Abstract

Using finite sample simulation methods, we assess the power of long-horizon predictive tests and compare them to their short-run counterparts, when the true underlying model contains financial asset bubbles. Our results indicate that long-run predictive tests using valuation predictors – specifically the dividend price ratio – do pick up the return predictability inherent in the asset bubble. However, after size-adjustment, the long-run predictive framework has no particular advantage over its short-run counterpart. The short and long-run predictive regression tests have nearly the same size-adjusted power in a present value model with asset bubbles.

Key words: asset bubbles, predictive regression, long-horizon regression, stock return predictability

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1 Introduction

In popular policy discussions, financial bubbles have become closely associated with the term “irrational exuberance” coined in 1996 by former Federal Reserve Chairman Alan Greenspan two days after a meeting in which Professors John Campbell and Robert Shiller famously warned him of the high stock market valuations. Their warning was supported by a regression scatter plot showing a negative relationship between ten year market returns and the long-term price equity ratio. Based on this negative long-horizon predictive relationship, they cautioned that historical periods of high stock valuations had often been followed by below average returns over the next decade - a warning that Carroll (2008) finds to be prescient.

Thus, in the policy context, financial bubbles and valuation based long-horizon predictive regression have been tightly linked. Yet, the academic literature on these two topics remains primarily distinct. As outlined in the following section, one large literature has developed to debate and test for the existence of, model, and date financial bubbles. A second, essentially distinct, literature debates the size and power properties of long-horizon tests and proposes new test procedures.

Our paper is an attempt to bridge these two literatures. We examine the ability of long-horizon predictive tests to detect the predictability inherent in a present value model with asset bubbles. In particular, we assess the statistic power of an appropriately size-corrected test based on a long-horizon predictive regression specification using a valuation based predictor, when return predictability is due to the presence of periodic financial bubbles. We also compare the predictive power of the long-horizon test to that of its short-horizon counterpart, in which returns are predicted just one period ahead.

The two primary (preliminary) findings emerge from our analysis. First, we confirm that valuation based linear predictive regression models have the power to detect return predictability due to more complicated models involving financial bubbles. Provided that properly sized tests are employed, this provides some support to the consideration of market valuations in assessing the possibility of a financial bubble. On the other hand, we find no distinct advantage to long-horizon predictive regression models, even in this setting in which intuition might suggest that they would prove more powerful than their short-run counterpart. For example, our simulations suggest that a predictive regression with a ten year horizon has no more ability to detect bubble driven return predictability than a predictive regression with a one-year horizon.\footnote{In addition to the ten year return horizon, the analysis of Campbell and Shiller’s referred to above used ten years of smoothed earnings to calculate their valuation ratio. This latter smoothing arguably has important benefits that are not analyzed here, where we focus on the length of the return horizon and employ the lagged dividend price ratio as the valuation based predictor.} The size-adjusted power of the two specifications are almost indistinguishable.

These results also contribute to the long-standing debate over the power of long-horizon
predictive regressions. Much of this debate has taken place in a linear predictive model, which is less simple than it appears due to the persistence of the lagged valuation predictors and their correlation with contemporaneous returns. However, an argument can be made that with the use of a powerful short-run test, there can be no inherent advantage to long-horizon regressions when the linear short-horizon model is a correct specification (Hjalmarsson 2011, Maynard and Ren 2014). This paper joins a smaller literature which has considered the debate in the context of non-linear models, in which the short-horizon regression is misspecified and there may therefore be stronger a priori reason to suspect improvements at longer horizons (Kilian and Taylor 2003, Wohar and Rapach 2005, Ang and Bekaert 2007, Maynard and Ren 2014).

The long ten-year return horizon was a seemingly critical component of Campbell and Shiller’s analysis. When an asset bubble is present it may continue to expand for many periods before eventually bursting or correcting. Thus they were not arguing that a correction would be immediate, but rather that it was likely to occur over the longer-horizon. The implicit intuitive argument is that valuation based predictors will have greater power to detect bubble driven return predictability at longer horizons. Yet, even in this non-linear setting in which intuition might appear to strongly favour the long-horizon specification, we find no inherent advantage to it.

The remainder of the paper is organized as follows. A discussion of the literatures on financial bubbles and long-horizon return predictability is given in the next section. Section 3 presents and derives the present value model with the financial bubble component. Section 4 describes the simulation framework and parameter settings. Section 5 presents and interprets our main results and findings. Section 6 concludes. Tables and figures are included in the appendix.

2 Literature, Background, and Motivation

What are asset bubbles? Do they exist and is it appropriate to integrate them into asset pricing models? These issues present challenges to both asset pricing and econometric theory. In order to answer these questions, many scholars provide different models and explanations for financial bubbles. There is no consensus regarding the uniform definition of bubbles in the literature. However, historical experiences include the dot.com bubble and 2008 subprime crisis appears to many observers to confirm the existence of asset bubbles.

