Investment Dynamics: Good News Principle*

Talat S. Genc
Department of Economics, University of Guelph, Guelph, ON, Canada

Georges Zaccour
Chair in Game Theory and Management, GERAD, HEC Montréal, Canada

11th February 2010

Abstract

We study a dynamic Cournot game with capacity accumulation under demand uncertainty, in which the investment is perfectly divisible, irreversible, and productive with a lag. We characterize equilibrium investments under closed-loop and S-adapted open-loop information structures. Contrary to what is established usually in the dynamic games literature with deterministic demand, we find that the firms may invest at a higher level in the open-loop equilibrium (which in some cases coincides with Markov perfect equilibrium) than in the closed-loop Nash equilibrium. The rankings of the investment levels obtained in the two equilibria actually depend on the initial capacities and on the degree of asymmetry between the firms. We also observe, contrary to the bad news principle of investment, that firms may invest more as demand volatility increases and they invest as if high demand (i.e., good news) will unfold in the future.

Key Words: Capacity Investment, Dynamic Games, S-adapted Open-Loop Equilibrium, Closed-loop Equilibrium.

JEL Codes: C73, L13.

1 Introduction

In many industries capital or capacity investments are made under uncertainty. Uncertainty may stem from the nature of production characteristics, demand, cost and macroeconomic conditions. Some uncertainties are industry specific and the degree of uncertainty may vary from industry to industry. Production capacity investments under uncertainty have been studied extensively in the literature. The recent studies revisit and extend the early contributions to incorporate different demand models and behavioral assumptions to study the new capital intensive markets including, e.g., restructured electric power generation, natural gas transportation, ethanol, and hot spot industries. The main objectives of these articles are to provide insights for equilibrium investment behavior, entry-exit decisions and explain policy relevant topics such as effects of mergers, the role of excess market capacity on market power and price caps on equilibrium predictions. However, the capacity competition over time, in which capacity is subject to a time-to-build constraint

*We wish to thank Roberto Cellini, Mike Hoy, Peter Kort, Stanley Reynolds, Thanasis Stengos and Henry Thille for valuable comments on previous drafts. Research supported by SSHRC and NSERC, Canada.

and firms face demand uncertainties over time, has not been adequately analyzed.[1] In particular, how firms would adjust their incremental capacity investments over time under different behavioral assumptions (precommitment versus no commitment, or open-loop versus closed-loop) is an important question to be addressed. For example, in the electricity production industry competing power generation firms can invest incrementally in some technologies under demand uncertainty either using some precommitment policies or using some state-dependent policies.

We study a finite-horizon discrete-time duopoly game with capacity accumulation under demand uncertainty. Investment is irreversible and can be accumulated over time. Investment is not productive instantly, and there is a lag between investment and production. Production is subject to total available capacity. We analyze the dynamics of capacity investments, characterize and compare closed-loop Nash equilibrium and S-adapted open-loop Nash equilibrium investment strategies (and therefore investment expenditures and profits). There is a significant literature in dynamic games focusing on the comparison of feedback and open-loop strategies.[2] In the capacity expansion literature, to which this article naturally belongs, it has been established in Reynolds (1987), in an infinite horizon differential game, that Markov strategies increase competition, i.e., Markov-perfect equilibrium investments exceed the open-loop ones. The same qualitative result has been obtained in different articles (and topics), e.g., Dockner (1992), Driskill and McCafferty (1989), Long et al. (1999), and Driskill (2001), whereas other articles find that Markov behavior softens competition (see, e.g., van der Ploeg and de Zeeuw (1990), Melese and Michel (1991), Piga (1998), Figuières (2002)). In a model where investment is reversible (a crucial assumption), Figuières (2009) shows that these contradictory findings are related to the concept of strategic substitutability and complementarity. A common feature in this literature is that the model is deterministic. Recently some authors, e.g..., Ruiz-Aliseda and Wu (2008), Wu (2007), and Garcia and Stacchetti (2008) studied capacity investment games under various assumptions including demand uncertainty. However, these articles do not focus on the role of uncertainty on the different equilibrium types (or information structures), nor do they explicitly study the role of state variables on equilibrium predictions.

The main objective of this article is to study capital accumulation under open-loop and closed-loop behavior in a context where demand is uncertain. We start by considering the simplest possible setting, that is a two-stage deterministic model involving one investment decision. We find that open-loop and closed-loop Nash equilibria coincide. This unsurprising result holds because state vectors match at each stage for both equilibrium types. This simple setting has however an interesting benchmark (or experimental) value. Indeed, departing from this setting by assuming that demand in the second period is uncertain, allows (i) to show that the two equilibrium concepts do not coincide any longer; and, interestingly, (ii) to characterize the observed differences in investment strategies in terms of the differences in initial capacities of the players (or their degree of asymmetry). More specifically, we find that when firms are symmetric in terms of initial capacities and costs, open-loop equilibrium investment exceeds that of closed-loop for all firms. The intuition for the result in the symmetric case is that a firm's output in the following period is increasing in the rival firm's investment. This creates collusive-like behaviour in which when a firm reduces its investment the rival also decreases its investment. Hence closed-loop investment levels will be lower than

[1] Time-to-build decision is empirically observable. Pacheco-de-Almeida and Zemsky (2003) observe that time-to-build differs across products and countries in petrochemical industry. Koeva (2000) measures average time-to-build in some industries and finds that it ranges from 13 to 86 months.

[2] Indeed, many papers have dealt with the comparison of open-loop and feedack strategies and equilibria in different areas. See, e.g., Dockner et al. (2000) and Figuières (2002, 2009) for capital accumulation games, Kossioris et al. (2008) and Long et al. (1999) for examples in environmental and resource economics, and Piga (1998) and Breton et al. (2006) for examples of advertising investments.

the open-loop counterparts. Whenever firms are asymmetric in terms of initial capacities and the larger firm does not make any investment in equilibrium then the small firm's investment under closed-loop information structure is higher than under its open-loop counterpart. The reason for this asymmetric case is that small firm's investment is a decreasing function of rival firm's output under the closed-loop structure. It invests strategically and preemptively, hence its investment is higher under the closed-loop information structure.

When demand is known with certainty and the time period extends to many stages then investments under both equilibrium types are expected to be different. In our model it appears that our findings for two stages extend to finitely many stages under demand uncertainty. In the three stage extension of the model, we focus on the symmetric players case and provide characterization of equilibrium under both information structures. We find that each firm invests more under open-loop equilibrium than under the closed-loop counterpart. This result is in the opposite direction of the one in Reynolds (1987) for the comparison of open-loop and Markov-perfect equilibrium. Further, another significant result of the article is that under no circumstances the players can achieve a higher payoff under an open-loop information structure than under a closed-loop information structure. This conclusion, which holds true for any number of finite stages, is a strong argument (to be added to the usual one stating that closed-loop strategies, that are state-contingent, are more conceptually appealing that here open-loop counterparts) in defense of the closed-loop information structure.

When investment is irreversible and future demand is stochastic, one expects, according to the "bad news" hypothesis of Bernanke (1983), that firms will invest as if the low demand scenario will unfold in the future. Our findings are just the opposite. Indeed, we obtain that the firms invest in equilibrium as if the high demand scenario will be realized.

1.1 A Brief Look at Relevant Literature

Because of its analytical tractability many models on capacity accumulation games assume a linear-quadratic (LQ) framework in optimization settings without uncertainty and capacity constraints.[3] They analyze, including seminal articles by Spence (1979) and Dixit (1980), the commitment value of capital investments. They find that firms invest strategically to preempt rival firms' capacity investments to secure large market shares and higher profits. Dixit (1980) studies a duopolistic Cournot-Nash game, in which the incumbent firm chooses a capacity investment level before the play of "post-entry" game. Hence the incumbent firm changes initial capacity states of the game and secures higher outputs. Spence (1979) studies first-in advantage and strategic capital investments in an infinite horizon model of duopoly, in which investment is completely irreversible. He finds that, in the equilibrium the advantaged firm invests strategically to preempt the rival firm capacity investment and secures a higher market share and greater profitability in the long term. Fershtman and Muller (1984) analyze a duopolistic model and characterize conditions for the existence of Nash equilibrium (NE) investment strategies and the asymptotic stability of capital trajectories. They find that there are a unique stationary NE capital stocks which are independent of initial capital stocks. This result is based on the open-loop Nash equilibrium concept. However, in our article initial capacities play important roles in equilibrium predictions. Cellini and Lambertini (1998) extend Fershtman and Muller (1984) by studying an infinite horizon continuous time symmetric differential game with capital accumulation of symmetrically differentiated goods (in which there is no capacity investment, but the excess output is reintroduced into the production process). They investigate optimal capital accumulation and study the influence of demand conditions on market equilibrium. The equilibrium solution concept is open-loop Nash

[3] In this paper we consider "hard" capacity constraints that cannot be relaxed at a cost.

equilibrium (OLNE). They find that when the equilibrium is driven by demand conditions social planning is more efficient than any competition setting. When equilibrium is dictated by capital accumulation, social planning and oligopoly lead to the same allocation.

The Spence and Dixit articles suggest that when capacity investment is completely irreversible then investment can have a commitment value for the duopolists. Reynolds (1987) analyzes commitment value of investment in a two-player linear-quadratic, infinite-horizon differential game when capacity is reversible with adjustment costs. He explains the preemptive effect of capacity investment under different behavioral assumptions on committing investment paths. He finds that when each firm precommits to the investment path (i.e., OLNE) the equilibrium is unique and is the pair of asymptotically stable equal investment capacities. When firms do not precommit to investment paths (i.e., Markov perfect equilibrium (MPE)) feedback equilibrium strategies are linear and unique. In the OLNE, each firm's investment strategy is independent of the rival firm's stock of capacity. In the MPE, each firm's current investment aims to seize the rival firm's future capacity expansion plan. The stationary capacity levels under MPE exceed the stationary capacity levels under OLNE.

Recent literature on capital accumulation games has incorporated uncertainty into the models. For example, Wu (2007) has studied a continuous-time endogenous Stackelberg leadership game in which identical firms choose entry timing and capacity investment in a new market with evolving uncertainty. In his model capacity investment is lumpy and there are two capacity states, low and high, at each time. Demand is stochastic and grows until some time then it declines to zero. He shows that in equilibrium, in most cases, the leader enters the market with smaller capacity than the follower's capacity in the ultimately declining industry. Ruiz-Aliseda and Wu (2008) use a real options approach to examine optimal entry and exit behavior of a single firm in a market with demand that cycles between growth and decline phases. They use the "bad news principle of irreversible investment" hypothesis of Bernanke (1983), which says a firm only cares about the arrival of bad news and their adverse effect on payoffs before making investment decision, to interpret firms' investment policies.

We study quantity competition in capacity investments and production. In a similar article, Garcia and Stacchetti (2008) analyze a dynamic extension of Kreps and Sheinkman's (1983) two-period Bertrand game with capacity investments. Duopolists have several equal-sized plants and the marginal cost of production is constant. Demand is inelastic and increases or stays the same with some probability from one period to the next one. They characterize Markov-perfect equilibrium of bidding and investment strategies. They find that in some equilibria excess market capacity is low and market prices are equal to the price cap. They argue that increasing the price cap causes high market prices and low consumer surplus. In a similar model to Garcia and Stacchetti (2008), Garcia and Shen (2010) study a dynamic oligopoly Cournot model in which market demand grows stochastically and capacity additions take place over long time lags. They confirm that oligopoly underinvests relative to the social optimum. They measure the rate of change of investment as demand growth probabilities, discount factors, depreciation rates, and production and investment costs vary.

In terms of the modeling assumptions and results this article has similarities with Pacheco-de-Almeida and Zemsky (2003), and Genc et al. (2007), who assume that investment does not become productive instantaneously but has a lag with the production decision. As Pacheco-de-Almeida and Zemsky point out there are many factors creating a lag between investment decision and production process in several industries, and time-to-build constraint is not commonly studied in IO literature. Pacheco-de-Almeida and Zemsky provide an interesting analysis of the impact of time-to-build on equilibrium in a three-period game where uncertainty about demand is resolved after the first period. One of the results is that firms will tend to invest incrementally instead of investing once. This result contrasts with some prior work, in which firms were allowed to make investment only once, or in equilibrium firms made investments only once when investment was productive immediately. In our model, investment also becomes productive in the following period, and in equilibrium (under both open-loop and closed-loop information structures) firms invest incrementally over time. Pacheco-de-Almeida and Zemsky (2003) assume that firms face demand uncertainty only once, however we assume uncertainty evolving over time and firms face uncertainty all the time before they make investment decisions. Also, contrary to Genc, Reynolds and Sen who provide an implementation of open-loop approach via stochastic programming for solving a large-scale oligopoly, we provide characterization of equilibrium for both open-loop and closed-loop information structures.

