The Impact of Spatial Price Differences on Oil Sands Investments

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Abstract

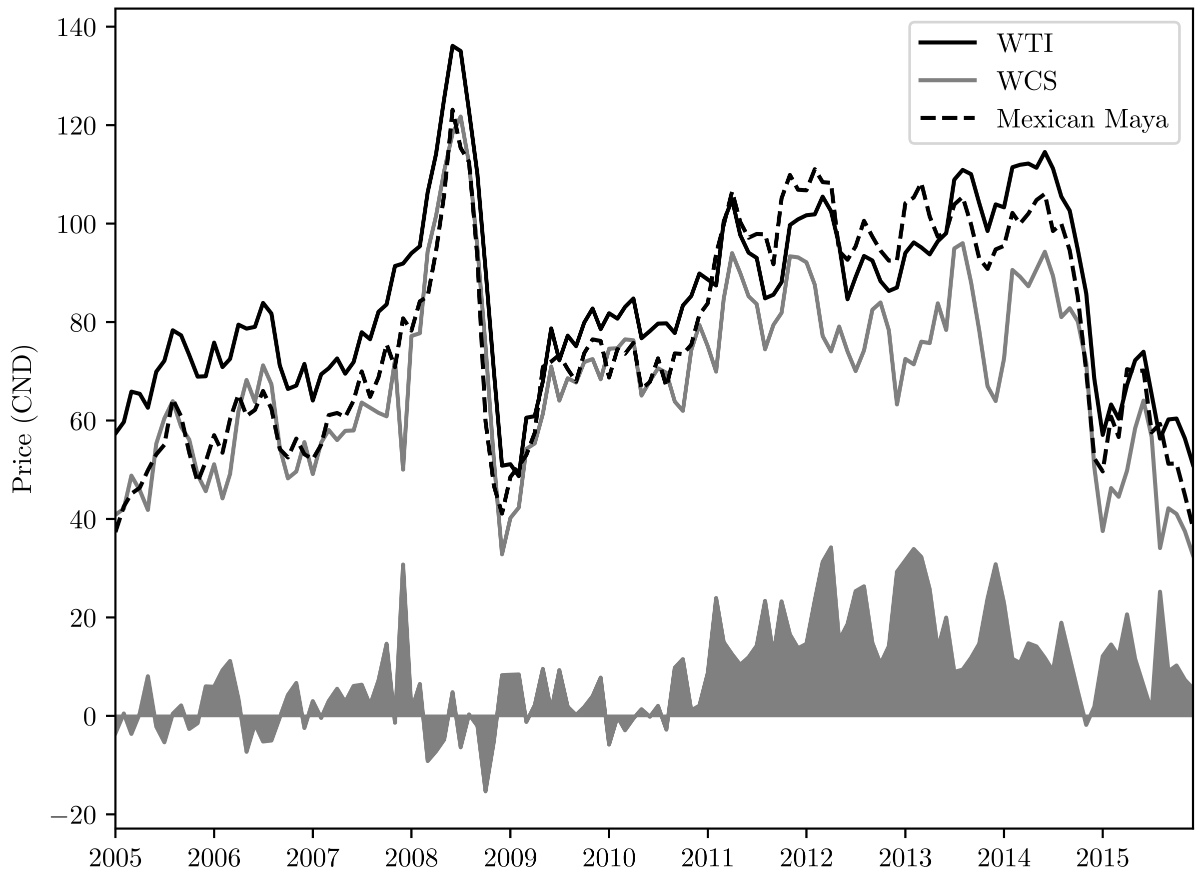
In this article, a two-factor real options model is developed to examine the impact spatial price differences have on the value of an oil sands project and the incentive to invest. Large, volatile price differences between locations can emerge when demand to ship exceeds capacity limits. This may have a significant impact on production, investment, and policy in exporting regions. Here, we assume the price difference between two locations follows a stationary process implying crude oil markets are integrated as oil prices in different locations move together. The investment decision is formulated as a linear complementarity problem that is solved numerically using a fully implicit finite difference method. Results show the value of an oil sands project and the incentive to invest in a new project will increase when the mean price difference decreases. Surprisingly, the standard deviation of the price difference has very little impact on project value or the incentive to invest.

1 Introduction

The feasibility of a natural resource investment critically depends on its access to markets. Spatial arbitrage models have shown, the more remote a natural resource is the lower its net price will be (Samuelson (1952) and Takayama and Judge (1971)). Consequently, improving market access has been the motivation behind the decision to build additional pipeline capacity to export crude bitumen and its derivatives from Alberta. Figure 1 plots monthly spot price data for West Texas Intermediate (WTI), Western Canadian Select (WCS), Mexican Maya, and the price difference between Mexican Maya and WCS from January 2005 to December 2015.[1] Prior to 2011, WCS and Mexican Maya tracked one another closely with Mexican Maya receiving a small location premium over WCS and large price differences were

[1] WCS is the benchmark for heavy crude oil in Canada and it is located in Hardisty, Alberta. It is a blend of heavy crude oil, crude bitumen and diluents with an API gravity of 20.5°. Mexican Maya is a heavy crude oil similar in quality to WCS located in the Gulf Coast.

Figure 1: Monthly crude oil spot prices and Mexican Maya-WCS price difference in Canadian dollars from January 2005 to December 2015. WTI data was collected from the EIA, Mexican Maya data was collected from Bloomberg, and WCS data was collected from Natural Resources Canada.



short lived. However, beginning in 2011, WCS and Mexican Maya diverged and WCS was heavily discounted relative to Mexican Maya. Proponents of additional pipeline capacity argue this large price difference is mostly attributed to inadequate transportation infrastructure and claim that both firms and governments would benefit from expanding pipeline capacity. Firms would gain access to international markets, higher world prices, and lower transport costs and governments would receive more tax revenue from higher royalties and income taxes.