The existence and econometric test of asset bubbles constitute a much debated subject of research in financial economics. Much of the theoretical literature focuses on the question of whether bubbles in asset price exist. The argument for the absence of bubble in asset price builds on the assumption of rational investors as stated by the efficient markets hypothesis of Fama (1965) who posits that the presence of informed traders will correct any overpricing or underpricing errors created by speculators. The justification for the non-existence of bubbles
is also provided by neoclassical theorists who claim that bubbles can be precluded if there are a maximum possible price, backward inductions and transversality conditions as in Santos and Woodford (1997). The views against bubbles are enforced by the use of nonlinear utility function and also the general equilibrium framework. However, Abreu and Brunnermeier (2003) challenges the traditional view by proposing that the persistence of bubbles is supported by the failure of arbitrageurs to synchronize or coordinate their trading strategies. Despite these rational bubbles, speculative bubbles model have been developed in which heterogeneous beliefs of agents generate overvaluation of asset prices and thus create speculative bubbles in both static and dynamic framework. Miller (1977), Chen, Hong and Stein (2002), Scheinkman and Xiong (2003) and others discuss speculative bubbles in detail.


Despite numerous previous studies devoted to the research on existence and econometric test of asset bubbles, there is not much academic research done to build a link between bubbles and the power of long-run predictability of stock returns. Firstly, researchers disagree over how or whether to integrate asset bubble component into asset pricing models (the discussion of Cochrane, 2005, pp. 402 -404) and what is the implication for empirical research if asset bubbles are incorporated into the asset pricing models. These different opinions are expressed in Pastor and Veronesi (2006), Cooper (2008). It appears that there is a disconnect between asset bubble and asset return literatures as a result of these controversial debates.

Recently, Lee, Phillips(2015) address this gap in the literature by incorporating a financial bubble into a consumption based asset pricing model. They derive implications for the risk premium and employ a GMM based approach to estimate the model parameters. Working independently, we also incorporate a financial bubble component to into a present value model, this but use this to address a different question. Specifically, what is the relation between bubbles and the statistical power of long run predictive regression? Can price bubbles help to improve

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2The main derivation in the next section was derived before coming across Lee and Phillips (2015)’s newly posted working paper. We also did not share our results with them ahead of their work. Thus both sets of results were arrived at independently.
the statistical power of long horizon predictability of stock returns? These questions, which are not answered in previous studies, motivate our paper.

The finance literature generally models a bubble as an increasing overvaluation of an asset relative to its fundamental evaluation that becomes self-fulfilling because investors bid the price of the asset up on the expectation that future investors will bid it up even further. Previous work also indicates that the dividends and earning price ratio should provide indicators of bubbles. They spike during bubbles and then subside. The goal of our paper is to explore whether there is a connection between the presence of bubbles and the power of long-horizon predictive regression.

In the empirical literature of predictive regression of stock returns, the statistical power of long-horizon predictability remains a topic of debate. As Boudoukh et al. (2008) note, the strong empirical results and large R-squared values from long-horizon predictive results have been hugely influential, cited by Cochrane (1999, “New Facts in Finance”) as one of the three most important new facts in finance. In fact, the 2013 Nobel Press release Royal Swedish Academy of Sciences (2013) made specific reference to the (partial) forecastability of stock and bond returns at long-horizons. This follows highly influential empirical applications to long-horizon stock predictability for stock returns (Fama and French 1988, Campbell and Shiller 1988), bonds yields (Fama and Bliss 1987, Cutler et al. 1991), and exchange rates (Mark 1995, Chinn and Meese 1995).

These strong empirical results were originally thought be due to higher statistical power obtained at longer horizons. However, this conclusion became less clear-cut as size distortions in long-horizon regressions became apparent. These were first noted in simulation studies (Kim and Nelson 1993, Goetzmann and Jorion 1993) and later formalized asymptotically (Valkanov 2003, Hjalmarsson 2012). Boudoukh et al. (2008) demonstrate that the empirical evidence of predictability appears to strengthen as the horizon increases even under the null hypothesis that returns are unpredictable.

This has led to a debate over the power of long-horizon regressions: is there really higher power at long-horizons or are the strong and influential empirical results simply a result of size distortion – a question first asked explicitly by Campbell (2001). Boudoukh et al. (2008) refer to the claim of higher statistical power at longer horizons as a “myth” and several other papers report evidence against it (Ang and Bekaert 2007, Boudoukh et al. 2008, Hjalmarsson 2008, Hjalmarsson 2012, Maynard and Ren 2014). Others works provide somewhat more favorable evidence (Campbell 2001, Kilian and Taylor 2003, Wohar and Rapach 2005, Mark and Sul

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3 Similar, though less severe size distortion, has been found in short-horizon regression when the lagged predictors are persistent and contemporaneously correlated with return innovations (Mankiw and Shapiro 1986, Cavanagh et al. 1995, Stambaugh 1999). This has led to a large financial econometric literature too vast to review here, where our focus is on longer horizon prediction.

