DEPARTMENT OF ECONOMICS AND FINANCE

DISCUSSION PAPER 2012-13

NON-MONOTONICITY OF FERTILITY IN HUMAN CAPITAL ACCUMULATION AND ECONOMIC GROWTH

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NOVEMBER, 2012

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Abstract

This paper investigates the relationship between per-capita human capital investment and the birth rate. Since the consequences of higher fertility (birth rate) on per-capita human capital accumulation (the so-called dilution effect) are not the same (in sign and magnitude) across different groups of countries with different birth rates, we analyze the growth impact of a non-linear dilution-effect. The main predictions of the model (concerning the relationship between population and economic growth rates) are then compared with those of a standard model in which the exogenous birth rate affects linearly and negatively (as postulated by most of the existing theoretical literature) human capital investment at the individual level. By using non-parametric techniques, we find evidence of strong nonlinearities in the total effect of fertility on human capital accumulation. This supports the idea that fertility plays a non-monotonic role in the accumulation of human capital and hence in the growth rate of an economy. The non-monotonic effect of fertility on human capital appears to be valid for OECD, as well as non-OECD countries according to our empirical results.

KEY-WORDS: Fertility; Population Growth; Economic Growth; Human Capital Investment; Dilution Effects

JEL Codes: *O41; J13; J24.*

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1. INTRODUCTION

The analysis of the impact of demographic change (population growth) on the growth rate of real per capita income represents an old, but still unsettled topic of research. Malthus (1798) was among the first to recognize that a higher population growth rate would ultimately (in the very long-run) have led to economic stagnation. According to his view, in a world in which economic resources are in fixed supply and technological progress is very slow or totally absent, the food-production activity would, sooner or later, have been overwhelmed by the pressures of a rapidly growing population. In this scenario, the available diet of each single individual in the population would have fallen below a given *subsistence level*, so leading to a fall of the productivity growth rate as well.[1] Unlike Malthus, proponents of the optimistic view[2] emphasize, instead, the positive effect that a larger population can exert on the rate of technological progress (an endogenous variable) and, thus, on economic growth**:** "…*More people means more Isaac Newtons and therefore more ideas. More ideas, because of nonrivalry, mean more per capita income. Therefore, population growth, combined with the increasing returns to scale associated with ideas delivers sustained long-run growth*" (Jones, 2003, p. 505).[3] Besides the pessimistic and optimistic ones, there also exists another belief about the long-run effects of population growth on economic growth: the neutralist one. The advocates of this view claim that population growth has in general only little significant impact on economic growth and that such impact can be either positive, or negative, or else wholly inexistent (Srinivasan, 1988; Bloom *et al.*, 2003, p. 17).

The pessimistic prediction of Malthus has fortunately never become reality. Nonetheless, it is now well known (Bloom *et al.*, 2003, Fig.1.1, p. 13; United Nations, 2004) that, unlike the industrialized world, the less (and especially the least) developed regions of the planet are rapidly and increasingly gaining shares of the world population. Since these regions are those that actually exhibit the highest fertility and the lowest literacy and economic growth rates, the following question becomes of paramount importance: what is the effect that a further increase in the fertility rate (thus, in the population growth rate) may have on human capital accumulation and, through this channel, on sustained long-run economic growth?

[1] More recent examples of the view that population growth is detrimental to economic development include the neoclassical growth theory with exogenous technological progress (Solow, 1956), some empirical applications of this theory (notably, Mankiw *et al.*, 1992), and Barro and Becker (1988, 1989). The last two authors consider an environment in which fertility is endogenous. Under the assumption that children are *normal goods* and parents are altruistic (they gain utility from having kids), they conclude that a faster population growth, by implying a *dilution* of the capital endowment of each individual in the population, harms long-run growth in real per capita incomes.

[2] See Kuznets (1960, 1967), Simon (1981), Boserup (1981), Kremer (1993), Jones (2001) and Tamura (2002, 2006), just to mention a few examples.

[3] In models with endogenous technological change population can affect economic growth in two distinct ways. In some papers (Romer, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991) real per capita income growth depends positively on *population size*. This result (known as *strong scale effect*) is rejected on empirical grounds (Jones, 1995). In other papers, notably the *semi-endogenous* growth models (Kortum, 1997; Segerstrom, 1998), it is *population growth* (as opposed to population size) to sustain economic growth in the very long-run. Another branch of endogenous growth theory has recently analyzed the impact of population growth on economic growth in environments in which there is also, along with technological progress, human capital accumulation. Such papers (Dalgaard and Kreiner, 2001; Strulik, 2005 and Bucci, 2008) find that population growth is not necessary for long-run economic growth and that the effect of population growth on economic growth can be either non-positive (Dalgaard and Kreiner, 2001), or ambiguous (Strulik, 2005 and Bucci, 2008). In all these papers, however, population (size and/or growth) is an exogenous variable.

The main objective of the present paper is to tackle this issue, from a theoretical as well as an empirical perspective. In order to achieve this objective we briefly present the main results of a simple benchmark model in which individuals can invest solely in human capital, the only input in the aggregate production function. We also assume that final output (*i.e.*, aggregate GDP) can be only consumed, that the economy is closed (there is no international trade in goods and services and no international migration of people) and, finally, that there exists no governmental activity. Thus, in such an environment there cannot be any investment in physical capital. In this model the birth rate, and as a consequence population growth, is exogenous and affects linearly and negatively per-capita human capital investment (linear dilution effect). This is a rather standard assumption in the growth literature with human capital accumulation. The main prediction of the model is that the impact of population growth on economic growth is monotonic and negative.[4]

Then we extend the benchmark model both by allowing for an endogenous birth rate, and for a nonlinear effect of this rate on per-capita human capital investment (nonlinear dilution effect). We see that, through these changes, the impact of population growth on economic growth becomes itself nonlinear, implying that the growth-effect of higher fertility may well differ (in sign and in magnitude) across different groups of countries at different levels of birth rate. Kelley (1988, p. 1686) was among the first to admit that the relationship between population and economic growth rates might have been somehow nonlinear.[5]

Finally, in the last section of the paper we test empirically the hypothesis of exogeneity of the birth rate and the hypothesis of linearity of its relationship with per-capita human capital investment. In order to do so we use semi-parametric and fully non-parametric methods. Our empirical investigation finds strong evidence against the exogeneity of the birth rate, as well as against linearity (the two building–blocks of the simple benchmark model).[6]

Kalaitzidakis *et al.* (2001), using non-parametric techniques, were among the first to search empirically for a nonlinear effect of human capital accumulation on economic growth. In their paper, the authors split human capital by gender (male *vs.* female human capital) and by category (primary *vs*. tertiary education). They maintain that the nonlinearities come especially from the distinction of human capital by gender (as

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4 At most, a rise of the growth rate of population has no effect on economic growth. This occurs when agents are perfectly altruistic towards future generations, a very special case.

5 *"[…] In some countries population growth may on balance contribute to economic development; in many others, it will deter development; and in still others, the net impact will be negligible"* (Kelley, 1988, p. 1686).

6 The economic consequences of demographic change are at the hearth of many empirical studies. While in some of them the focus is on the economy as a whole (for instance, Brander and Dowrick, 1994; Kelley, 1988 and Kelley and Schmidt, 2003), in others country-based, household-level surveys are presented in order to shed light on the link between family size/fertility and human capital investment/children's performance at school (see, among others, Rosenzweig and Schultz, 1987 for Malaysia; Goux and Maurin, 2004 for France and Stafford, 1987 for the US). In comparison to Brander and Dowrick (1994), who analyze the impact of the birth rate on physical capital investment, we study the possible nonlinear effect of the birth rate on human capital accumulation. Contrary to Kelley (1988) and Kelley and Schmidt (2003) we use data covering a larger time-span (1960/2000) and non-parametric methods. Finally, unlike the household-level, country-based surveys (that in general predict a negative and linear relationship between the two variables being analyzed), our main empirical motivation in the present paper is to search for possible nonlinearities in the relation between birth rate and per-capita skill investment.

a result of the discrimination between males and females in the labor market), and suggest that overall human capital investment has a negative impact on economic growth (due to the fact that, particularly in countries with low levels of human capital, acquiring skills is generally considered as a rent-seeking economic activity). In our paper, we follow a similar strategy as in Kalaitzidakis *et al*. (2001), since we try to understand the presence of possible nonlinearities of demographic factors on human capital accumulation and, hence, economic growth. Starting from the idea that a change in the birth rate affects the decision of how much to invest in per-capita human capital, and realizing that a higher birth rate leads to depletion of available resources and so investing in education is more costly, we claim that it is the relation between birth rate and per-capita human capital accumulation to be nonlinear. The presence of this nonlinearity, in turn, implies a nonlinearity in the relationship between the birth rate and economic growth as well, and suggests that the opportunity cost of having children is different across countries which have different birth rates.

The article is organized as follows. In section 2 we present briefly the main assumptions and predictions of a simple (benchmark) model in which all the demographic components of population growth are taken as exogenous and summarized in a single parameter, n. This benchmark model represents a useful theoretical starting point and offers itself for comparison with a richer model (section 3) in which the birth rate is made endogenous and affects in a nonlinear manner per-capita human capital accumulation. This model is compatible with the existence of a nonlinear relationship between population and economic growth rates. In section 4, we empirically estimate the effect of the birth rate on human capital accumulation. According to this analysis, the impact of the birth rate on human capital accumulation at the individual level is found to be nonlinear, a result that is contrary to the linear relationship assumed in the simple benchmark model. Furthermore, there is strong evidence that the birth rate is endogenous. The last section summarizes and concludes.

2. Analytical Framework: A simple (bemchmark) Model with Exogenous Fertility and Linear Dilution in Per-Capita Human Capital investment

2.1 The Environment

Consider an economy in which households purchase consumption goods and choose how much to invest in human capital. Each individual in the population offers inelastically one unit of labor-services per unit of time. Hence, population (N) coincides with the available number of workers and grows at an exogenous and constant rate $\gamma = n - d$. Population growth (γ) depends on three fundamental variables: fertility (the birth rate, n), mortality (the death rate, d), and migration. To start with, in this model we abstract from any fertility decision (whereby rational agents choose the number of their descendants by weighing costs and benefits of rearing children), we neglect migration (the economy is closed to international trade in goods and services and to international mobility of people) and consider the mortality rate as exogenous. Therefore, we take the growth rate of population as given. Moreover, since in this paper we focus on the birth rate as the fundamental variable affecting agents' investment in education (in the empirical section of this work we estimate the effect that a change in the birth rate has on per-capita human capital investment[7]), we simplify further the analysis by setting $d = 0$.[8] Hence, in the remainder of the paper the population growth rate equals the birth rate $\dot{N}_t / N_t \equiv \gamma = n$.[9]

Following Barro and Sala-i-Martin (2004, Chap.5, p. 240) the total stock of human capital existing in the economy at time *t* ($H_t \equiv N_t h_t$) changes not only because population size can change, but also because the average quality of each worker (or per capita human capital, $h_t = H_t / N_t$) may increase over time.

Consumption goods (or final output) are produced competitively by using solely human capital (H_Y) as an input. The aggregate technology for the production of final output is:

$$Y_t = AH_{Yt}, \qquad H_{Yt} = u_t H_t \tag{1}$$

with Y_t denoting the GDP of the economy at time *t*, $A > 0$ representing a positive productivity parameter (total factor productivity), and u_t being the share of the total stock of human capital devoted to the production of goods at *t*.

