.al [3] on the effects of inflow uncertainty on prices in the Nordic power market.

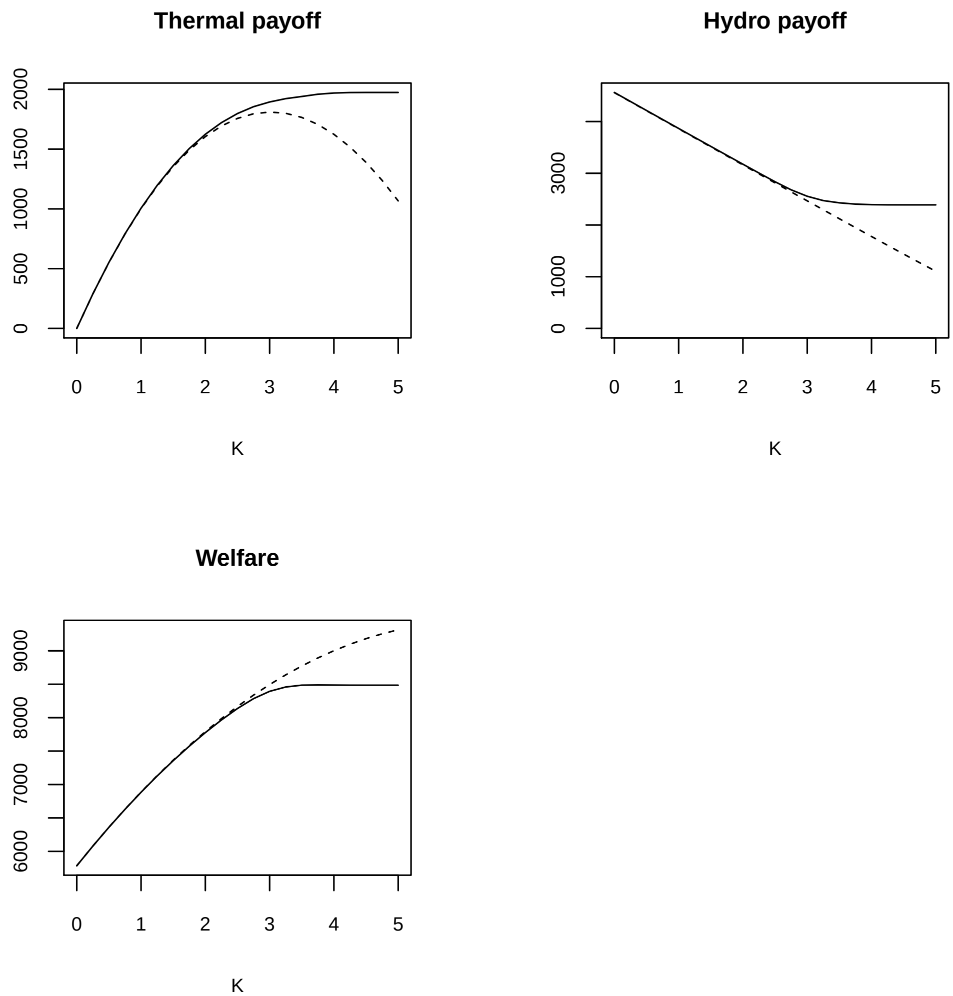

Figure 2: Payoffs for alternative thermal capacities: Duopoly (solid line) and Efficient (dashed line)

state space is reduced to one. Since the random inflow is observed only after production decisions have been made, both hydro and thermal strategies are now functions of W_t alone. This means that the hydro producer's value function is also a function of only one state variable, which simplifies the approximation routine.

Our solution strategy is similar to that described above for the case of demand uncertainty, but now we approximate the value function, $V(W_t)$, directly. Since $V(W_t)$ is likely to be kinked, we use splines for the collocation functions rather than Chebyshev polynomials. For our solution we now set $\mu_\omega = 3.5$, $\sigma_\omega^2 = 0.35$, and $\sigma_\alpha^2 = 0$. All other parameter values remain the same as in the previous subsection.

In order to compare with the results from the demand uncertainty case in Table 1, we present the simulated statistics for the duopoly outcome in Table 2 for the medium water inflow ($\mu_\omega = 3.5$) case.