4 Several new methods have been developed in an effort to overcome the size distortion in long horizon regressions, including Valkanov (2003), Liu and Maynard (2007), Hjalmarsson (2012), and Phillips and Lee (2013).
2006, Cochrane 2008), and Cochrane (2008) argues strongly in defense of the proposition of stronger predictability at longer horizons.

Maynard and Ren (2014) note that much of this debate takes place within the context of linear models, in which the short-horizon regression is correctly specified. As they note, due to the persistence of the valuation predictors and their contemporaneously correlated residuals standard short-horizon tests may be lose power even if size corrected (Campbell and Yogo 2006), opening the possibility of power gains at longer horizons (Cochrane 2008). However, once more powerful short-horizon tests are employed there is no obvious reason to expect increased power at longer horizons in linear models for which the short-horizon predictive regression is a correct specification (Hjalmarsson 2011, Maynard and Ren 2014).

More interestingly, in most models with non-linear dynamics or relations, the linear predictive regression is simply a linear approximation to the true underlying model at all horizons. If the longer-horizon regression provides an improved approximation, power improvements at longer horizons may exist even after controlling size. Kilian (1999) first suggests that power at longer horizons may be due to the presence of non-linearities. Kilian and Taylor (2003) and Wohar and Rapach (2005) study the power of long-horizon predictive regressions in the ESTAR model, (Ang and Bekaert 2007) considers the non-linear present value model, while (Maynard and Ren 2014) address the question in a model of regime switching.

By analysing the power of long-horizon regressions in present value models with non-linear price bubble dynamics, we contribute to this small but growing literature. In the empirical asset pricing literature, a common starting point characterizing their relation is

\[
dp_t = \lim_{i \to \infty} \rho^i dp_{t+i} + \sum_{i=0}^{\infty} \rho^i (r_{t+1+i} - \delta_{t+1+i}) + \frac{k}{1 - \rho},
\]

where \( dp_t, r_{t+1+i}, \delta_{t+1+i} \) are dividend price ratio, stock returns and dividend growth ratio, \( \rho = \frac{1}{1 + \exp(d-p)} \) is the discount factor and \( k = -\rho \log \rho - (1 - \rho) \log(1 - \rho) \). The transversality condition ensures that the first term \( \lim_{i \to \infty} \rho^i dp_{t+i} = 0 \). In this rational expectation present value model, bubbles are assumed away and price dividend ratio is interpreted as the discounted value of future cash flows. In our framework, we do not impose the transversality condition and re-derive stock returns as a function of dividend ratio and bubbles in the log linear approximation frame work. In our specification, we characterize the true model with this non linear present value model and provide the power analysis for both long and short horizon predictive tests.

### 3 Structural Present-Value Model with Bubble

This section develops the structural relations between long-run predictability of stock returns and bubbles in the log-linearized framework of Campbell and Shiller(1988). However, we propose an extended specification where the bubble is not assumed away due to the finite horizon. In
this setting, the bubble is assumed to be latent variable following Evans (1991) and Phillips, Shi, Yu (2014, 2015) and we allow for it to be incorporated into dividend price dynamics. Thus it has a non-trivial effect on expected stock returns.

Let \( B_t \) denote a periodically explosive bubble component in the aggregate stock market, following lognormal distribution,

\[
B_{t+1} = \begin{cases} 
\rho_b^{-1} B_t \varepsilon_{B,t+1}, & \text{if } B_t < b \\
[\zeta + (\pi \rho_b) \varepsilon_{B,t+1}] \varepsilon_{B,t+1}, & \text{if } B_t > b
\end{cases}
\]

where \( \rho_b > 1 \), \( \rho_b \) is the discount factor, \( \varepsilon_{B,t} = \exp(y_t - \tau^2/2) \) and \( y_t \) is i.i.d. \( N(0, \tau^2) \). \( \theta_t \) is assumed to follow a Bernoulli process which takes the value 1 with probability \( \pi \) and 0 with probability \( 1 - \pi \), and \( \zeta \) is the remaining size after the bubble collapses. In this case, bubble has a constant crash probability. To capture the predictive role of price dividend ratio in predicting future crashes, we also consider a logit specification for bubble crash probability. Now \( \pi_t \) is a function of price dividend ratio, which in turn depends on scaled bubbles, or more specifically, \( \pi_t = \frac{\exp(c + d * B_{t-1} + D_{t-1})}{1 + \exp(c + d * B_{t-1} + D_{t-1})} \), where \( \frac{B_t}{D_t} = c + d * \frac{B_{t-1}}{D_{t-1}} \), \( c \) and \( d \) are the parameters provided by Froot and Obstfeld (1991). Specially when \( c \) and \( d \) are equal to zero, time varying crash probability is reduced to constant crash probability. The bubble has the conditional expectation \( E_t(B_{t+1}) = \rho_b^{-1} \) and its collapse is triggered by the Bernoulli process if the bubble is bigger than the cut-off size \( b \).