This paper incorporates spatial price differences into a real options model to study the impact improved market access will have on the value of an oil sands project and the incentive to invest. Here, the value of an oil sands project is contingent upon uncertain oil prices and transport costs. We refer to the spatial price difference as transport costs to avoid confusion over price and spatial price differences. Transport costs include all factors that affect the spatial price difference including pipeline and rail tariffs, exchange rates, and capacity constraints. We assume oil prices follow a geometric Brownian motion (GBM) and transport costs follow a Ornstein-Uhlenbeck (OU) mean-reverting process. These assumptions are consistent with real options and oil price cointegration literature. Stationary process for transport costs implies the world oil market is 'one great pool' (Adelman (1984)) as crude oil prices in different geographical locations move together. Optimal stopping is used to identify the threshold prices when it is optimal to invest in a new project and abandon an operating project.[2] The optimal stopping problems result in free boundary problems that do not have known analytical solutions. Following Wilmott et al. (1993) and Insley and Rollins (2005), the free boundary problems are redefined as linear complementarity problems and we approximate the solutions numerically using a fully implicit finite difference method (IFDM). Model parameters are chosen to approximate a typical *in situ* oil sands project in Northern Alberta.

To preview the results, we find that a decrease in transport costs increases the value of the oil sands project, investments in new projects happen earlier, and operating projects are abandoned later. These results are consistent with the claims made by supporters of the policy to expand pipeline capacity. Surprisingly, we also find that changes in transport cost uncertainty has virtually no effect on the value of the oil sands project or on the decision of when to invest and when to abandon. Typically, the value of an option increases as uncertainty increases as upside potential increases while the option limits downside loses.

1.1 Literature Review

Evaluating natural resource investments using real options analysis is a standard approach in the literature. Brennan and Schwartz (1985) apply option pricing theory to the problem of valuing uncertain investments. They determine the combined value of the options to shut down and restart a copper mine when spot prices are uncertain and the convenience yield is constant. Paddock et al. (1988) combine option-pricing techniques with a model of equilibrium in the market for the underlying asset to value offshore petroleum leases. Bjerksund and Ekern (1990) value a Norwegian oil field with options to defer and abandon. Clarke and Reed (1990) consider the option to abandon a currently producing oil-well when oil prices and extraction rates are uncertain. Conrad and Kotani (2005) determine the trigger prices to initiate investment in the Arctic National Wildlife Refuge under different assumptions about the evolution of crude oil prices. Morck et al. (1989) value forestry resources under stochastic inventories and prices. Insley (2002) and Insley and Rollins (2005) consider the optimal tree harvest problem when tree harvesting can be delayed and output prices follow known stochastic processes. Conrad (2000) determines the order and timing of wilderness preservation, resource extraction, and development when amenity value, the value of the resource, and return from development all follow known stochastic processes.

Recently, a number of papers have analyzed the management of oil sands projects and the rate of oil sands development using real options analysis. Almansour and Insley (2016) extend the Brennan and Schwartz (1985) model to include cost uncertainty and study the optimal management of an oil sands project. *In situ* oil sands projects face high levels of

[2] Insley and Wirjanto (2010) compare dynamic programming and contingent claims approaches for valuing risky investments. They find contingent claims is preferred when data exists that allows for the estimation of the market price of risk or the convenience yield. However, in this setting, it might not be possible to create a perfect hedge as transport costs risk (i.e. crude oil price spreads) may not be actively traded in markets.

cost uncertainty from fluctuations in natural gas prices, natural gas is an important input in the extraction process. Almansour and Insley (2016) extend the Schwartz and Smith (2000) two factor commodity price model by incorporating a deterministic seasonality component. In their paper, commodity prices follow a non-stationary stochastic process made up of three factors: a long-run factor (non-stationary process), a short-run factor (stationary process), and a deterministic function that represents seasonality in the prices. Surprisingly, they find the value of the oil sands project is significantly negatively affected by stochastic costs and the value of the project decreases as cost volatility increases.

Kobari et al. (2014) evaluate the rate of oil sands expansion under different environmental cost scenarios in a dynamic, game-theoretic model. Their model considers a multi-plant/multi-agent setting with price and cost uncertainty. Like Almansour and Insley (2016), cost uncertainty is driven by uncertainty in natural gas prices. The price of oil follows a mean-reverting process with an increasing long-run average price. The cost of natural gas depends on a deterministic seasonality component and a mean-reverting stochastic component. They consider two environmental cost scenarios: an increasing environmental cost scenario and a decreasing environmental cost scenario. Their results show that decreasing environmental costs cause new investments to be delayed compared to increasing environmental costs but decreasing environmental costs have little effect on projects that have already been constructed.

Almansour and Insley (2016) and Kobari et al. (2014) both assume that the price of crude oil and natural gas in Northern Alberta follows the same dynamics as international crude oil and natural gas benchmarks.[3] These assumptions ignore crude oil price differences and factors that affect price differences such as the availability of pipeline capacity, exchange rates, weather, and the cost of diluent.[4] Carney et al. (2013) expect Canadian crude oil prices to remain depressed and more volatile than international benchmarks until sufficient capacity is in place. They believe this is an important issue facing Canadas energy sector and a major factor restraining business investment. This paper hopes to contribute to this literature by focusing on the effect spatial price differences have on a firm's investment decision. Due to the cost of investing in new pipeline projects, understanding how oil sands producers will respond to a decrease in spatial price differences is important for oil transport firms proposing new pipeline projects and for policymakers weighing the cost and benefit of these new pipeline projects.