Comparison of the equilibrium behavior (closed-loop versus open-loop) for capital investment dynamic games under demand uncertainty is one of the main objectives of this article. The open-loop equilibrium concept has been utilized by many authors, such as Cellini and Lambertini (1998), Fershtman and Muller (1984), for predicting market outcomes of deterministic dynamic games. [4] because we allow uncertainty in the model the appropriate equilibrium concept with the features of an open-loop information structure would be S-adapted open-loop Nash equilibrium (see Haurie et al. (1990) for this equilibrium concept). This approach has the advantage of tractability and is particularly useful for computing equilibrium outcomes of large scale games (see Genc et al. (2007), Genc and Sen (2008)). We assume random walk type demand uncertainty, whose continuous time version is the Brownian motion. This type of demand structure is used by, for example, Dixit and Pindyk (1992), and Genc et al. We assume that investment is perfectly divisible, irreversible (i.e., net investment is non-negative) and made under demand uncertainty. Investment becomes productive with a one period lag, as in Pacheco-de-Almeida and Zemsky (2003) and Garcia and Stacchetti (2008). In some industries investment may be perfectly divisible and it may require time to build for future use. For instance, in the electric power generation industry, a firm may invest on more flexible generators such as gas-fired generators with varying degrees of capacity choices and the investment takes some time to be productive.

The plan of the rest of the article is as follows. Section 2 introduces the model, and Section 3 states some general results and Section 4 deals with two-period models, with one investment decision. In Section 5, we generalize some results to $T > 2$ periods and provide an illustration in a three-period setting. Section 6 briefly concludes.

2 Model

We study a dynamic duopoly game, in which firms make capacity investment and production decisions over time. Firms produce a homogeneous good. For a given demand and capacity state vector in a time period, firms make capacity investments under demand uncertainty. The stochastic process we consider is discretized and described by an event tree. An investment made at time $t = 0, \ldots, T$, will become productive in the following period. After the demand uncertainty is revealed, firms make production decisions simultaneously and independently. We first introduce the model in general terms and next specify the functional forms.

Let i denote a player and J be the set of players, $i \in J = \{1, 2\}$. Let S^t be the set of possible realizations of the stochastic process that affects market demand at period t. The set S^0 has only one element, s^0, which is the root of the event tree. At any subsequent period, i.e., $t \geq 1$, the set S^t contains N_t elements

[4] See Reynolds (1987), and Deneckere and de Palma (1998) for a defense of using open-loop equilibrium in dynamic games.

(nodes), i.e., $S^t = \left\{s_1^t, \ldots, s_{N_t}^t\right\}$.. Let $a\left(s_k^t\right) \in S^{t-1}$ be the unique predecessor of $s_k^t \in S^t$, and denote by $B\left(s_k^t\right) \subset S^{t+1}, t = 0, \ldots, T-1$, the set of successors of node s_k^t in the event tree.

The stochastic inverse demand is given by

$$\mathfrak{p}\left(t, s_k^t\right) = P\left(\sum_i q_i\left(t, s_k^t\right)\right),$$

where $q_i\left(t, s_k^t\right)$ is the output of player i, and $\mathfrak{p}\left(t, s_k^t\right)$ is the market price for the realization s_k^t. Denote by $I_i\left(t, s_k^t\right)$ the investment in the production capacity $K_i\left(t, s_k^t\right)$ of player i. Assuming away obsolescence and taking into account the one-period delay for investment to become productive, the capacity accumulation dynamics is then given by

$$K_i\left(t, s_k^t\right) = K_i\left(t-1, a\left(s_k^t\right)\right) + I_i\left(t-1, a\left(s_k^t\right)\right), \quad t = 0, \ldots, T, \ \ \forall s_k^t \in S^t, \tag{1}$$

with $K_i\left(0, s^0\right) = K_{i0}$. Each player must satisfy the production capacity constraint at each production node, i.e.,

$$q_i\left(t, s_k^t\right) \leq K_i\left(t, s_k^t\right), \qquad t = 0, \ldots, T, \ \ \forall s_k^t \in S^t. \tag{2}$$

We assume that the investment cost $F_i(I_i)$ is convex, increasing and satisfying $F_i(0) = 0$. The production cost $C_i\left(q_i\right)$ is also convex, increasing and there is no fixed production cost, i.e., $C_i(0) = 0$. Denote by $\theta\left(s_k^t \mid a\left(s_k^t\right)\right)$ the conditional probability associated with the arc $\left(a\left(s_k^t\right), s_k^t\right)$ in the event tree with

$$\sum_{s_k^t \in S^t} \theta\left(s_k^t \mid a\left(s_k^t\right)\right) = 1.$$

Assuming profit maximization behavior, player i's optimization problem reads

$$\begin{gathered}
\max \pi_i = P\left(Q\left(0, s^0\right)\right) q_i\left(0, s^0\right) - C_i\left(q_i\left(0, s^0\right)\right) - F_i\left(I_i\left(0, s^0\right)\right) \\
+\sum_{t=1}^{T} \delta^t \sum_{s_k^t \in S^t} \theta\left(s_k^t \mid a\left(s_k^t\right)\right)\left[P\left(Q\left(t, s_k^t\right)\right) q_i\left(t, s_k^t\right) - C_i\left(q_i\left(t, s_k^t\right)\right) - F_i(I_i\left(t, s_k^t\right))\right] \\
\text{subject to (1)-(2)} \\
q_i\left(t, s_k^t\right) \geq 0, \quad I_i\left(t, s_k^t\right) \geq 0, \quad t = 1, \ldots, T, \forall s_k^t \in S^t,
\end{gathered}$$

where $Q\left(t, s_k^t\right) = \sum_i q_i\left(t, s_k^t\right)$, δ is the discount factor, $0 < \delta < 1$, and also $I_i\left(T, s_k^T\right) = 0$.

To compute and compare S-adapted open-loop and closed-loop strategies, we need to specify the forms of the cost and demand function. In the sake of keeping the computations as simple as possible while obtaining interesting qualitative insights, we adopt a quadratic investment cost function and a linear production cost, i.e.,

$$F_i\left(I_i\right) = 1/2 f\, I_i^2, \qquad C_i\left(q_i\right) = c q_i,$$

where $f > 0$ and $c > 0$. We assume that the inverse demand is affine and analyze the simple case where, for any $s_k^t \in S^t$, the set of successors is $B\left(s_k^t\right) = \left\{s_1^{t+1}, s_2^{t+1}\right\} = \{u, d\}$, where u stands for demand shifting up and d for demand shifting down. That is, from any non-terminal node, the demand distribution is a simple binary random walk, with demand shifting up with probability p or down with probability $1-p$. Formally, the inverse demand is given by

$$P\left(\sum_i q_i\left(t, s_k^t\right)\right) = 1 + \phi\left(t, s_k^t\right) - Q\left(t, s_k^t\right),$$

where

$$\phi\left(t, s_k^t\right) = \phi\left(a\left(t, s_k^t\right)\right) + \tilde{\xi}, \quad \text{where } \tilde{\xi} = \begin{cases} \xi, & \text{if } s_k^t = u \\ -\xi, & \text{if } s_k^t = d \end{cases},$$

and ξ is a nonnegative parameter and $\phi(0,0) = 0$.

Admittedly our model is far from being the most general one. Indeed, the selected demand distribution is a simple one, and the cost functions could have been more general. However, our parsimonious model still possess the required attributes to allow for a full analysis of the dynamics of investments in production capacities in the context of imperfect competition, with uncertain demand and under different information structures. For the sake of completeness, we provide the formal definition of S-adapted open-loop and closed-loop Nash equilibria.[5]

Definition 1 *S-adapted open-loop information: At any time each player's information set includes the current calendar time, the current demand state, the distribution of future demand, and the initial values of capacity states.*

Definition 2 *S-adapted closed-loop information: At any time each player's information set includes the current calendar time, the current states involving demand and capacity states, the distribution of future demand, and the history of the states.*

Here we use the term S-adapted (i.e., sample adapted) to reflect the fact that the game is stochastic and the demand distribution is modeled by event tree. Both S-adapted open-loop equilibrium (or simply open-loop equilibrium) and S-adapted closed-loop equilibrium (or simply closed-loop equilibrium) are Nash equilibrium in investment and production strategies. The former is obtained under the S-adapted open-loop information structure, and the latter is obtained under the S-adapted closed-loop information structure.

3 Some General Results

We report in this section some general results pertaining to production decisions at any given node in the demand event tree. We also show some relationships between output and investment decisions. These results provide some valuable first insights for the characterization and analysis of both open-loop and closed-loop equilibria.

Consider production decisions at any given node in the demand tree. Because the investment decision at a given node is independent of the quantity decision at the same node (because of the lag between investment and production), then at any node $s_k^t \in S^t, t = 0, \ldots, T$, each player chooses the production quantities by solving the following problem

$$\begin{gathered} \max P\left(Q\left(t, s_k^t\right)\right) q_i\left(t, s_k^t\right) - C_i\left(q_i\left(t, s_k^t\right)\right), \\ s.t., \quad 0 \leq q_i\left(t, s_k^t\right) \leq K_i\left(t, s_k^t\right). \end{gathered}$$

[5] We note that in the literature (see, Basar and Olsder, 1995) there are several forms of closed-loop equilibrium concepts. We use the one defined below.

Assume symmetric capacities so that $K_i(t, s_k^t) = K_j(t, s_k^t) \equiv K(t, s_k^t)$, $i \neq j$. The solution of the above problem produces three equilibrium candidates: (i) The interior Cournot solution, that is, $q_i(t, s_k^t) = q_j(t, s_k^t) = (1 + \phi(t, s_k^t) - c)/3$; (ii) The corner solution that is $q_i(t, s_k^t) = q_j(t, s_k^t) = K(t, s_k^t)$; or (iii) The asymmetric solution with (say) player i producing at full capacity, i.e., $q_i(t, s_k^t) = K(t, s_k^t)$ and the rival player j plays its best (interior) response strategy $q_j(t, s_k^t) = (1 + \phi(t, s_k^t) - c - K(t, s_k^t))/2$. The following lemma, however, shows that the asymmetric solution is ruled out.

Lemma 1 *At any node $s_k^t \in S^t, t = 0, \ldots, T$, whenever capacities of the players are symmetric, Nash equilibrium outputs are unique and symmetric.*

Proof. See Appendix. □

In the next lemma, we show that it can never occur that a player's output in downstate demand d exceeds his production in upstate demand u. Note that the result is independent of production capacities.

Lemma 2 *In any set $B(s_k^t)$, $q_i(t+1, d) \leq q_i(t+1, u)$.*

Proof. See Appendix. □

The following result states that if a player invests at a node at period t, then this player will produce at full capacity in the descendent upstate node.

Lemma 3 *If at any node $s_k^t \in S^t, t = 0, \ldots, T-1, I_i(t, s_k^t) > 0$, then in any set $B(s_k^t)$, player i produces at maximal capacity in the upstate demand case, i.e., $q_i(t+1, u) = K_i(t+1, u)$. Further, if $I_i(t, s_k^t) = 0$, then $q_i(t+1, u) < K_i(t+1, u)$.*

Proof. See Appendix. □

In this article we define a realization of high demand scenario as "good news". Good news principle of investment is that investment is made to meet future high demand (i.e., good news), and this investment is fully utilized in the production process (i.e., capacity is binding or excess capacity is zero). [6]

The result of this Lemma is an illustration of the good news principle, stipulating that a decision-maker is investing as if the optimistic scenario will materialize in the following period. Note that this result will not necessarily hold if we had a large fixed cost or indivisibility of investment. The second part of the lemma deals with the case where it is optimal not to invest at a node. The result then states that the player will not produce at full capacity in the upstate successor, whatever is the already available capacity. Further, by Lemma 2, we have that $q_i(t+1, d) < q_i(t+1, u) < K_i(t+1, u)$. Therefore, combining the two lemmas, we have that if a player does not invest at any given node, then he will not use his full capacity in all successors of that node.