Let \( D_t \) denote dividend of the stock market. We model it as the random walk process

\[ D_t = u + D_{t-1} + \varepsilon_{D,t}, \]

where \( \varepsilon_{D,t} \sim \text{i.i.d.} N(0, \sigma_D^2) \). Let \( P_t \) denote stock price which contains a market fundamental component \( P_t^f \), where

\[ P_t^f = \frac{u \rho}{(1 - \rho)^2} + \frac{\rho}{1 - \rho} D_t. \]

The actual price is then assumed to depend on both the fundamental price and the asset bubble as:

\[ P_t = P_t^f + \kappa B_t, \]

where \( \kappa \) controls the relative magnitudes of these two components, and then we characterize stock price process.

Given the aggregate stock price equation, following Froot and Obstfeld (1988), we can divide both sides by \( D_t \), where \( D_t \) is real dividend process to derive

\[ \frac{P_t}{D_t} = \frac{P_t^f}{D_t} + \kappa \frac{B_t}{D_t}, \]

Taking logs on both sides of the equation and then carrying out a Taylor expansion around \( \frac{B_t}{D_t} \), we obtain

\[ dp_{t+1} = \alpha + \phi dp_t + \varphi \log \left( \frac{B_t}{D_t} \right) + \varepsilon_{t,dp}, \]
where \( dp_t \) denotes the log dividend price ratio and \( \log \left( \frac{B_t}{D_t} \right) \) denotes the log scaled bubble.

Let \( r_{t+1} \) denote the total log return on the aggregate stock market, \( \Delta d_{t+1} \) denote dividend growth rate,

\[
\begin{align*}
  r_{t+1} & = \log \left( \frac{P_{t+1} + D_{t+1}}{P_t} \right) \\
  \Delta d_{t+1} & = \log \left( \frac{D_{t+1}}{D_t} \right).
\end{align*}
\]

We can relate expected stock returns to \( \log \left( \frac{B_t}{D_t} \right) \), \( \Delta d_{t+1} \), through log-linearized return as

\[
r_{t+1} = a + \Delta d_{t+1} - \rho dp_{t+1} + dp_t
\]

\[
\Delta d_{t+1} = u_d + b_d dp_t + \varepsilon_{t,d}
\]

where \( \rho = \frac{\exp(dp)}{1 + \exp(dp)} \), \( \bar{dp} = E(dp_t) \), as in Campbell and Schiller (1988). If we use this formula and substitute forward, it follows that

\[
\sum_{j=1}^{m} \rho^{j-1} r_{t+j} = \sum_{j=1}^{m} \rho^{j-1} \alpha + dp_t + \rho^m (p_{t+m} - d_{t+m}) + \sum_{j=1}^{m} \rho^{j-1} \Delta d_{t+j}.
\]

Note that previous literature sets \( \lim_{m \to \infty} \rho^m (p_{t+m} - d_{t+m}) = 0 \) for the purpose of exploring the relation between returns and various predictors. In our specification, we consider finite horizons, thus this term will not disappear. We iterate \( dp_t \) and \( \Delta d_{t+1} \) forward \( m \) periods, and then substitute them into the formula to obtain

\[
\sum_{j=1}^{m} \rho^{j-1} r_{t+j} = \text{const} + dp_t + \phi^m (p_t - d_t) + \rho^m \sum_{j=1}^{m} \phi^{j-1} \varphi \log \left( \frac{B_{t+m-j}}{D_{t+m-j}} \right) + \sum_{j=1}^{m} \rho^{j-1} b_d dp_{t+j-1} + \bar{u}_{t+j}
\]

\[
= \text{const} + (1 - \phi^m) (p_t - d_t) - \sum_{j=1}^{m} \phi^{j-1} \varphi \log \left( \frac{B_{t+m-j}}{D_{t+m-j}} \right) + \bar{u}_{t+j},
\]

where

\[
\text{const} = \sum_{j=1}^{m} \rho^{j-1} \alpha + \rho^m \sum_{j=1}^{m} \phi^{j-1} a + \rho^m \sum_{j=1}^{m} \phi^{j-1} a_d \quad \text{and}
\]

\[
\bar{u}_{t+j} = \rho^m \sum_{j=1}^{m} \phi^{j-1} \varepsilon_{t+m+1-j} + \sum_{j=1}^{m} \rho^{j-1} \varepsilon_{t+j,d}.
\]

This function provides the long run relation between the expected stock returns and the bubble. When \( m = 1 \), it reduces to the short run equation

\[
r_{t+1} = a + a_d - a_{dp} \rho + (1 + b_d - \rho \phi) dp_t + \rho \varphi \log \left( \frac{B_t}{D_t} \right) + \varepsilon_{t+1,d} - \rho \varepsilon_{t+1,dp} \quad (1)
\]
We extend the data generating process proposed by Cochrane (2008) to incorporate the bubble as follows:

\[ r_{t+1} = \Delta d_{t+1} - \rho dp_{t+1} + dp_t \]
\[ \Delta d_{t+1} = b_d dp_t + \epsilon_{t+1,d} \]
\[ dp_{t+1} = \phi dp_t + \varphi \log \left( \frac{B_t}{D_t} \right) + \epsilon_{t+1,dp}. \]