There is no physical capital investment and final output (the *numeraire* in this economy) can be only consumed. The aggregate production function (Eq. 1) is linear in the amount of human capital devoted to goods manufacturing. However, unlike *"AK"-type* growth models, individuals choose endogenously at each time *t* how to allocate the existing stock of (human) capital between production of consumption goods (u_t) and production of new (human) capital (1-u_t). Under our assumptions, the economy-wide budget-constraint reads as:

$$Y_t = AH_{Yt} = Au_t H_t = Au_t N_t h_t = C_t, \tag{2}$$

where C is aggregate consumption.

Concerning human capital accumulation, we follow Uzawa (1965) and Lucas (1988) and assume that the law of motion of human capital at the economy-wide level is:

$$\dot{H}_t = \sigma(1 - u_t) H_t, \qquad \sigma > 0, \tag{3}$$

[7] The birth rate is the main source of the human capital *dilution-effect* according to some empirical surveys in which it is clearly shown that an increase in the family size leads to a reduction in children's participation to education (Radyakin, 2007; Booth, 2005).

[8] For a comprehensive analysis of the effects of declining mortality (rising longevity) on investment in education and economic growth see, among others, Ehrlich and Lui (1991), Zhang and Zhang (2005), Kalemli-Ozcan (2002).

[9] In the subsequent model we endogenize the birth rate while continuing to leave migrations out of the analysis and to normalize the death rate to zero. Therefore, in the next model population growth is endogenous, as well.

where σ is a technological parameter representing the productivity of human capital in the production of new human capital.[10] For the sake of simplicity we assume that human capital is not subject to any form of material obsolescence. Given $\dot{H}_t$ and the definition of per capita human capital h_t, the law of motion of human capital in per-capita terms is:

$$\dot{h}_t = \frac{\dot{H}_t}{N_t} - nh_t = \left[\sigma\left(1-u_t\right)-n\right]h_t .$$

In the last equation, with $n>0$, the term $-nh_t$ represents the cost (in terms of per-capita human capital investment) of upgrading the degree of education of the newborns (who are uneducated) to the average level of education of the existing population (linear and negative dilution in human capital accumulation at the individual level).

With a *Constant Intertemporal Elasticity of Substitution* (CIES) instantaneous utility function, the objective of the family-head is to maximize under constraint the household's inter-temporal utility deriving from per-capita consumption:

$$\underset{\{c_t,u_t,h_t\}_{t=0}^{+\infty}}{Max}\ U \equiv \int_0^{+\infty}\left(\frac{c_t^{1-\theta}}{1-\theta}\right)N_t^{\nu}e^{-\rho t}dt\,, \qquad \rho>0\,; \qquad \nu\in\left[0;1\right]; \qquad \theta>0 \tag{4}$$

$$s.t.: \quad \dot{h}_t = \left[\sigma\left(1-u_t\right)-n\right]h_t\,, \qquad \sigma>0\,; \qquad n\geq 0\,; \qquad u_t\in\left[0;1\right], \quad \forall t \tag{5}$$

along with the transversality condition: $\lim_{t\to\infty}\lambda_{ht}h_t = 0$ (6)

and the initial condition:[11] $h\left(0\right)>0$. (7)

The household decides on the amount of per-capita consumption and on the share of human capital to be devoted to production activity (respectively $c_t \equiv C_t / N_t$ and u_t). Eq. (4) is the household's inter-temporal utility function and Eq. (5) represents the per-capita human capital accumulation function. We denoted by $1/\theta$ the constant inter-temporal elasticity of substitution in consumption. Following many existing examples in the literature (for instance, Razin and Sadka, 1995, Chap. 13, footnote 1, among others) we make a formal distinction between two types of altruism: *inter-temporal* (the pure rate of time preference, ρ) and *intra-temporal altruism,* ν. The limiting case of $\nu=1$ defines the situation of *perfect altruism* (the family-head maximizes the discounted value of total utility, *i.e.* per capita utility multiplied by the aggregate family size), whereas the opposite limiting case of $\nu=0$ defines the minimal degree of

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[10] *"…Uzawa (1965) worked out a model very similar to this one. The striking feature of his solution, and the feature that recommends his formulation to us, is that it exhibits sustained per-capita income growth from endogenous human capital accumulation alone: no external 'engine of growth' is required"* (Lucas, 1988, p. 19).

[11] We normalize population size (and, therefore, the number of workers) at time zero, $N\left(0\right)$, to one. Under this assumption the objective function can also be written as: $U \equiv \int_0^{+\infty}\left(\frac{c_t^{1-\theta}}{1-\theta}\right)e^{-(\rho-\nu n)t}dt$. In this case $\rho>\nu n$ ensures that U is bounded away from infinity if c remains constant over time.

altruism (the family-head maximizes solely the discounted value of per capita utility). Clearly, $\nu \in (0;1)$ describes an intermediate degree of intra-temporal altruism. Since u is a fraction, it must belong to the closed set $[0;1]$. Finally, in Eq. (5) the exogenous birth rate (n) is set at a value being positive or, at most, equal to zero.

By solving[12] the benchmark model, it is possible to show that:

$$u = \frac{\sigma(\theta-1)+\rho-(\theta+\nu-1)n}{\sigma\theta}$$

$$g = \frac{(\sigma-\rho)-(1-\nu)n}{\theta}$$

Hence, exogenous population growth (n) may have either a negative or no effect on real per-capita income growth (g). Intuitively, the impact of population growth on per-capita income growth can be explained as follows. Along the balanced growth path (BGP), economic growth is driven by human capital accumulation:

$$\frac{\dot{h}_t}{h_t} \equiv g = \sigma(1-u)-n$$

So, an increase in n has two opposing effects:

- On the one hand, it deters human capital investment and economic growth through the direct, linear and negative *'dilution'* effect (the term $-n$ in the equation above).
- On the other hand, for $\theta+\nu>1$, something that can be justified empirically, it fosters human capital investment and economic growth through the indirect and positive *'accumulation'* effect – the term $(1-u)$ in the equation above. Alternatively, if $\theta+\nu=1$, then there is no positive effect of population growth on human capital accumulation.[13]

The above argument suggests that in the benchmark model the linear and negative dilution effect always prevails over (or, at most, is equal to) the positive accumulation effect, e.g. $\partial g / \partial n \leq 0$, $\forall n \geq 0$.

[12] The solution of the model is very simple, and available upon request.

[13] *"…The above literature review is by no means complete; it is included here to point out that, although the assumption of $\gamma=1$ is knife-edge and disputable, the alternative assumption of $\gamma<1$ is also quite troublesome. Empirical investigations bring somewhat convincing evidence, that $\gamma>1$"* (Growiec, 2006, p.18). The author also states: *"…We note that for CRRA utility functions, u can be everywhere positive only if $\gamma<1$"* (Growiec, 2006, p.7). In Growiec (2006)'s framework $1/\gamma$ denotes the intertemporal elasticity of substitution in consumption. In order to ensure the positivity of the instantaneous utility function $u(c_t)=c_t^{1-\theta}/(1-\theta)$, we (like Growiec, 2006) assume in the rest of the analysis that $0<\theta<1$. Moreover, since we also know that (according to empirical studies) θ is sufficiently large, in principle assuming $\theta+\nu\geq 1$ does not appear so unrealistic.

3. The Model with Endogenous Fertility and Nonlinear Dilution in Per-Capita Human Capital Investment

In this section we extend the previous model along two directions: *i)* We endogenize the birth rate (n) and, more importantly, *ii)* We generalize our analysis by considering a nonlinear *dilution-effect* of the birth rate on the human capital investment equation expressed in per-capita terms. The main implication of this assumption is that at low birth rate levels children are more likely to be educated, whereas at high birth rate levels children enter more quickly into the labor force in order to contribute to their family income. Furthermore, in countries with low growth rates and shorter life expectancy, parents prefer to invest in the number of children as a form of security for their retirement. As far as we know, this is the first attempt in the literature at analyzing and characterizing a similar model.[14] To be more concrete, we postulate that the dilution-effect of the birth rate on human capital accumulation is represented by a generic nonlinear function of n, $f(n_t)$, in the law of motion of per-capita human capital:

$$\dot{h}_t = \left[\sigma(1-u_t) + f(n_t)\right] h_t \tag{8}$$

We assume that $f'(n_t) < 0$ and $f''(n_t) < 0$, which guarantees that we have a maximum solution for the agent's problem. The structure of the economy remains the same as in the benchmark model. In particular, the aggregate production function and the economy's budget constraint (Eqs. 1 and 2, respectively) are unchanged, and we continue to make the following main assumptions: agents devote their own time-endowment in part (u_t) to production activities and in part ($1-u_t$) to produce new human capital; the economy is closed; there is no migration of people across countries; we still set the death rate equal to zero ($d = 0$). Therefore, the rate of population growth coincides again with the birth rate, now being an endogenous variable.

Because the birth rate is endogenous, we follow Palivos and Yip (1993) in considering the birth rate (net population growth), along with per-capita consumption, as an argument of the individual instantaneous utility function. However, we depart from Palivos and Yip (1993) in that we analyze the predictions of a model with endogenous allocation of human capital between production and education sectors. More specifically, the instantaneous utility function of each individual in the economy is:

$$u(c_t; n_t) \equiv \frac{\left(c_t^{\beta} n_t^{1-\beta}\right)^{1-\theta}}{1-\theta}, \qquad \beta \in (0;1], \qquad \theta > 0 \qquad \text{and} \qquad \theta \neq 1 \tag{9}$$

[14] The original Lucas' (1988, Eq. 13, p. 19) formulation does not include any *dilution-effect*. Lucas' assumption (newborns enter the work-force endowed with a skill-level proportional to the level already attained by older members of the family, so population growth *per se* does not reduce the current skill level of the representative worker) is based on the *social nature* of human capital accumulation, which has no counterpart in the accumulation of physical capital and of any other form of tangible assets. On the other hand, the assumption of a linear and negative *dilution-effect* of population growth on per-capita human capital investment can be found in Strulik (2005, p. 135, Eq. 24) and Bucci (2008, p. 1134, Eq. 12'), just to mention a few examples.

where $c_t \equiv C_t / N_t$ is per-capita consumption, n_t is the net population growth rate (the birth rate), β and $(1-\beta)$ determine, respectively, the weights by which c and n enter an individual's utility function. As it is well-known, the utility function of Eq. (9) exhibits a constant elasticity of marginal utility for both c and n.[15] The hypothesis $\beta \in (0;1]$ suggests that per-capita consumption is a fundamental argument of each individual's instantaneous utility. As in Palivos and Yip (1993), θ must be different from one for an equilibrium value of n to exist (see next Eq. 14). For positive values of c_t and n_t, and with $\beta \in (0,1)$, the assumption $0 < \theta < 1$[16] guarantees that the second order conditions for a maximum are satisfied and that $u(\cdot)$ remains always strictly positive. The evolution of the population size (*N*) over time is governed by:

$$\dot{N}_t = n_t N_t, \qquad n_t \geq 0. \tag{10}$$

The inter-temporal problem faced by the representative family-head is now:

$$\underset{\{c_t, u_t, n_t, h_t, N_t\}_{t=0}^{+\infty}}{Max} U \equiv \int_0^{+\infty} \frac{\left(c_t^{\beta} n_t^{1-\beta}\right)^{1-\theta}}{1-\theta} e^{-\rho t} N_t^{\nu} dt, \qquad \rho > \nu n \geq 0; \qquad \nu \in [0;1]; \qquad \beta \in (0;1]; \qquad \theta \in (0;1) \quad (11)$$

s.t.: $$\dot{N}_t = n_t N_t, \qquad n_t \geq 0 \quad \forall t \tag{10}$$

$$\dot{h}_t = \left[\sigma(1-u_t) + f(n_t)\right] h_t, \qquad \sigma > 0; \qquad u_t \in [0;1], \ \forall t \tag{8}$$

along with the two transversality conditions: $\lim_{t\to\infty} \lambda_{ht} h_t = 0$; $\lim_{t\to\infty} \lambda_{Nt} N_t = 0$.

and the initial condition: $h(0) > 0$.

