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Uncertainty in an Interconnected Financial System, Contagion, and Market Freezes

Mei Li   Frank Milne Junfeng Qiu

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Mei Li, Frank Milne, Junfeng Qiu*

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Abstract

This paper studies contagion and market freezes caused by uncertainty in financial network structures and provides theoretical guidance for central banks. We establish a formal model to demonstrate that, in a financial system where financial institutions are interconnected, a negative shock to an individual financial institution could spread to other institutions, causing market freezes because of creditors' uncertainty about the financial network structure. Central bank policies to alleviate market freezes and contagion, such as information policy, bailout policy and the lender of last resort policy, are examined.

Keywords: Interconnection, Market Freezes, Contagion, Financial Crises

JEL Classifications: D82 G2

*Mei Li: Department of Economics and Finance, College of Management and Economics, University of Guelph, Guelph, Ontario, Canada N1G 2W1. Email: mli03@uoguelph.ca. Frank Milne: Department of Economics, Queen's University, Kingston, Ontario, Canada K7L 3N6. Email: milnef@qed.econ.queensu.ca. Junfeng Qiu: China Economics and Management Academy, Central University of Finance and Economics, 39 College South Road, Beijing, China 100081. Email: qiujunfeng@cufe.edu.cn.

1 Introduction

An important contagion mechanism emerged in the recent subprime mortgage crisis: with the rapid expansion of credit derivatives, financial institutions are interconnected through an increasingly complicated financial network. Consequently, when one of the financial institutions goes bankrupt, general uncertainty about the losses of other financial institutions in the network arises. This is because the complexity of the financial network makes it difficult for market participants to assess these losses. As a result, market participants can stop lending to one another; in other words, all the financial markets freeze. This contagion mechanism played an important role in the recent crisis following Lehman Brothers' bankruptcy. Market participants stopped lending to one another, as they were all perceived to have been exposed to some counterparty risk because of Lehman Brothers' bankruptcy.

This paper establishes a formal model of this phenomenon. In our model, financial institutions are interconnected through interbank loans.[1] Meanwhile, these financial institutions are financing their long-term investments through short-term liabilities. As a result, there is a maturity mismatch between their assets and liabilities, which could potentially lead to bankruptcy of these institutions resulting from a lack of liquidity. We demonstrate that, due to short-term creditor uncertainty about the interconnections among these financial institutions, a negative shock to one financial institution can spread to all the other financial institutions in the system, leading to systematic market freezes.

To illustrate the mechanism in the most transparent fashion, we assume that all financial institutions are divided into an equal number of borrowers and lenders. Each borrower is linked to a single lender, and each lender has only one borrower. (Later in the paper this assumption can be modified in a more realistic fashion.) Short-term creditors do not have perfect information about the financial institution lending network. Instead, they believe that each lending institution could lend to each of the borrowing institutions in the system with an even probability. This assumption introduces uncertainty about interconnections among financial institutions into the model and plays a key role in our contagion mechanism. In addition, we assume that a negative shock hits the long-term investments of one of the borrowing institutions that we call the distressed institution.

[1] Here we alter the commonly understood definition of "interbank loans" and use it to refer instead to financial transactions between any financial institutions that are not necessarily commercial banks.

The shock spreads to the rest of the financial institutions in the following way:

First, the financial institution that has lent to the distressed institution may suffer a loss because its interbank loans may default. Second, short-term creditors of the lending institutions may charge a higher interest rate (we call this case a partial market freeze) or even refuse to roll over their short-term loans (we call this case a complete market freeze) because of their concern that the financial institution they are lending to may be the one lending to the distressed institution and may thus fail to repay its debts. As a result, the lending institutions may be forced to liquidate their long-term investments and recall their interbank loans. Third, due to the interbank loan recall of the lending institutions, the healthy borrowing institutions unaffected by the shock could incur a loss. As a result, their short-term creditors may refuse to roll over their loans, forcing the borrowing institutions to liquidate part or all of their long-term investments.

Our model produces the following major results:

First, depending on the magnitude of the negative shock to the distressed institution, the severity of market freezes varies. When the negative shock is large enough, a systematic collapse can happen where complete market freezes occur in all the short-term markets. When the negative shock is moderate, partial market freezes may occur to some financial institutions. When the negative shock is low enough, a market freeze may not happen at all.

Second, in our model, contagion occurs among financial institutions who need not be interconnected through *actual* financial transactions. Instead, as long as a financial institution is *perceived* by the market to be connected to the distressed institution, it becomes part of the contagion. Thus, our model reveals that short-term creditor uncertainty over network structures can significantly increase the magnitude of contagion compared to a situation where there is no short-term creditor uncertainty.

Third, our model reveals that a small loss to an individual financial institution can be magnified through the contagion mechanism and can lead to a large social loss.

Fourth, we find that the maximum total social loss has a non-monotonic relationship with the number of financial institutions in the network. This is because of the trade-off between the number of financial institutions affected by contagion and the loss of each affected institution when the number of institutions in the network increases. When the number is small, complete market freezes are more common, leading to a higher loss for each individual institution. However, a small number of affected institutions lowers the total social loss. When the number is large, partial market freezes tend to happen, leading to a lower loss for each institution. However, a large number of affected institutions increases the total social loss.

Moreover, our model produces the following major policy implications.

First, we examine the information policy of a central bank. In a perfect information case where the central bank, because of its close cooperation with the prudential regulator, can signal credibly to short-term creditors that there is only one distressed bank, we find that contagion can be effectively prevented. The crucial assumption here is that the markets are confident that the central bank is fully informed and credible. In an imperfect information case where the central bank does not know for sure the identity of the distressed institution, we find that more information provided by a central bank, to reduce the number of financial institutions that might be connected to the distressed institution, does not necessarily improve social welfare.

Second, we examine bailout policies and find that both direct guarantees for the distressed institution and injections of capital to all the lending institutions can alleviate market freezes if the moral hazard problem is ignored.

Third, we examine the Lender of Last Resort (LOLR) policy and find that a central bank loan with an interest rate lower than the prevailing market rate will lower the market rate, and makes complete market freezes less likely.

Our paper is most closely related to Caballero and Simsek (2009) and Pritsker (2010), two studies that examine financial contagion caused by uncertainty in an interconnected financial system. However, both papers resort to the concept of Knightian uncertainty. In addition, the basic setup of their models is quite different from ours. For example, our model assumes that financial institutions finance their long-term investments through both interbank loans and small short-term creditors. As a result, one of the important theoretical contributions of our paper is to introduce a large creditor in the presence of a continuum of small creditors to a Diamond and Dybvig bank run model. Consequently, our model can provide insights into how the presence of a large creditor will affect bank run dynamics.

Moreover, our paper contributes to the literature on financial contagion. The existing literature on financial contagion focuses mainly on two contagion mechanisms. The first works through herding behavior caused by information externalities.[2] The second contagion mechanism works through credit chains. That is, when financial institutions are linked through financial transactions, a failure of one institution can spread to other institutions in the link through balance-sheet effects, leading to a systematic failure.[3] In particular, our paper is closely related to the network literature, which studies how complex financial networks cause contagion in a financial system through credit chains.[4] Our paper differs in that contagion in our model does not rely on the *actual* connection to the distressed institution, but the *perceived* connection. This perceived connection arises because short-term creditors cannot distinguish between lending institutions and their different exposures to the distressed institution, due to their uncertainty over network structures. This major difference originates from the introduction of incomplete information, rather than complete information (as in the existing literature) about financial interconnections. Our model demonstrates that the magnitude of contagion may be much larger with short-term creditor uncertainty over bank exposures than without it. More importantly, our model provides theoretical guidance for a central bank to tackle financial contagion caused by short-term creditor uncertainty over network structures.

Note that contagion in our model does not rely on complex network structures, but is caused by short-term creditor *uncertainty* about network structures. In our model, we adopt a very simple financial network structure in which all banks are paired as borrower and lender. This simple network structure is sufficient to convey the core contagion mechanism in our model.

Finally, our paper also contributes to the literature on market freezes. Explanations for market freezes in the existing literature include adverse selection caused by asymmetric information, Knightian uncertainty, gambling for resurrection, and preemptive runs of short-term creditors due to future rollover risk.[5] In our model, market freezes in the

[2]The related work includes Chen (1997) and King and Wadhwani (1990) among many others.

[3]The related work includes Allen and Gale (2000), Dasgupta (2004), Kiyotaki and Moore (1997, 2002), and Rochet and Tirole (1996) among many others. Besides these two major mechanisms, Kodres and Pritsker (2002) study contagion between countries due to the portfolio re-balancing effect.

[4]There is a large body of literature on this topic that usually involves complex financial networks. The related work includes Allen and Babus (2009), Allen et al. (2010), Anand et al. (forthcoming), and Gai and Kapadia (2010, 2011) among many others.

[5]The related work includes Acharya et al. (2009), Bolton et al. (2011), Brunnermeier and Oehmke (2009), Caballero and Krishnamurthy (2008), Diamond and Rajan (2011), Easley and O'Hara (2010),

short-term financial markets are caused by asymmetric information: short-term creditors cannot identify the lending institution actually connected to the distressed institution. As a result, they will charge a higher interest rate to all the lending institutions, or even refuse to roll over their loans. The major contribution of our paper is that we study *systemic* market freezes. Our model studies a financial *system* with multiple financial institutions and markets. As a result, our model reveals how market freezes spread from one institution to the rest of the institutions in the system and from the short-term financial market to the interbank loan market.

The rest of the paper is organized as follows. Section 2 describes the environment of the model with imperfect information. Section 3 studies the special case with perfect information. Section 4 characterizes the equilibrium of the model with imperfect information. Section 5 generalizes the model from the two-connection case to the N-connection case. Policy implications are examined in Section 6. Section 7 discusses possible extensions of the paper. Section 8 concludes. All the proofs are given in the appendix.

2 The environment

This is a two-period model with three dates denoted by $t = 0,\ 1$ and 2. There are four banks denoted by L_1, L_2, B_1, and B_2 respectively.[6] Banks L_1 and L_2 are the lending banks, and banks B_1 and B_2 are the borrowing banks with an interbank loan market structure as follows:

and He and Xiong (2009) among many others.

[6]The word "bank" is used for convenience. It can be interpreted as a non-bank financial institution as well.

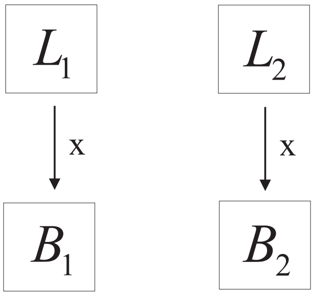


Figure 1: The interbank loan market structure. x is the position of interbank loans that the lending bank makes to the borrowing bank.

2.1 Initial balance sheets

The balance sheets of the lending and borrowing banks at date 0 are as follows:

Table 1: Lending banks' balance sheet

Lending banks' balance sheet at $t = 1$	
Interbank loan: x	Deposit : $D_0 + x$
Long-term Project: L	Equity: e_0

Table 2: Borrowing banks' balance sheet

Borrowing banks' balance sheet at $t = 1$	
	Deposit : $D_0 - x$
	Interbank loan: x
Long-term Project: L	Equity: e_0

From the above tables, we can tell that each lending bank has a total deposit of D_0+x and equity of e_0.[7] On the asset side, each lending bank has an interbank loan of x and a long-term project of $L = D_0 + e_0$. Each borrowing bank has a total deposit of $D_0 - x > 0$

[7]Note that here a "deposit" could be any short-term debt borrowed by a financial institution and should not be interpreted literally as a deposit issued by a commercial bank.

and equity of e_0. In addition, they borrow an interbank loan of x. It is straightforward to see that the size of the long-term project of each borrowing bank is also $L = D_0 + e_0$.

We assume that the long-term project will mature at date 2 with a net return rate of $R > 0$. If it is liquidated at date 1, the liquidation technology is as follows. For y units of *date 2* output, the liquidation income at date 1 is

$$\lambda y - \frac{1}{2}\gamma y^2$$

where $0 < \lambda < 1$ and $\gamma > 0$ are constants. Note that $\frac{1}{2}\gamma y^2$ is convex, which captures the increasing marginal liquidation cost. Moreover, note that y denotes *date 2* output. Let l denote the long-term project liquidated at *date 1*. Then $y = (1+R)l$, and the liquidation income could also be written as

$$\lambda l(1+R) - \frac{1}{2}\gamma[l(1+R)]^2$$

We assume that the marginal liquidation income of $\lambda - \gamma y$ is positive for all the values of y so that a bank will always earn positive income by liquidating an additional unit of the project.

We assume that banks can access only one-period short-term deposits. Thus, they will have to roll over their deposits at $t = 1$ if they invest in long-term projects at $t = 0$. This assumption captures the maturity mismatch between assets and liabilities in a real financial institution. Finally, we assume that both the lending banks and depositors are risk neutral and expect at $t = 0$ that their lending will be repaid for sure. Thus, both the interbank loan rate and deposit rate at $t = 0$ equal the riskless rate of zero.

One key assumption in our model is that banks L_1 and L_2 depositors know that banks L_1 and L_2 are the lending banks and banks B_1 and B_2 are the borrowing banks. However, they do not know who is lending to whom. As a result, from the perspective of the lending banks' depositors, there are two possible states. In one state, L_1 is lending to B_1 and L_2 is lending to B_2. In the other state, L_1 is lending to B_2 and L_2 is lending to B_1. The depositors believe that each state could happen with an even probability. This assumption introduces the short-term creditor uncertainty about the financial network structure into our model.

2.2 The timing of the model

At date 1, an unanticipated negative shock hits the long-term project of one of the borrowing banks. Without loss of generality, let bank B_1 be the one hit by the shock. As a result, its net project return becomes $\hat{R} = R - R_{shock}$, where $1 + \hat{R} > 0$. Here R_{shock} measures the magnitude of the negative shock. We assume that the identity of the bank hit by the shock is publicly known.

At date 1, after observing the shock to bank B_1, depositors and banks make their decisions in the following sequence.

First, the *lending banks' depositors* decide whether to roll over their deposits to the banks or not, and if they do, what interest rate (denoted as $\hat{r}$) is required. Note that both banks' depositors believe with a 50% probability that their bank is the one lending to B_1.