The rest of the paper is organized as follows. Section 2 presents the general valuation model when price and transport costs are uncertain. Section 3 values a typical *in situ* oil sands project and discusses the results. Section 4 summarizes and concludes the paper.

[3]Almansour and Insley (2016) use weekly WTI futures and Henry Hub (HH) natural gas futures data from January 1995 to August 2010 to calibrate their model and Kobari et al. (2014) use daily WTI futures and HH natural gas futures data from February 2, 2009 to May 10, 2012 to calibrate their model.

[4]Diluent is any lighter hydrocarbon added to heavy crude oil or bitumen in order to facilitate its transportation in crude oil pipelines.

2 General Model

This section presents a model for the valuation of a nonrenewable resource asset with nested real options subject to uncertain prices and transport costs. Here, an oil sands project has two stages: the development stage where a firm holds an option to develop an oil sands project and the operating stage where a firm operates an oil sands project and has an option to abandon for scrap value. Similar to Paddock et al. (1988), the value of the oil sands project in the development stage is contingent on the value of the oil sands project in the operating stage and is therefore a compound option (i.e. option on an option).

The motivation for this model is an *in situ* oil sands project located in Northern Alberta that must transport its output from remote production areas to consuming markets thousands of kilometers away,[5] but we believe it can be applied to any nonrenewable resource project that faces price and transport cost uncertainty.

2.1 Option to Develop an Oil Sands Project

Consider a firm that holds a lease to a previously undeveloped parcel of land that contains a known quantity of crude oil.[6] We assume all expenditures relating to exploration have been made. The lease gives the firm the proprietary right to extract and sell the crude oil from the parcel of land for a specified period of time. If, by the end of the lease, production has not begun the lease expires and the land is returned to the leasee.[7] If production has begun the lease is extended indefinitely, meaning the lease is extended until reserves are exhausted or the project is abandoned.

The lease is viewed as an option to develop an oil sands project. The underlying asset is an operating oil sands project whose value is contingent on the price of crude oil, the cost of transporting crude oil to market, and the amount of reserves in place. The exercise price is the cost of building the required production facilities and transportation infrastructure. The firm's problem is to determine the value of the option to develop and decide at what point in time they will exercise the option to develop given price and transport costs follow known stochastic processes.

Assume price, $S(t)$, follows a GBM and transport cost, $C(t)$, follows an OU mean-reverting process.

$$dS = \mu S dt + \sigma_S S dW_S, \tag{1}$$

$$dC = \kappa(\bar{C} - C)dt + \sigma_C dW_C. \tag{2}$$

Where μ is the drift and σ_S is the standard deviation in price, κ is the speed of reversion, $\bar{C}$ is the long-run average transport cost, and σ_C is the standard deviation in transport cost. dW_S and dW_C are increments of a correlated Brownian motion with correlation coefficient of $\rho_{S,C}$.

[5]The distance between Hardisty, Alberta and Cushing, Oklahoma, two major transportation hubs, is over 2500 kilometers.

[6]The standard term of a primary lease is 15 years.

[7]In Canada, the leasee, generally, refers to the provincial Crown as it own 97 percent of oil sands mineral rights.

Assuming GBM in commodity prices is a standard assumption in the real options literature (Brennan and Schwartz (1985) for copper prices, Paddock et al. (1988) for the value of developed reserves, Clarke and Reed (1990) and Conrad and Kotani (2005) for crude oil prices). Schwartz and Smith (2000) consider a two-factor model for commodity prices that incorporates short-term deviations from the equilibrium price (stationary factor) and long-term random fluctuations in the equilibrium price (nonstationary factor). They show that for long-term investments, short-term deviations from the equilibrium price have little effect on the value of the investment. Therefore, they argue, to simplify analysis a single-factor model that considers uncertainty in the equilibrium price can be used to value long-term investments.

Assuming prices follow a GBM and transport costs follow an OU mean-reverting process is consistent with the literature on crude oil price differentials. A number of authors have examined the co-movement of crude oil prices using cointegration analysis (Gülen (Gülen (1997) and Gülen (1999)), Hammoudeh et al. (2008), and Fattouh (2010)) and have found that crude oil prices differences are stationary. More recently, Wilmot (2013) found that secondary crude oil blends of similar and differing qualities are cointegrated with a structural break. A necessary condition for cointegration analysis is that crude oil prices are integrated of order one; meaning, GBM in crude oil prices is a consistent model with this literature. Empirical evidence of a unit root in crude oil prices has been mixed, therefore, we also consider crude oil prices follows a OU mean-reverting process in subsection 3.4.

Let $G(S, C, \tau)$ be the value of the option to develop an oil sands project at the current price, S, current transport cost, C, and with τ time remaining on the lease. Where $\tau = \bar{T} - t$, t is the current date and $\bar{T}$ is the expiration date of the lease. If $F(S, C, \bar{Q})$ is the value of an operating oil sands project with initial reserves $\bar{Q}$ and IC is the required investment cost then the firm's payoff from exercising the option to develop is $F(S, C, \bar{Q}) - \mathrm{IC}$. If the firm decides not to exercise the option, they receive a payoff of $\mathrm{M}(t)$ per unit of time from the undeveloped parcel of land,[8] and the option to develop an oil sands project in the next period.