3.1 BGP ANALYSIS

We employ the following definition of BGP equilibrium.

Definition: BGP Equilibrium

A BGP equilibrium is an equilibrium path along which: (i) All variables depending on time grow at constant (possibly positive) exponential rates; (ii) The allocation of human capital between production of consumption goods and production of new human capital is constant ($u_t = u$, $\forall t$).

PROPOSITION 1

Along the BGP equilibrium, we have:

[15] The elasticity of marginal utility of consumption equals $\left[1-\beta(1-\theta)\right]$, whereas n has an elasticity of marginal utility equal to $\left[\beta+\theta(1-\beta)\right]$. When $\beta = 1$ the utility function becomes $u(c_t; n_t) \equiv \frac{c_t^{1-\theta}}{1-\theta}$ and $1/\theta > 0$ is the usual elasticity of substitution in consumption.

[16] See Growiec (2006), pp.10,16 and 17 and Jones (2001), p. 4, Eq. (1).

$$u = \frac{\beta(\theta-1)\left[\sigma + f(n)\right] + \rho - vn}{\sigma\left[\beta(\theta-1)+1\right]} \tag{12}$$

$$g_c = g_y = g_h \equiv g = \frac{(\sigma-\rho) + vn + f(n)}{\left[\beta(\theta-1)+1\right]} \tag{13}$$

$$f^{'}(n) = \frac{v}{\beta(\theta-1)} - \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1) + \beta(\theta-1)f(n) + \rho - vn}{\beta(\theta-1)+1}\right] \tag{14}$$

Proof: See *Appendix A*. ■

Equations (12) and (13) provide respectively the fraction of human capital (u) devoted by each individual in the population to non-educational activities (namely, production of consumption goods), and the common growth rate (g) attained by the economy in per-capita terms over the very long-run (BGP equilibrium), whereas (14) gives, by knowing the exact form of the nonlinear function $f(n)$, the BGP equilibrium values of the endogenous birth rate, $n \geq 0$. Note that in this model (as in the benchmark case) economic growth is driven by human capital accumulation (that is, $\dot{h}_t / h_t = g$).

It is apparent from Eq. (14) that the current model may display multiplicity of equilibria. Indeed, it is immediate to see that if $f^{'}(n)$ is a quadratic function of n, in general Eq. (14) is satisfied (for given β, v, θ, σ and ρ) for different values of n.[17] However, it is not the objective of this paper to build a theory of the role of multiplicity in economic growth, since we are mainly focused here in highlighting the possible non-linear effects of the birth rate on real per-capita income growth through per-capita human capital investment. In this respect, it is worth observing that now economic growth is a nonlinear function of n (Eq. 13). This result is entirely explained by the nonlinearity of the effect of population growth (the net birth rate) on per-capita human capital investment.

Proposition 2 ensures that the endogenous variables of the model undertake economically meaningful values along the BGP.[18] The condition found in the following proposition is very important for conducting comparative statics.

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[17] In Appendix A (Table A.1) we show that, under specific combinations of the parameter values, multiple (real and positive) roots for n coming from Eq. (14) do exist.

[18] The birth rate (n) is an endogenous variable in this economy. In particular, Eq. (14) provides the endogenous value(s) of n as a function of β, ρ, v, σ and θ. Since at the moment we do not do any assumption about the specific functional form undertaken by $f(n)$, we cannot compute in closed form the possible solutions of $n(\beta, \rho, v, \sigma, \theta)$. Hence, inequality (15) provides ultimately restrictions on the relations between the parameters of the model.

PROPOSITION 2

Assume $\sigma > \rho - vn > 0$ *. The following inequality:*

$$-\left(\sigma - \rho + vn\right) < f\left(n\right) < \frac{\rho - vn - \sigma\beta\left(1-\theta\right)}{\beta\left(1-\theta\right)} \tag{15}$$

ensures that along the BGP equilibrium $g > 0$ *and* $u \in \left(0;1\right)$ *hold simultaneously.*

Proof: Immediate from (12) and (13). ■

In this economy the key policy-parameter is σ which captures the productivity of the education sector. A high value of σ ensures that the growth rate of the economy is positive. However, the total effect of σ as well as the other parameters β, ρ, v and θ on the growth rate of the economy can be broken down into a direct and an indirect effect due to the endogenous nature of the birth rate.

Proposition 3 shows the direct and indirect effects of the parameters on the balanced growth rate of the economy.

PROPOSITION 3

In this economy we have the following results:

DIRECT EFFECT	**INDIRECT EFFECT**	**TOTAL EFFECT**
$\frac{\partial g}{\partial \sigma} > 0$	$\frac{\left[v + f'\left(n\right)\right]\frac{\partial n}{\partial \sigma}}{\left[\beta\left(\theta-1\right)+1\right]}$>0; $\frac{dn}{d\sigma} < 0$	$\frac{dg}{d\sigma} > 0$
$\frac{\partial g}{\partial v} > 0$	$\frac{\left[v + f'\left(n\right)\right]\frac{\partial n}{\partial v}}{\left[\beta\left(\theta-1\right)+1\right]}$<0; $\frac{dn}{dv} > 0$	$\frac{dg}{dv} > 0$
$\frac{\partial g}{\partial \rho} < 0$	$\frac{\left[v + f'\left(n\right)\right]\frac{\partial n}{\partial \rho}}{\left[\beta\left(\theta-1\right)+1\right]}$<0; $\frac{dn}{d\rho} > 0$	$\frac{dg}{d\rho} < 0$
$\frac{\partial g}{\partial \beta} > 0$	$\frac{\left[v + f'\left(n\right)\right]\frac{\partial n}{\partial \beta}}{\left[\beta\left(\theta-1\right)+1\right]}$>0; $\frac{dn}{d\beta} < 0$	$\frac{dg}{d\beta} > 0$
$\frac{\partial g}{\partial \theta} < 0$	$\frac{\left[v + f'\left(n\right)\right]\frac{dn}{d\theta}}{\left[\beta\left(\theta-1\right)+1\right]}$>0; $\frac{dn}{d\theta} < 0$	$\frac{dg}{d\theta} < 0$, for small altruism $\frac{dg}{d\theta} > 0$, for high altruism

Proof: See Appendix A. ■

Proposition 3 highlights a number of interesting issues. First of all, the productivity parameter in the education sector (σ) has a positive effect on the growth rate of the economy, and the higher it is the lower is the birth rate. This implies that in more developed countries, where the education sector is probably more productive, parents would prefer to have fewer children as the opportunity cost of having children is quite high. When the altruistic parameter (v) increases, for a given level of population, parents will care more both of their own utility and that of their descendants, and hence the higher this parameter the higher the birth rate in equilibrium. In that case, the direct effect of (v) dominates the negative dilution-effect on economic growth.

The other two results are quite intuitive. It is well known that an increase in the time-preference parameter ρ reduces growth because people are more impatient and they do not save enough. Normally, the expected sign of $\frac{\partial n}{\partial \rho}$ would be negative because people with high (ρ) are not taking into account future generations. However, in our model it is positive, something that suggests that the motivation of having children goes beyond the simple consideration of available resources. The higher the weight individuals assign to consumption per capita (β) relatively to the number of births, the higher will be the positive impact of (β) on economic growth, and the lower the birth rate in equilibrium. Finally θ, which describes the relative risk aversion of individuals, has a negative effect on growth for small values of altruism (high β and low v), and a positive effect for high values (low β and high v). Children are considered to be a risky asset and the higher the value of θ, the lower the birth rate in equilibrium**.**

4. From Theory to Evidence: Empirical Methodology and Data Description

By focusing on the effect of the birth rate n on human capital accumulation,[19] our main interest in this section is to estimate Eq. (8):

$$\dot{h}_t / h_t = \sigma(1-u_t) + f(n_t).$$

Since there are no available data for u (which is a function of n according to Eq. 12), simply replacing in the equation above the equilibrium value of u (Eq. 12), yields:

[19] Even though in the theory presented above we set $d=0$, in our empirical analysis we include the death rate as a control variable. More specifically, we use life expectancy as a proxy for the death rate. Alternatively, and for the sake of robustness, we also employed the crude death rate. Contrary to life expectancy, the crude death rate displays negative sign and is statistically significant. In both cases, however, we do find a nonlinear effect of the birth rate on per-capita human capital accumulation. The results with the death rate as a control variable are available upon request.

$$\dot{h}_t / h_t = \underbrace{\sigma\left(1-u_t\right)}_{\text{accumulation}} + \underbrace{f\left(n_t\right)}_{\text{dilution}} = \underbrace{\sigma - \frac{\beta\left(\theta-1\right)\left[\sigma + f\left(n\right)\right] + \rho - \nu n}{\left[\beta\left(\theta-1\right)+1\right]}}_{\text{accumulation}} + \underbrace{f\left(n_t\right)}_{\text{dilution}} \equiv \phi\left(n_t\right)$$

Hence, the equation we ultimately estimate is of the form:

$$\frac{\dot{h}_t}{h_t} = \phi\left(n_t\right).$$

This equation captures at the same time the combined effect of the *accumulation* and *dilution* effects. If the total effect of a change in n on $\dot{h}_t / h_t$ is positive, then the dilution effect is smaller than the accumulation effect, whereas if it is negative then the dilution effect of having a higher fertility rate dominates the accumulation effect. We use an estimation method more flexible than OLS in order not only to check whether the relationship written above is mis-specified, but also to search for nonlinearities in the link between the birth rate and human capital investment at the individual level. In addition, we also examine the possible endogeneity of n. However, before going into the details of the econometric techniques and the specification tests used, we start by describing briefly the variables employed and the sources of our data.

Our sample consists of a panel of ninety-nine countries (both OECD and non-OECD) for the period 1960–2000. The observations are averages over a 5 years-interval.[20] Hence, we have 9 time periods. For human capital accumulation (which is our dependent variable) we use enrollment rates for population aged between 15 and 65 years.[21] The data about this variable come from the Barro–Lee (2000) dataset. Barro and Lee (2000) fill the missing observations for some specific years and countries by using information taken from United Nations on school enrollment ratios and the structure of population by age-groups. Furthermore, they use adjusted gross enrollment ratios in order to take into account a significant fraction of students who either start earlier school attendance, or change their own level of education with delay. It is well-known that the Barro–Lee dataset for human capital has an important limitation in the fact that it does not explicitly take into account differences in schooling-quality across countries. For this reason we use group dummy variables for controlling for any potential difference between groups of countries. Despite its limitations, the Barro–Lee (2000) dataset has been extensively used in recent years

[20] The use of averages over periods of 5-years, as opposed to the use of annual data, has been proposed in the literature as a strategy for reducing possible business cycle effects.

[21] When working with fertility it is better to use human capital data for population which is above 15 years old. We have tried another dataset for human capital from the Barro-Lee (2000) data base for population above 25 years and the nonlinearities are also present there. The results for population over 25 years old are available upon request.

and as such allows us to make direct comparisons with other empirical studies that explore the role of human capital in economic growth.[22]

According to de la Fuente and Domenech (2000), the stock measure of human capital (total mean years of schooling data) suffer from serious measurement error problems, something that would be exacerbated if we were to obtain growth rates from differencing the stock series. Hence, instead of measuring human capital accumulation in growth rates as it is the case for per-capita income, we prefer to use enrollment rates. Human capital accumulation for country *i* at time *t* is denoted by $HUM_{i,t}$. The data for the crude birth rate (*cbr*) come from the UN dataset (2008). The crude birth rate is measured as the number of births over 1000 people.[23] This is the variable that we estimate by using both OLS and semi/non-parametric methods.[24] All the other variables enter linearly in our econometric specification.