In the following lemma, we show that, in any pair of nodes sharing the same history, it cannot occur in a symmetric game that a player invests in the downstate node and does not invest in the upstate one.

Lemma 4 *In a symmetric game, in any set $B(s_k^t)$, $s_k^t \in S^t, t = 0, \ldots, T-1$, if $I_i(t, u) = 0$ then necessarily $I_i(t, d) = 0$.*

Proof. See Appendix. □

[6] Bernanke's bad news principle (1983) relies on the assumptions that there is a single optimizer (i.e., no competition), there is a menu of projects at each time and investment projects are lumpy, and new information relevant to long-run returns arrives over time. Under these assumptions postponing is desirable, since by waiting the investor may improve his chances of making a correct decision. This principle is totally based on the option value approach.

4 Equilibria in Two-Period Model

We explore in this section the effects of uncertainty and initial capacities on the equilibrium investment behaviour in a two-period model, that is in a setting where there is only one investment decision to be made at the root node. To start, consider the simplest possible setting of rivalry investment decisions, that is, the case where demand is known with certainty at period 1 ($\tilde{\xi} = 0$). We have the following result.

Proposition 1 *In the absence of uncertainty, closed-loop Nash equilibrium and open-loop Nash equilibrium investments coincide.*

Proof. See Appendix. □

The result holds because the closed-loop and open-loop Nash equilibrium state vectors at each stage coincide, and, therefore, the rollback solution is identical to the forward solution. Alternatively, as the investment cost is sunk for the second period and the effect of investment is to provide an upper bound for the production level, the two types of equilibria coincide in the two-period model with deterministic demand. Note that this result holds for any given initial production capacities, and in particular for equal ones. Further, as one can expect, total industry investment is lower than the welfare-maximizing level.[7]

We now switch to a stochastic demand (i.e., $\tilde{\xi} > 0$). Note that in period 1, the production capacity is the same in both states u and d... We hence simplify the notation and write $K_i\left(1, s_k^1\right) = K_{i1}$ and $I_i\left(0, s^0\right) = I_{i0}$. Depending on the model parameters' values, different cases may arise, namely:

Case 1: $I_{i0} = 0$ and (by Lemmas 2 and 3) $q_i\left(1, d\right) < q_i\left(1, u\right) < K_{i1}$.

Case 2: $I_{i0} > 0$ and $q_i\left(1, d\right) < q_i\left(1, u\right) = K_{i1}$.

Case 3: $I_{i0} > 0$ and $q_i\left(1, d\right) = q_i\left(1, u\right) = K_{i1}$.

Case 1 occurs when the player's initial capacity is "too large" and there is no need, at least in the short run, to increase it. The equilibrium solution when both players do not invest in capacity is trivial and does not present much interest. We are interested in the case where the initial capacity K_{i0} is large enough so that the capacity constraints do not always bind, but also low enough that the firms have an incentive to invest in capacity. Guided by Lemmas 2 and 3, we restrict our attention to equilibria where capacity is binding in the u state ($q_i\left(1, u\right) = K_{i1}$), and not in the d state ($q_i\left(1, d\right) < K_{i1}$). However, exceptionally in the fully symmetric case analyzed below, we shall also consider the scenario where capacity is binding in both states in period 1, i.e., $q_i\left(1, u\right) = q_i\left(1, d\right) = K_{i1}$. Our results are reported in the following propositions stated under different assumptions regarding initial capacities.

Assumption A1: $K_{i0} = K_{j0} = K_0$, $i \neq j$, and $I_{i0} > 0$, $i = 1, 2$.

Proposition 2 *Under assumption A1 and if $q_i\left(1, d\right) < q_i\left(1, u\right) = K_{i1}$, then*

1. *Symmetric S-adapted open-loop (OL) and closed-loop (CL) Nash equilibrium investments are given by*

$$I_{i0}^{OL} = \frac{\delta p\left(1 + \xi - c - 3K_{i0}\right)}{f + 3\delta p}, \quad I_{i0}^{CL} = \frac{\delta p\left(1 + \xi - c - 4K_{i0}\right)}{f + 4\delta p}, \quad i = 1, 2.$$

[7] Using the simple notation $I_i\left(0, s^0\right) = I_{i0}$ and $I_j\left(0, s^0\right) = I_{j0}$, it is shown in the Appendix that the total investments made in the market will be $I_{i0} + I_{j0} = \delta[2 - 3K_0 - 2c]/(f + 3\delta)$, where $K_0 = K_{i0} + K_{j0}$. Welfare maximizing efficient investment would be $I_0 = \delta[1 - K_0 - c]/(f + \delta)$, obtained through the solution of the problem

$$\max\{\int (1-q)dq - cq_0 - fI_0^2/2 + \delta[\int (1-q)dq - cq_1] + \alpha_0(K_0 - q_0) + \alpha_1(K_0 + I_0 - q_1)\}...$$

Clearly $I_0 > I_{i0} + I_{j0}$, that is duopoly underinvests relative to the efficient.

2. The equilibrium production quantities at time 1 are given by

$$\begin{aligned} q_i^{CL}(1,u) &= K_{i1}^{CL} = \frac{fK_{i0} + \delta p(1+\xi-c)}{f+4\delta p}, \quad q_i^{CL}(1,d) = \frac{1-\xi-c}{3}, \quad i=1,2, \\ q_i^{OL}(1,u) &= K_{i1}^{OL} = \frac{fK_{i0} + \delta p(1+\xi-c)}{f+3\delta p}, \quad q_i^{OL}(1,d) = \frac{1-\xi-c}{3}, \quad i=1,2. \end{aligned}$$

3. The equilibrium profits compare as follows

$$\pi_i^{OL} < \pi_i^{CL}, \quad i=1,2.$$

4. An asymmetric equilibrium in investment strategies is not possible.

Proof. See Appendix. □

Corollary 1 *Under assumption A1 and assuming* $q_i(1,d) = q_i(1,u) = K_{i1}, i=1,2,$ *then*

1. Symmetric S-adapted open-loop (OL) and closed-loop (CL) Nash equilibrium investments are given by

$$I_{i0}^{OL} = \frac{\delta(1-\xi-c-3K_{i0}+2p\xi)}{f+3\delta}, \quad I_{i0}^{CL} = \frac{\delta(1-\xi-c-4K_{i0}+2p\xi)}{f+4\delta}, \quad i=1,2.$$

2. The equilibrium production quantities at time 1 are given by

$$\begin{aligned} q_i^{CL}(1,u) &= q_i^{CL}(1,d) = K_{i1}^{CL} = \frac{fK_{i0} + \delta(1+\xi-c+2p\xi)}{f+4\delta}, \quad i=1,2, \\ q_i^{OL}(1,u) &= q_i^{OL}(1,d) = K_{i1}^{OL} = \frac{fK_{i0} + \delta(1+\xi-c+2p\xi)}{f+3\delta}, \quad i=1,2.. \end{aligned}$$

3. The equilibrium profits compare as follows

$$\pi_i^{OL} < \pi_i^{CL}, \quad i=1,2.$$

4. An asymmetric equilibrium in investment strategies is not possible.

Proof. Similar to the proof of Proposition 2 and is omitted. □

Contrasting Proposition 2 and its corollary with Proposition 1 confirms the known conclusion that under uncertainty the two information structures do not produce the same investment equilibrium strategies. Further, a simple comparison of the investment strategies in Proposition 2, as well as in its corollary, shows that $I_{i0}^{OL} > I_{i0}^{CL}$, and, therefore, the S-adapted open-loop Nash equilibrium capacity per firm exceeds its closed-loop Nash equilibrium counterpart. The economic intuition for this result is as follows. Under the closed-loop structure firms at the upstate node know that initial capacities of both players are identical and initial node investments will be identical due to symmetry. Because both players' capacities will be binding in this upstate node (because investments are positive), they will have the identical capacities. Therefore, a firm's output in the upstate is increasing in the rival firm's investment. This creates collusive-like behaviour in which when a firm reduces its investment the rival also decreases its investment. Hence closed-loop investment levels will be lower than the open-loop counterparts. In terms of output decisions, when capacity is binding, each player produces more in open-loop equilibrium than in closed-loop equilibrium. When capacity is not binding, i.e., in downstate demand of Proposition 2, open-loop and closed-loop equilibrium quantities are equal and correspond to the interior Cournot solution. Further, each player realizes a higher profit in the closed-loop equilibrium than in the open-loop counterpart. Finally, we note that under assumption A1, there is no room for an asymmetric equilibrium in investments strategies. This holds true under both information structures.

As can be seen from optimal investment expressions in Proposition 2 (and Corollary 1), investment is a function of demand probabilities. It can be easily shown that investment expressions (for both open-loop and closed-loop behavior) are increasing functions of (up-state) demand probability p. As the probability p approaches to zero, investment gets closer to zero in both open-loop and closed-loop cases. This, in turn, implies that if upstate demand is not likely to unfold, no investment occurs in the previous period.

We note that open-loop Nash equilibrium (OLNE) in Proposition 2 coincides with Markov-perfect Nash equilibrium (MPNE) investment levels.[8] The reason is that under the conditions of Proposition 2, both firms are capacity constrained in the high demand state in period 1. This implies that the MPNE output strategy for firm j for the high demand state in period 1 is, $q_j(K_{ju}, K_{iu}) = K_{ju}$, and $\partial q_j/\partial K_{iu} = 0$. Using this zero derivative, the OLNE and MPNE results are the same. The economic intuition for this is that a firm's period zero investment does not have any strategic value (over and above its value in an OLNE strategy) because it does not have an impact on its rival's period one output choice.

For completeness of the analysis of this symmetric game, it is easy to check that if both players do not invest in capacity (this is the case when initial capacities are large enough to cover the next-period upstate demand), then open-loop and closed-loop outputs coincide at each node, and are given by

$$\begin{aligned} q_i^{OL}(1,u) &= q_i^{CL}(1,u) = \frac{1+\xi-c}{3}, \quad i=1,2, \\ q_i^{OL}(1,d) &= q_i^{CL}(1,d) = \frac{1-\xi-c}{3}, \quad i=1,2. \end{aligned}$$

Consequently, individual profits are the same under both information structures.

Remark 1 *It is interesting to check under which conditions an interior solution is not observed in both equilibria at node* $(1,d)$*, i.e., capacity constraints do always bind in both periods. It is easy to verify that this will occur in*

$$D^x = \{(K_{i0}, f, \xi) \mid K_{i2}^x < q_{i2}^c, f, \xi \in (0,1), K_{i0} \geq 0\}, \qquad x = CL, OL,$$

where $K_{i2}^x = K_{i0} + I_{i0}^x$*, and* $I_{i0}^x > 0$*. In this set, Cournot outputs satisfy* $q_{i2}^c = (1-c-\xi)/3$*. Note that the investment quantities* I_{i0}^x *calculated above will be functions of the model parameters that belong to the set* D^x*.*

Assumption A2: Suppose that $K_{i0} < K_{j0}$, and $I_{i0} > 0$, $I_{j0} = 0$, $i \neq j$. That is, at the outset of the game duopolists have different initial capacities and one duopolist makes positive investment and the other has enough capacity and does not make any investment.

Proposition 3 *Under assumption A2, firm i's OLNE and CLNE investments are given by*

$$I_{i0}^{OL} = \frac{\delta p[1+\xi-c-3K_{i0}]}{2f+3\delta p}, \qquad I_{i0}^{CL} = \frac{\delta p[1+\xi-c-2K_{i0}]}{2f+2\delta p}.$$

[8] OLNE and MPNE investment levels also coincide in Corollary 1.

Further,

$$\begin{aligned}\pi_i^{OL} &< \pi_i^{CL},\\ \pi_j^{OL} &> \pi_j^{CL}.\end{aligned}$$

Proof. See Appendix. □

An interpretation of the above proposition is that facing a rival firm with large initial capacity, a player will invest less and realize lower profit in open-loop equilibrium than in closed-loop equilibrium. The closed-loop Nash equilibrium capacity for firm i exceeds its open-loop Nash equilibrium counterpart. Note that in both equilibria, player i produces at full capacity in upstate next-period demand and player j less than his capacity.