The firm's problem of valuing the option to develop an oil sands project and determining the development threshold can be formulated as an optimal stopping problem

$$G(S, C, \tau) = \max \left\{ F(S, C, \bar{Q}) - \mathrm{IC},\ \mathrm{M}dt + \frac{\mathrm{E}_t[G(S + dS, C + dC, \tau + d\tau)]}{1 + \delta_G dt} \right\}. \tag{3}$$

Where E_t is the conditional expectations operator and δ_G is the risk-adjusted constant discount rate.

The development threshold defines a surface that divides the (S, C, τ)-space into two regions: the continuation region and the development region. Let $\hat{S}(C, \tau)$ be the development threshold. The development threshold specifies the price at which the payoff from exercising the option to develop is equal to the payoff from waiting for a given amount of time remaining, τ, and transport cost, C. The continuation region lies below the development threshold, $S < \hat{S}(C, \tau)$. In this area it is optimal to delay development of an oil sands project as the value of delaying exceeds the payoff from development. The development region lies above

[8] The payoff from the undeveloped parcel of land can be either positive or negative.

the development threshold, $S > \hat{S}(C, \tau)$. In this area it is optimal to exercise the option to develop immediately. When $S = \hat{S}(C, \tau)$, the continuation payoff equals the exercise payoff.

In the continuation region, $S \leq \hat{S}$, the value of the option to develop an oil sands project satisfies the following Bellman equation

$$\delta_G G = \mathrm{M} + (1/dt)\mathrm{E}_t[dG]. \tag{4}$$

The Bellman equation requires the firm's payoff from waiting to exercise the option to develop, the right hand side of (4), to equal the required return from holding the option to develop.

Apply Ito's Lemma to $G(S, C, \tau)$ and substitute equations (1) and (2) and rearrange to get

$$\begin{aligned} dG =& (\mu G_S + \kappa(\bar{C} - C)G_C - G_\tau + \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S G_{SC}))dt \\ &+ \sigma_S S G_S dW_S + \sigma_C G_C dW_C. \end{aligned} \tag{5}$$

Equation (5) is the stochastic differential equation for the option to develop an oil sands project. Substitute (5) into the Bellman equation (4) and pass it through the expectations operator to obtain the partial differential equation for the value of the option to develop an oil sands project in the continuation region,

$$\delta_G G = \mathrm{M} + \mu S G_S + \kappa(\bar{C} - C)G_C - G_\tau + \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S G_{SC}). \tag{6}$$

This partial differential equation is subject to the following boundary condition,

$$G(S, C, 0) = \max\{F(S, C, Q) - \mathrm{IC}, 0\}. \tag{7}$$

If the lease reaches the expiration date and the oil sands project has not yet been developed, the option to develop an oil sands project is exercised if the value of the operating oil sands project exceeds the investment cost otherwise the option to develop expires unused.

The development threshold is determined by the following value-matching condition

$$G(\hat{S}(C, \tau), C, \tau) = F(\hat{S}(C, \tau), C, Q) - \mathrm{IC}, \tag{8}$$

and smooth-pasting conditions

$$G_S(\hat{S}(C, \tau), C, \tau) = F_S(\hat{S}(C, \tau), C, Q), \tag{9.1}$$
$$G_C(\hat{S}(C, \tau), C, \tau) = F_C(\hat{S}(C, \tau), C, Q). \tag{9.2}$$

The value-matching condition matches the value of the option to develop to the value of the operating oil sands project minus the investment cost on the optimal stopping boundary. The smooth-pasting conditions are required to jointly solve for the unknown function G and the unknown development threshold $\hat{S}$. On the boundary the functions, G and $F-\mathrm{IC}$, must meet tangentially for $\hat{S}$ to be the optimal stopping boundary.[9]

[9] See Dixit and Pindyck (1994) for a detailed discussion on value-matching and smooth-pasting conditions.

2.1.1 Option to Develop as a Linear Complementarity Problem

Equation (6) and conditions (7), (8), and (9) define a free boundary problem, the solution to the problem determines the value of the option to develop an oil sands project as well as the development threshold. We follow Wilmott et al. (1993) and Insley and Rollins (2005) and redefine the free boundary problem as a linear complementarity problem (LCP).[10] A solution to the LCP is a solution of the free-boundary problem and *vice versa*.[11] A benefit of redefining the free boundary problem as a LCP is that the complications caused by the free-boundary are eliminated and the free boundary can be recovered after the LCP has been solved.

The free boundary problem for the option to develop can be redefined as the following LCP

$$\begin{aligned}
&\delta_G G - \mathrm{M} - \mu S G_S - \kappa(\bar{C} - C)G_C + G_\tau \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S G_{SC}) \geq 0, && (11.1)\\
&G - F + \mathrm{IC} \geq 0, && (11.2)\\
&\big(\delta_G G - \mathrm{M} - \mu S G_S - \kappa(\bar{C} - C)G_C + G_\tau \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S G_{SC})\big) \times \big(G - F + \mathrm{IC}\big) = 0. && (11.3)
\end{aligned}$$

The option to develop, like all American-type options, defined as LCPs has the intuitive interpretation of a rational individual's strategy with regard to holding versus killing the option. For the option to develop, equation (11.1) holds with an equality when it is optimal to hold the option to develop and equation (11.2) is a weak inequality. Equation (11.1) holds with a weak inequality and equation (11.2) holds with an equality when it is optimal to exercise the option to develop. Equation (11.1) can be interpreted as the difference between the required return for holding the option to develop and the actual return from holding the option. When the required return equals the actual return it is optimal to hold the option to develop. When the required return exceeds the actual return it is optimal to exercise the option to develop. Equation (11.1) is nonnegative as realized returns cannot be consistently greater than required returns in equilibrium. Equation (11.2) is nonnegative, if negative it is optimal to exercise the option.