Most of the statistics are roughly similar to those in Table 1 with the exception of price skewness. In periods with a relatively high inflow realization, the amount of water available often exceeds the hydro producer's Cournot level of output. In these periods, neither producer varies output much with the water level, consequently price does not vary much. However, when there is a low realization of the random water inflow, the hydro producer becomes constrained and the adjustment of output to water level is more significant. These periods of low water availability result in a price "spike" which shows up as the high skewness in the price distribution.

The increased price skewness will is not as pronounced under the planner's solution. As in the demand uncertainty case, it is efficient to use hydro generation as much as possible in most cases. This results in hydro generation constrained much more frequently than occurs under the duopoly. Hence, prices will be less skewed, although more volatile.

4 Conclusion

We have studied dynamic competition between thermal and hydroelectric producers under both demand and water inflow uncertainty. In an infinite horizon game between the two producers, we have demonstrated that the hydro producer does have a strategic incentive to withhold water for sufficiently large water inflows, but with low inflow the hydro producer may overuse water. We find that when capacities of both producers are frequently binding the duopoly outcome is not far from the efficient one. Examination of the payoffs to the thermal producer at various capacity levels suggests that if capacity were to be chosen by the thermal player, it would not choose a capacity so that it is rarely constrained. This results in a welfare loss lower

	Duopoly	Efficient
Quantities:		
$E(h)$	3.27	3.50
$E(q)$	3.11	5.85
$\%(h = W)$	1.40	100.00
b_W	0.07	4.82
$\%(q = K)$	0.00	49.66
a_K	0.00	2.76
Price:		
$E(p)$	72.35	12.91
st.dev.(p)	0.60	4.07
skew.(p)	3.86	1.64
Water:		
$E(W)$	4.06	3.50
st.dev.(W)	0.42	0.35
min(W)	2.57	1.84
max(W)	5.37	4.93
Payoffs:		
$E\Pi^H$	2375.69	441.50
$E\Pi^T$	1923.06	172.30
E(Welfare)	8390.12	9364.97
Open Loop:		
$E(h)$	3.48	
$E(q)$	3.00	

Table 2: Simulated Descriptive Statistics: Inflow uncertainty

than that under unconstrained Cournot duopoly.

Our examination of uncertain water flows suggests that prices will be more skewed than when there is relatively certain water inflow. This effect is stronger under the duopoly market structure than under a more competitive one. Price skewness also varies by market structure under demand uncertainty. These results on price skewness are particularly interesting given concerns about price spikes in electricity markets.

5 Appendix

Figure 3 shows the approximation residuals for the model in Table 1 with the largest approximation error, which is the duopoly with the medium inflow ($\mu = 3.5$). These residuals are the difference between our approximation, $\widetilde{V}(W_+)$, and the computed $EV(\alpha_+, W_+)$ for a set of W_+ 20 times larger than is used in computing the approximation. The residuals display the oscillations expected with Chebyshev approximation. Although a smaller maximal residual would be preferred, attempts at using larger values of n resulted in a lack of convergence of the algorithm.

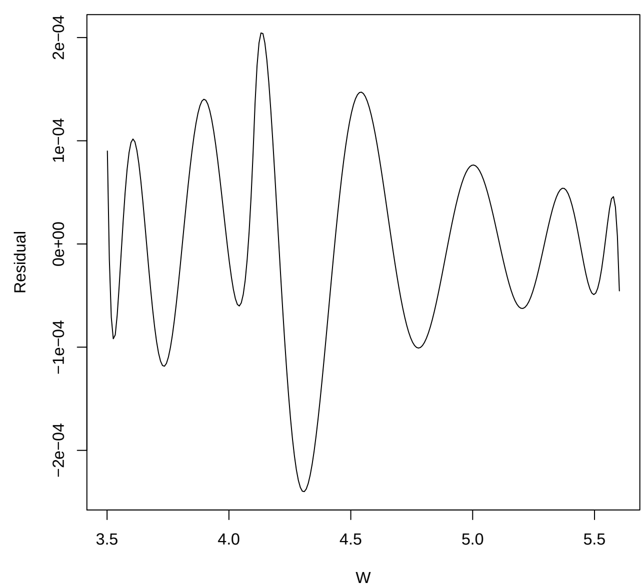

Figure 3: Approximation residual plot for case with demand uncertainty and medium inflow

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