Contrary to the Proposition 2, OLNE does not coincide with the MPNE investments in Proposition 3. Here CLNE coincides with MPNE outcomes, because firm i's period zero investment has a strategic value and it has an impact on firm j's period one output choice.

Propositions 2–3 show the role of the initial conditions and the degree of asymmetry on equilibrium predictions. In a fully symmetric game (and equilibrium), Proposition 2 indicates that each player invests more in the open-loop equilibrium than in its closed-loop counterpart. In Proposition 3, when we assume one firm has higher initial capacity than the other, and the firm with higher capacity does not make any investment, we find that the firm with low initial capacity invests and its investment would be higher under closed-loop information structure than under the open-loop structure. A first conclusion emerges from these propositions: when comparing investments made by a player under the two different information structures, our results show that everything can go either way, depending on the circumstances, i.e., initial capacity levels and their degree of asymmetry. However, in the deterministic capacity investments literature (e.g., Reynolds (1987)), it is observed that firms overinvest under the Markov perfect information structure (which is also a state-dependent structure) relative to the open-loop structure. The intuition is based on "strategic investment": a firm's investment is a decreasing function of the rival's output, and the investing firm seizes the rival firm's capacity expansion. All players behave in the same manner, hence they overinvest.

The second conclusion is that player i who makes the strategic investment is better off by considering the role of its investment on rival player's output choice and will realize a higher profit in the closed-loop equilibrium than in the open-loop one. Therefore, on top of being conceptually more appealing, this result provides a profit-grounded justification for the adoption of the state-dependent closed-loop equilibrium information structure.

5 A Generalization and an Example

In the two-period setting, we have shown in Proposition 2 that if the players start out with identical capacities and invest positively at each period, then they invest more in an open-loop equilibrium than in a closed-loop equilibrium. The generalization of the comparative investment result to multi-period games is given in the following proposition.

Proposition 4 *Assume the T stage extension of Assumption 1. For $T \geq 2$ period extension of the game, T is finite, equilibrium investment under the open-loop structure is higher than the one under the closed-loop structure; that is $I_i^{OL}(T-1, s_k^{T-1}) > I_i^{CL}(T-1, s_k^{T-1})$ at any node s_k^{T-1} on the event tree.*

Proof. See the Appendix. □

Although the setting, i.e., equal initial capacities and positive investment at each period, may look restrictive, the above result is still interesting for mainly two reasons. The symmetry assumption, of which the economics literature is replete, is necessary here to be sure that any difference in the investment strategies is due, and only due, to the information structure. Put differently, given our focus on comparing open-loop and closed-loop investment behaviors, it makes sense, as in any experimental design, to control for all other variables than the one studied that might affect the result. The investment positivity assumption is not severe in a context where investment is divisible and the cost is quadratic.

As a corollary to this proposition, we argue that for any finite number of periods, expected payoff under the closed-loop equilibrium is higher than the expected payoff under the open-loop equilibrium; that is $\pi_i^{CL} > \pi_i^{OL}$. We do not offer a formal proof for that, however we proved this result for $t = 2$ in Proposition 2. Because positive investments happen and open-loop investments exceed closed loop investments each period, and this T-stage extension is the recurrence of the 2 period game, we expect this equilibrium payoff ranking. Also, in this proposition we treat the symmetric case only. The analysis in the asymmetric case, that is allowing different initial capacities and/or taking into account of one player may invest and the other may not at a particular time, could also be analyzed. However, analytic solution may not be tractable due to the (curse of) dimensionality.

5.1 An Example with Three Periods

To obtain some additional insights into the impact of the model's parameters (especially initial capacities) on investment decisions, we consider a three-period model, and determine open-loop and closed-loop symmetric equilibria[9]. The event tree is depicted in Figure 1.[10] The root of the tree is node 0, at period 0. In period 1, we have two demand states; upstate demand at node $(1, u)$ and downstate demand at node $(1, d)$. In period 2 we have four demand states, i.e., $(2, uu), (2, ud), (2, du)\,(2, dd)$, where the first letter in the second argument refers to the parent node belonging to stage 1, and the second letter to the state of demand in period 2. With this notation, one knows at a glance the full history of each node in period 2.

Given the result in Lemma 4, we have six possible cases for investments:

$$\begin{aligned}
\text{Case 1} &: \quad I_i(0,0) = 0, \quad I_i(1,u) = 0, \quad I_i(1,d) = 0; \\
\text{Case 2} &: \quad I_i(0,0) > 0, \quad I_i(1,u) > 0, \quad I_i(1,d) = 0; \\
\text{Case 3} &: \quad I_i(0,0) > 0, \quad I_i(1,u) > 0, \quad I_i(1,d) > 0; \\
\text{Case 4} &: \quad I_i(0,0) = 0, \quad I_i(1,u) > 0, \quad I_i(1,d) = 0; \\
\text{Case 5} &: \quad I_i(0,0) = 0, \quad I_i(1,u) > 0, \quad I_i(1,d) > 0; \\
\text{Case 6} &: \quad I_i(0,0) > 0, \quad I_i(1,u) = 0, \quad I_i(1,d) = 0;
\end{aligned}$$

In cases 1, 4 and 5, the players do not invest at the initial node, implying that capacity exceeds the interior Cournot solution at upstate node in period 1. In the other three cases, the players do invest at the initial node. We summarize in Table 1 the equilibrium results for these different cases. Note that depending on initial production capacity, the equilibrium output at node 0 will be either at capacity (i.e., $q_i(0,0) = K_{i0}$) or interior (i.e., $q_i(0,0) = \frac{1-c}{3}$) in both open-loop and closed-loop equilibria, yielding the same profit. Given

[9] Note that asymmetric equilibria are also tractable but involve very long mathematical formulae without adding much more qualitative insight.

[10] As time period increases, it is possible that corner solution occurs and the production quantity is zero due to the nature of random walk demand distribution. In that case, clearly no investment is made at that decision node, and the accumulated capacity will be carried to the following periods to be used for future high-demand states.

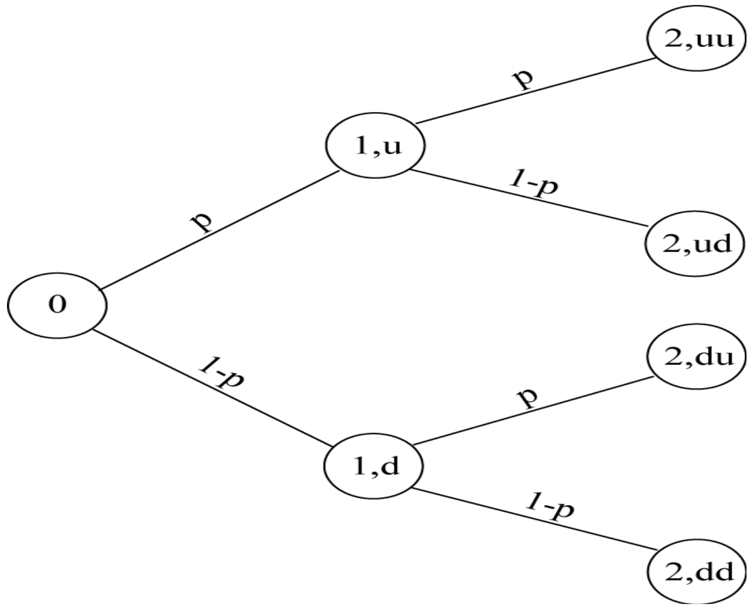

Figure 1: Event tree

the lack of strategic interest, we do not print the corresponding line in Table 1, nor in the other tables to follow. Also note that we drop the player index in this fully symmetric game. Regarding investments, this table is qualitatively valid for open-loop and closed-loop equilibria, i.e., in terms of whether investment is positive or zero. The actual value depends on the information structure, the case considered and whether capacity is binding or not in downstate demands. This last feature is captured in cells having two rows, with the first one corresponding to an output equal to capacity and the second line gives the equilibrium output when capacity is not binding. What remains to be seen is for which set of parameter values each of the six cases occurs.

First, note that cases 5 and 6 can be disregarded because they involve a contradiction. Indeed, in case 5, we simultaneously need to satisfy that $K_0 \geq \dfrac{1+\xi-c}{3}$ and $K_0 < \dfrac{1-c}{3} - I(1,d)$, which is impossible. Similarly, case 6 requires that $K_0 + I_0 < \dfrac{1+\xi-c}{3}$ and $K_0 + I_0 > \dfrac{1+2\xi-c}{3}$, which is again infeasible. Further, case 4 cannot be part of an open-loop equilibrium. Indeed, the first-order conditions include, among others, the following two conditions:

$$\begin{aligned} fI(0) &= \lambda(1,u) + \lambda(1,d) + \lambda(2,uu) + \lambda(2,ud) + \lambda(2,du) + \lambda(2,dd), \\ \delta pfI(1,u) &= \lambda(2,uu) + \lambda(2,ud). \end{aligned}$$

As the multipliers must be non negative, it is not possible to have simultaneously $I(0) = 0$ and $I(1,u) > 0$. We are therefore left with cases 1-3. To illustrate, let us assume that the capacity constraint is not active in the downstate demand nodes, i.e., in a two-row cell, we select the interior value.[11] A simple inspection of the cells leads to the following bounds for initial capacity in the remaining cases:

[11] The results for the scenario where the capacity is also binding at downstate demand nodes do not provide much additional qualitative insight. They are available from the authors upon request.

Table 1: Investments and outputs in the six possible cases

Node	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
I_0	0	>0	>0	0	0	>0
$I(1,u)$	0	>0	>0	$>$	$>$	0
$I(1,d)$	0	0	>0	0	$>$	0
$q(1,u)$	$\frac{1+\xi-c}{3}$	K_0+I_0	K_0+I_0	$\frac{1+\xi-c}{3}$	$\frac{1+\xi-c}{3}$	K_0+I_0
$q(1,d)$	$\frac{1-\xi-c}{3}$	K_0+I_0 $\frac{1-\xi-c}{3}$	K_0+I_0 $\frac{1-\xi-c}{3}$	$\frac{1-\xi-c}{3}$	$\frac{1-\xi-c}{3}$	K_0+I_0 $\frac{1-\xi-c}{3}$
$q(2,uu)$	$\frac{1+2\xi-c}{3}$	$K_0+I_0+I(1,u)$	$K_0+I_0+I(1,u)$	$K_0+I(1,u)$	$K_0+I(1,u)$	$\frac{1+2\xi-c}{3}$
$q(2,ud)$	$\frac{1-c}{3}$	$K_0+I_0+I(1,u)$ $\frac{1-c}{3}$	$K_0+I_0+I(1,u)$ $\frac{1-c}{3}$	$K_0+I(1,u)$ $\frac{1-c}{3}$	$K_0+I(1,u)$ $\frac{1-c}{3}$	$\frac{1-c}{3}$
$q(2,du)$	$\frac{1-c}{3}$	$\frac{1-c}{3}$	$K_0+I_0+I(1,d)$	$\frac{1-c}{3}$	$K_0+I(1,d)$	$\frac{1-c}{3}$
$q(2,dd)$	$\frac{1-2\xi-c}{3}$	$\frac{1-2\xi-c}{3}$	$K_0+I_0+I(1,d)$ $\frac{1-2\xi-c}{3}$	$\frac{1-2\xi-c}{3}$	$K_0+I(1,d)$ $\frac{1-2\xi-c}{3}$	$\frac{1-2\xi-c}{3}$

Table 2: Investment levels in Case 2

Node	Open-loop equilibrium investments
0	$\frac{\delta p\left\{(f+3\delta p)[1+\xi-c+\delta p(1+2\xi+3c)-3K_0]-3f\delta pK_0-\delta^2p^2(1+2\xi-c)\right\}}{\left[(f+3\delta p)^2+3f\delta^2p^2\right]}$
$(1,u)$	$\frac{\delta p(1+2\xi-3K_{i0}-c)}{(f+3\delta p)}-\frac{3\delta p}{(f+3\delta p)}I_i(0)$
	Closed-loop equilibrium investments
0	$\frac{\delta p[1+\xi-c-4K_0]}{f+4\delta p}$
$(1,u)$	$\frac{\delta p[f(1-c+2\xi-4K_0)+4\xi\delta p]}{(f+4\delta p)^2}$

$$\begin{aligned}
\text{Case 1} \quad &: \quad K_0 \geq \frac{1+2\xi-c}{3}, \\
\text{Case 2} \quad &: \quad \frac{1-c}{3}-I_0 < K_0 < \min\left(\frac{1+\xi-c}{3}-I_0, \frac{1+2\xi-c}{3}-I_0-I(1,u)\right), \\
\text{Case 3} \quad &: \quad K_0 < \min\left(\frac{1+\xi-c}{3}-I_0, \frac{1-c}{3}-I_0-I(1,d), \frac{1+2\xi-c}{3}-I_0-I(1,u)\right),
\end{aligned}$$

Case 1 will yield unique equilibrium if the initial capacity is large enough to cover the highest possible demand, i.e., demand in state uu in period 2. If the initial capacity is sufficient along the downstate demand path but not along the upstate path, then case 2 will yield the equilibrium. Table 2 provides the equilibrium investments under closed-loop and open-loop information structures in Case 2.