2.2 Operating Oil Sands Project

In subsection (2.1) we determined a free boundary problem for the development of an oil sands project and derived the corresponding LCP for a given value function for an operating oil sands project, $F(S, C, \bar{Q})$. Now we turn to the problem of valuing an operating oil sands project with the option to abandon for scrap value.

[10] A LCP has the following form

$$\begin{aligned} x, F(x) &\geq 0, \\ x^T F(x) &= 0. \end{aligned} \tag{10}$$

Where x is a vector and $F(x)$ is a linear vector valued function.

[11] See Elliot and Ockendon (1982), Friedman (1988), and Kinderlehrer and Stampacchia (1980) for proofs of the existence and uniqueness of the solutions.

After exercising the option to develop, the firm receives an operating oil sands project with the option to abandon for scrap value. While it is operating, crude oil is extracted, transported, and then sold in a perfectly competitive market. The after-tax cash flows from operations, $\pi(q; S, C, Q, z)$, are affected by the amount of output sold, q, the current price and transport cost, and the amount of reserves remaining and other factors including taxes, z. The payoff to the firm from the operating oil sands project are the cash flows from operations and the expected discounted future value of the operating oil sands project. If the firm decides to exercise the option to abandon the firm receives the scrap value of the oil sands project, $\Omega(S, C, Q)$. Here, scrap value represents all the costs associated with abandoning the project and restoring the land to its previous state and is likely to be negative.[12]

The firm's problem of valuing the operating oil sands project with the option to abandon for scrap value can be represented by the following optimal stopping problem

$$F(S, C, Q) = \max \left\{ \Omega(S, C, Q), \max_{q \in [\underline{q}, \bar{q}]} \pi(q; S, C, Q)dt + \frac{\mathrm{E}_t[F(S + dS, C + dC, Q + dQ)]}{1 + \delta_F dt} \right\}. \tag{12}$$

The value of an operating oil sands project is the larger of either exercising the option to abandon immediately or continuing to operate the project. Where δ_F is the risk-adjusted constant discount rate for the operating oil sands project. The firm chooses the flow of output overtime to maximize the expected discounted value of the operating oil sands project. Due to technological and capacity constraints management cannot produce output below $\underline{q}$ or above $\bar{q}$.

The abandonment threshold defines a surface that divides the (S, C, Q)-space into two regions: the continuation region and the abandonment region. Let $S^*(C, Q)$ be the abandonment threshold. The threshold specifies a price for a given amount of reserves, Q, and transport cost, C, where the payoff from abandonment equals the payoff from continuing operations. The continuation region lies above the surface, $S > S^*(C, Q)$. In this area it is optimal to continue operating the project as expected discounted cash flows exceed the scrap value of the project. The abandonment region lies below the surface, $S < S^*(C, Q)$. In this area it is optimal to abandon the project for scrap value as expected discounted cash flows do not justify continued operations. When $S = S^*(C, Q)$, the continuation payoff is equal to the abandonment payoff.

In the continuation region the value of an operating oil sands project satisfies the following Bellman equation

$$\delta_F F = \max_{q \in [\underline{q}, \bar{q}]} \pi(q) + (1/dt)\mathrm{E}_t[dF]. \tag{13}$$

Similar to equation (4), the Bellman equation here requires the firm's payoff from operations to be equal to the required return from operations.

Let $q(t)$ represent the quantity of reserves extracted at a particular point in time so that changes is reserves are

$$dQ = -q dt. \tag{14}$$

[12]Scrap Value may be positive if the option to abandon is exercised before reserves are exhausted and the restored land has some value to other oil producers or another purposes.

This assumption ensures that if the option to abandon is not exercised reserves will be exhausted in finite time and the option to abandon expires unused.

Apply Ito's Lemma to $F(S,C,Q)$ and make the appropriate substitutions to get the stochastic differential equation for an operating oil sands project.

$$\begin{aligned} dF =& (\mu S F_S + \kappa(\bar{C} - C)F_C - qF_Q + \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC}))dt \\ & + \sigma_S S F_S dW_S + \sigma_C F_C dW_C \end{aligned} \tag{15}$$

Substitute (15) in to the Bellman equation (13) and pass through the expectations operator to obtain the following partial differential equation for the value of an operating oil sands project with the option to abandon in the continuation region,

$$\delta_F F = \max_{q\in[\underline{q},\bar{q}]} \pi + \mu S F_S + \kappa(\bar{C} - C)F_C - qF_Q + \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC}) \tag{16}$$

The optimal flow of output is determined by differentiating the right hand side of equation (16) with respect to q. The firm will produce at an interior solution if the marginal cash flow from selling an extra unit of output, π_q, is equal to the shadow price of production an extra unit of output, F_Q. The firm will produce at the lower boundary output boundary if the shadow price exceeds the marginal cash flow at $\underline{q}$. The firm will produce at the upper boundary output boundary if the marginal cash flow exceeds the shadow price at $\bar{q}$.

$$q^* = \begin{cases} \underline{q} & \text{if } \pi_q(\underline{q}) < F_Q \\ q^* & \text{if } \pi_q(q^*) = F_Q \\ \bar{q} & \text{if } \pi_q(\bar{q}) > F_Q \end{cases}$$

Let $q^*(S,C,Q)$ be the optimal output level. At the optimal output level the partial differential equation becomes

$$\delta_F F = \pi(q^*) + \mu S F_S + \kappa(\bar{C} - C)F_C - q^* F_Q + \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC}) \tag{17}$$

The partial differential equation is subject to the following boundary condition,

$$F(S,C,0) = \Omega(S,C,0). \tag{18}$$

When reserves are exhausted the value of an operating oil sands project is equal to the remaining scrap value of the project and the option to abandon expires unused.