Shocks to expected dividend growth rate \( \epsilon_{t+1,d} \) and dividend price ratio \( \epsilon_{t+1,dp} \) are assumed to be jointly independent and identically distributed over time with mean zero and covariance matrix

\[ \Sigma = \begin{bmatrix} \sigma_d^2 & \sigma_{d,dp} \\ \sigma_{d,dp} & \sigma_{dp}^2 \end{bmatrix}. \]

4 Simulation Design

Based on the model development in the previous section, we now describe the simulation design and parameter settings. Our simulations are designed to provide a (partial) answer to the following hypothetical questions. Suppose that the present value model with bubbles describes the true model from which the data were generated. Suppose also that the econometrician does not know the true model and therefore tests for the presence of return predictability using either the standard short or long-horizon linear predictive regression specification. Would she uncover predictability when present? If so, which specification, the long or short-run specification, would be more likely to uncover this predictability?

To address this question, we repeatedly simulate pseudo return and dividend data from the structural present value model of the previous section. Then on each simulated pseudo data series, we test for return predictability using the standard short and long-horizon predictive tests described below. The average rejection rate over five thousand simulations is used to calculate the test power.\(^5\) By repeating this exercise over a range of values for \( \varphi \), the parameter that controls the strength of the bubble component, we form a power curve. As a comparison, we also perform a similar exercise for the case in which returns are predictable in the absence of financial bubbles.

In order to undertake this analysis, we need to define the null and alternative hypotheses in both the ‘true’ present value model and the empirical specifications employed by the econometrician. The underlying non-predictability hypothesis that we impose in both cases is given by

\[ H_0 : E_t r_{t+1} = \mu. \]  

\(^5\)Both regression specifications involve size distortions. As discussed below, we address this both by comparing size corrected power in standard regression t-tests and by calculating power in predictive tests designed to control size.
However, this implies different parameter restrictions in each model. In Section 4.1 we first discuss the restrictions and parameter settings of the present value model which forms the data generating process for the pseudo return and dividends data. In Section 4.2, we then describe the empirical specifications and tests employed by the econometrician.

### 4.1 Present value model: parameter settings and restrictions

In the present value model of the previous section, the non-predictability null hypothesis in (2) implies the joint restriction:

$$H_0^{PV} : \varphi = 0, \quad b_d = \phi \rho - 1 \quad \text{and} \quad E_t \tilde{u}_{t+1} = 0. \quad (3)$$

This both eliminates the financial bubble component by setting $\varphi = 0$ and sets the coefficient $1 - \phi \rho + b_d$ on the dividend price ratio in (1) equal to zero. Thus, the null hypothesis eliminates both the traditional linear source of predictability that arises in the present value model without bubbles and also eliminates the financial bubble.

The traditional alternative in a present value model without bubbles is given by

$$H_A^{TRAD} : b_d > \phi \rho - 1 \quad \text{and} \quad \phi = 0, \quad (4)$$

in which the dividend price ratio has predictive power for future returns, despite the fact that no bubble is present ($\phi = 0$). In order to provide a simulated power curve we simulate the rejection rates for a range of values for $b_d$. In particular, we specify the local alternative

$$b_d = \phi \rho - 1 + \frac{\gamma}{T} \quad (5)$$

where we vary $\gamma \geq 0$, while holding $\phi = 0$ in order to vary the distance from the null hypothesis. When $\gamma = 0$, $b_d = \phi \rho - 1$ and the dividend price ratio has no predictive content for returns. As we increase $\gamma$, we increase the strength of the predictive relation between the returns and lagged dividend price ratio. We consider this alternative first as a basis of comparison.

Our primary focus is on the alternative in which predictability is due to the presence of a bubble process. We specify this alternative as

$$H_A^B : \varphi > 0 \quad \text{and} \quad b_d = \phi \rho - 1 \quad (6)$$

Under this alternative, the only source of predictability is the bubble process. The traditional linear predictability in (4) is eliminated by the restriction that $b_d = \phi \rho - 1$. We simulate over a range of values for

$$\varphi = \frac{\gamma}{T} \quad (7)$$
where we vary $\gamma \geq 0$ in order to vary $\varphi$ and adjust $b_d$ accordingly in order to maintain the restriction $b_d = \phi \rho - 1$. When $\varphi = \gamma = 0$, there is no bubble component. As we increase $\varphi$, we increase the strength of the bubble component.\(^6\)