Finally, the third case in which the players invest at all investment decision nodes emerges when the initial capacity is low. In this case, the open-loop and closed-loop investment levels are those given in Table 3. Observe that in Table 3 closed-loop investment formulations in the nodes 0 and $(1,u)$ are the same as the corresponding ones in Table 2, however the exact levels of the equilibrium investments may be different

Table 3: Investment levels in Case 3

Node	Open-loop equilibrium investments
0	$\dfrac{\delta p[3\delta p(1+\xi-c-3K_0)+f((1+\delta)(1-c-3K_0)+\xi+2\xi p\delta)]}{(f+3\delta p)^2+3\delta^2 fp}$
$(1,u)$	$\dfrac{\delta p[f^2(1+2\xi-c-3K_0)+3\delta pf(1-c-3K_0+\xi(3-2\delta(p-1)))+9\xi\delta^2p^2]}{(f+3\delta p)\left[(f+3\delta p)^2+3\delta^2 fp\right]}$
$(1,d)$	$\dfrac{\delta p[f^2(1-c-3K_0)+3\delta pf(1-\xi-c-3K_0-2\xi\delta p)-9\xi\delta^2p^2]}{(f+3\delta p)\left[(f+3\delta p)^2+3\delta^2 fp\right]}$
	Closed-loop equilibrium investments
0	$\dfrac{\delta p[1+\xi-c-4K_0]}{f+4\delta p}$
$(1,u)$	$\dfrac{\delta p[f(1-c+2\xi-4K_0)+4\xi\delta p]}{(f+4\delta p)^2}$
$(1,d)$	$\dfrac{\delta p[f(1-c-4K_0)-4\xi\delta p]}{(f+4\delta p)^2}$

because they satisfy different parameter regions as defined above for Cases 2 and 3. When we compare capacity levels in nodes $(2,ud)$ and $(2,du)$, even though the demand states are identical, we obtain different capacities in these nodes as can be seen from equilibrium investment levels in Table 3. The reason is that these two nodes differ in terms of histories and expectations.

Although we above explain how as initial capacity decreases, the equilibria switch from Case 1 to Case 2 and then Case 3, we do not formally formulate the regions of initial capacity under which Cases 1, 2, and 3 hold. It is possible to have an expression for the thresholds that determine the different cases. However, we will explain why we do not need to define them, and hence for the sake of briefness we do not report these bounds of initial capacities. We describe, for example, how the bounds for initial capacity can be obtained for Case 2. Under Case 2, investment only benefits up-state demands. In that case upper bound for the initial capacity will be upstate Cournot output, and the lower bound will satisfy the property that the upstate production capacity (investment plus initial capacity) is greater or equal to the down-state Cournot output. As investment quantities in both types of equilibria are different, the bounds of the initial capacities under both equilibria will be different. But one interval will subsume the other interval of initial capacities, because investment in one equilibrium type (OL) is greater than the investment in other equilibrium type (CL). Therefore, our comparison of investments will hold true without specifying the bounds of initial capacities.

Table 4 collects the result of the sensitivity analysis (i.e., comparative statics) of investment levels with respect to the model's parameters. We note that the open-loop and closed-loop investment levels mostly vary in the same manner with respect to each of the parameters. The following observations can be made: (i) The higher the marginal production cost, the lower are the investments in both equilibria and at all nodes. When it becomes more expensive to produce, there is less incentive to increase the capacity. (ii) As one can expect, the higher the initial capacity, the lower are the investment levels in both equilibria and at all nodes. (iii) At the upstate demand node $(1,u)$, as well as at the initial node, the investment level in both equilibria increases with ξ; the higher the rate of increase in demand, the better is the reward from investing in capacity. The parameter ξ is playing the reverse role at downstate demand node $(1,d)$, and hence the negative sign. (iv) The same result is observed for the discount factor δ which is simply scaling the revenues at the different

Table 4: Sensitivity Analysis for Case 3

	$I^{OL}(0)$	$I^{OL}(1,u)$	$I^{OL}(1,d)$	$I^{CL}(0)$	$I^{CL}(1,u)$	$I^{CL}(1,d)$
c	$-$	$-$	$-$	$-$	$-$	$-$
K_0	$-$	$-$	$-$	$-$	$-$	$-$
ξ	$+$	$+$	$-$	$+$	$+$	$-$
δ	$+$	$+$	$-$	$+$	$+$	$-$
f	$-$	$-$	$+/-$	$-$	$-$	$+$

periods (and nodes). (v) Finally, the higher the investment cost parameter f, the lower is the investment level at the initial period 0 and the upstate node $(1, u)$ for both types of equilibria. This is rather intuitive. However, at node $(1, d)$ the rate of change of downstate investment with respect to the investment cost is puzzling for both types of equilibria. Note that investment at node $(1, u)$ will be higher than the one at $(1, d)$ due to higher demand. In the S-adapted open-loop equilibrium, at the outset of the game firms may increase or decrease their investments at node $(1, d)$ as a response to an increase in investment cost parameter f. There is a threshold value of f, below which firms in equilibrium reduce their investments as a response to the increase in f. Above that threshold value, however, firms raise their investments as f increases. The intuition for that is firms balance their investments under uncertainty, in which demand fluctuates between up and down states. In the upstate demand scenario $(1, u)$ they reduce their investments as investment costs increase, in the downstate demand $(1, d)$ they may increase their investments as a response to the investment cost increases with the expectation that demand with some probability will increase in the following period and investment made in the node $(1, d)$ will be totally used in the node $(2, du)$. That is, firms maximizing their expected profits at the initial node and making investment plans for the future will make sure that expected profits are maximized through "balanced" investment decisions (+ and -) as investment costs change. Also, as time moves (from t to $t + 1$) they keep following this balanced investment approach. This approach is clear for the state dependent decision making process (i.e., CLNE), however it may not be consistent with the information structure in which firms precommit their investment decisions (i.e., OLNE). Hence the sign of rate of change of investment $I^{OL}(1, d)$ with respect to f can be positive or negative.

6 Conclusions

The main objective of this article was to characterize and compare OL and CL investment strategies in a dynamic game with a stochastic demand described by an event tree. Assuming (most of the cases) symmetry, the main conclusions are:

1. The dynamics of investment is governed by the good news principle, i.e., the players invest in their productive capacities as if the upstate-demand scenario is going to unfold in the next period. As long as the probability of realization of this scenario is positive this result holds true under both S-adapted open-loop and closed-loop information structures. Further, at each node where players invest, they do so incrementally, i.e., they choose to increase the capacity by the exact value that is needed in the upstate demand node.

2. The ranking of open-loop and closed-loop investment equilibrium levels depend on initial capacities. In a discrete-time dynamic game where the randomness in demand is represented by an event tree, one can think of any decision node as the root of a subgame starting at that node. This means that "initial

capacities" can be interpreted, for this comparative purpose, in a more general way than strictly as the capacities at hand at the very beginning of the game.

3. In the symmetric game, under no circumstances can a player achieve a better outcome in an open-loop equilibrium than in its closed-loop counterpart. As pointed out earlier, this result constitutes a strong defense in favor of the closed-loop information structure. However, in terms of welfare, under open-loop behavior prices are lower and the production quantities are higher, and this benefits consumer surplus.

We made in this article, as in any modelling effort, some restrictive assumptions that are worthwhile relaxing in future investigations to assess their impact on the equilibrium results.

We assumed that demand either shifts up or shifts down by a positive quantity. The implication, at least in the very long run, is that demand may become zero or negative along the downstate path(s). If for any reason this market exit is undesirable, then one should adopt a demand distribution that prevents this from occurring. One easy way out is to assume, as in Garcia and Stacchetti (2008), that demand can either shift up or stay the same. Note that, especially if ξ is "very" small, our assumption would not have a significant qualitative impact on the equilibrium results of the first periods, which are actually more important for immediate decisions than distant ones in terms of both profits and (our understanding of) investment strategies. Further, following many contributions in the literature, we supposed that the investment cost is quadratic. Adding a linear term, i.e., having a positive marginal cost for zero investment, would surely alter quantitatively and possibly qualitatively the results. Similarly, the addition of a fixed cost may have an interesting impact on the incremental investment result obtained here.

Appendix 1

Proof of Lemma 1

Because no ambiguity may arise, we omit the variable's argument $\left(t, s_k^t\right)$. It is clear that if player i plays $q_i = K$ then the best response of player j is $q_j = (1+\phi-c-K)/2$ by the profit maximization. In that case, $P(Q) = (1+\phi+c-K)/2$, and the profit of player i is $\pi_i = (1+\phi-c-K)K/2$. However, player i can do better, namely, its best response to player j strategy q_j is $q_i^* = (1+\phi-c+K)/4$. Then, $P^*(Q) = (1+\phi+3c+K)/2$, and player i's profit is $\pi_i^* = ((1+\phi-c+K)/4)^2$. Then, clearly, $\pi_i^* \geq \pi_i$ if and only if $(1+\phi-c-3K)^2 \geq 0$, but this inequality holds because the production constraint must satisfy $q_i^* = (1+\phi-c+K)/4 \leq K$. Hence, asymmetric outcomes are not part of the equilibrium. If the capacity K is lower than the symmetric Cournot level then the capacity constraints must be binding. If capacity K is greater than the symmetric Cournot outputs then the solution is the interior one. If K is equal to the Cournot outputs then the interior solution coincides with the corner solution. Therefore depending on the capacity level, the equilibrium will be unique.

Proof of Lemma 2

We will show that $q_i(t,u) \geq q_i(t,d)$, for $i = 1, 2$. Suppose that there exists a player j for whom $q_j(t,u) < q_j(t,d)$. The Lagrangian of the profit maximization problem is

$$L_j = P\left(Q\left(t, s_k^t\right)\right) q_j\left(t, s_k^t\right) - C_j\left(q_j\left(t, s_k^t\right)\right) + \lambda_j\left(t, s_k^t\right)\left(K_j\left(t, s_k^t\right) - q_j\left(t, s_k^t\right)\right).$$

The first order conditions lead to

$$P\left(Q\left(t,s_k^t\right)\right)+P'\left(Q\left(t,s_k^t\right)\right)q_j\left(t,s_k^t\right)-C_j'\left(q_j\left(t,s_k^t\right)\right)-\lambda_j\left(t,s_k^t\right)=0,\quad s_k^t=u,\,d.$$

Because $q_j(t,u)<q_j(t,d)$, $\lambda_j(t,u)=0\le\lambda_j(t,d)$ holds, then

$$\begin{aligned}P(Q(t,u))+P'(Q(t,u))q_j(t,u)-C_j'(q_j(t,u))-\lambda_j(t,u)&=0,\\ P(Q(t,d))+P'(Q(t,d))q_j(t,d)-C_j'(q_j(t,d))&\ge 0.\end{aligned}$$

It follows that

$$\begin{aligned}P(Q(t,u))&=-P'(Q(t,u))q_j(t,u)+C_j'(q_j(t,u))\\ &< -P'(Q(t,d))q_j(t,d)+C_j'(q_j(t,d))\le P(Q(t,d)).\end{aligned}$$

Because the inverse demand is monotone and linear, we have $Q(t,d)<Q(t,u)$. This implies that $q_i(t,d)<q_i(t,u)$ for $i\neq j$. Then, using the same reasoning above, one can obtain that this inequality implies that $Q(t,d)>Q(t,u)$, which is a contradiction. Therefore, there cannot exist a player such that his production in down-state is higher than its production in up-state.