Second, the *lending banks* make their decisions. If their depositors are willing to roll over their deposits, then the two lending banks will decide how many deposits to roll over and how many deposits to repay. Banks may repay part of their deposits by (1) recalling interbank loans and (2) liquidating the long-term project. We examine the general case where banks can recall part of interbank loans or liquidate part of the long-term project. We also assume that banks L_1 and L_2 aim at maximizing their net value at date 2, even when the net value is negative. We believe that this assumption is realistic, because it would be more difficult for a bank manager to find a new job following a more substantial loss in his previous job. Thus, the manager has an incentive to minimize losses when the bank is insolvent. Given this assumption, bank L_1, which suffers the interbank loan loss, will still have an incentive to roll over its deposits.

When depositors are unwilling to roll over their deposits, we assume that the lending banks must meet withdrawal demand by recalling all the interbank loans and liquidating the long-term project. The proceeds are equally shared by depositors.

Third, the *borrowing banks' depositors* decide whether to withdraw their deposits or not at date 1, after they observe the interbank loans recall decision of the lending banks. Banks must use all available resources to repay recalled interbank loans and depositor withdrawal. If resources are not enough to meet all the withdrawal demand from the lending bank and depositors, then each withdrawer will receive a payment proportional to his withdrawal amount. Creditors who do not withdraw will get nothing at date 2.

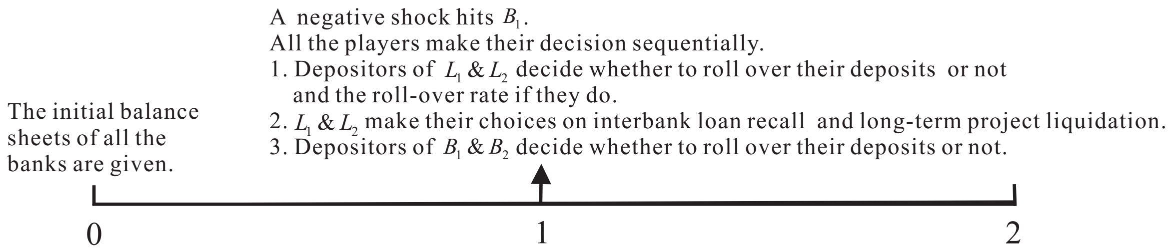


Figure 2: The timeline

Note that here we assume that the lending banks cannot front run the borrowing banks' depositors, but only make the interbank loan recall decision before the borrowing banks' depositors.

Finally, at date 2, banks repay their debts. If a bank's assets are not sufficient to repay all the debts, then they will be allocated to creditors proportionally.

Figure 2 gives the timeline of this model.

Thus at date 1, there is a sequential game where the lending banks' depositors move first, the lending banks move second, and the borrowing banks' depositors move last. In this game, we assume that as long as the no-run equilibrium is a feasible Nash equilibrium, depositors will choose not to run. That is, we rule out the equilibrium in which depositors run at date 1 because of their self-fulfilling beliefs. Here we focus on "essential bank runs" caused by economic fundamentals as in Allen and Gale (1998).

Next, we will examine a special case with perfect information where the identity of bank L_1 who has lent to bank B_1 is publicly known. This simpler case will help us better understand an imperfect information case where the identity of bank L_1 is not publicly known. Later in Section 4, we will return to the imperfect information case introduced in this section.

3 A special case with perfect information

In this case, after the negative shock hits bank B_1, only banks L_1 and B_1 are affected. Banks B_2 and L_2 are unaffected. Without any uncertainty, each depositor either withdraws his deposits at date 1 or rolls over his deposits at the riskless rate.

We use backward induction to find the subgame perfect Nash equilibrium as follows.

First, given α, the proportion of interbank loans recalled by bank L_1, bank B_1 depositors decide whether to withdraw their deposits at date 1 or not. Second, we find the optimal proportion of interbank loans that bank L_1 will recall after taking into account the best responses of bank B_1 depositors. This is conditional on bank L_1 depositors not withdrawing at date 1. Third, given bank L_1's optimal choice, bank L_1 depositors decide whether to withdraw their deposits at date 1 or not.

3.1 Optimal choices for bank B_1 depositors

We start with bank B_1 depositors, who move last in the sequential game. Let α denote the proportion of bank L_1's interbank loans that is recalled. Given α, we need to first find out whether bank B_1 depositors will roll over their deposits or not. It turns out that the key variable that determines the depositors' decision is the net value of bank B_1 at date 2, conditional on bank B_1 depositor not running at date 1, which we denote by $NV_{B_1}^{nr}$. Given that $NV_{B_1}^{nr} \geq 0$,

$$NV_{B_1}^{nr} = (L - l(\alpha))(1 + \hat{R}) - [D_0 - x + (1 - \alpha)x] = (L - l(\alpha))(1 + \hat{R}) - (D_0 - \alpha x) \quad (1)$$

where l is determined by

$$\alpha x = \lambda l(1 + \hat{R}) - \frac{1}{2}\gamma[l(1 + \hat{R})]^2 \quad (2)$$

Equation (2) determines l, the units of the long-term project liquidated to repay the recalled interbank loans of αx. Equation (1) gives the net asset value of bank B_1 at date 2, which is its unliquidated long-term project return at date 2 minus its total liabilities (unpaid interbank loans and deposits) at date 2.

Let α_1 be the solution to Equation (1)=0. Thus, given that bank B_1 depositors do not withdraw at date 1, bank B_1's asset value at date 2 exactly covers its liabilities to depositors and the remaining interbank loans when its interbank loan recall is $\alpha_1 x$. We have the following lemma:

Lemma 1. *$NV_{B_1}^{nr}$ is strictly decreasing in α.*

Proof: See the appendix. ■

Lemma 1 implies that $NV_{B_1}^{nr}$ is positive when $\alpha < \alpha_1$ and negative when $\alpha > \alpha_1$. Thus we have the following results.

Lemma 2. *(1) If $\alpha \leq \alpha_1$, bank B_1 depositors will not withdraw their deposits at date 1. (2) If $\alpha > \alpha_1$, bank B_1 depositors will withdraw at date 1.*

Proof: See the appendix. ■

3.2 Optimal choices for bank L_1

Next, we move backward through the sequence to find out bank L_1's optimal choice of α, conditional on its depositors not withdrawing at date 1. However, we need not consider bank L_1's optimal choice conditional on its depositors withdrawing at date 1, because we are confining our attention to "essential bank runs" in which depositors run a bank if, and only if, a no-run equilibrium is infeasible. As a result, we need only to determine whether a no-run equilibrium is feasible here or not. Note that given that bank L_1 depositors roll over their deposits, bank L_1 will never liquidate its long-term project because liquidation is costly. Thus, bank L_1 needs only to choose the optimal level of α to maximize its net value, given the best responses of bank B_1's depositors. We prove that in equilibrium, bank L_1 will recall either all or none of its interbank loans, even when it is allowed to recall its interbank loans partially.

We define bank L_1's payoff from interbank loans as its *date 2* value of total proceeds from interbank loans. Thus, if bank L_1 recalls αx of interbank loans at *date 1*, its payoff from this proportion of interbank loans is $\alpha x(1+r)$, where r is the interest rate charged by depositors. Since r is zero in this perfect information case, the payoff equals the sum of cash payments that bank L_1 receives from bank B_1 over dates 1 and 2. Let us denote the value of bank B_1 when its whole long-term project is liquidated at date 1 as $V_{B_1,liquidation}$. Moreover, let α_2 be the solution to

$$V_{B_1,liquidation} = \lambda L(1+\hat{R}) - \frac{1}{2}\gamma(L(1+\hat{R}))^2 = (D_0 - x) + \alpha_2 x = D_0 - (1-\alpha_2)x \qquad (3)$$

Thus, when bank L_1 recalls $\alpha_2 x$ of interbank loans and all the depositors withdraw at date 1, bank B_1 needs to liquidate all of its long-term project to repay its deposits and bank L_1's recalled interbank loans. That is, bank B_1 has no assets left for date 2. As a result, when $\alpha > \alpha_2$ and all of bank B_1's depositors withdraw at date 1, bank B_1's liquidation value at date 1 is not enough to repay its liabilities and, thus, this amount will be proportionally shared by bank L_1 and the depositors.

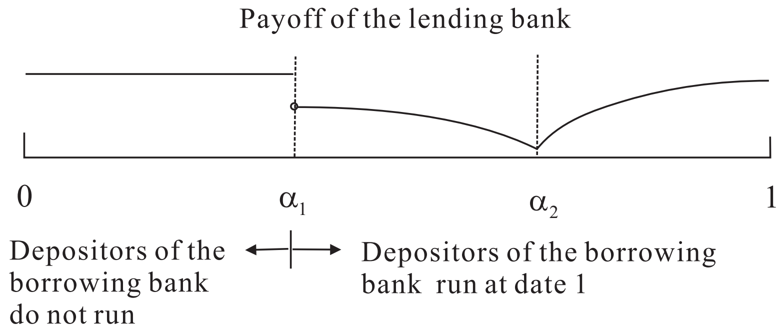


Figure 3: The lending bank's payoff from interbank loans in the case of $\alpha_1 \in [0, 1]$ and $\alpha_2 > \alpha_1$

It turns out that bank L_1's payoff from interbank loan recall depends crucially on α_1 and α_2. Note that α_2 can be either lower or higher than α_1. When the liquidation cost is high (or λ is low and γ is high), α_2 tends to be small. In addition, if $\alpha_1 \leq 1$, we have $\alpha_2 \leq 1$.[8]

Lemma 3 summarizes bank L_1's payoff from interbank loan recall in a special case where $0 \leq \alpha_1 < \alpha_2 \leq 1$. Appendix A.5 gives a general proof about all the other cases with different combinations of α_1 and α_2.

Lemma 3. *Given that $0 \leq \alpha_1 < \alpha_2 \leq 1$, bank L_1's payoff from interbank loans is a constant of x when $\alpha \in [0, \alpha_1]$, is strictly decreasing in α when $\alpha \in (\alpha_1, \alpha_2]$, and is strictly increasing in α when $\alpha \in (\alpha_2, 1]$. Moreover, bank L_1's payoff has a downward jump at α_1 and is continuous at α_2.*

Proof: See the appendix. ■

Figure 3 illustrates the results in lemma 3. The intuition behind the results is as follows. When $\alpha \in [0, \alpha_1]$, bank B_1 depositors do not run because bank B_1 has enough resources to repay its liabilities at date 2. Thus, the total payment to bank L_1 is x. When $\alpha \in (\alpha_1, \alpha_2]$, bank B_1 depositors will run. However, because $\alpha < \alpha_2$, bank B_1 still has positive resources left for date 2, but the resources are less than its liabilities. Thus bank L_1 will seize all bank B_1's remaining assets at date 2. In this case, when bank L_1 increases α, it is the equivalent of bank L_1 liquidating its own long-term project at date 1, which is costly by assumption. Thus bank L_1's payoff is decreasing in α. However, once α reaches α_2, bank B_1 has no resources left for date 2. In this case, when bank L_1 increases α, it will

[8]See the appendix for the proof.

increase its share in the liquidation value of bank B_1 at date 1. Thus, in this case, bank L_1's payoff is increasing in α. At α_1, there is a downward jump in bank L_1's payoff. This is because, given that bank L_1 recalls $\alpha x > \alpha_1 x$ of its interbank loans, bank B_1 depositors switch from a no-run equilibrium to a run equilibrium, incurring additional liquidation costs that lower bank L_1's date 2 payoff.

We assume that when bank L_1 receives the same payoff from recalling different proportions of interbank loans, it always chooses to recall the minimum proportion of interbank loans. By considering all the possible combinations of α_1 and α_2, we arrive at the following proposition:

Proposition 1. *In a perfect information case, if $\alpha_1 \geq 0$, bank L_1 will not recall any interbank loans, and bank B_1 depositors will not withdraw at date 1. Otherwise, bank B_1 depositors will withdraw at date 1, and bank L_1's optimal choice is to recall either no loans ($\alpha = 0$) or all the loans ($\alpha = 1$).*

Proof: See the appendix. ■

The above result describes the optimal choice for bank L_1 that maximizes its payoff from interbank loans. The choice also maximizes bank L_1's asset value at date 2, conditional on its depositors not running at date 1, which, denoted by $V_{L_1}^{nr}$, is given by $L(1+R)$ plus its maximum payoff from interbank loans.

3.3 Optimal choices for bank L_1 depositors

Given the maximum value of bank L_1, bank L_1 depositors then decide whether to withdraw at date 1 or not. Following a similar argument as the one for the decisions of bank B_1 depositors, we arrive at the following proposition:

Proposition 2. *If $V_{L_1}^{nr} \geq D_0 + x$, bank L_1 depositors will not withdraw at date 1. The optimal interbank loan recalling strategy of bank L_1 and the corresponding best responses of bank B_1 depositors are characterized in proposition 1. If $V_{L_1}^{nr} < D_0 + x$, banks L_1 and B_1 depositors will withdraw at date 1. Both banks B_1 and L_1 will be insolvent and thus will be forced to liquidate all of their long-term projects at date 1.*

Proof: See the appendix. ■

4 Equilibrium in an imperfect information case

In this section, we study the imperfect information case where the identity of bank L_1, who has lent to the bank hit by the shock, is not publicly known. The imperfect information case differs from the perfect information case in that the two lending banks' depositors will now make the same decisions about deposit withdrawing. This is because they cannot identify the bank suffering a loss due to the shock to bank B_1 and believe with a 50% probability that their bank may suffer a loss. As a result, banks L_1 and L_2 face the same decisions from their depositors and, consequently, banks L_2 and B_2 are also affected by the shock to bank B_1. In the perfect information case, however, they are not affected at all. Thus, our model provides a contagion mechanism in which contagion spreads to banks L_2 and B_2 because of uncertainty regarding the interconnections among financial institutions.

4.1 Optimal choices for the lending banks

We start with the optimal choices for the lending banks when the market rate charged by their depositors, $\hat{r}$, is given. Our later analysis reveals that there are two possible cases. In the first case, the lending banks' depositors are willing to roll over their deposits and ask for an interest rate of $\hat{r} \geq 0$. When $\hat{r} > 0$, we call it a partial market freeze. In the second case, there exists no $\hat{r}$ at which the depositors are willing to roll over their deposits. As a result, depositors of banks L_1 and L_2 will withdraw at date 1. We call this case a complete market freeze.

When $\hat{r} = 0$, both lending banks behave the same as in the perfect information case conditional on their depositors not withdrawing at date 1. That is, they will optimally choose the proportion of interbank loans to recall, α, as characterized in section 3.2, and will never liquidate any long-term projects.