We use time dummies (d_t) in order to capture any time specific effects and group dummies ($dOECD_i$) to capture any difference between OECD and non-OECD countries. Following Kalaitzidakis *et al.* (2001), we also employ regional dummies (dRegion_i) – such as dummies for Latin America (*la*) and Sub-Saharan Africa (*af*) countries – to control for possible heterogeneity in our sample.

Finally, we use a vector $X_{i,t}$ = (lifexp, hum60, infmort) of control variables that are used at a later stage in order to check for the robustness of our results. These variables are life expectancy (*lifexp*), human capital at 1960 (*hum60*),[25] and infant mortality (*infmort*), respectively. The data concerning the two demographic variables (life expectancy and infant mortality), like those on the birth rate, come from the UN (2008) dataset. Life expectancy (*lifexp*) is defined as *"the average number of years of life expected by a hypothetical cohort of individuals who would be subject during all their lives to the mortality rates of a given period"* (United Nations, 2008). This variable is expected to have a positive impact on human capital accumulation (intuitively, when expecting a higher chance of surviving, rational individuals invest more in education to enhance earnings for consumption over a longer life. Moreover, the longer life expectancy, the longer the time-horizon within which the costly investment in human capital can be re-paid). Infant mortality (*infmort*) is the probability of dying between birth and age one. It is expressed as the number of deaths per 1000 births and it is expected to have a negative impact on human capital accumulation, since a rise of infant mortality would imply that the *quantity-effect* (having more children)

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[22] We use total human capital, which is the sum of primary, secondary and higher education. This is done for three major reasons. The first is mainly theoretical in nature and relies on the fact that in the model of the previous section there is no distinction of human capital by category of education. Secondly, non-OECD countries exhibit very small participation rates at higher levels of education. Finally, because in every country participation at primary and secondary level of education is a necessary requirement in order for people to proceed into higher levels of schooling, we believe it is preferable to include also primary and secondary education in our measure of human capital.

[23] The crude birth rate is expressed in percentage terms for comparability with the enrollment rates.

[24] We have also used the *adjusted birth rate* proposed by Brander and Dowrick (1994):*"…Adjusted births are simply crude or gross births net of infant mortality"* (Brander and Dowrick, 1994, p. 5). The results do not change and are available upon request.

[25] Human capital at 1960 (*hum60*) is the stock of human capital in the year 1960. It is measured as the average years of schooling in that year. We use this variable, and not the enrollment rate at 1960, in order to reduce any possible bias due to the correlation with the dependent variable. However, we have also used enrollment rates at 1960, but results are not affected at all by this change.

dominates the *quality-effect* (having lower fertility, but more education per child). The average years of schooling at 1960 (*hum60*) is a stock measure of human capital and is used in order to capture the difference in initial conditions across countries. Contrary to initial GDP per capita (used in empirical exercises on the neoclassical growth theory to predict convergence), in our endogenous growth model in the long-run (the BGP equilibrium) there exists no convergence across countries starting from different levels of development. Hence this variable reveals the preference of a country towards human capital accumulation and as such it is expected to have a positive impact on our dependent variable.

The equation we estimate by OLS is the following:

$$HUM_{i,t} = b_0 + b_1\left(dOECD_i\right) + b_2\left(dRegion_i\right) + b_3 d_t + b_4 n_{i,t} + \varepsilon_{i,t}\ ,$$

with $\varepsilon_{i,t}$ being an *iid* error term. The equation we estimate in the semi-parametric framework is:

$$HUM_{i,t} = b_0 + b_1\left(dOECD_i\right) + b_2\left(dRegion_i\right) + b_3 d_t + \phi\left(n_{i,t}\right) + \varepsilon_{i,t}\ .$$

In the above equation, $\phi(n_{i,t})$ is used to capture possible nonlinearities in the relationship between birth rate and human capital accumulation (all the other variables enter linearly in our specification). We also augment the model to include additional available explanatory variables to guard against omitted variable bias and we further test within the linear framework for possible endogeneity of n, using a Durbin-Wu-Hausman test. Having established the presence of a nonlinear relationship, we will try to best approximate it by different polynomials both in the model with and without the control variables, selecting the one with the highest explanatory power. Subsequently, we use the specification with the extra control variables for conducting sensitivity analysis. The following equation is the one used for checking the robustness of the linear specification estimates, and a similar equation is used when we include the optimal polynomial of birth rate:

$$HUM_{i,t} = b_0 + b_1\left(dOECD_i\right) + b_2\left(dRegion_i\right) + b_3 d_t + b_4 n_{i,t} + b_5 X_{i,t} + \varepsilon_{i,t}$$

where $X_{i,t}$ = (lifexp, hum60, infmort) is the vector of control variables. Finally, the equation we estimate by considering all the variables non-parametrically is:

$$HUM_{i,t} = f(b_0, dOECD_i, dRegion_i, d_t, n_{i,t}) + \varepsilon_{i,t}\ .$$

The fully non-parametric method that is used in this paper was proposed first by Racine and Li (2004) and is appropriate for mixed data (discrete and continuous variables). We also use the Hsiao *et al.* (2007) test to check whether the linear model is well specified against semi/non parametric alternatives. Finally, we use a significance test[26] for the nonparametric regression model in order to ensure that the birth rate is significant in the nonparametric framework.

4.1 Results

A first glance at Table B.1 (all Tables are in *Appendix B*) reveals that the birth rate (*cbr*) is statistically significant and appears in the regression with a negative sign.[27] This is consistent with the benchmark theoretical model (see Eq. 5). The dummies both for Latin America and Sub-Saharan Africa countries are statistically significant and positive in all the specifications, except when human capital is measured at the secondary-higher level. In this case the region dummies have negative sign. This is an indication that in these groups of countries the *dilution-effect* is stronger at the secondary-higher level of education.[28] We proceed to estimate a semi-parametric and a fully nonparametric version of the above benchmark model. Clearly, the semi-parametric and the fully non-parametric specifications[29] increase the explanatory power of the model (both have a very similar explanatory power). Furthermore, the Hsiao *et al.* (2007)[30] test strongly rejects the hypothesis of a linear specification against the semi-parametric and non-parametric alternatives, see the p-values of P(Specific) in Tables B.1 and B.4. We proceed in Table B.2 to use polynomial terms for the birth rate and also a dummy-variable for OECD countries which interacts with the birth rate in order to control for any heterogeneity between OECD and non-OECD group of countries. Clearly, using polynomial terms increases the explanatory power of the model. We only present here results concerning the inclusion of a third degree polynomial, since higher-order polynomials do not appear to be statistically significant. In column B of Table B.2 the polynomial terms are statistically significant, both individually and jointly (see the *F - Joint test*). In column C of the same Table we add an extra variable which captures the interaction between the OECD-dummy and the birth rate. This term is statistically significant and positive. Since the negative coefficient on the birth rate is larger than the positive coefficient on the interaction term, we infer that the *dilution-effect* (*i.e.*, the negative impact of the birth rate on human capital investment at the individual level) in OECD countries is smaller than in non-OECD countries. Finally, in column D we consider the interaction between the OECD-dummy and the higher-power terms of the birth rate. By looking at the correspondent t-statistics and the F-Joint test, all the interaction terms are statistically insignificant both individually and jointly, which means that the birth

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26 For this test see Racine (1997).

27 As we are going to mention in a moment, the birth rate (*cbr*) appears with negative sign in the linear specification and there exist nonlinearities (a polynomial of the 3rd power). Furthermore, we have the same kind of nonlinearities also when we use total fertility (instead of the crude birth rate), which confirms our hypothesis of a nonlinear effect of newborns on per capita human capital accumulation in per-capita terms. We do not present the results with total fertility here, but they are available upon request.

28 The results for primary and tertiary level of education are not presented but are available upon request.

29 The optimal bandwidths, for these methods are found by cross validation and are available upon request. The choice of kernel is the Gaussian kernel and the method of estimation relies on a constant kernel approach.

30 The Hsiao test results are showed in the row P(Specific.). In column A, there is the comparison between OLS and fully-non parametric and in column B the comparison is between OLS and semi parametric.

rate behaves similarly across the different groups of countries.[31] We further proceed to test for possible endogeneity of the birth rate in the extended linear model with the extra control variables at our disposal (lifexp, hum60, infmort). The results are presented in Table B.3. In columns C and D we have similar results as in Table B.2 (without the control variables). Again the interaction terms of the polynomial with the OECD dummy, is not statistically significant both individually and jointly and also the third power polynomial remains statistically significant jointly and the signs are the same as in Table B.2. Finally, the effect of the birth rate is still nonlinear. This result appears to be robust and is the same for OECD as well as for non-OECD countries. Since the specification with the control variables displays by far the best fit we use it as the basis to conduct the test of endogeneity. We find strong evidence against the null hypothesis of no endogeneity, a result that contradicts the main premise of the benchmark model that was based on an exogenous birth rate.[32] Note that as instruments we use the lagged values of the birth rate**.** We also estimated the semi-parametric model using the lagged birth rate variable in place of the current birth rate in the semi-parametric specification, see the semi-parametric graphs (Figures 1 and 2). Figure 1 is the semi-parametric graph for the contemporaneous value of birth rate and Figure 2 for the lagged birth rate**.** Interestingly, both graphs are quite similar, suggesting that the nature of the nonlinear relationship is preserved for different measures of the birth rate, its current and lagged forms.[33] A closer look at either figure reveals that a low birth rate has a small positive contribution if any at all but after a given point the dilution effect appears to become strong. This is in line both with theories that support birth control in overpopulated countries and also with those that claim that rich countries need to do the opposite.

4.2 Sensitivity Analysis

In order to check for the robustness of our results we follow two different ways. Firstly, we use for the initial sample (792 observations) the extra control variables mentioned earlier (*i.e.,* life expectancy, infant mortality and human capital in 1965) to test the significance of the polynomial terms (compare Tables B.2 and B.3). Secondly, we exclude the *OPEC* countries as income earned from oil production potentially distorts the fertility–human capital nexus and we also exclude the *Ex-socialist* countries (Eastern Europe and Cuba), since for these countries the existence of a centrally-planned regime might have influenced more effectively the dynamics of the birth rate in the past. Results appear in Table B.4.

Table B.3 allows us to draw some additional conclusions. First of all, from column B, we observe that the polynomial for the birth rate preserves the same order and its terms are jointly statistically significant.

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31 We have also done separate regressions for OECD and non-OECD countries and still the same third power polynomial of the birth rate appears to be the correct specification. These results are available upon request.

32 The Prob>chi2 value of Durbin-Wu-Hausman test for endogeneity is 0.0476 which implies that the null hypothesis for the absence of correlation between birth rate "*n*" and the error term is rejected.

33 We have also tried additional lagged values of the birth rate and the results remained qualitatively the same.

Furthermore, in columns C and D we have similar results as in Table B.2 (without the control variables). Finally, the effect of the birth rate still appears nonlinear. This result seems to be robust and the same for OECD as well as for non-OECD countries.

From Table B.4 we see that some of the time-dummies are statistically significant. All the other results are qualitatively the same as in Table B.1. In particular, we observe that the birth rate is statistically significant and still has a negative sign. Yet, as before the Hsiao *et al.* (2007) test strongly rejects the linear specification of the benchmark model. However, the current nonlinear effect appears stronger (see Figure 3). This is in line with our conjecture that the excluded countries have been able to control more effectively their own birth rates, hence mitigating the presence of nonlinear effects.