Proof of Lemma 3

Consider the optimization problem of player i at any node $s_k^t\in S^t, t=1,\ldots,T-1$, with the two successor nodes being u and d. It is straightforward to verify that the S-adapted OL Nash equilibrium (OLNE) conditions include

$$\begin{aligned}\frac{\partial\pi_i}{\partial I_i\left(t,s_k^t\right)}&=-fI_i\left(t,s_k^t\right)+\lambda_i(t+1,u)+\lambda_i(t+1,d)=0,\\ \frac{\partial\pi_i}{\partial q_i(t+1,u)}&=\delta p\left[1+\phi(a(t+1,u))+\xi-2q_i(t+1,u)-q_j(t+1,u)-c\right]-\lambda_i(t+1,u)=0,\\ \lambda_i(t+1,u)&\ge 0,\qquad K_i\left(t,s_k^t\right)+I_i\left(t,s_k^t\right)-q_i(t+1,u)\ge 0,\\ 0&=\lambda_i(t+1,u)\left[K_i\left(t,s_k^t\right)+I_i\left(t,s_k^t\right)-q_i(t+1,u)\right],\\ \frac{\partial\pi_i}{\partial q_i(t+1,d)}&=\delta(1-p)\left[1+\phi(a(t+1,d))-\xi-2q_i(t+1,d)-q_j(t+1,d)-c\right]-\lambda_i(t+1,d)=0,\\ \lambda_i(t+1,d)&\ge 0,\qquad K_i\left(t,s^t\right)+I_i\left(t,s^t\right)-q_i(t+1,d)\ge 0,\\ 0&=\lambda_i(t+1,d)\left[K_i\left(t,s_k^t\right)+I_i\left(t,s_k^t\right)-q_i(t+1,d)\right].\end{aligned}$$

For $I_i(t,s_k^t)>0$, we have $\lambda_i(t+1,u)+\lambda_i(t+1,d)>0$. We have the following possibilities

$$\begin{aligned}\lambda_i(t+1,u)&>0\quad\text{and}\quad\lambda_i(t+1,d)>0,\\ \lambda_i(t+1,u)&>0\quad\text{and}\quad\lambda_i(t+1,d)=0,\\ \lambda_i(t+1,u)&=0\quad\text{and}\quad\lambda_i(t+1,d)>0.\end{aligned}$$

The last possibility is excluded by Lemma 2. Hence, in all events we have $\lambda_i(t+1,u)>0$, and, from complementarity conditions, $q_i(t+1,u)=K_i(t,s_k^t)+I_i(t,s_k^t)$.

For $I_i(t,s_k^t)=0$, we have $\lambda_i(t+1,u)+\lambda_i(t+1,d)=0$. The nonnegativity of the multipliers imply that

$\lambda_i (t+1,u) = \lambda_i (t+1,d) = 0$, and hence $q_i (t+1,u) < K_i (t+1,u)$ and $q_i (t+1,d) < K_i (t+1,d)$.

The proof of the result for the closed-loop Nash equilibrium is also similar. The structure of the proof is available in the following propositions.

Proof of Lemma 4

Recall that any set $B(s_k^t)$ contains two nodes u and d, sharing the same history. Denote by $I_i (t,u)$ and $I_i (t,d)$ the investment decisions at these two nodes. Denote by uu and ud the descendants of node u and by du and dd the descendants of node d. The equilibrium output conditions at the upstate descendents node uu and du for player i are given by

$$\begin{aligned}
\frac{\partial v_i}{\partial q_i (t+1,uu)} &= \theta\left(s_u^{t+1} \mid a\left(s_u^{t+1}\right)\right)\left(1+\phi\left(t,s_u^t\right)+\xi-2q_i (t+1,uu)-q_j (t+1,uu)-c\right)-\lambda_i (t+1,uu)=0,\\
&\lambda_i (t+1,uu)\geq 0,\ K_i (t,u)+I_i (t,u)-q_i (t+1,uu)\geq 0,\quad \lambda_i (t+1,uu)\left[K_i (t,u)+I_i (t,u)-q (t+1,uu)\right]=0,\\
\frac{\partial v_i}{\partial q_i (t+1,du)} &= \theta\left(s_d^{t+1} \mid a\left(s_d^{t+1}\right)\right)\left(1+\phi\left(t,s_d^t\right)-\xi-2q_i (t+1,du)-q_j (t+1,du)-c\right)-\lambda_i (t+1,du)=0,\\
&\lambda_i (t+1,du)\geq 0,\ K_i (t,d)+I_i (t,d)-q_i (t+1,du)\geq 0,\quad \lambda_i (t+1,du)\left[K_i (t,d)+I_i (t,d)-q_i (t+1,du)\right]=0,
\end{aligned}$$

where v_i is the value function.

Suppose that $I_i (t,u) = 0$ and $I_i (t,d) > 0$. By Lemma 3, we must have $q_i (t+1,uu) < K_i (t+1,uu)$ implying $\lambda_i (t+1,uu) = 0$, and $q_i (t+1,du) = K_i (t+1,du)$. Further, because nodes u and d at time t share the same history, then $K_i (t,u) = K_i (t,d)$. The above conditions become

$$\begin{aligned}
q_i (t+1,uu) &= \frac{(1+\phi(t,s_u^t)+\xi-q_j (t+1,uu)-c)}{2} < K_i (t,u) = K_i (t,d),\\
q_i (t+1,du) &= K_i (t,d)+I_i (t,d) = \frac{(1+\phi(t,s_d^t)-\xi-q_j (t+1,du)-c)}{2}-\frac{\lambda_i (t+1,du)}{\theta\left(s_u^{t+1} \mid a\left(s_d^{t+1}\right)\right)},\\
\lambda_i (t+1,du) &> 0.
\end{aligned}$$

Invoking symmetry, we then have

$$\begin{aligned}
q_i (t+1,uu) &= \frac{(1+\phi(t,s_u^t)+\xi-c)}{3} < K_i (t,u) = K_i (t,d),\\
q_i (t+1,du) &= K_i (t,d)+I_i (t,d) = \frac{(1+\phi(t,s_d^t)-\xi-c)}{3}-\frac{\lambda_i (t+1,du)}{\theta\left(s_u^{t+1} \mid a\left(s_d^{t+1}\right)\right)},\\
\lambda_i (t+1,du) &> 0.
\end{aligned}$$

These conditions are incompatible. Indeed, $q_i (t+1,du)$ is at the same time larger than $K_i (t,u)$ and less than $q_i (t+1,uu)$, which is less than $K_i (t,u)$... Therefore, if $I_i (t,u)$ is zero then $I_i (t,d)$ cannot be positive.

Proof of Proposition 1

In this deterministic case, there is only one node in each period and therefore there is no need to distinguish between periods and nodes. For a variable x, we write x_{it} instead of $x_i (t,s_k^t)$, $t = 0,1$. Consider first the open-loop case. Player i maximizes

$$L_i = q_{i0}(1-q_{i0}-q_{j0})-cq_{i0}-f\,I_{i0}^2/2+\delta[q_{i1}(1-q_{i1}-q_{j1})-cq_{i1}]+\lambda_{i0}(K_{i0}-q_{i0})+\lambda_{i1}(K_{i0}+I_{i0}-q_{i1}).$$

At time 0, the first order necessary conditions for production decisions (that are irrelevant of investment decisions) might yield several possibilities due to capacity constraints. It might produce interior Cournot solution: $\lambda_{i0} = 0$, $i = 1, 2$, implying $q_{i0} = (1 - c)/3$. Or, it might lead to one interior one corner solution: $\lambda_{i0} = 0$ and $\lambda_{j0} > 0$ yielding $q_{i0} = (1 - K_{j0} - c)/2$ and $q_{j0} = K_{j0}$, $i \neq j$. Or, both players are at the capacity: $\lambda_{i0} > 0$ and $\lambda_{j0} > 0$ implying $q_{i0} = K_{i0}$, $i = 1, 2$.

At time 1, the production quantities are the same as the ones above, except the state variable at that period might change with the possible capacity expansion made in earlier period. The optimum investment must solve the first order necessary conditions, which imply $I_{i0} = \lambda_{i1}/f$. Assuming positive investments by both firms means $\lambda_{i1} > 0$, which in turn implies, $K_{i1} + I_{i0} = q_{i1}$. The derivative of the objective function with respect to q_{i1} results in $\lambda_{i1} = \delta[1 - 2q_{i1} - q_{j2} - c]$. Plugging this into the investment expression yields

$$fI_{i0} = \delta[1 - 2(K_{i0} + I_{i0}) - (K_{j0} + I_{j0}) - c], i, j = 1, 2, i \neq j.$$

The OLNE investment will satisfy this equality.

To characterize the closed-loop Nash equilibrium (CLNE) investment levels we solve the problem backwards and start from the final stage. At time 1, the value function is

$$v_{i1} = q_{i1}(1 - q_{i1} - q_{j1}) - cq_{i1} + \lambda_{i1}(K_{i0} + I_{i0} - q_{i1}).$$

The complementarity condition is, $\lambda_{i1}(K_{i0} + I_{i0} - q_{i1}) = 0$. Assuming that $\lambda_{i0} > 0$ (if we assume $\lambda_{i1} = 0$, we will obtain zero investment level in equilibrium), we obtain the corner solution $q_{i1} = K_{i0} + I_{i0}$. Next we plug this expression into the value function and write the value function at time 0:

$$v_{i0} = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - fI_{i0}^2/2 + \delta w_{i1}(I_{i0}) + \lambda_{i0}(K_{i0} - q_{i0}).$$

Taking the derivative with respect to the investment results in, assuming positive investments by both firms,

$$fI_{i0} = \delta[1 - 2(K_{i0} + I_{i0}) - (K_{j0} + I_{j0}) - c], i, j = 1, 2, i \neq j.$$

Clearly this expression is the same as the one obtained for OLNE. Hence, investment levels coincide under both equilibrium concepts.

Proof of Proposition 2

First we characterize closed-loop Nash equilibrium investments. At time 1 on node u player i maximizes

$$v_{iu} = [q_i(1, u)(1 + \xi - q_i(1, u) - q_j(1, u)) - cq_i(1, u)] + \lambda_{iu}(K_{iu} - q_i(1, u)),$$

where $K_{iu} = I_{i0} + K_{i0}$. The optimum output will satisfy $q_i(1, u) = K_{iu}$ because of the assumption that K_{i0} is low and $I_{i0} > 0$.

At time 1 on node d player i maximizes

$$v_{id} = [q_i(1, d)(1 - \xi - q_i(1, d) - q_j(1, d)) - cq_i(1, d)] + \lambda_{id}(K_{iu} - q_i(1, d)).$$

The optimum output will satisfy $q_i(1, d) < K_{id}$, where $K_{id} = K_{iu}$, because of the assumption that K_{i0} is large enough so that the capacity constraints do not always bind.

At initial node, player i maximizes

$$v_{i0} = q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f\,I_{i0}^2/2 + \delta p w_{iu}(K_{iu}, K_{ju}) + \delta(1-p)w_{id}(.) + \lambda_{i0}(K_{i0} - q_{i0}),$$

where $w_{iu}(K_{iu}, K_{ju})$ is the profit for player i at node u in period 1 when it has capacity of $K_{iu} = I_{i0} + K_{i0}$ and the rival has the capacity of $K_{ju} = I_{j0} + K_{j0}$. Also $w_{id}(.) = q_i\,(1,d)\,(1 - \xi - q_i\,(1,d) - q_j\,(1,d)) - cq_i\,(1,d)$ is the profit for player i at node d in period 1. The optimal investment must satisfy

$$-fI_{i0} + p\delta \frac{\partial w_{iu}}{\partial K_{iu}} \frac{\partial K_{iu}}{\partial I_{i0}} = 0,$$

or

$$-fI_{i0} + p\delta[1 + \xi - q_j\,(1,u)\,(K_{iu}) - 2K_{iu} - K_{iu} q_j'\,(1,u)\,(K_{iu}) - c] = 0.$$

When $q_i\,(1,u) = K_{iu},\ q_j\,(1,u)\,(K_{iu}) = K_{iu}$ must hold because of symmetry. (It could be possible that $q_j\,(1,u)\,(K_{iu}) = (1 + \xi - c - K_{iu})/2$. We analyze this case for asymmetric equilibrium.) Substituting $q_j\,(1,u)\,(K_{iu}) = K_{iu}$ and $K_{iu} = I_{i0} + K_{i0}$ and simplifying we have

$$I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 4K_0]}{f + 4\delta p}, i = 1, 2.$$

The equilibrium production quantities at time 1 will satisfy $q_u = (K_{iu}, K_{ju})$ at the upstate demand, and $q_d = ((1 - \xi - c)/3,\ (1 - \xi - c)/3)$ at the downstate demand.