When $\hat{r} > 0$, a lending bank will optimally choose the proportion of interbank loans to recall, α, and the amount of the long-term project to liquidate, l. First, we define a variable Z as the total resources that the lending bank collects to repay its depositors at date 1, which is given by

$$Z = \lambda l(1 + R) - \frac{1}{2}\gamma[l(1 + R)]^2 + F(\alpha x) \tag{4}$$

where $\lambda l(1+R)-\frac{1}{2}\gamma[l(1+R)]^2$ are the proceeds the bank receives from liquidating l of the long-term project, and $F(\alpha x)$ is the proceeds the bank receives from recalling αx of its interbank loans. When NV of the borrowing bank taking the interbank loans is positive, or $NV<0$ but $V_{liquidation}>\alpha x+D_0-x$, $F(\alpha x)=\alpha x$. When for the borrowing bank, $NV<0$ and $V_{liquidation}<\alpha x+D_0-x$, $F(\alpha x)=\frac{\alpha x}{D_0-x+\alpha x}V_{liquidation}$. Recall that $V_{liquidation}$ is the asset value of the borrowing bank after liquidating its entire long-term project at date 1.

If $Z<D_0+x$, the lending bank will roll over a positive amount of $D_0+x-Z>0$ of deposits, and its net asset value is given by

$$NV=(L-l)(1+R)-(D_0+x-Z)(1+\hat{r})+H((1-\alpha)x) \tag{5}$$

where $(L-l)(1+R)$ is the proceeds the bank receives at date 2 from the unliquidated long-term project, $(D_0+x-Z)(1+\hat{r})$ is the repayment to depositors, and $H((1-\alpha)x)$ is the proceeds from the remaining interbank loans. When NV of the borrowing bank taking the interbank loans is positive, $H((1-\alpha)x)=(1-\alpha)x$. When for the borrowing bank, $NV<0$ but $V_{liquidation}>\alpha x+D_0-x$, $H((1-\alpha)x)$ equals the asset value of the borrowing bank at date 2. When for the borrowing bank, $NV<0$ and $V_{liquidation}<\alpha x+D_0-x$, $H((1-\alpha)x)=0$.

If $Z\geq D_0+x$, the bank chooses not to roll over any deposits, and its net asset value is given by

$$NV=(L-l)(1+R)+Z-(D_0+x)+H((1-\alpha)x) \tag{6}$$

It is difficult to give a general analytical solution to the above problem. We focus on the more interesting case where $Z<D_0+x$ (that is, the lending bank chooses to roll over a positive amount of deposits) in equilibrium. In this case, we find the following results:

Lemma 4. *Given that in equilibrium $\hat{r}>0$, and $Z<D_0+x$, we find that: (1) The lending banks will liquidate their own projects if, and only if, $1+\hat{r}>\frac{1}{\lambda}$. (2) Given that $0\leq\alpha_1<\alpha_2\leq 1$, the optimal amount of recalled interbank loans, αx, can be chosen from three local optimal points in the three regions of $[0,\alpha_1]$, $[\alpha_1,\alpha_2]$, and $[\alpha_2,1]$ respectively. The bank will recall at least $\alpha_1 x$ of its interbank loans.*

Proof: See the appendix. ■

4.2 Equilibrium interest rate

To find the equilibrium rate that the lending banks' depositors will charge, recall that the decision-making sequence is assumed as follows. First, depositors promise to roll over their deposits at the rate of $\hat{r}$. Second, the lending banks choose the optimal amount of the long-term project to liquidate, the optimal amount of interbank loans to recall, and the optimal amount of deposits to roll over. Let V_{L_1} and V_{L_2} denote the maximum asset value at date 2 under the optimal choices of banks L_1 and L_2 respectively at a given level of $\hat{r}$. Let D_{L_1} and D_{L_2} denote the deposits that the two banks choose to roll over at a given level of $\hat{r}$. We described above the rules for the two lending banks to maximize the net value at date 2 that determine V_{L_1}, D_{L_1}, V_{L_2}, and D_{L_2} at each given level of $\hat{r}$. Note that both V and D are functions of $\hat{r}$ and are endogenously chosen by the banks. The conditional probability that the deposits will be rolled-over by a bank is also endogenous. For bank L_2, the conditional probability is $\pi_{L_2} = D_{L_2}/(D_0 + x)$, and for bank L_1, it is $\pi_{L_1} = D_{L_1}/(D_0 + x)$.

An individual depositor of bank L_1 or L_2 knows that his bank will be good or bad with a 50% probability. In addition, he will take into account the conditional probability of his deposit being rolled over by the bank given that the bank is good or bad. Given the market interest rate, a risk-neutral depositor will be willing to roll over his deposit if his expected rate equals the riskless rate. If his expected rate is lower than the riskless rate, he will withdraw at date 1. The general equation is

$$1 = \frac{1}{2}\left[\pi_{good}\min(\frac{V_{good}}{D_{good}}, 1+\hat{r}) + (1-\pi_{good})\right] + \frac{1}{2}\left[\pi_{bad}\min(\frac{V_{bad}}{D_{bad}}, 1+\hat{r}) + (1-\pi_{bad})\right] \quad (7)$$

The left-hand side is the gross riskless rate, which is the payoff if the depositor withdraws at date 1. The right-hand side is the expected return for promising to roll over the deposit at $\hat{r}$. With a 50% probability, the depositor's bank is good. In this case, with a probability of $1 - \pi_{good}$, the bank will repay its depositor at date 1, and the depositor receives 1 unit of payment; with a probability of π_{good}, his deposit is rolled over. In this case, if $1+\hat{r}$ is smaller than $\frac{V_{good}}{D_{good}}$, he will receive the promised payoff of $1+\hat{r}$ for each unit of his deposits at date 2. Otherwise, all the assets of the bank at date 2 are evenly allocated to the remaining depositors, and the depositors receive the recovery rate of $\frac{V_{good}}{D_{good}}$. A similar argument is applied to the second term when the bank is bad.

If R_{shock} is low enough that the depositors know that the bad lending bank can pay the riskless rate for sure (that is, $\frac{V_{bad}}{D_{bad}} \geq 1$), then the good bank should also be able to pay it. In this case, the depositors will simply charge the riskless rate of zero, that is, $\hat{r} = 0$. This case is identical to the perfect information case where bank L_1 depositors do not run and offer to roll over the deposit at a zero interest rate.

For the remaining analysis, we will focus on the more interesting case where R_{shock} is high enough such that $\frac{V_{bad}}{D_{bad}} < 1$. We find that the equilibrium condition (7) allows for two types of case for the good bank: $\frac{V_{good}}{D_{good}} \geq 1 + \hat{r}$ and $\frac{V_{good}}{D_{good}} < 1 + \hat{r}$. In the first case, Equation (7) can be written as

$$1 = \frac{1}{2}\left[\pi_{good}(1+\hat{r}) + (1-\pi_{good})\right] + \frac{1}{2}\left[\pi_{bad}\frac{V_{bad}}{D_{bad}} + (1-\pi_{bad})\right] \tag{8}$$

$$s.t \quad 1+\hat{r} \leq \frac{V_{good}}{D_{good}} \tag{9}$$

In the second case, the condition can be written as

$$1 = \frac{1}{2}\left[\pi_{good}\frac{V_{good}}{D_{good}} + (1-\pi_{good})\right] + \frac{1}{2}\left[\pi_{bad}\frac{V_{bad}}{D_{bad}} + (1-\pi_{bad})\right] \tag{10}$$

$$s.t \quad 1+\hat{r} > \frac{V_{good}}{D_{good}} \tag{11}$$

In the special case where the banks choose not to roll over any deposits such that D_{good} or D_{bad} is zero, the corresponding π_{good} or π_{bad} will be zero, and we define the term $\pi\frac{V}{D}$ as zero.

The above equations suggest the following steps for looking for the numerical solutions of the equilibrium. Given a value for $\hat{r}$, let $\Gamma(\hat{r})$ denote the actual return rate that must be received from the good bank in order to satisfy the equilibrium condition:

$$1 \;=\; \frac{1}{2}\left[\pi_{good}(1+\Gamma(\hat{r})) + (1-\pi_{good})\right] + \frac{1}{2}\left[\pi_{bad}\frac{V_{bad}}{D_{bad}} + (1-\pi_{bad})\right] \tag{12}$$

If for $1+\hat{r} \leq \frac{V_{good}}{D_{good}}$, we can find a value for $\hat{r}$ such that $\Gamma(\hat{r}) = \hat{r}$, then an equilibrium exist. In this case, the required rate is smaller than the maximum payment $\frac{V_{good}}{D_{good}}$ that can be paid by the good bank, so the actual payment is $1+\hat{r}$. Alternatively, if for $1+\hat{r} > \frac{V_{good}}{D_{good}}$, we find $1+\Gamma(\hat{r}) = \frac{V_{good}}{D_{good}}$, then an equilibrium also exists. In this case, the required payment $1+\Gamma(\hat{r})$ is equal to the maximum payment that can be paid by the good bank, and it gives the depositors an expected net return rate of zero. We call the former case the type I equilibrium, and the latter case the type II equilibrium. If for all the values of $\hat{r}$, the required rate, $1+\Gamma(\hat{r})$, is higher than $1+\hat{r}$ or $\frac{V_{good}}{D_{bad}}$, then an equilibrium $\hat{r}$ will not exist, and a complete market freeze will occur.

Note that in the above analysis, we use subscripts "good" and "bad" to denote the good and bad lending banks. This is because a particular bank's depositors do not know whether their bank is good or not, and they only know the optimal actions that will be taken by each type of bank. In our example reflecting the true state, the actual good bank is L_2 and the actual bad bank is L_1. Thus, π_{good}, V_{good} and D_{good} equal the actual values of π_{L_2}, V_{L_2} and D_{L_2}, and π_{bad}, V_{bad} and D_{bad} equal the actual values of π_{L_1}, V_{L_1} and D_{L_1}.

4.3 The equilibrium

Based on the above analysis, the equilibrium in the imperfect information case can be characterized as follows.

Proposition 3. *An equilibrium $\hat{r}$ exists when (1) given $\hat{r}$, the two lending banks maximize their net asset value at date 2, $V - D(1+\hat{r})$, by optimally choosing the amount of long-term projects to liquidate and the proportion of interbank loans to recall, and (2) given the expected optimal choices of the lending banks, $\hat{r}$ satisfies either Equations (8) and (9) or Equations (10) and (11). If the required rate $\Gamma(\hat{r})$ is always higher than $\hat{r}$ or $V_{L_2}/D_{L_2}-1$, then an equilibrium with a solution to $\hat{r}$ does not exist. In this case, the lending banks' depositors will not roll over their deposits.*

We call the case with a positive equilibrium $\hat{r}$ a partial market freeze and the case where the lending banks' depositors refuse to roll over any deposits a complete market freeze.

From propositions 1 and 3, we derive the following implications.

Corollary 1. *If $\hat{r}^* > 0$ or if $\hat{r}^*$ does not exist, $\hat{R}$ must be so low that for bank L_1, $\alpha_1 < 0$, and bank B_1 depositors run at date 1 in equilibrium.*

Proof: See the appendix. ■

4.4 Numerical examples

In this section, we provide numerical examples to illustrate the model with imperfect information. In the numerical examples throughout this paper, we will use the same baseline parametrization in which $e_0 = 0.08$, $L = 1$, $D_0 = L - e_0 = 0.92$, $R = 0.05$, $\lambda = 0.92$, $\gamma = 0.4$, and $x = 0.6$. Note that here we do not intend to calibrate the economy. Instead, we use the numerical examples to illustrate the qualitative results of our model. We let e_0 be 0.08 to ensure that the capital/assets ratio equals the capital adequacy ratio of 8% required by Basel Accords. We let $\lambda = 0.92$ and $\gamma = 0.4$ in order that an entirely liquidated project is worth approximately 70% of its unliquidated value. Note that changes of these parameter values will not affect our qualitative results. Nonetheless, the choice of the values of two key parameter, e_0 and x, does affect the magnitude of contagion greatly. We tend to find more severe contagion with a lower e_0 or a higher x.

Our examples produce the following major results:

First, there may exist multiple equilibria in which the lending banks' depositors charge different interest rates. We focus on the equilibrium with the smallest interest rate, which implies that contagion is, at the very least, as severe as our results reveal.

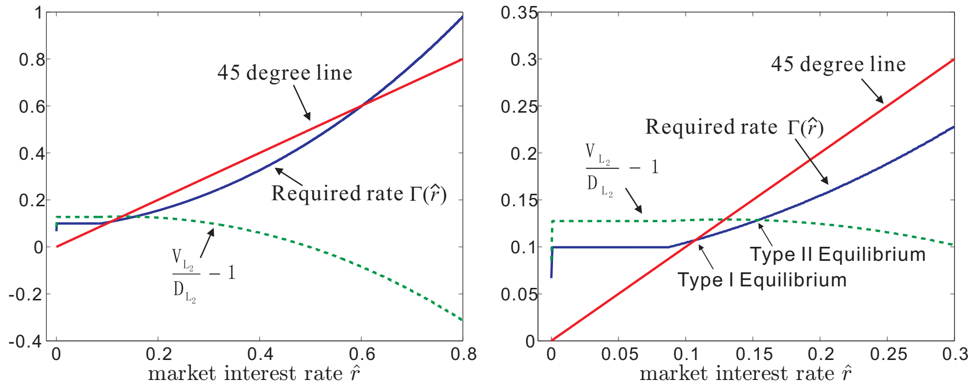
Second, the severity of market freezes increases in the magnitude of the negative shock to the distressed bank, B_1. The lending banks' depositors charge the riskless rate of zero when R_{shock} is lower than a threshold level. Afterwards, they start to charge a positive interest rate that is increasing in R_{shock}. When R_{shock} is high enough, equilibrium interest rates may not exist, and depositors may refuse to roll over their deposits at all.

Third, contagion spreads as follows: First, bank L_1 may suffer a loss because of its interbank loans to troubled bank B_1. Second, the lending banks' depositors may charge a positive interest rate or even refuse to roll over their deposits because they suspect that their lending bank may lend to bank B_1 and incur a loss. Third, the healthy borrowing bank, B_2, may suffer a loss because its lending bank L_2 may recall its interbank loans when facing a higher rollover rate or even a withdrawal of its depositors. In the worst case scenario, a systematic bank run occurs in which all the banks are run by their depositors and are forced to liquidate all of their long-term projects.

4.4.1 Equilibrium market rate given R_{shock}

We first present a benchmark case at $R_{shock} = 0.32$ to illustrate how the equilibrium interest rate charged by the lending banks' depositors is determined. Figure 4 shows the result. Here panel (b) is a closeup of part of panel (a) for a clearer presentation.