The abandonment threshold is determined by the value-matching

$$F(S^*(C,Q),C,Q) = \Omega(S^*(C,Q),C,Q), \tag{19}$$

and smooth-pasting conditions

$$F_S(S^*(C,Q),C,Q) = \Omega_S(S^*(C,Q),C,Q), \tag{20.1}$$
$$F_C(S^*(C,Q),C,Q) = \Omega_C(S^*(C,Q),C,Q). \tag{20.2}$$

2.2.1 Operating Oil Sands Project as a Linear Complementarity Problem

Equation (16) and conditions (18), (19), and (20) define a free boundary problem that determines the value of an operating oil sands project and the abandonment threshold. The free boundary problem for the operating oil sands project can be redefined as the following LCP

$$\begin{aligned}
&\delta_F F - \pi(q^*) - \mu S F_S - \kappa(\bar{C} - C)F_C + q^* F_Q \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC}) \geq 0, && (21.1)\\
&F - \Omega \geq 0, && (21.2)\\
&\big(\delta_F F - \pi(q^*) - \mu S F_S - \kappa(\bar{C} - C)F_C + q^* F_Q \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC})\big) \times \big(F - \Omega\big) = 0. && (21.3)
\end{aligned}$$

Equation (21) has the same intuitive interpretation as equation (11).

3 Results

In this section we use an IFDM to approximate the value of a typical *in situ* oil sands project in Northern Alberta and determine the development and abandonment thresholds. Given the specification of after-tax cash flows in equation (22), we simplify the numerical scheme and reduce the dimensionality of the domain by defining net price, $P = S - C$, so that the value of the option to develop is $g(P, \tau) = G(S, C, \tau)$, and the value of an operating oil sands project is $f(P, Q) = F(S, C, Q)$. With this change, the partial derivatives in equation (11) can be replaced with

$$\begin{aligned}
G_S &= g_P, & G_{SS} &= g_{PP},\\
G_C &= -g_P, & G_{CC} &= g_{PP},\\
G_\tau &= g_\tau, & G_{SC} &= -g_{PP}.
\end{aligned}$$

Similarly for equation (21)

$$\begin{aligned}
F_S &= f_P, & F_{SS} &= f_{PP},\\
F_C &= -f_P, & F_{CC} &= f_{PP},\\
F_Q &= f_Q, & F_{SC} &= -f_{PP}.
\end{aligned}$$

The domain on which the oil sands project is defined is now two dimensional instead of three dimensional. For a fixed transport cost value, the option to develop is defined on the two dimensional discretized domain $\{-C, S_1 - C, \ldots, S_i - C, \ldots, S_M - C\} \times \{0, \tau_1, \ldots, \tau_n, \ldots, \tau_N\}$ and the operating project is defined on $\{-C, S_1 - C, \ldots, S_i - C, \ldots, S_M - C\}$ $\times \{0, Q_1, \ldots, Q_j, \ldots, Q_K\}$.[13] A detailed explanation of the IFDM used in this paper can be found in Appendix A.1.

Oil sands are a complex mixture of sand and other rock material containing crude bitumen. Crude bitumen is a heavy, viscous oil that will not flow in its natural state; as a

[13]The values for S_M, Δ_S, τ_N, Δ_τ, Q_K, and Δ_Q appears to give a good approximation of the value of an oil sands project. Doubling the number of grid nodes has an average approximation error of roughly 3 percent the value of an oil sand project.

result, traditional extraction methods are not appropriate. There are two main methods for recovering crude bitumen from the oil sands mixture, the choice of extraction method depends on the depth of the oil sands deposits. Open-pit mining is used to recover crude bitumen from shallow deposits while *in situ* methods are used to extract crude bitumen from deep deposits.[14] We focus on a project that uses *in situ* methods in this paper for two reasons. First, production from *in situ* projects has exceeded the production from mining projects since 2012 (Canadian Association of Petroleum Producers Canadian Association of Petroleum Producers (2015)). Second, approximately 80 percent of oil sands deposits are too deep to be recovered from open-pit mining and must be extracted using *in situ* methods.

Table 1: *In situ* Oil Sands Project Design Parameters (Canadian Dollars)

Option to Develop	
Length of Lease, years (T)	15
Investment Costs, millions of dollars (IC)	\$1050
Benefits(Costs) from lease (M)	0
Discount Rate (δ_G)	10%
Operating Project	
Production life, years	30
Initial Reserves, millions of barrels ($\bar{Q}$)	328.5
Annual Production, millions of barrels (q)	10.95
Average cost, per barrel (AC)	\$35.00
Scrap Value, (Ω)	0
Royalty Rate (λ_R)	30%
Income Tax Rate (λ_I)	40%
Property Tax Rate (λ_P)	10%
Discount Rate (δ_F)	10%