Next we characterize open-loop Nash equilibrium investments. We write the objective function to be maximized by firms $i, j = 1, 2,\ i \neq j$,

$$\begin{aligned} z_{i0} \;=\;& q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f\,I_{i0}^2/2 + \delta p[q_i\,(1,u)\,(1 + \xi - q_i\,(1,u) - q_j\,(1,u)) - cq_i\,(1,u)] \\ & + \delta(1-p)[q_i\,(1,d)\,(1 - \xi - q_i\,(1,d) - q_j\,(1,d)) - cq_i\,(1,d)] \\ & + \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{iu}(K_{i0} + I_{i0} - q_i\,(1,u)) + \lambda_{id}(K_{i0} + I_{i0} - q_i\,(1,d)). \end{aligned}$$

Taking the derivative of the above objective function (z_{i0}) with respect to the investment will yield to $I_{i0} = (\lambda_{iu} + \lambda_{id})/f$, where

$$\lambda_{iu} = \delta p[1 + \xi - c - 2q_i\,(1,u) - q_j\,(1,u)] = \delta p[1 + \xi - c - 3(K_{i0} + I_{i0})],$$

because upstate production constraints are binding, and $\lambda_{id} = 0$ because downstate production constraints are non-binding by assumption. Then, the OLNE strategy as a function of the model parameters is

$$I_{i0}^{OL} = \frac{\delta p[1 + \xi - c - 3K_{i0}]}{f + 3\delta p}, i = 1, 2.$$

We now show that $\pi_i^{CL} > \pi_i^{OL}$. The CLNE and OLNE profits at initial node and node d in period 1 are clearly the same. Therefore, we need to compare the profits at node u in period 1. The difference in profits is given by

$$\pi_i^{OL} - \pi_i^{CL} = A + B,$$

where, dropping the player index,

$$\begin{aligned} A &= -f((I^{OL})^2 - (I^{CL})^2)/2 = -f(I^{OL} - I^{CL})(I^{OL} + I^{CL})/2, \\ B &= \delta p[(K_0 + I^{OL})(1 + \xi - 2(K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - 2(K_0 + I^{CL}) - c)]. \end{aligned}$$

Because $I^{OL} > I^{CL}$, A is negative. If the sign of B is negative, then we are done. Otherwise, we need to determine the sign of $|A| - B$. We have

$$\begin{aligned} B &= \delta p[(K_0 + I^{OL})(1 + \xi - 2(K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - 2(K_0 + I^{CL}) - c)] \\ &= \delta p[-2K_0 I^{OL} + 2K_0 I^{CL} + I^{OL}(1 + \xi - 2(K_0 + I^{OL}) - c) - I^{CL}(1 + \xi - 2(K_0 + I^{CL}) - c)] \\ &= \delta p[2K_0(I^{CL} - I^{OL}) + (1 + \xi - 2K_0 - c)(I^{OL} - I^{CL}) - 2(I^{OL})^2 + 2(I^{CL})^2] \\ &= \delta p[(I^{OL} - I^{CL})(1 + \xi - 4K_0 - c) - 2(I^{OL} - I^{CL})(I^{OL} + I^{CL})] \\ &= \delta p[(I^{OL} - I^{CL})(1 + \xi - c - 4K_0 - 2(I^{OL} + I^{CL}))]. \end{aligned}$$

In the expression $\pi_i^{OL} - \pi_i^{CL} = A + B$, we will show that $|A| > B$. Indeed,

$$\begin{aligned} |A| - B &= (I^{OL} - I^{CL})[\frac{f(I^{OL} + I^{CL})}{2} - \delta p(1 + \xi - c - 4K_0 - 2(I^{OL} + I^{CL}))] \\ &= (I^{OL} - I^{CL})[(I^{OL} + I^{CL})(2\delta p + f/2) - \delta p(1 + \xi - c - 4K_0)] \\ &= (I^{OL} - I^{CL})[(I^{OL} + I^{CL})(2\delta p + f/2) - I^{CL}(f + 4\delta p)] \\ &= (I^{OL} - I^{CL})[(I^{OL}(2\delta p + f/2) - I^{CL}(2\delta p + f/2)] \\ &= (I^{OL} - I^{CL})^2(2\delta p + f/2) > 0. \end{aligned}$$

Hence, $\pi_i^{OL} - \pi_i^{CL} < 0$.

Next we show that asymmetric equilibrium in investment strategies is not possible under Assumption A1. That is whenever $K_{i0} = K_0 = K_{j0}$ and investment is positive then $I_{i0}^{CL} = I_{j0}^{CL}$, and $I_{i0}^{OL} = I_{j0}^{OL}$, $i \neq j$. To see this in the OLNE we look at the investment expression, $I_{i0} = \lambda_{i1}/f$, where

$$\lambda_{iu} = \delta p[1 + \xi - 2q_i(1, u) - q_j(1, u)] = \delta p[1 + \xi - c - 2(K_0 + I_{i0}) - (K_0 + I_{j0})].$$

Then, we will have

$$\begin{aligned} I_{i0}^{OL} &= \frac{\delta p[1 + \xi - c - 3K_0 - 2I_{i0}^{OL} - I_{j0}^{OL}]}{f}, \\ I_{j0}^{OL} &= \frac{\delta p[1 + \xi - c - 3K_0 - 2I_{j0}^{OL} - I_{i0}^{OL}]}{f}, \end{aligned}$$

which are clearly symmetric expressions and the only solution is $I_{i0}^{OL} = I_{j0}^{OL}$.

In the CLNE at initial node player i maximizes

$$v_{i0} = v - f\, I_{i0}^2/2 + \delta p[(K_0 + I_{i0})(1 + \xi - c - 2K_0 - I_{i0} - I_{j0})],$$

where v is the portion of the profit not involving the investment term. Taking the derivative of this expression with respect to I_{i0} and equating it to zero yield

$$I_{i0}^{CL} = \frac{\delta p[1 + \xi - c - 3K_0 - I_{j0}^{CL}]}{f + 2\delta p}.$$

Similarly, for player j we obtain

$$I_{j0}^{CL} = \frac{\delta p[1 + d - c - 3K_0 - I_{i0}^{CL}]}{f + 2\delta p}.$$

Clearly these best response functions admit a unique symmetric solution. Hence $I_{i0}^{CL} = I_{j0}^{CL}$.

Proof of Proposition 3

We write the objective function to be maximized by firms:

$$\begin{aligned} v_{i0} &= q_{i0}(1 - q_{i0} - q_{j0}) - cq_{i0} - f\, I_{i0}^2/2 + \delta p[q_i\,(1,u)\,(1 + \xi - q_i\,(1,u) - q_j\,(1,u)) - cq_i\,(1,u)] + \\ &\quad \delta(1-p)[q_i\,(1,d)\,(1 - \xi - q_i\,(1,d) - q_j\,(1,d)) - cq_i\,(1,d)] \\ &\quad + \lambda_{i0}(K_{i0} - q_{i0}) + \lambda_{i1}(K_{i0} + I_{i0} - q_i\,(1,u)) + \lambda_{i2}(K_{i0} + I_{i0} - q_i\,(1,d)). \end{aligned}$$

Without loss of generality label the firms such that firm i makes investment, and the firm j does not make investment. First we characterize CLNE investments. At the upstate demand $q_i\,(1,u) = K_{i0} + I_{i0}$, and $q_j\,(1,u) = (1 + \xi - c - K_{i0} - I_{i0})/2$ will hold. At the downstate demand, we have $q_i\,(1,d) = (1 - \xi - c)/3 = q_j\,(1,d)$. We plug these expressions into the above objective function and maximize with respect to I_{i0} for firm i. The closed-loop investment strategy will be equal to $I_{i0}^{CL} = \dfrac{p\delta[1 + \xi - c - 2K_{i0}]}{2f + 2p\delta}$.

Next we characterize open-loop investment strategy. We optimize the above objective function and obtain that $I_{i0} = \lambda_{i1}/f$, and $\lambda_{i1} = \delta p[1 + \xi - c - 2q_i\,(1,u) - q_j\,(1,u))]$, where $q_i\,(1,u) = K_{i0} + I_{i0}$, and $q_j\,(1,u) = (1 + \xi - c - K_{i0} - I_{i0})/2$. Then the OLNE investment will be equal to $I_{i0}^{OL} = \dfrac{p\delta[1 + \xi - c - 3K_{i0}]}{2f + 3p\delta}$. Clearly, $I_{i0}^{CL} > I_{i0}^{OL}$ holds.

We now show that $\pi_i^{CL} > \pi_i^{OL}$. We have

$$\begin{aligned} \pi_i^{CL} &= \Pi - f\left(I_{i0}^{CL}\right)^2/2 + \delta p[(K_{i0} + I_{i0}^{CL})(1 + \xi - (K_{i0} + I_{i0}^{CL}) - (1 + \xi - c - K_{i0} - I_{i0}^{CL})/2 - c)], \\ \pi_i^{OL} &= \Pi - f\left(I_{i0}^{OL}\right)^2/2 + \delta p[(K_{i0} + I_{i0}^{OL})(1 + \xi - (K_{i0} + I_{i0}^{OL}) - (1 + \xi - c - K_{i0} - I_{i0}^{OL})/2 - c)], \end{aligned}$$

where Π is the profit term involving initial node and node d in period 1. The profit difference is thus given by

$$\pi_i^{OL} - \pi_i^{CL} = -f(\left(I^{OL}\right)^2 - \left(I^{CL}\right)^2)/2 + \delta p[(K_0 + I^{OL})(1 + \xi - (K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - (K_0 + I^{CL}) - c)]/2.$$

Let

$$\begin{aligned} A &= -f(\left(I^{OL}\right)^2 - \left(I^{CL}\right)^2)/2, \\ B &= \delta p[(K_0 + I^{OL})(1 + \xi - (K_0 + I^{OL}) - c) - (K_0 + I^{CL})(1 + \xi - (K_0 + I^{CL}) - c)]/2. \end{aligned}$$

A is positive because $(I^{OL} - I^{CL})(I^{OL} + I^{CL}) < 0$ because $I^{OL} < I^{CL}$. It is easy to check that B reduces to

$$B = \delta p[(I^{OL} - I^{CL})(1 + \xi - c - 2K_0 - (I^{OL} + I^{CL}))].$$

Now, note that

$$\begin{aligned}\pi_i^{OL} - \pi_i^{CL} &= A + B, \\ &= (I^{MP} - I^{OL})(I^{OL} - I^{CL})(\delta p/2 + f/2),\end{aligned}$$

which is negative, and hence $\pi_i^{OL} < \pi_i^{CL}$.

We next show that $\pi_j^{CL} < \pi_j^{OL}$ for player j. Similar to the profit difference for player i, the profit difference for player j under both equilibria boils downs to

$\pi_j^{OL} - \pi_j^{CL} = (I^{CL} - I^{OL})(2 + 2\xi - 2c - 2K_0 + I^{CL} - I^{OL})$.

Note that the investment levels I^{CL}, $I^{OL}$are the investments made by player i. The difference is positive because both the first term and the second term on the right hand side are positive.

Proof of Proposition 4

In Proposition 2 we prove that, for $T = 2$, $I_0^{OL} > I_0^{CL}$. Assume that at each node on the event tree OLNE investment exceeds CLNE investment in the $T-1$ stage game. By induction, we will show that this result extends to T-stage game and the nodes on the event tree. First we will compare the investments in T stage game. We start with the open-loop analysis.