We next focus our attention on the remaining parameters of the present value model of the previous section. The coefficient $\phi$ denotes autoregressive (AR(1)) coefficient for the dividend price ratio under null hypothesis. Following Cavanagh et al. (1995), it is common in the prediction regression literature to model this parameter as a local to unity process of (Phillips 1987, Chan and Wei 1987):\(^7\)

$$\phi = 1 + \frac{c}{T},$$

where $T$ denotes the available sample size. This near unit root process is a statistical device designed to model a process that has its largest root close or equal to one. Thus it captures the fact that the dividend price ratio has been observed to be persistent without imposing an exact unit root. Although normally employed in asymptotic theory, the parameter $c$ provides a useful way to describe the persistence of the simulated dividend price ratio in a manner that is independent of the sample size. Since the value of the local-to-unity coefficient cannot be consistently estimated in the time series context, we plan to assess the robustness of our results across a range of choices for $c$ and thus $\phi$. In this preliminary draft of the paper, we set $c = -2.5$ and $T = 500$, corresponding to a value of $\phi = 0.995$.

The remaining parameters are held fixed, staying the same under the null and both alternative hypotheses. Their values, provided in Table 1, are set realistically based on those reported by Evans (1991) and Cochrane (2008). As defined in the previous section $\sigma_f, P_f^0, \rho, b, \pi, \kappa, \sigma_d, \sigma_{dp}, \sigma_{d,dp}$ denote the standard deviation of fundamental price, initial fundamental price, discount factor, cutoff bubble, initial bubble, remaining collapsing size of bubble, standard deviation of bubble, the relative magnitudes of fundamental price and bubble, standard deviation of dividend growth rate and dividend price yields ratio and covariance of dividend growth rate and dividend yield ratio. All these numbers are annualized to generate yearly data series for the simulation purpose.

### 4.2 Empirical specifications and tests

Specifically, the empirical specification we assume has the form

$$r_{t+k}^k = \beta_0(k) + \beta_1(k)dp_t + \epsilon_{t+k}^k$$ (9)

\(^6\)Note that the meaning of $\gamma$ in the two alternative specifications is not equivalent since it is used to set different parameters. Mixtures of these two alternative could also be considered. We do not pursue this here since we obtain fairly clear results from these two polar alternatives.

\(^7\)More recently Phillips and Magdalinos (2007) generalize the local-to-unity model to include mildly integrated processes and Magdalinos and Phillips (2009) employ this framework in the predictive regression literature to provide a novel solution for the predictive regression problem.
in which the dependent variable is defined as
\[ r_{t+k}^k = \sum_{j=1}^{k} r_{t+j}. \] (10)

When \( k = 1 \) this specializes to the standard short-horizon predictive regression using the dividend price ratio as the predictor. When \( k > 1 \) it corresponds to what is arguably the most common long-horizon specification employed in practice. We calibrate our simulation to a one-year data frequency and employ values of \( k \) equal to one, five, and ten year horizons. In the empirical specification in (9), the null hypothesis is imposed as
\[ H^E_0 : \beta_1(k) = 0 \text{ and } E_t \varepsilon_{t+k} = 0, \] (11)
although it is the restriction on the regression coefficient \( \beta_1(k) \) that is normally tested in practice. The empirically relevant alternative is
\[ H^{EMP}_E : \beta_1(k) > 0, \] (12)
in which case a low dividend price ratio implies higher expected future returns.

Although influential, standard OLS based tests of (11) are size distorted even at short horizons (Mankiw and Shapiro 1986, Cavanagh et al. 1995, Stambaugh 1999), due to both the persistence of dividend price process and its contemporaneous correlation with return innovations. As shown by Valkanov (2003), the size distortion becomes even worse at longer horizons \((k > 1)\). Although our primary concern is the test power, a size-distorted test may have power for the wrong reason. We therefore address this in two ways. First we size correct the standard OLS tests by recalculating their critical values via simulation to ensure that all tests have the correct rejection rate under the null hypothesis. Secondly, we provide a power comparison using a modified test by Hjalmarsson (2011) that both improves power and corrects size for tests of (11) under the local-to-unity specification in (8).

One attractive feature of Hjalmarsson (2011)’s test in this context is that it is designed to work without modification at both long \((k > 1)\) and short-horizons \((k = 1)\). At the short-horizons it essentially equivalent to the test of (Campbell and Yogo 2006) and its feasible implementation uses a similar Bonferroni bounds procedure based on the inversion of a unit root test to address the inherent uncertainty regarding the value of \( c \) in (8). On the other hand, a short-coming of the test, is that without modification, this Bonferroni procedure is invalid when the predictor is stationary or mildly integrated Phillips (2014). In order to ensure that none of our results are driven by the use of Bonferroni procedure, we also provide results using an infeasible or ‘oracle’ version Hjalmarsson (2011)’s test, in which the true value of \( c \) is employed in place of the Bonferroni bound.\textsuperscript{8}

\textsuperscript{8}Phillips and Lee (2013) provide a long-horizon IVX test of (12) that remains valid for a re-arranged version
5 Results and Interpretation

As a point of comparison, in Figures 1-3, we first compare the power of short and long-horizon regression predictive tests using the empirical specification in (9), when the true alternative hypothesis is specified by the present value model without a bubble component as in (4). The vertical axis of each figure provides the simulated rejection rate using five thousand simulations and a simulated sample size of \( T = 500 \). The horizontal axis of each figure provides the value of \( \gamma \) in (5). Power curves for predictive tests at a one, five, and ten year horizon are represented by the blue, green, and red curves respectively. The intersection of these curves with the vertical axis (\( \gamma = 0 \) in (5)) show the rejection rates under the null hypothesis in (3) under which returns are unpredictable.