Here the movement of $\Gamma(\hat{r})$ is determined by the optimal choices of banks L_1 and L_2 on interbank loan recall and long-term project liquidation that consequently determine variables such as D_{L_1}, V_{L_1}, D_{L_2}, V_{L_2}, π_{L_2}, π_{L_1}, $\frac{V_{L_2}}{D_{L_2}}$, and $\frac{V_{L_1}}{D_{L_1}}$. The details about the optimal choices of banks L_1 and L_2 and about the determination of $\Gamma(\hat{r})$ are given in appendix B.



(a) $\Gamma(\hat{r})$

(b) $\Gamma(\hat{r})$ when $\hat{r} \leq 0.3$

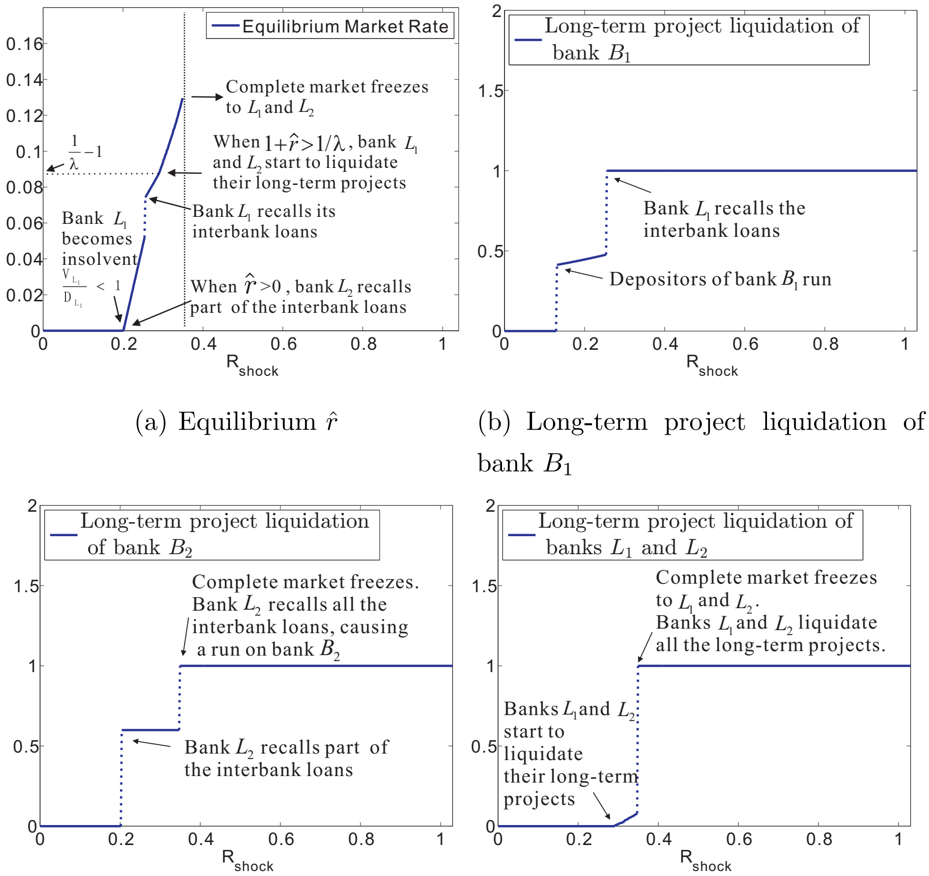
Figure 4: Determination of the equilibrium interest rate charged by the lending banks' depositors at $R_{shock} = 0.32$

In the example, we have both type I and type II equilibria. In the type I equilibrium, $\Gamma(\hat{r})$ crosses the 45 degree line below $\frac{V_{L_2}}{D_{L_2}} - 1$. More specifically, $\Gamma(\hat{r}) = \hat{r} = 0.1080 < \frac{V_{L_2}}{D_{L_2}} - 1 = 0.1287$. In the type II equilibrium, $\Gamma(\hat{r})$ crosses $\frac{V_{L_2}}{D_{L_2}} - 1$ below the 45 degree line. More specifically, $\Gamma(\hat{r}) = \frac{V_{L_2}}{D_{L_2}} - 1 = 0.1285 < \hat{r} = 0.1544$.

A partial market freeze occurs in this example. Appendix B shows that at the equilibrium $\hat{r}$, banks L_1 and L_2 liquidate part of their own long-term projects. Bank L_1 recalls all its interbank loans, and bank L_2 recalls part of its interbank loans.

4.4.2 Equilibrium market rate with different levels of R_{shock}

Figure 5 shows how equilibrium outcomes change when R_{shock} varies from 0 to 1.04.



(a) Equilibrium $\hat{r}$

(b) Long-term project liquidation of bank B_1

(c) Long-term project liquidation of bank B_2

(d) Long-term project liquidation of banks L_1 and L_2

Figure 5: How equilibrium outcomes vary in R_{shock}

Panel (a) of figure 5 illustrates how the equilibrium market rate, $\hat{r}^*$, varies in R_{shock}. It turns out that when $R_{shock} \leq 0.2018$, $\hat{r}^* = 0$. This is because $R_{shock} = 0.2018$ is the maximum shock at which bank L_1 can still make the full promised payment to depositors at the riskless rate.

When $R_{shock} > 0.2018$, $NV_{L_1} < 0$. This implies that $\frac{V_{L_1}}{D_{L_1}} < 1$, and, consequently, $\hat{r}^*$ becomes positive. In general, $\hat{r}^*$ is increasing in R_{shock} when $R_{shock} > 0.2018$, because a higher R_{shock} leads to a higher loss of bank L_1 from interbank loans to bank B_1 and, consequently, a lower $\frac{V_{L_1}}{D_{L_1}}$.

An upward jump of $\hat{r}^*$ occurs at $R_{shock} = 0.2538$ where $\hat{r}^* = 0.0518$. This is because bank L_1 starts to recall all of its interbank loans and uses the proceeds to repay its depositors. This will lead to a lower return rate for the remaining depositors, $\frac{V_{L_1}}{D_{L_1}}$. So depositors will charge a higher rate to compensate for the expected loss. When $R_{shock} < 0.2538$, bank L_1 recalls no interbank loans.

At $\hat{r}^* = \frac{1}{\lambda} - 1 = 0.087$ (with $R_{shock} = 0.2912$), both banks L_1 and L_2 start to liquidate their long-term projects. When $R_{shock} > 0.3494$ ($\hat{r}^* > 0.1296$), there exists no equilibrium $\hat{r}$, implying a complete market freeze to banks L_1 and L_2. In this case, a systematic collapse occurs: depositors of all the banks (including banks B_1 and B_2) withdraw at date 1. In addition, all the banks completely liquidate their long-term projects. Banks L_1 and L_2 also recall all of their interbank loans.

Note that when $R_{shock} = 0.2018$ where $\hat{r}^*$ becomes positive, bank L_2 starts to recall $\alpha_1^{L_2} = 0.8344$ of its interbank loans to repay its depositors. Inefficiency arises here because as long as $\alpha \leq \alpha_1^{L_2}$, bank B_2 depositors do not run, and the private cost of bank L_2 to recall interbank loans is zero. As a result, bank L_2 will always recall $\alpha_1^{L_2} x$ of interbank loans when the market rate $\hat{r}^* > 0$. However, because of the liquidation cost incurred by bank B_2 due to bank L_2's interbank loan recall, the social cost of bank L_2's recall is positive. Bank L_2 will not internalize the liquidation cost because it is borne by bank B_2. Thus, at the social level, there is too much liquidation. We find this result important because it reveals one source of liquidity shortage and inefficiency during financial crises. When facing higher financial costs during a crisis, creditors start to recall loans from solvent borrowers, regardless of the high social costs of doing so.

So, contagion in our model occurs as follows. After a shock hits bank B_1, contagion spreads from bank B_1 to the lending banks: the lending banks face a partial or complete market freeze. The market freeze induces the lending banks to raise liquidity by recalling their interbank loans to the borrowing banks, leading to contagion spreading from the lending banks to other solvent borrowers and forcing those borrowers to liquidate part or all of their long-term projects. In the worst case scenario, a systematic bank run occurs in which all the banks are run by their depositors and are forced to liquidate all of their long-term projects.

5 The general case of N connections

5.1 Equilibrium

Now we extend the model to a more general case where bank B_1 is connected to N lending banks. Assume that there are N pairs of banks. A pair of banks includes one lending bank and one borrowing bank. We still call the bank that lends to bank B_1 bank L_1. The remaining $N-1$ lending banks are called L_2-type banks and the remaining $N-1$ borrowing banks are called B_2-type banks. Figure 6 gives the interbank loan market structure.

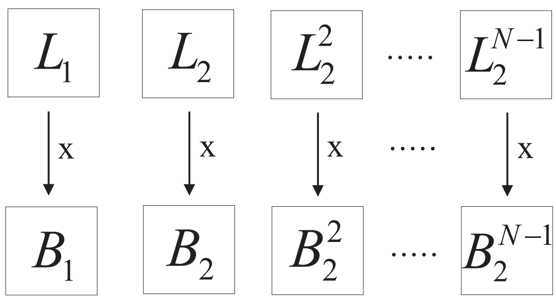


Figure 6: The interbank loan market structure with N pairs of banks. x is the position of interbank loans that the lending bank makes to the borrowing bank.

The sole difference between the 2-pair and N-pair cases is that the expected return to all the lending banks' depositors who roll over their deposits now changes to:

$$1 = \frac{N-1}{N}\left[\pi_{L_2}\min(\frac{V_{L_2}}{D_{L_2}}, 1+\hat{r}) + (1-\pi_{L_2})\right] + \frac{1}{N}\left[\pi_{L_1}\min(\frac{V_{L_1}}{D_{L_1}}, 1+\hat{r}) + (1-\pi_{L_1})\right] \quad (13)$$

That is, all the lending banks' depositors believe, with a probability of $\frac{N-1}{N}$, that their bank is a L_2-type bank, and, with a probability of $\frac{1}{N}$, that their bank is a L_1-type bank. This difference will lead to different equilibrium market rates for the lending banks. However, all the previous analysis about the optimal decisions of the lending and borrowing banks in the 2-pair case under a given $\hat{r}$ can be applied to the N-pair case, and we can find the equilibrium in a similar way.

5.2 A numerical example

Here we give numerical examples to illustrate the model, using the same baseline parametrization.

The major results that we find are as follows. (1) At each given level of R_{shock}, the equilibrium interest rate charged by the lending banks' depositors is decreasing in N. This is because the probability of being a bad bank assigned by the lending banks' depositors to their bank is decreasing in N. Complete market freezes disappear when N is high enough. (2) There may exist a non-monotonic relationship between the total liquidation cost and N at a given level of R_{shock}. This relationship is caused by the tradeoff in the liquidation cost when N increases: on one hand, a higher N alleviates market freezes, inducing a lower liquidation cost for an individual bank; on the other hand, contagion spreads to more banks with a higher N, inducing a higher aggregate liquidation cost. (3) A small shock to an individual bank can lead to a huge social welfare loss through contagion.

Figure 7 shows how $\hat{r}^*$ changes in R_{shock} for $N = 2$, 3, 4, 5, and 6. $\hat{r}^*$ becomes positive for all the Ns when R_{shock} exceeds 0.2018. Because $\frac{1}{N}$, the ex ante probability that a lending bank is bank L_1, is decreasing in N, the required rate $\Gamma(\hat{r})$ is also lower for the same R_{shock} when N becomes larger. As a result, $\hat{r}^*$ is lower for a higher N at the same level of R_{shock}. In our example, a complete market freeze for the lending banks occurs when $N = 2$, 3, and 4, after R_{shock} reaches 0.3494, 0.6198, and 0.8549 respectively. When $N > 4$, no complete market freeze occurs at any level of R_{shock}.

Figure 8 shows the long-term project liquidation for each type of bank when N changes from 2 to 4. Panel (a) of figure 8 illustrates how the total liquidation of bank B_1 changes in R_{shock} at different levels of N. In our numerical example, bank L_1 follows a trigger strategy in which it recalls all of its interbank loans if and only if R_{shock} is higher than a threshold level. This threshold level is higher with a larger N. This is because, given R_{shock}, when N is higher the market rate tends to be lower, which reduces the incentive of bank L_1 to recall its interbank loans and repay its depositors at date 1.

Panel (b) of figure 8 illustrates how the total liquidation of B_2-type banks changes in R_{shock} at different levels of N. The liquidation of B_2-type banks jumps at two threshold levels of R_{shock}. When R_{shock} reaches the first threshold level such that $\hat{r}^* > 0$, L_2-type banks start to recall $\alpha_1^{L_2} x = 0.8344x$ of their interbank loans. As a result, each B_2-type

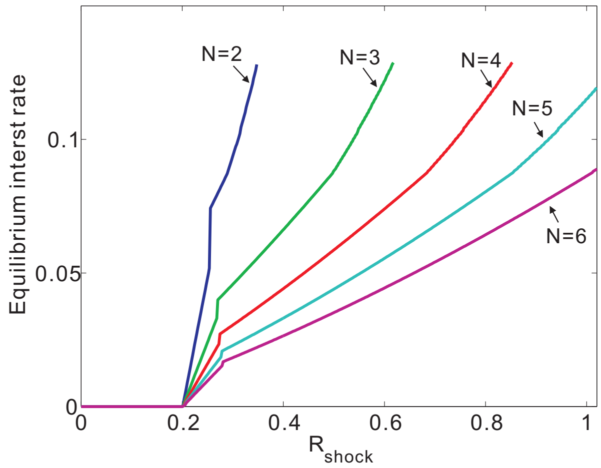


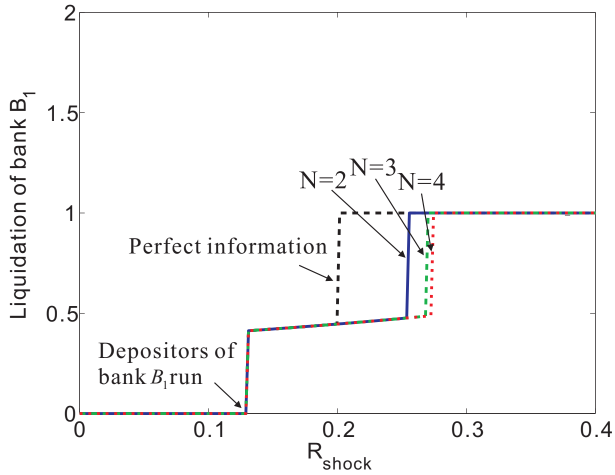
Figure 7: How $\hat{r}^*$ changes in R_{shock} when bank B_1 is connected to N lending banks.

bank will be forced to liquidate the long-term project to repay them. When R_{shock} reaches the second threshold level (it does not exist when $N = 5$ and 6), a complete market freeze occurs, and L_2-type banks are forced to recall all of their interbank loans. Note that in a perfect information case, B_2-type banks are unaffected, and there is no liquidation.