Let us take a look at a particular node in time $T-1$ and write down the expected payoff for firm i from that node to the nodes in the next period T

$$\begin{aligned}z_i(T-1, s_k^{T-1}) \;=\;& q_i(T-1, s_k^{T-1})(1 + \phi - c - q_i(T-1, s_k^{T-1}) - q_j(T-1, s_k^{T-1})) - f\, I_i^2(T-1, s_k^{T-1})/2 + \\ & \delta p[q_i(T, s_{k_u}^T)(1 + \phi + \xi - c - q_i(T, s_{k_u}^T) - q_j(T, s_{k_u}^T))] \\ & + \delta(1-p)[q_i(T, s_{k_d}^T)(1 + \phi - \xi - c - q_i(T, s_{k_d}^T) - q_j(T, s_{k_d}^T))] \\ & + \lambda_i(T-1, s_k^{T-1})(K_i(T-1, s_k^{T-1}) - q_i(T-1, s_k^{T-1})) + \lambda_i(T, s_{k_u}^T)(K_i(T-1, s_k^{T-1}) \\ & + I_i(T-1, s_k^{T-1}) - q_i(T, s_{k_u}^T)) + \lambda_i(T, s_{k_d}^T)(K_i(T-1, s_k^{T-1}) + I_i(T-1, s_k^{T-1}) - q_i(T, s_{k_d}^T)),\end{aligned}$$

Taking the derivative of the function $z_i(T-1, s_k^{T-1})$ with respect to the investment will yield to $I_i(T-1, s_k^{T-1}) = (\lambda_i(T, s_{k_u}^T) + \lambda_i(T, s_{k_d}^T))/f$

Case 1: Upstate production is binding and downstate production is interior.

Then

$$\begin{aligned}\lambda_i(T, s_{k_u}^T) &= \delta p[1 + \phi + \xi - 2q_i(T, s_{k_u}^T) - q_j(T, s_{k_u}^T)] = \\ & \delta p[1 + \phi + \xi - c - 3(K_i(T-1, s_k^{T-1}) + I_i(T-1, s_k^{T-1}))]\end{aligned}$$

and $\lambda_i(T, s_{k_d}^T) = 0$. The OLNE strategy would be,

$$I_i^{OL}(T-1, s_k^{T-1}) = \frac{\delta p[1 + \phi + \xi - c - 3K_i^{OL}(T-1, s_k^{T-1})]}{f + 3\delta p}.$$

Under closed-loop Nash equilibrium, the investments will satisfy

$$I_i^{CL}(T-1, s_k^{T-1}) = \frac{\delta p[1 + \phi + \xi - c - 4K_i^{CL}(T-1, s_k^{T-1})]}{f + 4\delta p},$$

as we compute in the proof of Proposition 2. We will show that $I_i^{OL}(T-1, s_k^{T-1}) > I_i^{CL}(T-1, s_k^{T-1})$. Observe that capacity states in the above equalities are functions of investments made in earlier periods.

Similar to the problem at time $T-1$, we could write the maximization problem at time $T-2$ and obtain the equilibrium investment levels under both types of equilibria:

$$\begin{aligned}
I_i^{OL}(T-2, a(s_k^{T-1})) &= \frac{\delta p[1+\phi-c-3K_i^{OL}(T-2, a(s_k^{T-1}))]}{f+3\delta p}, \\
I_i^{CL}(T-2, a(s_k^{T-1})) &= \frac{\delta p[1+\phi-c-4K_i^{CL}(T-2, a(s_k^{T-1}))]}{f+4\delta p}.
\end{aligned}$$

We know, by the induction assumption, that $I_i^{OL}(T-2, a(s_k^{T-1})) > I_i^{CL}(T-2, a(s_k^{T-1}))$, and $K_i^{OL}(T-2, a(s_k^{T-1})) > K_i^{CL}(T-2, a(s_k^{T-1}))$. Noting that

$$K_i^{OL}(T-1, s_k^{T-1}) = K_i^{OL}(T-2, a(s_k^{T-1})) + I_i^{OL}(T-2, a(s_k^{T-1})),$$

the open-loop investment at $T-1$ can be rewritten as

$$(f+3\delta p)I_i^{OL}(T-1, s_k^{T-1}) = \delta u[1+\phi+\xi-c-3K_i^{OL}(T-1, s_k^{T-1})],$$

which implies

$$I_i^{OL}(T-1, s_k^{T-1}) = \frac{\delta p[1+\phi+\xi-c-3K_i^{OL}(T-2, a(s_k^{T-1}))]}{f+6\delta p} = \frac{I_i^{OL}(T-2, a(s_k^{T-1}))(f+3\delta p)}{f+6\delta p}.$$

Similarly using

$$I_i^{CL}(T-1, s_k^{T-1}) = \frac{\delta p[1+\phi+\xi-c-4K_i^{CL}(T-1, s_k^{T-1})]}{f+4\delta p},$$

and

$$K_i^{CL}(T-1, s_k^{T-1}) = K_i^{CL}(T-2, a(s_k^{T-1})) + I_i^{CL}(T-2, a(s_k^{T-1})),$$

the CLNE investment at time $T-1$ reduces to

$$I_i^{CL}(T-1, s_k^{T-1}) = \frac{I_i^{CL}(T-2, a(s_k^{T-1}))(f+4\delta p)}{f+8\delta p}.$$

However, given that $I_i^{OL}(T-2, a(s_k^{T-1})) > I_i^{CL}(T-2, a(s_k^{T-1}))$ and $\dfrac{f+3\delta p}{f+6\delta p} > \dfrac{f+4\delta p}{f+8\delta p}$ we conclude that $I_i^{OL}(T-1, s_k^{T-1}) > I_i^{CL}(T-1, s_k^{T-1})$. This holds true for all k indexing the nodes s_k^{T-1} at time $T-1$, because of the demand structure.

Case 2: Upstate production is binding and downstate production is binding.

The proof for this case is similar to the above one. In the open loop one, we will have both the Lagrange multipliers positive, that is $\lambda_i(T, s_{k_d}^T) > 0$, and $\lambda_i(T, s_{k_u}^T) > 0$ so that $I_i(T-1, s_k^{T-1}) = (\lambda_i(T, s_{k_u}^T) + \lambda_i(T, s_{k_d}^T))/f$. We calculate the multipliers by maximizing $z_i(T-1, s_k^{T-1})$ and obtain that

$$\lambda_i(T, s_{k_u}^T) = \delta p[1+\phi+\xi-c-3(K_i(T-1, s_k^{T-1}) + I_i(T-1, s_k^{T-1}))]$$

and

$$\lambda_i(T, s_{k_d}^T) = \delta(1-p)[1+\phi-\xi-c-3(K_i(T-1, s_k^{T-1}) + I_i(T-1, s_k^{T-1}))].$$

Plugging them into the investment expression yields to

$$I_i^{OL}(T-1, s_k^{T-1}) = \frac{\delta[1+\phi-\xi+2p\xi-c-3K_i^{OL}(T-1, s_k^{T-1})]}{f+3\delta}.$$

Under closed-loop Nash equilibrium, the investments will satisfy

$$I_i^{CL}(T-1, s_k^{T-1}) = \frac{\delta[1+\phi-\xi+2p\xi-c-4K_i^{CL}(T-1, s_k^{T-1})]}{f+4\delta}.$$

Observe that these investment expressions are qualitatively similar to the ones in Case 1. Using the same induction procedure we used above, it is clear that $I_i^{OL}(T-1, s_k^{T-1}) > I_i^{CL}(T-1, s_k^{T-1})$ must hold for any s_k^{T-1} at time $T-1$.

References

[1] T. Basar, G.J. Olsder, Dynamic Noncooperative Game Theory, SIAM's Classics in Applied Mathematics, Philadelphia, 1995.

[2] B. Bernanke, Irreversibility, Uncertainty, and Cyclical Investment, Quarterly Journal of Economics 98 (1983), 85–106.

[3] M. Breton, R. Jarrar, G. Zaccour, A Note on Feedback Stackelberg Equilibria in a Lanchester Model with Empirical Application, Management Science 52, 5 (2006), 804–811.

[4] R. Cellini, L. Lambertini, A Dynamic Model of Differentiated Oligopoly with Capital Accumulation, Journal of Economic Theory 83 (1998), 145–155.

[5] R.J. Deneckere, A. de Palma, The Diffusion of Consumer Durables in a Vertically Differentiated Oligopoly, Rand Journal of Economics 29 (1998).

[6] A. Dixit, The Role of Investment in Entry-Deterrence, Economic Journal 90 (1980), 5–106.

[7] A. Dixit, R. Pindyk, Investment Under Uncertainty, Princeton University Press, Princeton, NJ, 1993.

[8] E.J. Dockner, A dynamic theory of conjectural variations, The Journal of Industrial Economics 40, 4 (1992), 377–395.

[9] E.J. Dockner, S. Jørgensen, N. Van Long, G. Sorger, Differential Games in Economics and Management Science, Cambridge University Press, Cambridge, UK, 2000.

[10] R.A. Driskill, Durable goods oligopoly, International Journal of Industrial Organization 19 (2001), 391–413.

[11] R.A. Driskill, S. McCafferty, Dynamic duopoly with adjustment costs: A differential game approach, Journal of Economic Theory 49 (1989), 324–338.

[12] C. Fershtman, E. Muller, Capital Accumulation Games of Infinite Duration, Journal of Economic Theory 33 (1984), 160–182.

[13] C. Figuières, Complementarity, substitutability and the strategic accumulation of capital, International Game Theory Review 4, 4 (2002), 371–390.

[14] C. Figuières, Markov interactions in a class of dynamic games, Theory and Decision 66 (2009), 39–68.

[15] A. Garcia, E. Stacchetti, Investment Dynamics in Electricity Markets, working paper, University of Virginia, 2008.

[16] A. Garcia, J. Shen, Equilibrium Capacity Expansion under Stochastic Demand Growth, Operations Research, 2010.

[17] T.S. Genc, S. Reynolds, S. Sen, Dynamic Oligopolistic Games under Uncertainty, Journal of Economic Dynamics and Control 31 (2007), 55–80.

[18] T.S. Genc, S. Sen, An Analysis of Capacity and Price Trajectories for the Ontario Electricity Market Using Dynamic Nash Equilibrium Under Uncertainty, Energy Economics, 30 (2008), 173–191.

[19] A. Haurie, G. Zaccour, Y. Smeers, Stochastic Equilibrium Programming for Dynamic Oligopolistic Markets, Journal of Optimization Theory and Applications 66, 2 (1990), 243–253.

[20] P. Koeva, The Facts About Time-to-Build, International Monetary Fund, working paper WP/00/138 (2000).

[21] G. Kossioris, M. Plexousakis, A. Xepapadeas, A. de Zeeuw, K.-G... Mäler, Feedback Nash equilibria for non-linear differential games in pollution control, Journal of Economic Dynamics and Control 32, 4 (2008), 1312–1331.

[22] D. Kreps, J. Sheinkman, Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes, Bell Journal of Economics 14 (1983), 326–337.

[23] N.V. Long, K. Shimomura, H. Takahashi, Comparing open-loop with Markov equilibria in a class of differential games. Japanese Economic Review 50, 4 (1999), 457–469.

[24] F. Melese, Ph. Michel, Reversing the arms race: A differential game model. Southern Economic Journal 57, 4 (1991), 1133–1143.

[25] G. Pacheco-de-Almeida, P. Zemsky, The effect of time-to-build on strategic investment under uncertainty, RAND Journal of Economics 34, 1 (2003), 166–182.

[26] C. Piga, A dynamic model of advertising and product differentiation, Review of Industrial Organization 13 (1998), 509–522.

[27] S. Reynolds, Capacity Investment, Preemption and Commitment in an Infinite Horizon Model, International Economic Review 28 (1987), 69–88.

[28] F. Ruiz-Aliseda, J. Wu, Irreversible Investment in Stochastically Cyclical Markets, working paper, Universitat Pompeu Fabra, 2008.

[29] A.M. Spence, Investment Strategy and Growth in a New Market, Bell Journal of Economics, 10 (1979), 1–19.

[30] F. Van der Ploeg, A.J. de Zeeuw, Perfect equilibrium in a model of competitive arms accumulation, International Economic Review 31, 1 (1990), 131–146.

[31] J. Wu, Capacity preemption and leadership contest in a market with uncertainty, working paper, University of Arizona, 2007.