We can summarize the findings of this section as follows. Superficially, at first there seems to be a negative relationship between the birth rate and human capital by using a linear specification corresponding to the benchmark model developed in section 2. The benchmark specification is strongly rejected when subjected to rigorous testing and re-estimation of the human capital–birth rate nexus using semi-parametric and nonparametric methods reveals the presence of a strong nonlinear relationship. However, even in the presence of nonlinearities the dilution effect dominates any positive effect that comes from the accumulation effect for most of the observations in our sample. The linear benchmark specification also suffers from endogeneity bias, as the birth rate cannot be taken as exogenous. Adding extra control variables does not change the overall nonlinear pattern of the relationship. The latter result invalidates the main premise of the benchmark model that the relationship between birth rate and per-capita human capital investment would be expected to be linear.

5. Summary and Conclusions

This paper has reassessed the long-run correlation between demographic change and economic growth from an empirical as well as a theoretical point of view. In order to accomplish this task, we focused on the fertility rate as our demographic variable of interest (the birth rate is, indeed, one of the most important demographic variables, as it affects directly population growth), and on human capital accumulation as the fundamental driver of sustained long-run economic growth. From the empirical point of view, our motivation was to search for possible nonlinearities in the (negative) relation between birth rate and human capital investment at the individual level. From the theoretical point of view, instead, our main objective has been to compare the results (concerning the long-run relationship between population and economic growth rates) of two completely different models: the first is a simple (although quite standard) model in which the birth rate is exogenous and affects linearly and negatively per-capita human capital accumulation (linear and negative *dilution-effect*); the second, instead, is a model in which the birth rate is endogenous and the *dilution-effect* is nonlinear. Since countries experience different birth rates and have different economic performances as well, our major conjecture in building the second model was that fertility can have a nonlinear impact on economic growth through its nonlinear effect on per-capita human capital investment. This result accords well with the idea that an increase in population growth may have a differential impact on economic growth across countries (Kelley, 1988).

The benchmark model is strongly rejected by the data, according to which there exist strong nonlinearities in the relationship between the fertility rate and per-capita human capital investment. Because of these nonlinearities, we can say, for any different value of n (and, hence, for any different country in the sample), which effect (whether the positive accumulation effect, or the negative dilution effect) dominates the other one.

As a whole, our analysis suggests that in more developed economies higher birth rates can have a positive growth-effect and that the main policy initiatives here should concentrate on easing the opportunity costs of having children, since the *accumulation-effect* is stronger than the *dilution-effect*. Of course, for most of the other countries, and especially for the least developed ones, birth rates should be reduced substantially. Our proposal for the group of countries with high birth rates and low life expectancy is that they need simultaneously to reduce the birth rate and to improve the health conditions of their people in order to be able to achieve higher growth rates**.**

This paper can be extended along different directions, three of which are listed here. The first would be to consider a human capital production technology different from the Lucas' (1988), and more similar to the Mincer (1974)'s specification (Bils and Klenow, 2000, Eq. 3, p. 1162). Secondly, in this article we treated human capital as the sole reproducible factor, entering the aggregate production function as the only input. Though, it is well known that economic agents (individuals and firms) do accumulate and produce by way of a variety of other factor-inputs (think of physical and technological capital, just to mention some notable examples). Building and testing a theory of endogenous fertility in which there is more than one reproducible factor-input and where there exist nonlinear *dilution-effects* of the birth rate in the law of motion of one or all of these factor-inputs still remains at the top of future theoretical research agenda and, thus, represents a further possible extension of the present paper. Finally, there is scope to develop a model based on micro foundations in which nonlinearities will be derived from individual behavior, something that we leave for future research.

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APPENDIX A:

1) EQS. (12) – (14)

Since $c_t \equiv \frac{C_t}{N_t} = \frac{Y_t}{N_t} \equiv y_t = Au_t h_t$, the Hamiltonian function (J_t) is:

$$J_t = \frac{\left[\left(Au_t h_t\right)^{\beta} n_t^{1-\beta}\right]^{1-\theta}}{1-\theta} e^{-\rho t} N_t^{\nu} + \lambda_{Nt} n_t N_t + \lambda_{ht}\left[\sigma\left(1-u_t\right) + f\left(n_t\right)\right] h_t .$$

The necessary FOCs read as:

(A1) $\frac{\partial J_t}{\partial n_t} = 0 \quad \Leftrightarrow \quad \left(1-\beta\right)\left(Au_t h_t\right)^{\beta(1-\theta)} n_t^{(1-\beta)(1-\theta)-1} e^{-\rho t} N_t^{\nu} + \lambda_{Nt} N_t + \lambda_{ht} h_t f'\left(n_t\right) = 0$

(A2) $\frac{\partial J_t}{\partial u_t} = 0 \quad \Leftrightarrow \quad \beta\left(Ah_t\right)^{\beta(1-\theta)} u_t^{\beta(1-\theta)-1} n_t^{(1-\beta)(1-\theta)} e^{-\rho t} N_t^{\nu} - \sigma \lambda_{ht} h_t = 0$

(A3) $\frac{\partial J_t}{\partial N_t} = -\dot{\lambda}_{Nt} \quad \Leftrightarrow \quad \nu \frac{\left(Au_t h_t\right)^{\beta(1-\theta)} n_t^{(1-\beta)(1-\theta)}}{1-\theta} e^{-\rho t} N_t^{\nu-1} + \lambda_{Nt} n_t = -\dot{\lambda}_{Nt}$

(A4) $\frac{\partial J_t}{\partial h_t} = -\dot{\lambda}_{ht} \quad \Leftrightarrow \quad \beta\left(Au_t\right)^{\beta(1-\theta)} n_t^{(1-\beta)(1-\theta)} h_t^{\beta(1-\theta)-1} e^{-\rho t} N_t^{\nu} + \lambda_{ht}\left[\sigma\left(1-u_t\right) + f\left(n_t\right)\right] = -\dot{\lambda}_{ht}$.

Along the BGP equilibrium all variables depending on time grow at constant exponential rates. Therefore, *n* and *u* are constant in the long-run (see Eqs. 10 and 8 in the text). With this in mind, after taking logs of both sides of (A2) and differentiating with respect to time, we obtain:

(A5) $-\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \left[\beta\left(\theta-1\right)+1\right]\left[\sigma\left(1-u\right) + f\left(n\right)\right] + \rho - \nu n$, $\qquad \frac{\dot{h}_t}{h_t} \equiv g_h = \left[\sigma\left(1-u\right) + f\left(n\right)\right]$.

We solve (A2) with respect to λ_h:

(A2') $\lambda_{ht} = \sigma^{-1}\beta A^{\beta(1-\theta)} h_t^{\beta(1-\theta)-1} n^{(1-\beta)(1-\theta)} u^{\beta(1-\theta)-1} e^{-\rho t} N_t^{\nu}$.

Using (A2') into (A4) and dividing both sides of the resulting equation by λ_{ht}, yields:

(A4') $-\frac{\dot{\lambda}_{ht}}{\lambda_{ht}} = \sigma + f\left(n\right)$.

We now divide both sides of (A3) by λ_{Nt}:

(A3') $-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = \frac{v\left(Auh_t\right)^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)} e^{-\rho t} N_t^{v-1}}{\left(1-\theta\right)\lambda_{Nt}} + n$,

and both sides of (A1) by λ_{ht}:

$$\frac{\left(1-\beta\right)}{\lambda_{ht}}\left(Auh_t\right)^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)-1} e^{-\rho t} N_t^{v} + \frac{\lambda_{Nt}}{\lambda_{ht}} N_t + h_t f^{'}\left(n\right) = 0 .$$

If we substitute (A2') for the first term of the last equation we obtain:

(A1') $\frac{\lambda_{Nt} N_t}{\lambda_{ht} h_t} = -\left[f^{'}\left(n\right) + \frac{\left(1-\beta\right)\sigma u}{\beta n} \right]$.

In the BGP equilibrium the right hand side of (A1') is a constant. Therefore, we can take logs of both sides of (A1'), and derive with respect to time:

$$\frac{\dot{\lambda}_N}{\lambda_N} + \frac{\dot{N}}{N} - \frac{\dot{\lambda}_h}{\lambda_h} - \frac{\dot{h}}{h} = 0 .$$

By using (A4') and Eqs. (8) and (10) in the main text, we can recast the last expression as:

(A1") $-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = n + \sigma u$.

By equating (A4') with (A5) we obtain the BGP equilibrium value of u:

(A6) $u = \frac{\beta\left(\theta-1\right)\left[\sigma + f\left(n\right)\right] + \rho - vn}{\sigma\left[\beta\left(\theta-1\right)+1\right]}$

Eq. (A2') can be recast as:

$$\frac{\sigma u \lambda_{ht} h_t}{\beta} = A^{\beta(1-\theta)} h_t^{\beta(1-\theta)} n^{(1-\beta)(1-\theta)} u^{\beta(1-\theta)} e^{-\rho t} N_t^{v} .$$

This expression can be replaced into (A3'), yielding:

(A3") $-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = \frac{\lambda_{ht} h_t}{\lambda_{Nt} N_t}\left[\frac{v\sigma u}{\beta\left(1-\theta\right)}\right] + n$.

Using (A1') into (A3") gives:

$$-\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}} = \frac{-nv\sigma u + n\left(1-\theta\right)\left[\beta n f^{'}\left(n\right) + \left(1-\beta\right)\sigma u\right]}{\left(1-\theta\right)\left[\beta n f^{'}\left(n\right) + \left(1-\beta\right)\sigma u\right]} .$$

After equating the last expression to (A1"), we get:

(A7) $f^{'}\left(n\right) = \frac{v}{\beta\left(\theta-1\right)} - \frac{\left(1-\beta\right)\sigma u}{\beta n}$,

Substituting *u* from (A6) into (A7):

(A7') $f^{'}\left(n\right) = \frac{v}{\beta\left(\theta-1\right)} - \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta\left(\theta-1\right) + \beta\left(\theta-1\right) f\left(n\right) + \rho - vn}{\beta\left(\theta-1\right)+1}\right]$.

From the fact that:

$$c_t \equiv \frac{C_t}{N_t} = \frac{Y_t}{N_t} \equiv y_t = Auh_t ,$$

we conclude:

$$g_c = g_y = g_h \equiv g = \sigma(1-u) + f(n).$$

After replacing in the last equation u from (A6) we finally obtain:

(A8) $$g_c = g_y = g_h \equiv g = \frac{(\sigma-\rho)+vn+f(n)}{\left[\beta(\theta-1)+1\right]}.$$

We now check that the two transversality conditions:

$$\lim_{t\to+\infty} \lambda_{ht} h_t = 0 \text{ and } \lim_{t\to+\infty} \lambda_{Nt} N_t = 0$$

do hold. Using Eq. (A4') and the fact that $g_h \equiv g = \sigma(1-u) + f(n)$, the first condition can be recast as:

$$\lim_{t\to+\infty} \lambda_{ht} h_t = \lambda_h(0) h(0) \lim_{t\to+\infty} e^{\left(\frac{\dot{\lambda}_{ht}}{\lambda_{ht}}+g\right)t} = \lambda_h(0) h(0) \lim_{t\to+\infty} e^{-(\sigma u)t} = 0.$$

Hence, for each $\lambda_h(0) > 0$, $h(0) > 0$ and $\sigma > 0$, the transversality condition is clearly satisfied for each

$$u > 0.$$

The second transversality condition, after employing Eqs. (A1") and (10) in the text, can be re-written as:

$$\lim_{t\to+\infty} \lambda_{Nt} N_t = \lambda_N(0) \lim_{t\to+\infty} e^{\left(\frac{\dot{\lambda}_{Nt}}{\lambda_{Nt}}+\frac{\dot{N}_t}{N_t}\right)t} = \lambda_N(0) \lim_{t\to+\infty} e^{-(\sigma u)t} = 0, \qquad N(0) \equiv 1.$$

Again, for each $\lambda_N(0) > 0$ and $\sigma > 0$, this transversality condition is also satisfied for each $u > 0$. In the main text we guarantee that $u > 0$ does hold.