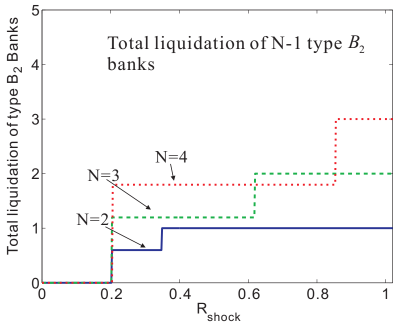
Panel (c) of figure 8 illustrates how the total liquidation of bank L_1 and L_2-type banks changes in R_{shock} at different levels of N. Both bank L_1 and L_2-type banks start to liquidate their long-term projects when R_{shock} is so large that $\hat{r}^* > \frac{1}{\lambda} - 1 = 0.087$, and are forced to liquidate all their long-term projects when a complete market freeze occurs. Note that in a perfect information case only bank L_1 will liquidate its long-term project when R_{shock} reaches a threshold level.

It turns out that at a given level of R_{shock}, the aggregate liquidation may have a non-monotonic relationship with N. This non-monotonic relationship is caused by the tradeoff in long-term project liquidation when N increases. On one hand, as we showed in figure 7, a higher N induces a lower $\hat{r}^*$, necessitating less liquidation of an individual bank's long-term project. On the other hand, the total liquidation of long-term projects may increase because more banks are involved in the contagion.

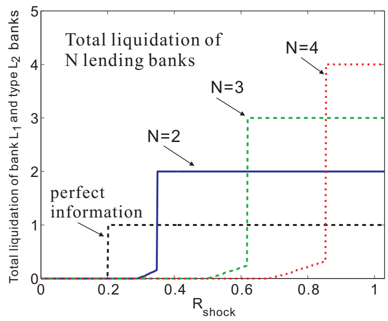
Figure 9 gives a numerical example at $R_{shock} = 0.88$ to illustrate the non-monotonic relationship between N and total liquidation and the associated liquidation cost of long-term projects. Here we exclude bank B_1 in order to focus on the social cost caused purely by contagion. In this example, a complete market freeze occurs at $N = 2$, 3



(a) Bank B_1



(b) $N-1$ B_2-type Banks

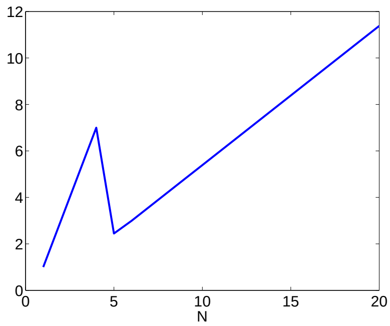


(c) N lending banks

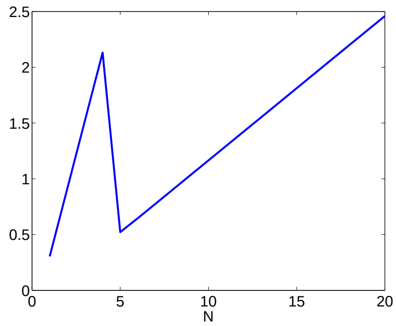
Figure 8: Long-term project liquidation of different types of banks when $N \leq 6$

and 4 with imperfect information and also occurs with perfect information. However, no complete market freeze occurs at $N = 5$ and 6 with imperfect information. As a result, the aggregate liquidation of all the banks except bank B_1 is $2N-1$ when $N \leq 4$, which is strictly increasing in N. The total liquidation cost is given by $0.3045 \times (2N-1)$, because the cost of liquidating a healthy bank's entire project is 0.3045. However, at $N = 5$, there is a downward jump of the aggregate liquidation because, in this case, bank L_1 and L_2-type banks are not run by their depositors. All the lending banks will liquidate a small amount of their long-term projects (0.01) because $\hat{r}^* > \frac{1}{\lambda} - 1 = 0.087$. In addition, L_2-type banks will recall $\alpha_1^{L_2} x = 0.8344x$ of their interbank loans, forcing all the B_2-type banks to liquidate 0.6006 of their long-term projects. Thus the total liquidation is 2.452. The associated liquidation cost is around 0.52. When $N \geq 6$, only B_2-type banks will liquidate their long-term projects to meet the interbank loan recall from L_2-type banks. As a result, the total liquidation is given by $0.6006 \times (N-1)$, and the associated liquidation cost is given by $0.13 \times (N-1)$, where 0.13 is the liquidation cost incurred by each B_2-type bank. Note that as long as $\hat{r}^* > 0$, L_2-type banks will recall $\alpha_1^{L_2} x = 0.8344x$ of their interbank loans, inducing the liquidation of B_2-type banks' long-term projects.

Note that the liquidation cost in an imperfect information case could be much higher than that in a perfect information case ($N = 1$), implying that contagion due to short-term creditor uncertainty about financial structures could greatly magnify the total loss across the whole financial system.



(a) total liquidation



(b) total liquidation cost

Figure 9: Total liquidation and the associated liquidation cost caused by contagion with different Ns at $R_{shock} = 0.88$. (Bank B_1 is excluded. $N = 1$ represents the perfect information case.)

In our numerical examples, we set parameters at intermediate values. We can raise the probability of a complete market freeze and, consequently, the contagion cost, if we use more extreme parameter values. In particular, we find that if we lower the equity level e_0 of banks, the probability of a complete market freeze increases. For example, if we decrease e_0 to 0.04, then a complete market freeze happens even when $N = 6$.

6 Policy implications

In this section, we explore the policy implications of our model. We examine three major policies: the information policy, the bailout policy, and the LOLR policy.

6.1 Information policy

Contagion in our model is caused by uncertainty. Because depositors do not know the identity of the exposed lending bank, they infer that all lending banks could be exposed to the distressed bank. As a result, they may run on all the lending banks, inducing runs on the healthy borrowing banks too. If we can reveal the identity of the exposed lending bank, then we can prevent market freezes from spreading to the healthy lending and borrowing banks.

Thus, our model demonstrates that it is critical for a central bank to keep track of the financial network structure and, in a time of crisis, reveal this structure to the market in a credible way. During a financial crisis, the banks perceived by the market to possibly be insolvent will have difficulty in credibly identifying themselves as solvent to the market, even though they have private information about their solvency. As a result, if the central bank can credibly identify the solvent banks, contagion can be eliminated. Moreover, no other central bank interventions such as bailouts and central bank loans are needed, and consequently no moral hazard problem will arise.

Note that it is important that the central bank is credible when it identifies the solvent banks: it must have a reliable record in supervising financial institutions. Meanwhile, the capability of a central bank to keep track of the financial network structure is greatly affected by different financial market structures. A sophisticated financial system with thousands of financial institutions, such as in the US, is obviously much more challenging than a simple financial system with a few major financial institutions, such as in Canada and Australia. Our model shows that a more competitive financial market, with all the benefits originating from perfect competition, may suffer the disadvantage of greater opacity (through the contagion caused by opacity) during a financial crisis, compared to a more concentrated banking system.

It is interesting to examine a case where a central bank can help reduce the uncertainty about the identity of the distressed lending bank, but does not have perfect information. In this case, less uncertainty does not necessarily improve social welfare. We demonstrated previously that the total liquidation cost is not a monotonic function of N, the number of possible distressed lending banks. A central bank can reduce the number of possible candidates by providing information about the economic fundamentals of healthy banks. However, as long as it cannot identify all the healthy banks, more information may lead to more long-term project liquidation and lower social welfare. For example, suppose that initially there are N lending banks perceived by the market as being the ones that might lend to the distressed bank. The central bank identifies one of them as healthy, so that the number of possible candidates for the distressed lending bank is reduced to $N-1$. Because the probability for each lending bank to be distressed now increases, a market freeze may be more severe. In an extreme case, no complete market freeze occurs to the lending banks with N candidates, but a complete market freeze occurs to the lending banks with $N-1$ candidates. In this case, the social cost could be much higher with less uncertainty.

Next we will examine the case where the central bank does not have perfect information about the financial network structure. In this case, they will resort to a bailout policy or LOLR policy to alleviate contagion.

6.2 Bailout policy

6.2.1 Guaranteeing bank B_1's debt

In our model, contagion originates from bank B_1, which is hit by a negative shock. A straightforward way to prevent contagion is to remove the originator; that is, to bail out bank B_1 by using taxpayers' money to pay the losses of bank B_1's creditors. As a result, the bank lending to B_1 is saved and the contagion is stopped. In our simple model, this method is effective and easy to implement. However, we have not taken into account the moral hazard caused by this policy, which is what makes the central bank reluctant to use this policy in reality.[9]

6.2.2 Injecting capital into the lending banks

The central bank can alleviate market freezes by buying preferred shares or stock issued by the lending banks. Suppose that the preferred stock injected into each bank is sufficient for bank L_1 to meet the promised payment to depositors at a zero interest rate at date 2. The market interest rate will be zero. As a result, bank L_1 will optimally choose a

[9]In the recent subprime mortgage crisis, the US government refused to bail out Lehman Brothers due to the concern of moral hazard, which led to severe market freezes in the financial system right after Lehman Brothers' bankruptcy.

proportion of its interbank loans to recall, conditional on its depositors not withdrawing at date 1. (We analyzed this situation in the perfect information case.) Note that although the central bank buys preferred shares in the N lending banks, only the preferred shares in bank L_1 will suffer a loss. So in our case, when N increases, the initial funds that the central bank needs to inject into the banks will increase, but the actual loss that the central bank incurs will remain constant. This is limited to the loss of bank L_1 that equals the loss due to interbank loan x, minus the loss that can be absorbed by bank L_1's own capital. So as N increases, the actual cost of saving the banks does not increase, while the cost of not saving the banks (i.e., the cost of liquidating healthy banks' long-term projects) could increase, creating a stronger incentive for a central bank to inject capital into all the banks. Of course, here we ignore moral hazard and the controversy associated with the nationalization of the banking industry.

The advantage of this policy, compared to directly bailing out bank B_1, is that the central bank will use less taxpayers' money to bail out the financial system. The central bank need not pay all bank B_1's creditors, and the loss to bank L_1 is absorbed first by bank L_1's capital. In this sense, the moral hazard problem is less severe under this policy than under the policy of direct bailout.

6.3 LOLR policy

In this section, we examine the central bank policy of emergency liquidity assistance, also known as the LOLR policy. LOLR policy discussions often cite the classic book by Bagehot (1873) in which he summarized the LOLR policy as a central bank lending freely against good collateral at a higher interest rate. According to Freixas and Rochet (2004), the classic Bagehot rules can be criticized on two grounds: (1) it is impossible for a central bank to distinguish between illiquidity and insolvency as a LOLR (see, e.g., Goodhart (1999)), and (2) with a well-functioning interbank loan market, the open market operation of central banks is enough to maintain an efficient market, rendering the LOLR policy unnecessary (see, e.g., Goodfriend and King (1988)). As Freixas and Rochet (2004) observed, although the classic Bagehot rules might have been considered obsolete, the current crisis has revealed that we do not have well-established rules to replace LOLR policies.

Our paper establishes a model to study aspects of the LOLR policy. This model produces the following major results:

First, a central bank has to implement LOLR only when it does not have perfect information about a financial network structure and cannot differentiate between solvent and insolvent banks. Our previous analysis on information policy indicates that central bank interventions are not needed if the central bank can credibly reveal perfect information about the financial network structure.[10] This result is consistent with Goodhart (1999), who argues that when the central bank becomes the LOLR to a commercial bank, it must be because the commercial bank is under the suspicion of insolvency, but the central bank cannot know for sure whether this suspicion is valid or not.

Our model demonstrates that, with imperfect information, both insolvent and solvent lending banks may face market freezes, and short-term lending contagion spreads from lending banks to solvent borrowing banks through the process of interbank loan recall. This gives the central bank the role of LOLR in order to improve social welfare. Losses will be inevitable for the central bank because it cannot distinguish between insolvent and solvent lending banks. However, as our model reveals, the total social loss incurred as a result of contagion without the LOLR policy is much higher, compared to the cost incurred by the LOLR policy.[11]

Second, we find that given that the central bank has no better information than market participants, the optimal LOLR policy should be to lend freely at the riskless interest rate. This policy can effectively stop complete and partial freezes and achieve maximum social welfare. Moreover, we find that any LOLR policy with limited lending at a rate lower than the prevailing market rate will generally alleviate market freezes and improve social welfare.

We make a simple extension of our basic model with $N = 2$. More specifically, we assume that (1) the central bank provides loans up to $\bar{L}_{CB}$ to each of the lending banks,

[10] Note that this result is reached in our model where contagion is caused solely by uncertainty in the financial network structure. In reality, contagion may be caused by other mechanisms, such as the actual financial interconnections, and central bank interventions may still be necessary even with perfect information.

[11] Note that we do not take into account moral hazard here. But, as argued by Goodhart (1999), there is always a tradeoff between "preventing panic now" and "inducing riskier activity later." When systemic risk is high, it is impossible for the central bank to eschew the LOLR policy together because of the concern of moral hazard.

at a fixed interest rate $r_{CB} \geq 0$, (2) $\bar{L}_{CB}$ is small so that the lending banks will still need to borrow from the market, and (3) the lending banks will choose to borrow from the market when the market rate $\hat{r} \leq r_{CB}$. As a result, only when $\hat{r} > r_{CB}$, will the banks borrow from the central bank, and the amount will be $\bar{L}_{CB}$. We make assumption (2) because if $\bar{L}_{CB}$ is large enough to cover all the liquidity need of the banks, then the banks may borrow only from the central bank, and the equilibrium market rate may not exist. We want to focus on the more interesting and realistic case where both lending banks still need to borrow from the market in addition to their central bank loans so that we can analyze how central bank lending will affect the market rate.

We find the equilibrium market rate using a similar method to the case without central bank lending. Proposition 4 gives the results.