Table 1 summarizes the assumptions we make about a typical *in situ* oil sands project. All costs are in Canadian dollars. The investment cost, initial reserves, and annual production are from Millington et al. (2014) who estimate the supply costs for various oil sand projects based on their type. We assume the deflated average cost for producing a barrel of oil is constant and equal to \$35. We feel this is a fair assumption as firms operating *in situ* projects in Alberta report average production costs ranging from \$25-49 in 2014 and Millington et al. (2014) estimate supply costs of \$50.89 (excluding transport and blending costs). They define supply costs as the constant dollar price needed to recover all capital expenditures, operating costs, royalties, and taxes and earn a specified return on investment. In Alberta, royalty rates are applied to gross revenue and net revenue and the rates, which range from 25 to 40 percent, depend on the price of WTI. To simplify the analysis we assume a constant royalty rate of 30 percent applied only to net revenue, $S - C$. Income tax rate includes both provincial and federal taxes.[15] We assume that the discount rate for the option to develop

[14] *In situ* methods involves drilling several wells into deep oil sands deposits then injecting steam to heat the bitumen so that it flows and can be pumped to the surface. The primary *in situ* methods used today are the thermal techniques of Cyclic Steam Stimulation (CSS) and Steam Assisted Gravity Drainage (SAGD).

[15] The general federal tax rate is 28 percent and the Alberta provincial corporate tax rate is 10 percent.

and the operating project are both 10 percent. We assume after-tax cash flows are given by

$$\begin{aligned}\pi(q^*; S, C, Q, z) =&((1-\lambda_R)(S-C)-\mathrm{AC})q^* \\ &+\max\{\lambda_I[((1-\lambda_R)(S-C)-\mathrm{AC})q^*], 0\} - \lambda_P F(S, C, Q).\end{aligned} \tag{22}$$

To estimate the parameters in equations (1) and (2) we collect monthly spot price data for WTI and WCS for the period January 2005 to December 2015. WTI data was collected from the EIA and WCS data was collected from Natural Resources Canada. The WTI price series were converted to Canadian dollars using Canada/U.S. exchange rates from the U.S. Federal Reserve and both price series were deflated using the CPI from Statistics Canada. Transport costs estimates were generated by subtracting WCS from WTI.

Table 2: Summary Statistics

	WTI	WCS	Transport Cost
Mean	71.94	58.52	13.42
St. Dev.	14.32	14.65	5.64
Min	40.44	25.88	3.55
Max	117.03	104.32	37.28
Skewness	0.3	0.3	0.93
Kurtosis	0.74	0.76	2.12
AR(1)	0.93	0.88	0.68
Obs.	132	132	132

Note: The data is comprised of monthly spot price data from January 2005 to December 2015. All prices are in real Canadian dollars.

Crude oil prices are assumed to be log-normally distributed with a mean of $\mu - \sigma_S^2/2$ and a variance of σ_S^2. Following Wilmott et al. (1993), the mean and variance are estimated with the following equations

$$\hat{m} = \frac{1}{ndt}\sum_{t=1}^{n}\log(\mathrm{WTI}_t/\mathrm{WTI}_{t-1}),$$

$$\hat{\sigma}_S^2 = \frac{1}{(n-1)dt}\sum_{t=1}^{n}(\log(\mathrm{WTI}_t/\mathrm{WTI}_{t-1}) - \hat{m})^2.$$

The drift, $\hat{\mu}$, is recovered by adding $\hat{\sigma}_S^2/2$ to $\hat{m}$. For the selected data period, the average annual growth rate in WTI is 1 percent with an annual standard deviation of 28 percent. The parameters for equation (2) are estimated by running the regression

$$C_t - C_{t-1} = a + bC_{t-1} + \epsilon_t$$

and then calculating

$$\hat{\kappa} = \frac{-\hat{a}}{\hat{b}},$$

$$\bar{C} = -\log(1+\hat{b}),$$

$$\hat{\sigma}_C = \sigma_\epsilon\sqrt{\frac{\log(1+\hat{b})}{(1+\hat{b})^2 - 1}},$$

where $\hat{\sigma}_\epsilon$ is the standard error of the regression. Over this period the long run average transport cost, $\bar{C}$, is \$13.38, the speed of reversion to the long run average, $\hat{\kappa}$, is 0.39, and the standard deviation, $\hat{\sigma}_C$, is \$3.53. Deviations from the long run average have a half life of $\log(2)/0.39 = 0.77$ years. The estimated correlation between oil prices and transport costs, $\hat{\rho}_{S,C}$, is 0.14.

To understand the effect a change in market access will have on the value of an oil sands project and the incentive to invest we consider changes in the mean and variance of transport costs. To focus on the first-order effects of a change in mean we set transport cost standard deviation equal to zero then solve the model for different mean transport cost values. To understand the second-order effects of transport cost uncertainty on the value of an oil sands project and the incentive to invest we solve the model for different transport cost standard deviation values and different transport cost starting values.