Going back to Eq. (A7') we see that this equation gives, by knowing the exact form of the nonlinear function $f(n)$, the BGP equilibrium values of the endogenous birth rate, $n \geq 0$. If $f'(n)$ were a square function of n, Eq. (A7') would be satisfied, for given β, v, θ, σ and ρ, for three different values of n.

Since empirically the relationship between fertility and human capital accumulation is mainly negative (the *dilution-effect* dominates the *accumulation-effect*), we assume that the empirical results we find provide a proxy for the magnitude of the dilution-effect. By looking at these empirical results (Table B2, column B), we observe that the dilution-effect can be approached by the following polynomial function: $f(n) = 26.35n - 1143n^2 + 9224n^3$. Then, Table A.1 in this Appendix provides the solutions (the real and positive roots) of Eq. (A7'), for specific combinations of the values of the parameters. For some of them we have, indeed, empirical estimates (or baseline specifications) coming from previous works. For others, we consider a set of possible parameterizations. More precisely, we use the following parameter-values:

- $\theta = 0.8$;

When $\beta = 1$, the utility function used in the text becomes $u(c;n) = \frac{\left(c^\beta n^{1-\beta}\right)^{1-\theta}}{1-\theta} = \frac{c^{1-\theta}}{1-\theta} = u(c)$ and $1/\theta > 0$ is the elasticity of substitution in consumption. So, θ represents the inverse of the intertemporal elasticity of substitution in consumption when $\beta = 1$. In the endogenous fertility literature many authors suggest θ to take values lower than one in order for the flow instantaneous utility function to be everywhere positive. A value of $\theta = 0.8$ is used by Growiec (2006, p.16, Table 1) who, unlike the present paper, considers a CRRA utility function which is separable in its two arguments: per capita consumption and the birth rate, respectively. When $u(c;n) = \frac{\left(c^\beta n^{1-\beta}\right)^{1-\theta}}{1-\theta}$, the elasticity of substitution in consumption becomes: $\frac{1}{\left[1-\beta(1-\theta)\right]}$. Therefore, the value of this elasticity depends both on θ and β.

- $\rho = 0.08$;

This parameter comes also from the numerical example discusses in Growiec (2006, p.16, Table 1).

- $\sigma = 0.12$.

This parameter-value come from Mulligan and Sala-i-Martin (1993, p.761).

- $\nu \in [0;1]$

As far as we know, Altonji *et al.* (1997) represents one of the very few attempts at obtaining a direct estimate of agents' degree of altruism. Their paper tests for a specific form of altruism, namely that of parents who make money transfers to their children. According to the theory of *inter-vivos* transfers, we would face perfect altruism if an increase, say, by one-dollar in the income of parents making transfers to a child, coupled with a simultaneous one-dollar decrease in that child's income, resulted in the parents' increasing their transfer to the child by exactly one dollar. To test for this hypothesis, the authors use the 1968-89 Panel Study of Income Dynamics (PSID) data-set, which contains a supplementary survey on family transfers. The PSID collects separate panel data on parents and most of their adult children. Consequently, the authors can control for the principal theoretical determinants of money transfers (the current and permanent incomes of the parents, the child, and the child's siblings). The effective sample consists in the end of 3402 parent-child pairs, including 687 pairs with positive transfers. The findings of this study say that redistributing one dollar from a recipient child to donor parents leads to only about a 13-cent increase in the parents' transfer to the child, far less than the one-dollar increase we would observe in the presence of perfect altruism. Using panel data on bequests, rather than *inter-vivos* transfers from parents to children, Laitner and Ohlsson (2001) obtain a similar result. On the basis of such literature, we use four different values of ν, namely $\nu = 0$; $\nu = 0.13$; $\nu = 0.5$; $\nu = 1$.

- $\beta(0;1]$

Like ν, we use three different values for β, as well: $\beta = 0.25$; $\beta = 0.5$; $\beta = 0.75$; $\beta = 1$ (this parameter cannot be equal to zero). Results are as follows:

	$\theta = 0.8$ $\rho = 0.08$ $\sigma = 0.12$
$\nu = 0$ $\beta = 0.25$	$n_1 = 0.0214$ $n_2 = 0.0653$
$\nu = 0$ $\beta = 0.5$	$n_1 = 0.0163$ $n_2 = 0.0673$
$\nu = 0$ $\beta = 0.75$	$n_1 = 0.0145$ $n_2 = 0.0682$
$\nu = 0$ $\beta = 1$	$n_1 = 0.0138$ $n_2 = 0.0688$
$\nu = 0.13$ $\beta = 0.25$	$n_1 = 0.0230$ $n_2 = 0.0634$
$\nu = 0.13$ $\beta = 0.5$	$n_1 = 0.0170$ $n_2 = 0.0665$
$\nu = 0.13$ $\beta = 0.75$	$n_1 = 0.0150$ $n_2 = 0.0676$
$\nu = 0.13$ $\beta = 1$	$n_1 = 0.0143$ $n_2 = 0.0683$
$\nu = 0.5$ $\beta = 0.25$	$n_1 = 0.0286$ $n_2 = 0.0572$

$\nu = 0.5$ $\beta = 0.5$	$n_1 = 0.0193$ $n_2 = 0.0640$
$\nu = 0.5$ $\beta = 0.75$	$n_1 = 0.0166$ $n_2 = 0.0660$
$\nu = 0.5$ $\beta = 1$	$n_1 = 0.0155$ $n_2 = 0.0671$
$\nu = 1$ $\beta = 0.25$	$n_1 = 0.0408$ $n_2 = 0.0439$
$\nu = 1$ $\beta = 0.5$	$n_1 = 0.0230$ $n_2 = 0.0600$
$\nu = 1$ $\beta = 0.75$	$n_1 = 0.0190$ $n_2 = 0.0635$
$\nu = 1$ $\beta = 1$	$n_1 = 0.0174$ $n_2 = 0.0652$

Table A1: *The roots ($n \geq 0$) of the equation* $f^{'}(n) = \frac{\nu}{\beta(\theta-1)} - \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-\nu n}{\beta(\theta-1)+1}\right]$ *for possible combinations of* ν *and* β *, and for given* $\theta = 0.8$ *,* $\rho = 0.08$ *and* $\sigma = 0.12$ *.*

For each single combination of the parameters reported in Table A1, it is possible to show that the restrictions $\rho - \nu n > 0$ and $\sigma > \rho - \nu n$ (see Proposition 2 in the text) do hold. Moreover, the elasticity of substitution in consumption, $\frac{1}{\left[1-\beta(1-\theta)\right]}$, is equal to: 1.05 (when $\beta = 0.25$); 1.11(when $\beta = 0.5$); 1.18 (when $\beta = 0.75$), and 1.25(when $\beta = 1$). ■

2) SECOND ORDER CONDITIONS FOR MAXIMIZED HAMILTONIAN

In our model we may have many interior solutions for *n*, due to the specific form taken by the nonlinear function $f(n)$. Hence, we cannot solve explicitly for *n* and as such it is quite hard to solve explicitly for the Arrow concavity condition. For this reason we use Corollary 3.1 from Caputo (1995) ch. 3, p. 55 **,** which requires the intertemporal utility function to be concave in u, n, h and N and the two dynamic constraints for population and human capital to be also concave on u, n, h and N :

$$U_{NN} = \frac{\nu(\nu-1)}{1-\theta}(Auh)^{\beta(1-\theta)}\, n^{(1-\beta)(1-\theta)} N_t^{\nu-2} \leq 0, \qquad \text{for } \theta < 1,$$

$$U_{hh} = \left[\beta(1-\theta)-1\right]\beta(Au)^{\beta(1-\theta)}\, h^{\beta(1-\theta)-2} n^{(1-\beta)(1-\theta)} N_t^{\nu} \leq 0,$$

$$U_{nn} = \left[(1-\beta)(1-\theta)-1\right](1-\beta)(Auh)^{\beta(1-\theta)}\, n^{(1-\beta)(1-\theta)-2} N_t^{\nu} \leq 0,$$

$$U_{uu} = \beta\left[\beta(1-\theta)-1\right](Ah)^{\beta(1-\theta)}\, u^{\beta(1-\theta)-2} n^{(1-\beta)(1-\theta)} N_t^{\nu} \leq 0.$$

The first constraint $g_1 = n_t N_t$ is linear in all the variables (u,n,h,N) and therefore concave, whereas the second constraint $g_2 = \left[\sigma(1-u_t) + f(n_t)\right]h_t$ is linear in (u,h,N) and for *n* takes the following value: $g_{2nn} = f''(n_t)h_t \le 0$ for $n \in [0, 0.0413]$, which contains the average value of birth rate in our sample. For very high values of *n* it is easy to show that the growth rate of the economy is negative and the human capital accumulation will decrease, so $g_{2nn} = f''(n_t)h_t \to 0$. That implies that the second constrain is linear and therefore concave in *n*. ■

3) CONDITIONS FOR PROPOSITION 3

By differentiating Eq. (13) with respect to σ, v, ρ, and θ we get the total effects as follows:

$$\frac{dg}{d\sigma} = \frac{1+\left[v+f'(n)\right]\frac{\partial n}{\partial \sigma}}{\left[\beta(\theta-1)+1\right]}; \quad \frac{dg}{dv} = \frac{n+\left[v+f'(n)\right]\frac{\partial n}{\partial v}}{\left[\beta(\theta-1)+1\right]}; \quad \frac{dg}{d\rho} = \frac{-1+\left[v+f'(n)\right]\frac{\partial n}{\partial \rho}}{\left[\beta(\theta-1)+1\right]};$$

$$\frac{dg}{d\beta} = \frac{-(\theta-1)\left[(\sigma-\rho)+vn+f(n)\right]}{\left[\beta(\theta-1)+1\right]^2} + \frac{\left[v+f'(n)\right]\frac{\partial n}{\partial \beta}}{\left[\beta(\theta-1)+1\right]}; \quad \frac{dg}{d\theta} = -\frac{\left[\sigma-\rho+vn+f(n)\right]\beta}{\left(\beta(\theta-1)+1\right)^2} + \frac{\left[v+f'(n)\right]\frac{dn}{d\theta}}{\left[\beta(\theta-1)+1\right]}$$

The direct effects are:

$$\frac{\partial g}{\partial \sigma} = \frac{1}{\left[\beta(\theta-1)+1\right]} > 0; \quad \frac{dg}{dv} = \frac{n}{\left[\beta(\theta-1)+1\right]} > 0; \quad \frac{dg}{d\rho} = \frac{-1}{\left[\beta(\theta-1)+1\right]} < 0;$$

$$\frac{dg}{d\beta} = \frac{-(\theta-1)\left[(\sigma-\rho)+vn+f(n)\right]}{\left[\beta(\theta-1)+1\right]^2} > 0; \quad \frac{dg}{d\theta} = -\frac{\left[\sigma-\rho+vn+f(n)\right]\beta}{\left(\beta(\theta-1)+1\right)^2} < 0$$

The indirect effects are:

For σ: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\sigma}}{\left[\beta(\theta-1)+1\right]}$;

For v: $\dfrac{\left[v+f'(n)\right]\frac{dn}{dv}}{\left[\beta(\theta-1)+1\right]}$;

For ρ: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\rho}}{\left[\beta(\theta-1)+1\right]}$;

For θ: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\theta}}{\left[\beta(\theta-1)+1\right]}$;

For β: $\dfrac{\left[v+f'(n)\right]\frac{dn}{d\beta}}{\left[\beta(\theta-1)+1\right]}$.