Proposition 4. *At a given level of $\hat{r} > r_{CB}$, with central banking lending, both lending banks' optimal choices on interbank loan recall and long-term project liquidation are the same as without central bank lending. In addition, for both lending banks, the maximum return rate that can be paid on deposits becomes higher, leading to a lower required rate $\Gamma(\hat{r})$ for any given deposit rollover probabilities of the lending banks.*

Proof: See the appendix. ■

We find that, in general, central bank loans at an interest rate lower than the prevailing market rate will lead to a higher maximum return for both good and bad banks, inducing the following two effects. First, the higher maximum rate from the good lending bank will reduce the probability of a complete market freeze. As explained before, for an equilibrium market rate to exist, the required rate, $\Gamma(\hat{r})$, cannot be higher than the maximum return rate of the good bank. The good bank having a higher maximum rate means that this condition can be satisfied for more values of $\hat{r}$, and an equilibrium is more likely to exist. Second, the higher maximum rate from the bad lending bank will reduce the required rate, $\Gamma(\hat{r})$. This is because when the bad bank is insolvent, all its remaining assets are allocated to creditors, and the actual return rate is equal to the maximum rate. With a higher return rate from the bad lending bank, the depositors will charge a lower required rate, $\Gamma(\hat{r})$, to compensate for the expected loss.

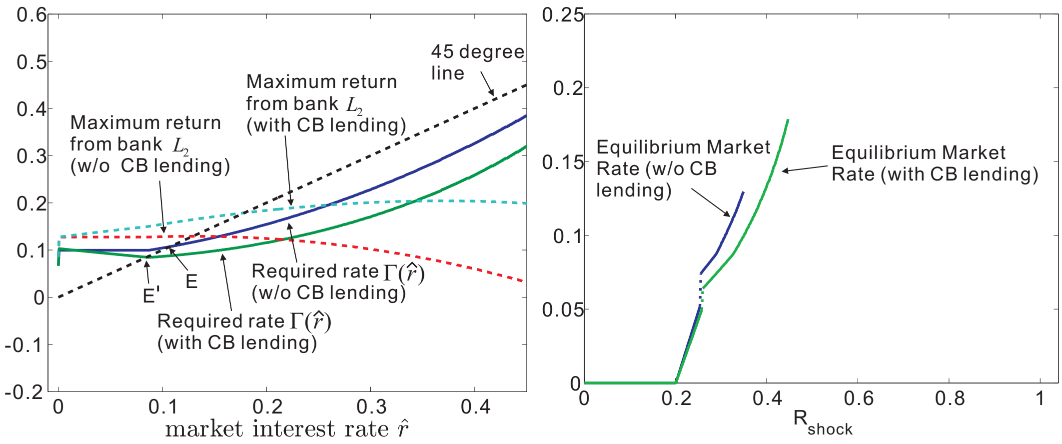
However, central bank loans also lead to a lower deposit rollover probability for both good and bad lending banks, inducing a higher required rate, $\Gamma(\hat{r})$. To see this, note that the good lending bank tends to have more cash resources than the bad one, because it can recall more interbank loans from its healthy borrowing bank. By the same token, it tends to borrow less than the bad bank. As a result, when both lending banks reduce their deposits by the same amount, $\bar{L}_{CB}$, the percentage drop in π_{good} is larger than that of π_{bad}. This effect will raise the required rate, $\Gamma(\hat{r})$, because there is a relatively larger probability that deposits will be rolled over by the bad lending bank.

We find that as long as the amount borrowed from the market is not extremely small (so that π_{good} is not extremely small), the overall effect of central bank loans is to lower equilibrium market rates. Thus, in general, central bank lending at a rate lower than the prevailing market rate tends to alleviate market freezes and contagion. The equilibrium market rate tends to be lower, and a complete market freeze is less likely to happen.

Below we provide a numerical example with the same baseline parametrization to illustrate the effects. We set $\bar{L}_{CB} = 0.25$ and $r_{CB} = 0$. Figure 10(a) compares the results with and without central bank lending at $R_{shock} = 0.32$. Here the case without central bank lending is the same as in figure 4. We can see that with central bank lending, the curve of the required rate, $\Gamma(\hat{r})$, shifts downward and crosses the 45 degree line at a lower equilibrium rate. The equilibrium rate without central bank lending is $\hat{r}^* = 0.1264$, while the equilibrium rate with central bank lending is $\hat{r}^* = 0.1$. In addition, the constraint imposed by the maximum return from the good bank L_2 is now shifted up. Note that this curve is independent of the level of R_{shock}, because the action of the good bank L_2 depends only on $\hat{r}$. This implies that the required rate, $\Gamma(\hat{r})$, is more likely to be below this curve, and a complete market freeze is less likely to happen.

Figure 10(b) shows the result for different values of R_{shock}. For values of R_{shock} that lead to a positive $\hat{r}$ in the case without central bank lending, the equilibrium rate is now lower with central bank lending. In addition, for values of R_{shock} that lead to the non-existence of equilibrium rates in the case without central bank lending, an equilibrium rate now exists. That is, for those values of R_{shock}, a partial market freeze now replaces a complete one.

In the above example, we focus on the more realistic case where the lending banks still need to borrow from the market. However, theoretically, the policy that minimizes the liquidation of assets and the associated social cost would be for the central bank to lend at the riskless rate and to meet all the liquidity need of the lending banks. Intuitively, if the lending banks can borrow enough liquidity from the central bank at the lowest possible



(a) Equilibrium at $R_{shock} = 0.32$ with and without central bank lending

(b) Equilibrium $\hat{r}$ with and without central bank lending

Figure 10: The effect of central bank lending on the market equilibrium rate when $\bar{L}_{CB} = 0.25$ and $r_{CB} = 0$.

rate, then they will not liquidate their own assets. The good lending bank will not need to recall any interbank loans, and the bad lending bank will have the weakest incentive to recall its interbank loans. Consequently, long-term asset liquidation is minimized. This also implies that the liquidation cost will be higher when the central bank lends less or charges a higher rate.

7 Interpretations and extensions of the model

In our model we assume that there are equal numbers of paired borrowers and lenders. This is an extreme assumption to make the game theoretical analysis symmetrical and relatively easy to solve. But we can reinterpret the model to allow for more realistic network credit exposures.

First, assume that the model describes a distressed bank and a group of identical creditor banks that are aggregated into one representative creditor bank. Then we could treat the remainder of the lending banks as institutions that are known to have been creditors of the distressed bank in the past; but short-term creditors are uncertain about their current exposures to the distressed bank. In this interpretation, the other banks have no exposure to the distressed bank, but could suffer a complete or partial freeze because of short-term creditor doubt.

Second, we can add a further group of institutions that are widely known to short-term creditors as having no exposure to the distressed bank. This group are immune to a freeze.

Third, one can think of this as a domestic banking system embedded in an international banking system. It is easy to reinterpret our model in a situation where a domestic banking system and domestic short-term money markets are potentially exposed to international credit risks through domestic bank exposures to international credit risks. Although only a subset of domestic banks may be vulnerable, domestic money markets may fear that there are undisclosed exposures.

A critical aspect of our model is that short-term creditors do not have accurate information about the network of bank exposures. We could create a richer theory if we allow short-term creditors to acquire additional information by observing some signals. For instance, in the current model, we implicitly assume that borrowing from the central bank does not change depositors' beliefs. A possible extension is to allow banks' borrowing activities to reveal information about their asset quality. As a result, banks may use central bank loans as a signaling tool, and it would be interesting to find out what the optimal central bank loan policy would be after this informational effect is taken into account. For example, central banks may have an incentive to hide the identity of the borrowing banks, if depositors believe that all the banks borrowing from the central bank are insolvent. In this case, borrowing from the central bank will greatly increase a bank's cost of borrowing from the market, resulting in a lower than socially optimal level of borrowing from the central bank.

8 Conclusions

This paper studies contagion and systemic market freezes caused by uncertainty regarding interconnections in the financial system. Our model demonstrates that a negative shock to an individual financial institution can spread to other financial institutions because of creditor uncertainty about the interconnections among the financial institutions. This can lead to partial or complete market freezes affecting all the financial institutions. Our model reveals that, because of the uncertainty regarding interconnections, all the financial institutions *perceived* by the market to be connected to the distressed institution can be involved in the contagion, even when they have no *actual* connection to it. Thus, our model shows that the magnitude of contagion could be greatly magnified because of short-term creditor uncertainty about interbank exposures in the financial system. Policy implications are also explored in our paper. Using our model, we find that it is crucial for a central bank to keep accurate information about the financial network structure in order to prevent contagion in a financial crisis. Moreover, given that the central bank does not have perfect information, better information provided by the central bank to market participants to refine their beliefs about interbank exposures does not necessarily improve social welfare. This is because the number of lending institutions involved in the contagion has a non-monotonic relationship with the social losses caused by the contagion. If we are to ignore moral hazard, (a) bailout policies guaranteeing the debts of the financial institution hit by the shock, and (b) capital injections to all the lending financial institutions are two policies that can check the contagion. The LOLR policy can also effectively alleviate market freezes by lowering market rates and reducing the incidence of complete market freezes.

A Proofs

A.1 Proof of lemma 1

Using (1), we get

$$\frac{\partial NV_{B_1}^{nr}}{\partial \alpha} = -(1+\hat{R})\frac{\partial l}{\partial \alpha} + x \tag{14}$$

Using (2), we get

$$\frac{\partial \alpha}{\partial l} = \frac{\lambda(1+\hat{R}) - \gamma l(1+\hat{R})^2}{x} \Rightarrow \frac{\partial l}{\partial \alpha} = \frac{x}{\lambda(1+\hat{R}) - \gamma l(1+\hat{R})^2} \geq \frac{x}{\lambda(1+\hat{R})} \tag{15}$$

Thus

$$\frac{\partial NV_{B_1}^{nr}}{\partial \alpha} < -(1+\hat{R})\frac{x}{\lambda(1+\hat{R})} + x = -(\frac{1}{\lambda} - 1)x < 0 \tag{16}$$

since $\lambda < 1$. ■

A.2 Proof of lemma 2

Proof of result (1): Given lemma 1, when $\alpha < \alpha_1$, $NV_{B_1}^{nr}$ is positive, implying that bank B_1 is able to repay all the liabilities. Given that other depositors roll over their deposits, each depositor does not have an incentive to deviate. Thus, all the bank B_1 depositors rolling over their deposits is a Nash equilibrium. Since we assume that whenever a no-run equilibrium is feasible, depositors will choose this equilibrium, the no-run equilibrium is the equilibrium.

Proof of result (2): Given lemma 1, when $\alpha > \alpha_1$, $NV_{B_1}^{nr}$ is negative. Thus, bank B_1 is unable to repay all the liabilities at date 2, given that all the depositors withdraw at date 2. We first prove that a no-run equilibrium is not feasible. Note that when $\alpha > \alpha_1$, a depositor receives less than 1 unit of the good at date 2 for each unit of the deposit. Suppose that an individual depositor deviates to withdrawing at date 1. Given that bank B_1's asset value is positive at date 2, he will receive 1 unit of the good by withdrawing at date 1. In this case, a depositor is better off by withdrawing at date 1. Given that bank B_1 has no assets left at date 2, a depositor will receive nothing by withdrawing at date 2, while withdrawing at date 1 will yield a positive payment. Thus, in this case, a depositor is also better off by withdrawing at date 1. Thus we prove that a no-run equilibrium is not feasible.

Next we prove that all the depositors withdrawing at date 1 is actually a Nash equilibrium. Given that all the other depositors choose to withdraw at date 1, and bank B_1's asset value is 0 at date 2, an individual depositor is strictly better off withdrawing at date 1. In this case, bank B_1's resources, which are strictly positive, will be proportionally allocated to depositors. Given that bank B_1's asset value is positive at date 2, then bank B_1 must have enough resources to meet the withdrawal at date 1. Thus, withdrawing at date 1 will yield a payoff of 1 unit of the good. This is no worse than the payoff from withdrawing at date 2, which is no greater than 1 unit of the good. This means that a depositor has no incentive to deviate from the strategy of withdrawing at date 1. Thus we prove that withdrawing at date 1 is a Nash equilibrium. ■

A.3 Proof of $\alpha_2 \leq 1$ given $\alpha_1 \leq 1$

We prove this result by contradiction. We show that if $\alpha_2 > 1$, then $\alpha_1 > 1$.

The meaning of $\alpha_2 > 1$ is that the liquidation value of B_1 at date 1 is more than enough to meet the withdrawal of bank B_1's depositors and bank L_1, given that bank L_1 recalls all of its interbank loans. Thus we have

$$V_{B_1,liquidation} = \lambda L(1+\hat{R}) - \frac{1}{2}\gamma(L(1+\hat{R}))^2 > (D_0 - x) + x = D_0 \tag{17}$$

The meaning of $\alpha_1 > 1$ is that when bank L_1 recalls all of its interbank loans at date 1, bank B_1 still has enough assets to meet the withdrawal of its depositors at date 2. Thus we have

$$x = \lambda l(1+\hat{R}) - \frac{1}{2}\gamma(l(1+\hat{R}))^2 \tag{18}$$

$$NV^{nr}_{B_1,\alpha=1} = (L-l)(1+\hat{R}) - (D_0 - x) > 0 \tag{19}$$

A simple transformation of (19) gives us $(L-l)(1+\hat{R}) + x > D_0$.

Given (17), if we prove that $(L-l)(1+\hat{R})+x > D_0$, then we prove that $NV^{nr}_{B_1,\alpha=1} > 0$ and, consequently, $\alpha_1 > 1$. Using (17), it is equivalent to proving that $(L-l)(1+\hat{R})+x > V_{B_1,liquidation} = \lambda L(1+\hat{R}) - \frac{1}{2}\gamma(L(1+\hat{R}))^2$. The proof is as follows. Using (17) and (18), we get

$$\begin{aligned}
& (L-l)(1+\hat{R}) + x - V_{B_1,liquidation} \\
= \;& (L-l)(1+\hat{R}) + \left[\lambda l(1+\hat{R}) - \frac{1}{2}\gamma(l(1+\hat{R}))^2\right] - \left[\lambda L(1+\hat{R}) - \frac{1}{2}\gamma(L(1+\hat{R}))^2\right] \\
= \;& (1+\hat{R})(1-\lambda)(L-l) + (L^2 - l^2)\frac{1}{2}\gamma((1+\hat{R}))^2 > 0
\end{aligned} \tag{20}$$

This is because $L > l$, $1 > \lambda$ and $1 + \hat{R} > 0$. Thus we prove that if $\alpha_2 > 1$, then $\alpha_1 > 1$. So, by contradiction, if $\alpha_1 \leq 1$, then $\alpha_2 \leq 1$.