The three figures differ in the testing procedure employed. Figure 1 shows the size adjusted power of the standard OLS tests based on the specification in (9). The size adjustment is implemented by replacing the standard critical values with their simulated counterparts. Without this adjustment all three tests would be size distorted, making test power more difficult to interpret. Furthermore, the size distortion would be more severe at longer horizons, so that the power of the tests would also be difficult to compare. Figure 2 shows the ‘oracle’ version of Hjalmarsson (2011). By design, it has correct size and maximal power, within the context of the linear model in (9) when the dividend price ratio is a local to unity process. However, it is infeasible in practice because it relies on knowledge of the true value of the local-to-unity parameter in (8). Finally, Figure 3 provides a feasible version of Hjalmarsson (2011) based on a first stage confidence interval for the local-to-unity parameter. Provided that the true process for the dividend price ratio is well described by a local-to-unity process, this provides a practical version of the test that can be applied at all three horizons.

The performance of these three tests across horizons are remarkably similar across horizons. After proper size-adjustment, the standard t-tests shown in 1 have almost identical power. There is a slight advantage at shorter horizons in the middle of the power curve, but this advantage is probably too small to matter in practice. Figure 2 similarly again shows a slight advantage to the of the long-horizon test in which a one-period ahead return is regressed on a multi-period predictor. A distinct advantage of their test is that it avoids the need for Bonferroni bounds and remains valid across a much wider range of persistent processes. However, we are not currently aware of a long-horizon IVX test that applies directly to the traditional alternative specification in (12) in the long-horizon case. Liu and Maynard (2007) propose a long-horizon predictive sign test with exact size, but also in the re-arranged long-horizon specification. Valkanov (2003) provides long-horizon tests in the traditional long-horizon specification, but these tests require a long-horizon and are less easily compared to the short-horizon case (\( k = 1 \)). There is an extensive financial econometrics literature proposing size corrective procedures in the short-horizon case when \( k = 1 \), many of which have not been extended to the long-horizon case.

In this preliminary draft, all three are given by solid and may be difficult to distinguish in a black and white printout. We suggest referring to the electronic version.
short-horizon specification. However, the overall message is again the same: there is essentially no difference in power between the short and long-run specifications, provided that a properly sized test is employed. Figure 3 is the only one to show any advantage to the long-horizon test, with it have a very slight advantage in the left portion of the power curve and a very slight disadvantage in the right portion. Yet, in all cases these differences are simply too small to matter in practice. For all practical purposes, the tests based on the one, five, and ten year horizons are all equally equipped to detect predictability.

These results support previous contentions that within the context of standard linear model there is no power gain at longer horizons and that the stronger empirical results observed at longer horizons may be solely a consequence of size distortion (Boudoukh et al. 2008, Hjalmarsson 2011). The result is supported by the intuition that when the model in linear, the short horizon empirical specification (9 with $k = 1$) is a correct or approximately correct specification. In this case, there are unlikely to be any advantages to changing the specification or horizon.

A more interesting question, in our view, concerns the difference power across the model when the true model is fundamentally non-linear. In this case, neither the short or long-horizons are correct specifications and they may be therefore be a stronger a priori argument for the possibility of power advantages at longer horizons. Figures 4-6 show same three power curve comparisons when predictability is driven by the presence of financial bubble component. Since the empirical specification in (9) does not explicitly allow for the possibility of a financial bubble, both the short and long-horizon tests, may be viewed as misspecifications, which may nonetheless detect a predictive relationship between the valuations (dividend price ratio) and future returns, due to the presence of the financial bubble. Since the bubble is persistent, it is an a priori conceivable conjecture that this relationship could be stronger at longer horizons.

All three figures confirm that, as a valuation based predictor, the dividend price ratio, has predictive ability for returns in the presence of a financial bubble. Given the presence of bubbles, this provides some support for the use valuation based predictors, such as the dividend price ratio. Of course, the predictive regression does not distinguish predictability due to the presence of a bubble component from other sources of predictability such as the more traditional alternative in (4). Moreover, if one is convinced that a bubble component is present and important, tests specifically designed to detect the bubble may likely be more powerful.