In order to find the signs for the indirect and total effects we consider the result from Proposition 2 (*i.e.*, $\rho - vn > 0$) and $f'(n) < 0$, $f''(n) < 0$, which is true for the range $n \in (0.0138;\ 0,0413)$ containing the average value of the birth rate ($n = 0.03123$).

From Eq. (14) in the text we have:

$$f'(n) = \frac{v}{\beta(\theta-1)} - \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]$$

or alternatively:

$$\Psi(n) = f'(n) - \frac{v}{\beta(\theta-1)} + \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right]$$

By using the *implicit function theorem*:

$$\frac{dn}{d\sigma} = -\left(\frac{d\Psi}{d\sigma} / \frac{d\Psi}{dn}\right) \qquad (3.1)$$

$$\frac{d\Psi}{d\sigma} = \frac{(1-\beta)\beta(\theta-1)}{\beta n(\beta(\theta-1)+1)} = \frac{(1-\beta)(\theta-1)}{n(\beta(\theta-1)+1)} < 0 \qquad (3.2)$$

$$\frac{d\Psi}{dn} = f''(n) - \left(\frac{1-\beta}{\beta n^2}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right] + \left(\frac{1-\beta}{\beta n}\right)\left[\frac{\beta(\theta-1)f'(n)-v}{\beta(\theta-1)+1}\right] \qquad (3.3)$$

We now prove that $\frac{d\Psi}{dn} < 0$.

Proof:

Use $f''(n) < 0$ and $\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right] > 0$ (see Proposition 2).

Hence:

$$-\left(\frac{1-\beta}{\beta n^2}\right)\left[\frac{\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn}{\beta(\theta-1)+1}\right] < 0 \qquad \text{(A)}$$

The term $\left(\frac{1-\beta}{\beta n}\right)\left[\frac{\beta(\theta-1)f'(n)-v}{\beta(\theta-1)+1}\right] > 0$ (B)

Then, we have to compare term (A) and term (B):

From Proposition 2:

$\sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho-vn > 0 \Rightarrow \sigma\beta(\theta-1)+\beta(\theta-1)f(n)+\rho > vn \geq 0$

(A) > 0 $\quad\Rightarrow\quad$ $\rho-\sigma\beta(1-\theta)-\beta(1-\theta)f(n) > 0$

(B) > 0 $\quad\Rightarrow\quad$ $-nf'(n)\beta(1-\theta) > 0$, for $\theta < 1$ and $f'(n) < 0$.

Then (A) > (B) iff: $\left|\rho-\sigma\beta(1-\theta)-\beta(1-\theta)f(n)\right| > \left|nf'(n)\beta(1-\theta)\right|$.

A (sufficient) condition for the last inequality to be checked is:

$\left|f(n)\right|/n \geq \left|f'(n)\right|$,

which implies that the average dilution effect is higher or equal than the marginal dilution effect. So:

$$\frac{d\Psi}{dn} < 0.$$

Since $\frac{\left[v + f'(n)\right]}{\left[\beta(\theta-1)+1\right]} < 0$, the indirect effect for σ is positive and the total effect is positive, as well.

In the same way, we can prove: the following:

i) $\frac{d\Psi}{dv} = -\frac{1}{\beta(\theta-1)} - \frac{(1-\beta)n}{\beta n\left(\beta(\theta-1)+1\right)} \Rightarrow \frac{1}{\beta(1-\theta)} - \frac{(1-\beta)}{\beta\left(\beta(\theta-1)+1\right)} = \frac{1-\beta(1-\theta)-(1-\beta)(1-\theta)}{\beta(1-\theta)\left(\beta(\theta-1)+1\right)} \Rightarrow$

$\frac{d\Psi}{dv} = \frac{\theta}{\beta(1-\theta)\left(\beta(\theta-1)+1\right)} > 0 \Rightarrow \frac{dn}{dv} > 0$

ii) $\frac{d\Psi}{d\rho} = \frac{1-\beta}{\beta n\left(\beta(\theta-1)+1\right)} > 0 \Rightarrow \frac{dn}{d\rho} > 0$.

For θ we have:

$\frac{d\Psi}{d\theta} = -\frac{v}{\beta(\theta-1)^2} - \left(\frac{1-\beta}{\beta n}\right)\frac{\left[(\sigma + f(n))\beta - (\rho - vn)\beta\right]}{\left(\beta(\theta-1)+1\right)^2}$ <0 since the numerator in the second term is:

$(\sigma + f(n))\beta - (\rho - vn)\beta = \beta\left[\sigma + f(n) - \rho + vn\right] > 0$ by using the result from Proposition 2. So, $\frac{d\Psi}{d\theta} < 0$

and $\frac{dn}{d\theta} < 0$. Hence the indirect effect is positive, while the direct effect is negative.

In order to check for the total effect we compare two extreme cases:

i) $v = 0;\ \beta = 0.75$, which denotes a situation with very low altruism towards children, and

ii) $v = 1;\ \beta = 0.25$, which denotes the case where there is more altruism towards children.

In the former case the total effect of θ is negative, whereas in the latter it is positive.

Finally, for β we also consider these two extreme cases:

i) $v = 0;\ \beta = 0.75$

ii) $v = 1;\ \beta = 0.25$.

$$\frac{d\Psi}{d\beta} = \frac{v}{\beta^2(\theta-1)} + \frac{\left\{\left[\sigma + f(n)\right](\theta-1) - 2\left[\sigma + f(n)\right](\theta-1)\beta - \rho + vn\right\}\left[\beta n\left(\beta(\theta-1)+1\right)\right]}{\left[\beta n\left(\beta(\theta-1)+1\right)\right]^2} -$$

$$\frac{(1-\beta)\left\{\left[\sigma + f(n)\right](\theta-1)\beta + \rho - vn\right\}\left[2\beta n(\theta-1) + n\right]}{\left[\beta n\left(\beta(\theta-1)+1\right)\right]^2}$$

After considerable amount of algebra and by using different parameter values one can show the following. For case i) $v = 0;\ \beta = 0.75$, $\frac{dn}{d\beta} < 0$, the indirect effect is positive, the direct effect is negative but the total effect is positive.

For case ii) $v = 1; \beta = 0.25$, one can show that $\frac{dn}{d\beta} < 0$, both direct and indirect effects are positive and the total is positive as well. ■

APPENDIX B

OECD COUNTRIES: Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Republic of Korea, Spain, Sweden, Switzerland, Turkey, U.K., U.S.

NON-OECD COUNTRIES: Afghanistan, Algeria, Argentina, Bahrain, Bangladesh, Barbados, Bolivia, Botswana, Brazil, Bulgaria, Cameroon, Central African Republic, Colombia, Costa Rica, Cuba, Cyprus, Dominican Republic, Ecuador, El Salvador, Fiji, Ghana, Guatemala, Guyana, Haiti, Honduras, Hong Kong, India, Indonesia, Iran (Islamic Republic of), Iraq, Israel, Jamaica, Jordan, Kenya, Kuwait, Lesotho, Liberia, Malawi, Malaysia, Mali, Mauritius, Mozambique, Myanmar, Nepal, Nicaragua, Niger, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Romania, Senegal, Sierra Leone, Singapore, South Africa, Sri Lanka, Sudan, Swaziland, Syrian Arab Republic, Thailand, Togo, Trinidad and Tobago, Tunisia, Uganda, Uruguay, Venezuela, Zambia, Zimbabwe.

OPEC COUNTRIES: Algeria, Ecuador, Indonesia, Iran (Islamic Republic of), Iraq, Kuwait, Venezuela.

EASTERN EUROPEAN/EX-SOCIALIST COUNTRIES: Bulgaria, Czech Republic, Cuba, Hungary, Poland, Romania.

LATIN AMERICA COUNTRIES: Argentina, Barbados, Bolivia, Brazil, Chile, Colombia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Trinidad and Tobago, Uruguay, Venezuela.

SUB-SAHARAN AFRICA COUNTRIES: Botswana, Cameroon, Central African Republic, Ghana, Kenya, Lesotho, Liberia, Malawi, Mali, Mauritius, Mozambique, Niger, Senegal, Sierra Leone, South Africa, Sudan, Swaziland, Togo, Uganda, Zambia, Zimbabwe.

TABLE B.1: *SEMI /NON-PARAMETRIC REGRESSION RESULTS*

	OLS	SP	NP
Variables	(A)	(B)	(C)
constant	1.1521*** (0.0176)	0.6132*** (0.0164)	-
doecd	0.0528*** (0.0115)	0.0883*** (0.0168)	-
la	0.1709*** (0.0136)	0.1547*** (0.0135)	-
af	0.0153 (0.0197)	0.0355*** (0.0162)	-
d1965	0.0044 (0.0221)	0.0078 (0.0216)	-
d1970	0.0250 (0.0211)	0.0281 (0.0214)	-
d1975	0.0226 (0.0194)	0.0219 (0.0209)	-
d1980	0.0280 (0.0185)	0.0252 (0.0204)	-
d1985	0.0274 (0.0176)	0.0268 (0.0268)	-
d1990	0.0317 (0.0175)	0.0318 (0.0318)	-
d1995	0.0181 (0.0172)	0.0168 (0.0168)	-
cbr	-16.3436*** (0.5328)	-	-
N	792	792	792
R2/R2adj.	72.37/71.98	74.2/73.7	77.66/-
F-test	185.8***		
Heterosked.$^{(1)}$	158.96***		
$\text{P}\left(\text{Specific.}\right)^{(2)}$	2.22e-16***	2.22e-16***	
$\text{NP}\left(Test\right)^{(3)}$	0.012531*		

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). Robust standard errors are in parentheses. (2): The P(specific.) shows the p-values if the null of the parametric linear (linear-OLS) is correctly specified in comparison to a fully *non-parametric* (NP) model, using the Hsiao *et al.* (2007) test for continuous and discrete data models after 399 Bootstrap replications. The P(specific.) at column B, checks if the parametric model (linear-OLS) is well specified when compared to the *semi-parametric* (SP) model. (3): The NP(Test) is a significance test for the explanatory variable (*cbr*) in the locally linear nonparametric specification. It is like the *t-test* in the parametric regression framework and is based on Racine (1997). We drop the time dummy for 2000.