The intuition behind this result is as follows. When $\alpha_2 > 1$, the liquidation value of B_1 at date 1 is more than enough to meet the liquidity demands from both its depositors and bank L_1, even when bank L_1 recalls all of its interbank loans. Now suppose that bank L_1 still recalls all of its loans, but the depositors wait until period 2. Then the bank's net asset value is definitely higher than in the previous case where depositors withdraw at date 1 and, consequently, is more than enough to meet the withdrawal of depositors. This is because with a zero interest rate, depositors still withdraw the same amount at date 2. Since liquidation is costly, the assets that otherwise must be liquidated to meet the withdrawal of depositors at date 1 can now be carried over to date 2, implying that bank

B_1's assets at date 2 must be more than enough to meet the withdrawal of its depositors. This means that $NV^{nr}_{B_1,\alpha=1} > 0$, and by definition, $\alpha_1 > 1$, which means that $\alpha_1 \leq 1$ is impossible. So, by contradiction, when $\alpha_1 \leq 1$, we must have $\alpha_2 \leq 1$. ■

A.4 Proof of lemma 3

Let Π denote the total payoff of bank L_1 from its interbank loans in terms of date 2 value. Thus we have the following results.

First, when $\alpha \in [0, \alpha_1]$, bank B_1 has enough resources at date 2 to repay all its creditors. Thus, we have

$$\Pi = \alpha x + (1-\alpha)x = x \tag{21}$$

Second, when $\alpha \in (\alpha_1, \alpha_2]$, we have

$$\Pi = \alpha x + (L-l)(1+\hat{R}) \tag{22}$$

$$\alpha x + D_0 - x = \lambda l(1+\hat{R}) - \frac{1}{2}\gamma[l(1+\hat{R})]^2 \tag{23}$$

It turns out that

$$\frac{\partial \Pi}{\partial \alpha} = x - (1+\hat{R})\frac{\partial l}{\partial \alpha} \tag{24}$$

Using (23), we have

$$\frac{\partial l}{\partial \alpha} = \frac{x}{(1+\hat{R})(\lambda - \gamma(1+\hat{R})l)} \tag{25}$$

Thus we have

$$\frac{\partial \Pi}{\partial \alpha} = x - \frac{x}{\lambda - \gamma(1+\hat{R})l} < 0 \tag{26}$$

because, by assumption, $\lambda < 1$ and $\lambda - \gamma(1+\hat{R})l > 0$. As a result, $0 < \lambda - \gamma(1+\hat{R})l < 1$.

When $\alpha \in [\alpha_2, 1]$, we have

$$\Pi = \frac{\alpha x}{\alpha x + (D_0 - x)}(\lambda L(1+\hat{R}) - \frac{1}{2}\gamma[L(1+\hat{R})]^2) \tag{27}$$

It is straightforward to see that Π is strictly increasing in α.

Note that there is a downward jump in Π at α_1, because $(L-l)(1+\hat{R}) < (1-\alpha)x$. This jump is caused by the liquidation cost incurred by the withdrawal of bank B_1's depositors at date 1.

In addition, Π is continuous at α_2, that is, $\Pi(\alpha \in (\alpha_1, \alpha_2], \alpha = \alpha_2) = \Pi(\alpha \in (\alpha_2, 1], \alpha = \alpha_2)$. Note that by definition, $\alpha_2 x + (D_0 - x) = \lambda L(1+\hat{R}) - \frac{1}{2}\gamma[L(1+\hat{R})]^2$. As a result, $\Pi(\alpha \in (\alpha_1, \alpha_2], \alpha = \alpha_2) = \Pi(\alpha \in (\alpha_2, 1], \alpha = \alpha_2) = \alpha_2 x$. ■

A.5 The proof of bank L_1's payoff from interbank loans with different combinations of α_1 and α_2

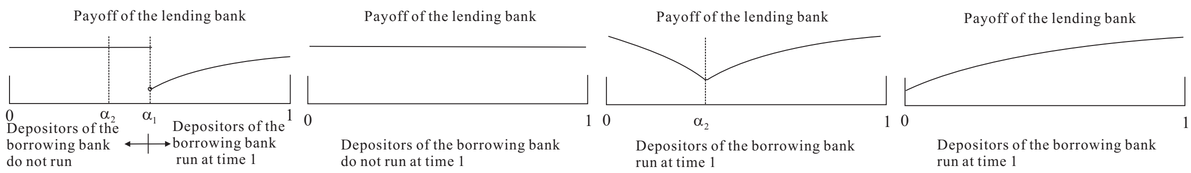
Appendix A.4 gives bank L_1's payoff when $0 \leq \alpha_1 < \alpha_2 \leq 1$. The following points elaborate on all the other possible cases.

First, $\alpha_1 \in [0,1]$ and $\alpha_2 \leq \alpha_1$. In this case, when $\alpha \in [0, \alpha_1]$, bank L_1's payoff, Π, is given by (21). This is because when $\alpha \leq \alpha_1$, a no-run equilibrium is feasible for bank B_1, and bank B_1 is solvent and able to repay all its creditors at date 2. When $\alpha \in [\alpha_1, 1]$, bank L_1's payoff is given by (27), which we proved is strictly increasing in α. This is because when $\alpha > \alpha_1$, bank B_1 depositors will withdraw at date 1. In addition, bank B_1 has no assets left at date 2 since $\alpha > \alpha_1 > \alpha_2$.

Note that given that $\alpha_1 \in [0,1]$, bank L_1's payoff from its interbank loans is maximized at $\alpha = 0$. This is because, given that $\alpha_1 \in [0,1]$, a no-run equilibrium is always feasible for bank B_1 at $\alpha = 0$.

Second, $\alpha_1 > 1$. In this case, bank L_1's payoff, Π, is given by (21) over $\alpha \in [0,1]$. This is because bank B_1's net value at date 2 is positive even when bank L_1 recalls all of its interbank loans. Thus, bank B_1 depositors will not withdraw at date 1 for all $\alpha \in [0,1]$.

Third, $\alpha_1 < 0$. In this case, bank B_1 does not have enough resources to repay its liabilities at date 2, even when bank L_1 does not recall any interbank loans at date 1. So bank B_1 depositors will always withdraw at date 1 for $\alpha \in [0,1]$. In the case of $\alpha_2 \in (0,1]$, when $\alpha \in [0, \alpha_2]$, bank L_1's payoff is given by (22), which we proved is strictly decreasing in α. When $\alpha \in [\alpha_2, 1]$, bank L_1's payoff is given by (27), which we proved is strictly increasing in α. As a result, bank L_1's payoff is maximized either at $\alpha = 0$ or at $\alpha = 1$. In the case of $\alpha_2 \leq 0$, bank L_1's payoff is given by (27) over $\alpha \in [0,1]$, which we proved is strictly increasing in α and is maximized at $\alpha = 1$.[12] Figure 11 illustrates the above results. ■



(a) $0 \leq \alpha_1 \leq 1$, $\alpha_2 < \alpha_1$ (b) $\alpha_1 > 1$ (c) $\alpha_1 < 0$, $\alpha_2 > 0$ (d) $\alpha_1 < 0$, $\alpha_2 \leq 0$

Figure 11: The lending bank's payoff from interbank loans in other cases

A.6 Proof of proposition 1

When $\alpha_1 \geq 0$, bank B_1 has enough resources to meet all the liabilities at date 2 at $\alpha = 0$. Thus at $\alpha = 0$, bank B_1 depositors will coordinate for the no-run equilibrium, and bank L_1 receives the maximum payoff from its interbank loans, x. So $\alpha = 0$ produces the first best allocation. When $\alpha_1 < 0$, bank B_1 does not have enough resources to meet its liabilities at date 2, even when bank L_1 does not recall any interbank loans ($\alpha = 0$). Thus, bank B_1 depositors will withdraw at date 1 for any α. Our previous analysis reveals that bank L_1's payoff is first strictly decreasing, and then strictly increasing (when $\alpha_2 < 0$, it will be strictly increasing over the whole region). So the optimal solution is either $\alpha = 0$ or $\alpha = 1$. ■

A.7 Proof of proposition 2

This argument is similar to lemma 2. Given that $V_{L_1}^{nr} \geq D_0 + x$, the no-run equilibrium is feasible. Given that other depositors do not run, an individual depositor will not run. Given that $V_{L_1}^{nr} < D_0 + x$, since liquidation is costly, $V_{L_1,liquidation} < V_{L_1}^{nr} < D_0 + x$. As a result, a no-run equilibrium is not feasible. Given that other depositors do not withdraw at date 1, an individual depositor will always be better off by withdrawing at date 1, because he will receive 1 unit of the good by withdrawing at date 1 and will receive less than 1 unit of the good by withdrawing at date 2. Moreover, a run equilibrium is actually

[12]Note that for $\alpha_1 < 0$, we need only to consider the case where $\alpha_2 \leq 1$, because we prove that if $\alpha_1 \leq 1$, then $\alpha_2 \leq 1$ (see Appendix A.3).

a Nash equilibrium: Given that other depositors run, an individual depositor will get nothing if he withdraws at date 2. While withdrawing at date 1 will yield a positive payoff of $V_{L1,liquidation}/(D_0 + x)$. So withdrawing at date 1 is a Nash equilibrium.

We prove that when bank L_1 depositors run, bank B_1 depositors will always run as follows. If bank B_1 depositors do not run, bank L_1 depositors will never run. This is because when bank B_1 depositors do not run, bank B_1 must have been solvent at date 2, implying that bank L_1 will receive full repayments of x at date 2. As a result, bank L_1 will not incur any losses, and bank L_1 depositors will be fully paid at date 2. Thus a no-run equilibrium is feasible for bank L_1. Thus we can infer that when bank L_1 depositors run, bank B_1 depositors must run as well. ■

A.8 Proof of lemma 4

If the lending bank collects goods at date 1 by liquidating its long-term project, the marginal cost of 1 unit of goods in terms of date 2 goods is at least $\frac{1}{\lambda}$. So the lending bank is always worse off by liquidating the long-term project to reduce the amount of rolled over deposits when $1+\hat{r} < \frac{1}{\lambda}$. More specifically, the optimal liquidation of a lending bank's own long-term project is determined as follows. Suppose the bank liquidates l units of its long-term project to repay deposits. The associated payoff is

$$[\lambda l(1+R) - \frac{1}{2}\gamma(l(1+R))^2](1+\hat{r}) + (L-l)(1+R) \tag{28}$$

The first term is debt reduction achieved by using the liquidated goods to repay deposits, and the second term is the value of the unliquidated long-term project. The first order derivative of the payoff w.r.t l is $(1+R)[(\lambda - \gamma l(1+R))(1+\hat{r}) - 1]$. When at $l = 0$, $\lambda(1+\hat{r}) < 1$, we have the corner solution of $l = 0$. When at $l = L$, $(\lambda - \gamma L(1+R))(1+\hat{r}) > 1$, we have the corner solution of $l = L$. Otherwise, we have the interior solution of $l = \frac{\lambda - \frac{1}{1+\hat{r}}}{\gamma(1+R)}$. Thus we prove result (1).

A lending bank's decision of α can be analyzed in a similar way to the perfect information case. Figure 12 illustrates the intuition behind this decision. In the general case, we can still separate α into three regions of $[0, \alpha_1]$, $(\alpha_1, \alpha_2]$, and $[\alpha_2, 1]$. The reactions of the borrowing banks' depositors given α are the same as in the perfect information case. The payoff for the lending bank is different, however, because the interest rate for deposits is now $1 + \hat{r}$.

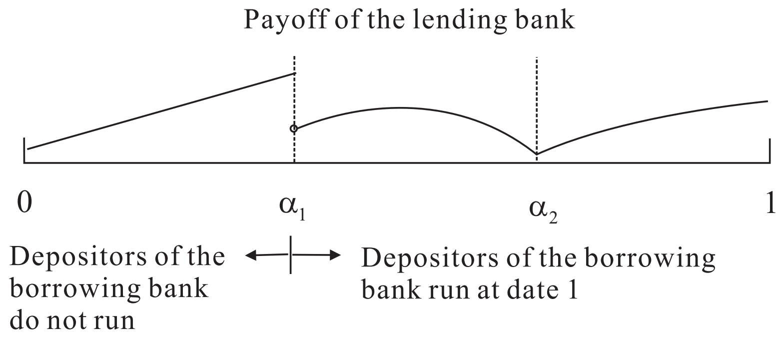


Figure 12: An example of the lending bank's payoff from recalling αx of interbank loans under imperfect information

Let Π^i be the total payoff from interbank loans in terms of date 2 value. Note when the market rate $\hat{r}$ is positive, the bank will use the recalled money to repay its deposits. Thus, in terms of date 2 value, the payoff from the recalled interbank loans of αx equals the proceeds from the recall multiplied by $1+\hat{r}$. When $\alpha \in [\alpha_2, 1]$, $\Pi^i = \frac{\alpha x}{\alpha x+(D_0-x)}(\lambda L(1+\hat{R}) - \frac{1}{2}\gamma[L(1+\hat{R})]^2)(1+\hat{r})$. It is strictly increasing in α. So the local optimal point in the region of $[\alpha_2, 1]$ is at $\alpha = 1$.

When $\alpha \in [0, \alpha_1]$,

$$\Pi^i = (1-\alpha)x + \alpha x(1+\hat{r}) \tag{29}$$

The first term on the right-hand-side is the payoff from the remaining interbank loans at date 2. The second term means the withdrawal of αx reduces the date 2 debts by $\alpha x(1+\hat{r})$. Since the payoff is increasing in α, the local optimal point is α_1, implying that the bank will recall at least α_1 of its interbank loans.