Turning to the comparison of power across test horizons, the striking result is again how closely the tests perform across horizons. The longer-horizon specification has slightly higher power close to the null hypothesis (left portion of power curve) for both the size adjusted OLS and Bonferroni tests shown in Figures 4 and 6 respectively. This power improvement is not found using the oracle test shown in Figure 5. In all three cases, the long-horizon test also suffers a slight disadvantage far from the null hypothesis (right portion of power curve). However, the overall impression which emerges is that the difference in power across the three horizons is fairly
6 Conclusion

The paper provides a comprehensive comparative evaluation of long horizon and short horizon powers of predictive regression when true data generating process is structural non linear present value models and also traditional linear model, and also in order to ensure that our results are robust, we incorporate time varying bubble collapse probability, that is, the probability is a function of bubbles. Therefore the effect of exogenous and endogenous bubble collapse on the predictability will be assessed equally. The reason for us to endogenize bubble collapse probability is that bubble magnitude will affect its bursting probability and then in turn affect asset returns. As a result, the long run predictability does hinge on bubble magnitude and collapse probability and our model accommodates these variables to provide more reasonable specification.

We compare the powers of predictive tests across three different horizons and test methods. In contrast to sized distorted methods applied in previous literature, size adjusted powerful tests such as Bonferonni and Oracle tests, have been implemented throughout the paper, generating a striking evidence that in the presence of financial bubbles power improvement has been detected in the long run specification executing standard Ols and Bonfferoni tests, and there is no distinct difference in the powers of long horizon and short horizon predictive tests. The results are confirmed to be robust even when we treat bubble collapsing probability as time varying. Even when different parameters controlling bubble magnitude are utilized, there appears that the overperformance of long-run power still remains with Ols and Bonfferoni tests. Interestingly, The same advantage disappears from Oracle tests regardless of probability setup.
References


### Table 1: Fixed Parameters Settings

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<th>$\sigma_f$</th>
<th>$P_f^0$</th>
<th>$\rho$</th>
<th>$b$</th>
<th>$B_0$</th>
<th>$\pi$</th>
<th>$\zeta$</th>
<th>$\tau$</th>
<th>$\kappa$</th>
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<th>$\sigma_{dp}$</th>
<th>$\sigma_{d,dp}$</th>
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<td>0.05</td>
<td>20</td>
<td>14</td>
<td>15.3</td>
<td>7.5</td>
</tr>
</tbody>
</table>

As defined in the previous section $\sigma_f, P_f^0, \rho, b, B_0, \pi, \zeta, \tau, \kappa, \sigma_d, \sigma_{dp}, \sigma_{d,dp}$ denote the standard deviation of fundamental price, initial fundamental price, discount factor, cutoff bubble, initial bubble, remaining collapsing size of bubble, standard deviation of bubble, the relative magnitudes of fundamental price and bubble, standard deviation of dividend growth rate and dividend price yields ratio and covariance of dividend growth rate and dividend yield ratio. All these numbers are annualized to generate yearly data series for the simulation purpose.

### 7 Figures
Size−adjusted power across return horizons $H_A$: $β_1 ≠ 0$ with $α=0.05$.

Figure 1: Size adjusted power comparison using the OLS based test in the present value model without bubbles. The vertical axis shows the rejection rate of the size adjusted test. The horizontal axis shows the distance from the null hypothesis of no predictability. The blue, green, and red lines respectively show the size-adjusted power curves for the one, five, and ten year horizon tests.
Figure 2: Power comparison using the infeasible ‘oracle’ test in the present value model without bubbles. The vertical axis shows the rejection rate of the size adjusted test. The horizontal axis shows the distance from the null hypothesis of no predictability. The blue, green, and red lines respectively show the size-adjusted power curves for the one, five, and ten year horizon tests.
Figure 3: Power comparison using the Bonferroni test in the present value model without bubbles. The vertical axis shows the rejection rate of the size adjusted test. The horizontal axis shows the distance from the null hypothesis of no predictability. The blue, green, and red lines respectively show the size-adjusted power curves for the one, five, and ten year horizon tests.
Figure 4: Size adjusted power comparison using the OLS based test in a present value model in which predictability is due to the presence periodic bubbles. The vertical axis shows the rejection rate of the size adjusted test. The horizontal axis shows the distance from the null hypothesis of no predictability. The blue, green, and red lines respectively show the size-adjusted power curves for the one, five, and ten year horizon tests.
Figure 5: Power comparison using the infeasible ‘oracle’ test in a present value in which predictability is due to the presence periodic bubbles. The vertical axis shows the rejection rate of the size adjusted test. The horizontal axis shows the distance from the null hypothesis of no predictability. The blue, green, and red lines respectively show the size-adjusted power curves for the one, five, and ten year horizon tests.
Figure 6: Power comparison using the Bonferroni test in a present value in which predictability is due to the presence periodic bubbles. The vertical axis shows the rejection rate of the size adjusted test. The horizontal axis shows the distance from the null hypothesis of no predictability. The blue, green, and red lines respectively show the size-adjusted power curves for the one, five, and ten year horizon tests.
References


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