TABLE B.2: *REGRESSION RESULTS BY USING POLYNOMIAL TERMS FOR THE BIRTH RATE* (cbr)

	OLS	OLS	OLS	OLS
Variables	(A)	(B)	(C)	(D)
constant	1.1521*** (0.0176)	0.6958*** (0.0699)	1.1926*** (0.0205)	0.6108*** (0.1305)
doecd	0.0528*** (0.0115)	0.0883*** (0.0115)	-0.0630 (0.02428)	0.1007 (0.1563)
la	0.1709*** (0.0136)	0.1551*** (0.0140)	0.1626*** (0.0140)	0.1546*** (0.0143)
af	0.0153 (0.0197)	0.0301 (0.0195)	0.0227*** (0.0198)	0.0312 (0.0195)
d1965	0.0044 (0.0221)	-0.0023 (0.0219)	0.0022 (0.0219)	-0.0018 (0.0222)
d1970	0.0250 (0.0211)	0.0180 (0.0209)	0.0236 (0.0210)	0.0194 (0.0211)
d1975	0.0226 (0.0194)	0.0139 (0.0193)	0.0222 (0.0193)	0.0147 (0.0195)
d1980	0.0280 (0.0185)	0.0199 (0.0182)	0.0280 (0.1844)	0.0237 (0.0172)
d1985	0.0274 (0.0176)	0.0234 (0.0171)	0.0290 (0.0175)	0.0237 (0.0172)
d1990	0.0317 (0.0175)	0.0300 (0.0170)	0.0335 (0.0173)	0.0307 (0.0171)
d1995	0.0181 (0.0172)	0.0165 (0.0166)	0.0195 (0.0169)	0.0163 (0.0166)
cbr	-16.3436*** (0.5328)	26.35** (7.670)	-17.4017*** (0.6085)	34.04* (13.3694)
$(cbr)^2$		-1143*** (265.99)		-1355** (419.65)
$(cbr)^3$		9224** (2868.025)		11050** (4147.205)
$oecd(cbr)$			5.1988*** (1.040)	4.665 (18.14)
$oecd(cbr)^2$				-403.77 (674.02)
$oecd(cbr)^3$				-6856 (7950)
N	792	792	792	792
R2/R2adj.	72.37/71.98	73.87/73.43	72.84/72.42	73.91/73.37
F-test	185.8***	169.2***	174.1***	137.2***
F-Joint	158.96***	22.269***	13.371***	0.4312
Heterosked.[(1)]	133.24***	183.37***	167.11***	185.55***

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). Robust standard errors are in parentheses. We drop the time dummy for 2000.

TABLE B.3: *REGRESSION RESULTS USING POLYNOMIAL TERMS FOR THE BIRTH RATE (CBR) AND CONTROL VARIABLES*

	OLS	OLS	OLS	OLS
Variables	(A)	(B)	(C)	(D)
constant	0.33319** (0.1454)	0.0449 (0.1504)	0.3824*** (0.1413)	0.0161 (0.1754)
doecd	0.0038 (0.0097)	0.0205 (0.0094)	-0.0816*** (0.0205)	0.1449 (0.1339)
la	0.1260*** (0.0093)	0.1158*** (0.0094)	0.1204*** (0.0094)	0.0113*** (0.1138)
af	0.0610*** (0.0119)	0.0704*** (0.0114)	0.0643*** (0.0118)	0.0709*** (0.0115)
d1965	-0.0180 (0.0164)	-0.0298* (0.0165)	-0.0233 (0.0163)	-0.0274 (0.0162)
d1970	-0.0132 (0.0162)	-0.0231* (0.0161)	-0.0172 (0.0160)	-0.0204 (0.0162)
d1975	-0.0154 (0.0155)	-0.0246 (0.0154)	-0.0179 (0.0153)	-0.0218 (0.0155)
d1980	-0.0098 (0.0146)	-0.0170 (0.0141)	-0.0113 (0.0144)	-0.0149 (0.0144)
d1985	-0.0110 (0.0139)	-0.0147 (0.0136)	-0.0108 (0.0137)	-0.0131 (0.0136)
d1990	-0.0029 (0.0133)	-0.0048 (0.0132)	-0.0020 (0.0131)	-0.0035 (0.0131)
d1995	-0.0021 (0.0135)	-0.0492*** (0.0133)	-0.0012 (0.0133)	-0.0030 (0.0132)
hum65	0.0466*** (0.0030)	19.17*** (0.0033)	0.0478*** (0.0030)	0.0496*** (0.0032)
infmort	-0.002*** (0.003)	-0.0016*** (0.003)	-0.002*** (0.0003)	-0.0017*** (0.0003)
lifexp	0.0048*** (0.0018)	0.0054** (0.0017)	0.0045*** (0.0017)	0.0051*** (0.0017)
cbr	-0.5468 (0.6897)	19.17*** (5.2137)	-1.4363*** (0.7230)	24.78* (9.9806)
$(cbr)^2$		-487.7*** (189.44)		-665* (306.6897)
$(cbr)^3$		3384 (1895)		5067 (2985.376)
$oecd(cbr)$			3.7724*** (0.8013)	-14.82 (15.7202)
$oecd(cbr)^2$				488.5 (585.4747)
$oecd(cbr)^3$				-4455 (6899.486)
N	792	792	792	792
R2/R2adj.	88.34/88.13	88.8/88.57	88.58/88.36	88.84/88.57
F-test	420.5***	384.2***	401.2***	322.6***
F-Joint			16.153***	0.9286
F-Joint 3rd Polyn.	-	16.049***	-	7.8147***
Heterosked.(1)	33.30***	27.01***	29.20***	22.99***

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). The standard errors are in parentheses and t-statistics are available upon request. We drop the time dummy for 2000,. Instead of life expectancy we have used also crude death rate. This variable has negative sign contrast to the sign of life expectancy, is statistically significant and the results for the variable of birth rate (*cbr*) remain the same.

FIGURE 1: *SEMI-PARAMETRIC PLOT FOR THE CONTEMPORANEOUS BIRTH RATE IN THE WHOLE SAMPLE*

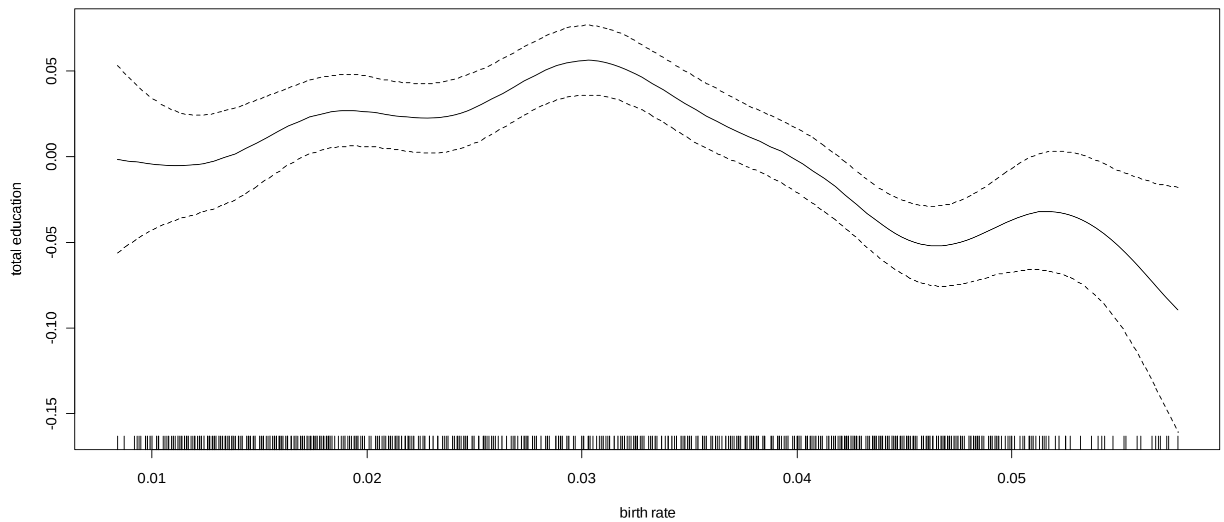

FIGURE 2: *SEMI-PARAMETRIC PLOT FOR THE LAGGED BIRTH RATE IN THE WHOLE SAMPLE*

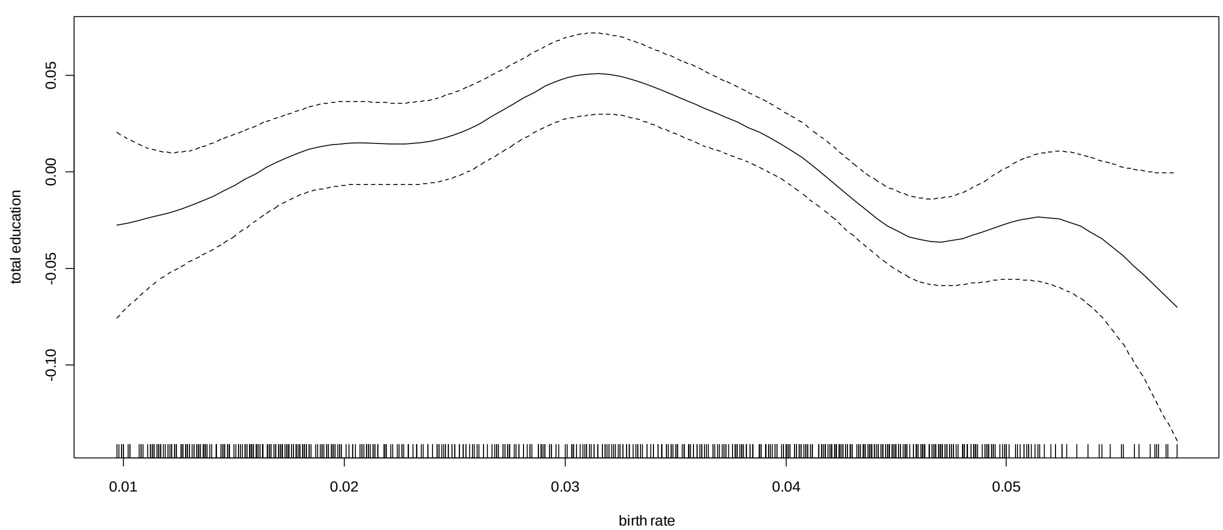

TABLE B.4: *SEMI/NON-PARAMETRIC REGRESSION RESULTS IN THE NEW SAMPLE*

	OLS	SP	NP
Variables	(A)	(B)	(C)
Constant	1.1559*** (0.0211)	0.6101*** (0.0188)	-
doecd	0.0503*** (0.0142)	0.0938*** (0.0199)	-
la	0.1644*** (0.0160)	0.1500*** (0.0152)	-
af	0.0101 (0.0222)	0.0316 (0.0181)	-
d1965	0.0015 (0.0248)	0.0035 (0.0240)	-
d1970	0.0214 (0.0236)	0.021 (0.0238)	-
d1975	0.0175 (0.0218)	0.0147 (0.0233)	-
d1980	0.0183 (0.0208)	0.0154 (0.0227)	-
d1985	0.0187 (0.0198)	0.0194 (0.0223)	-
d1990	0.0271 (0.0198)	0.0287 (0.0222)	-
d1995	0.0136 (0.0194)	0.0145 (0.0220)	-
cbr	-16.2055*** (0.6248)	-	-
N	688	688	688
R2/R2adj.	70.85/70.37	72.8/72.2	74.71/-
F-test	149.4***		
Heterosked.$^{(1)}$	130.38***		
$\mathrm{P}(\mathrm{Specific.})^{(2)}$	2.22e-16***	2.22e-16***	
$\mathrm{NP}(Test)^{(3)}$	0.0025063**		

Notes: ***, **, and * denote the 1%, 5% and 10% significance levels. (1): Heterosked. is the heteroskedasticity LM-test by Breusch and Pagan (1979). Robust standard errors are in parentheses. (2): The P(specific.) shows the p-values if the null of the parametric linear (linear-OLS) is correctly specified in comparison to a fully *non-parametric* (NP) model, using the Hsiao *et al.* (2007) test for continuous and discrete data models after 399 Bootstrap replications. The P(specific.) at column B, checks if the parametric model (linear-OLS) is well specified when compared to the *semi-parametric* (SP) model. (3): The NP(Test) is a significance test for the explanatory variable (*cbr*) in the locally linear nonparametric specification. It is like the *t-test* in the parametric regression framework and is based on Racine (1997). We drop the time dummy for 2000.

FIGURE 3: *SEMI-PARAMETRIC GRAPH IN THE RESTRICTED SAMPLE*