When $\alpha \in (\alpha_1, \alpha_2]$,

$$\Pi^i = \alpha x(1+\hat{r}) + (L-l)(1+\hat{R}) \tag{30}$$

$$\alpha x + D_0 - x = \lambda l(1+\hat{R}) - \frac{1}{2}\gamma[l(1+\hat{R})]^2 \tag{31}$$

In this case, the borrowing bank's depositors will run, and the lending bank owns all the remaining assets of the borrowing bank. It turns out that

$$\frac{\partial \Pi^i}{\partial \alpha} = x(1+\hat{r}) - \frac{x}{\lambda - \gamma(1+\hat{R})l} \tag{32}$$

Thus, when $1+\hat{r} > \frac{1}{\lambda-\gamma(1+\hat{R})l}$, Π^i is strictly increasing in α. Otherwise, it is strictly decreasing in α. Note that l is strictly increasing in α. Let $l(\alpha_1)$ and $l(\alpha_2)$ denote the liquidated long-term project at α_1 and α_2 respectively. Given that $l(\alpha_1) \geq \frac{\lambda - \frac{1}{1+\hat{r}}}{\gamma(1+\hat{R})}$, Π^i is always strictly decreasing in $\alpha \in (\alpha_1, \alpha_2]$. Given that $l(\alpha_2) \leq \frac{\lambda - \frac{1}{1+\hat{r}}}{\gamma(1+\hat{R})}$, Π^i is always strictly increasing in $\alpha \in (\alpha_1, \alpha_2]$. Given that $l(\alpha_1) < \frac{\lambda - \frac{1}{1+\hat{r}}}{\gamma(1+\hat{R})} < l(\alpha_2)$, Π^i is concave when $\alpha \in (\alpha_1, \alpha_2]$, and there is an optimal level of $\alpha \in (\alpha_1, \alpha_2)$ that maximizes Π^i. Thus we prove result (2).

Similar to the perfect information model, other cases with different combinations of α_1 and α_2 are simply special examples of our case above. We can find these payoffs in a similar way as we did for the perfect information model. ■

A.9 Proof of corollary 1

From proposition 1, we know that if $\alpha_1 \geq 0$, bank B_1 will have enough resources to repay all the debts and bank L_1 will get the full payment of x from its interbank loans. The assumption is if bank L_1 has not suffered any loss, it should be able to repay all its debts at the riskless rate. Since the bad lending bank can repay the riskless rate with no uncertainty, the riskless rate is the equilibrium rate. Conversely, if the equilibrium rate is not the riskless rate, then it must be the case that $\alpha_1 < 0$, so that bank B_1 depositors will run given any choice of α by bank L_1. ■

A.10 Proof of proposition 4

When $\hat{r} \leq r_{CB}$, the banks borrow only from depositors, so their decisions are the same as in the case without central bank lending. When $\hat{r} > r_{CB}$, the two banks will borrow $\bar{L}_{CB}$ from the central bank first and then borrow at $\hat{r}$ on the market. Remember that without central bank lending, bank L_1 will recall the optimal proportion of $\alpha(\hat{r})$ of interbank loans and liquidate $l(\hat{r})$ of its long-term project to maximize its net asset value at date 2, which is given by

$$H((1-\alpha)x) + (L-l)(1+R) - (D_0 + x - Z)(1+\hat{r}) \tag{33}$$

With central bank lending, bank L_1's net asset value changes into

$$\begin{aligned} &H((1-\alpha)x) + (L-l)(1+R) - (D_0 + x - Z - \bar{L}_{CB})(1+\hat{r}) - \bar{L}_{CB}(1+r_{CB}) = \\ &H((1-\alpha)x) + (L-l)(1+R) - (D_0 + x - Z)(1+\hat{r}) + \bar{L}_{CB}(\hat{r} - r_{CB}) \end{aligned} \tag{34}$$

which is Equation (33) plus a constant $\bar{L}_{CB}(\hat{r} - r_{CB})$. This implies that the solutions of α and l are the same as those for Equation (33). The decision of bank L_2 can be proved similarly.

Because the solutions of α and l are still the same, the assets at date 2, denoted as V', will be the same as in the case without central bank lending, $V' = V$. With central bank lending, the cash used to repay deposits will be increased by $\bar{L}_{CB}$. Let $D' = D - \bar{L}_{CB}$ denote the deposits that are rolled over. At date 2, each unit of date 1 deposit will turn into $1 + \hat{r}$ units, and the total outstanding debt of the lending bank will become $D'(1 + \hat{r}) + \bar{L}_{CB}(1 + r_{CB})$. The maximum return rate for each unit of date 1 deposit will be $\frac{V}{D'(1+\hat{r})+\bar{L}_{CB}(1+r_{CB})}(1 + \hat{r})$. Without central bank lending, the maximum rate is $\frac{V}{D(1+\hat{r})}(1 + \hat{r}) = \frac{V}{D}$. When $r_{CB} < \hat{r}$, we have

$$\frac{V}{D'(1 + \hat{r}) + \bar{L}_{CB}(1 + r_{CB})}(1 + \hat{r}) > \frac{V}{D'(1 + \hat{r}) + \bar{L}_{CB}(1 + \hat{r})}(1 + \hat{r}) = \frac{V}{D' + \bar{L}_{CB}} = \frac{V}{D} \tag{35}$$

so the maximum return rate becomes higher.

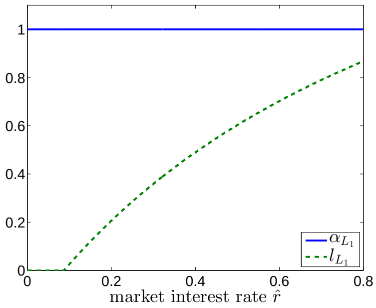
The required rate is decided according to

$$1 = \frac{1}{2}\left[\pi'_{good}(1 + \Gamma(\hat{r})) + (1 - \pi'_{good})\right] + \frac{1}{2}\left[\pi'_{bad}\frac{V_{bad}(1 + \hat{r})}{D'_{bad}(1 + \hat{r}) + \bar{L}_{CB}(1 + r_{CB})} + (1 - \pi'_{bad})\right] \tag{36}$$

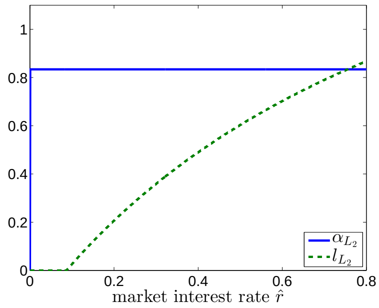
where $D' = D - \bar{L}_{CB}$ and $\pi' = D'/(D_0 + x)$ for each type of bank. If we take π'_{good} and π'_{bad} as given, then the higher maximum return rate from the bad bank, $\frac{V_{bad}(1+\hat{r})}{D'_{bad}(1+\hat{r})+\bar{L}_{CB}(1+r_{CB})} > \frac{V_{bad}}{D_{bad}}$, will lead to a lower $\Gamma(\hat{r})$. ■

B Numerical examples: optimal choices for banks L_1 and L_2 and the determination of $\Gamma(\hat{r})$

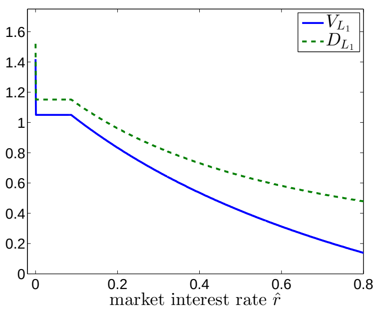
Panels (a) and (b) of figure 13 illustrate the optimal choices of banks L_1 and L_2 on interbank loan recall and long-term project liquidation at $R_{shock} = 0.32$ for different levels of $\hat{r}$. At this R_{shock} level, bank L_1 always chooses to recall all the interbank loans from bank B_1 for any $\hat{r} \geq 0$. Bank L_2 will always recall $\alpha_1^{L_2}x = 0.8334x$ of interbank loans when $\hat{r} > 0$, where $\alpha_1^{L_2}$ is determined by our previous analysis on α_1. Both L_1 and L_2 start to liquidate long-term projects when $1 + \hat{r} > \frac{1}{\lambda} \approx 1.087$. Given the parameter



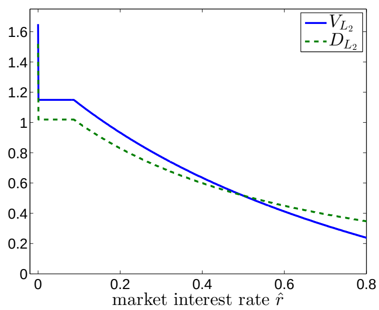
(a) Long-term project liquidation and interbank loan recall of bank L_1



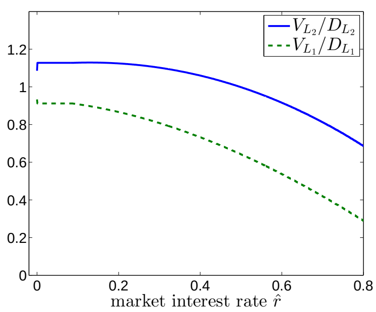
(b) Long-term project liquidation and interbank loan recall of bank L_2



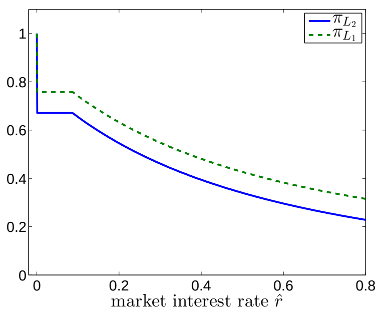
(c) V_{L_1} and D_{L_1}



(d) V_{L_2} and D_{L_2}



(e) Maximum feasible return



(f) Rollover probability

Figure 13: Optimal choices of banks L_1 and L_2 at $R_{shock} = 0.32$

values in our numerical example, $l = \frac{\lambda - \frac{1}{1+\hat{r}}}{\gamma(1+R)}$ (because $\frac{1}{\lambda} < 1 + \hat{r} < \frac{1}{\lambda - \gamma L(1+R)}$) is strictly increasing in $\hat{r}$.

Panels (c) and (d) of figure 13 illustrate how V and D of the lending banks change in $\hat{r}$. For both banks L_1 and L_2, a downward jump of V and D occurs when $\hat{r}$ changes from zero to positive. For bank L_1, when $\hat{r} = 0$, the bank is indifferent between keeping the proceeds from recalling the interbank loan and using the proceeds to repay its depositors. We assume that the bank will keep the proceeds. When $\hat{r} > 0$, the bank will use the proceeds to repay its depositors at date 1, causing a downward jump of both V_{L_1} and D_{L_1}. Similarly, when $\hat{r}$ becomes positive, bank L_2 will recall $\alpha_1^{L_2} x$ of the interbank loan and use the proceeds to repay its depositors, causing a downward jump of V_{L_2} and D_{L_2}. When $1 + \hat{r} > \frac{1}{\lambda}$, V and D decrease in $\hat{r}$. This is because, as $\hat{r}$ becomes higher, the banks will liquidate more long-term projects to repay its depositors at date 1.

Panel (e) of figure 13 illustrates how $\frac{V}{D}$ changes in $\hat{r}$. When $\hat{r}$ turns from zero to positive, the repayment to depositors by bank L_2 will cause $\frac{V_{L_2}}{D_{L_2}}$ to jump upward, while the repayment to depositors by bank L_1 will cause $\frac{V_{L_1}}{D_{L_1}}$ to jump downward. This is because in this example, at $\hat{r} = 0$, we have $\frac{V_{L_2}}{D_{L_2}} > 1$ and $\frac{V_{L_1}}{D_{L_1}} < 1$. It is straightforward to show that $\frac{V-Z}{D-Z}$ is strictly increasing in Z when $\frac{V}{D} > 1$, and is strictly decreasing in Z when $\frac{V}{D} < 1$, where Z is the cash used to repay the depositors, with $0 < Z < \min(V, D)$. So here repaying the depositors increases the maximum rate available to bank L_2 depositors, but reduces the maximum rate available to bank L_1 depositors. When $1 + \hat{r} < \frac{1}{\lambda}$, both $\frac{V_{L_2}}{D_{L_2}}$ and $\frac{V_{L_1}}{D_{L_1}}$ remain constant. When $1 + \hat{r} > \frac{1}{\lambda}$, both $\frac{V_{L_2}}{D_{L_2}}$ and $\frac{V_{L_1}}{D_{L_1}}$ are decreasing in $\hat{r}$. This is because the marginal cost of liquidating long-term projects is increasing, and a decrease in one additional unit of V leads to a less and less decrease in D.

Panel (f) of figure 13 illustrates how π changes in $\hat{r}$. There is a downward jump in both π_{L_1} and π_{L_2} when $\hat{r}$ turns positive, caused by the repayment to depositors explained before. Except for the jump at $\hat{r} = 0$, both π_{L_2} and π_{L_1} remain constant when $1 + \hat{r} \leq \frac{1}{\lambda}$. When $1 + \hat{r} > \frac{1}{\lambda}$, both π_{L_2} and π_{L_1} are decreasing in $\hat{r}$, because both banks liquidate more long-term projects to repay their depositors.

Next we give a detailed explanation for the movement of $\Gamma(\hat{r})$ in figure 4. The equilibrium condition of $\Gamma(\hat{r})$ (Equation (12)) can be written as

$$1 = \frac{1}{2}\left[\pi_{L_2}(1 + \Gamma(\hat{r})) + (1 - \pi_{L_2})\right] + \frac{1}{2}\left[\pi_{L_1}\frac{V_{L_1}}{D_{L_1}} + (1 - \pi_{L_1})\right]$$

$\Gamma(\hat{r})$ has a small upward jump when $\hat{r}$ turns positive. As we explained before, when $\hat{r}$ turns positive, there is a downward jump in both π_{L_2} and π_{L_1}. A lower probability that deposits will be rolled over by the good bank, π_{L_2}, will induce a higher $\Gamma(\hat{r})$, while a lower π_{L_1} will induce a lower $\Gamma(\hat{r})$. In addition, the maximum rate from the bad bank $\frac{V_{L_1}}{D_{L_1}}$ is lower, while the maximum rate from the good bank $\frac{V_{L_2}}{D_{L_2}}$ is higher. The lower $\frac{V_{L_1}}{D_{L_1}}$ will induce a higher $\Gamma(\hat{r})$, but the higher $\frac{V_{L_2}}{D_{L_2}}$ has no effect on $\Gamma(\hat{r})$. This is because, as long as $\frac{V_{L_2}}{D_{L_2}} > 1 + \hat{r}$, depositors receive only the promised interest rate of $1 + \hat{r}$ from the good bank. The overall effect is a small upward jump in $\Gamma(\hat{r})$. When $0 < \hat{r} < \frac{1}{\lambda}$, $\Gamma(\hat{r})$ remains constant because there are no changes in the choices of the two banks. When $\hat{r} > \frac{1}{\lambda}$, $\Gamma(\hat{r})$ is increasing in $\hat{r}$. This is because banks start to liquidate their long-term projects, incurring liquidation costs. As a result, $\frac{V_{L_1}}{D_{L_1}}$ decreases, causing depositors to require a higher interest rate, $\Gamma(\hat{r})$, from the good bank to compensate for the higher expected loss to the bad bank.

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