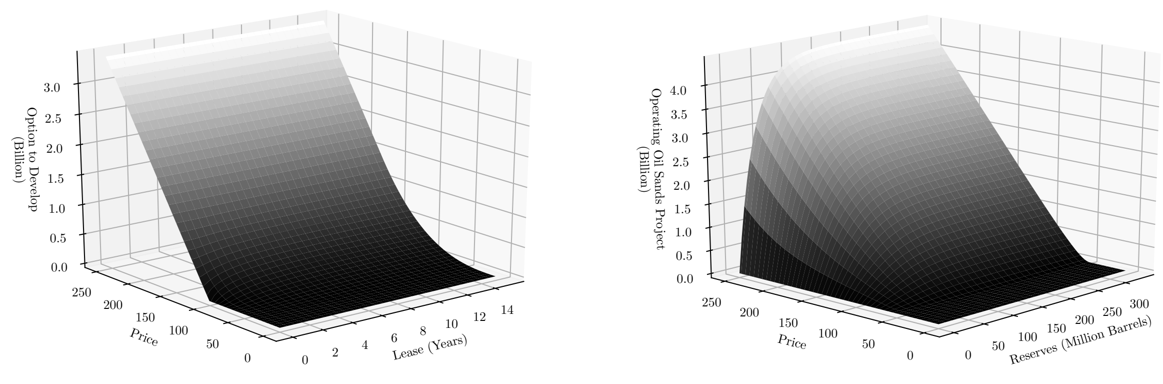
3.1 Value of an Oil Sands Project

Figure 2 plots the value of an oil sands project that faces fixed transport costs of \$13.38 per barrel (i.e. $\sigma_C = 0$). In the development stage shown in Figure 2b, the value of the option to develop is increasing in both price and lease. When the lease expires ($\tau = 0$), if the price of oil is above \$101.50 the option is exercised and the project is developed; this price is the net present value investment rule. If the price is below \$101.50, the lease expires unused and the value of the project is zero. The NPV investment rule in this paper exceeds the supply costs estimated by Millington et al. (2014). They estimate a supply costs of \$86.72 per barrel that adjusts for blending and transportation for a steam-assisted gravity drainage project.[16] The development threshold is shown in Figure 3a. When there are 15 years left on the lease the development threshold reaches its maximum value of \$162.50. The range of values the development threshold takes (\$101.50 to \$162.50) is comparable to some of the results found by Kobari et al. (2014). In their increasing environmental cost scenario, Kobari et al. (2014) find critical thresholds ranging from \$50 to \$150 per barrel. In their decreasing environmental cost scenario, they find critical thresholds ranging from \$150 to \$300 per barrel.

In the operating stage shown in Figure 2b, the value of an operating project is increasing in both price and reserves. When reserves are exhausted the value of the project equals scrap value, in this case zero. The abandonment threshold is shown in Figure 3b. As reserves are extracted the abandonment threshold increases. Projects with low reserves are abandoned before projects with high reserves (i.e. at a higher price) because there is less time for expected price to increase. On the abandonment threshold, cash flows from operations range from -\$5 to -\$16. In this example the project will have a negative net present value before it is abandoned because of the positive value of managerial flexibility. The abandonment threshold found in this example is similar to abandonment threshold found by Almansour and Insley (2016). In their paper, an oil sands project is closed when the price of bitumen is between US\$20 and US\$35 per barrel and a project is abandoned when the price of bitumen is between US\$10 and US\$20 per barrel. They assume the difference between crude oil prices and crude bitumen prices is about US\$30 per barrel. Adding the US\$30 to their closure and

[16] They assume a fixed exchange rate of US\$0.98.

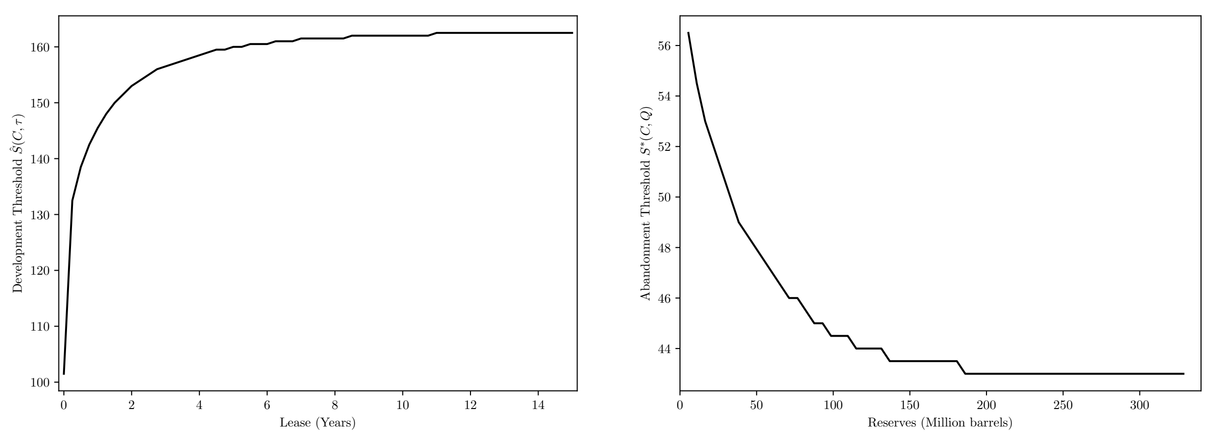
Figure 2: Value of an oil sands project that faces fixed transport costs of \$13.38



(a) Option to develop an oil sands project

(b) Operating oil sands project with an option to abandon for scrap value

Figure 3: Development and abandonment thresholds when transport costs are fixed at \$13.38



(a) Development threshold

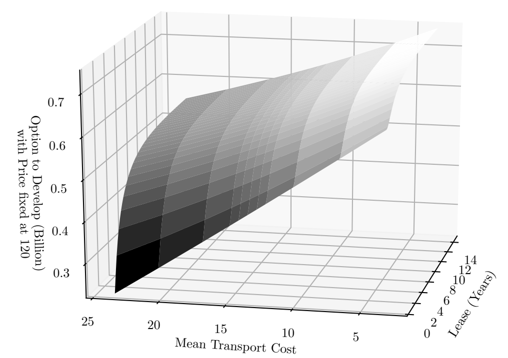
(b) Abandonment threshold

abandonment results and they look similar to the abandonment threshold found here.

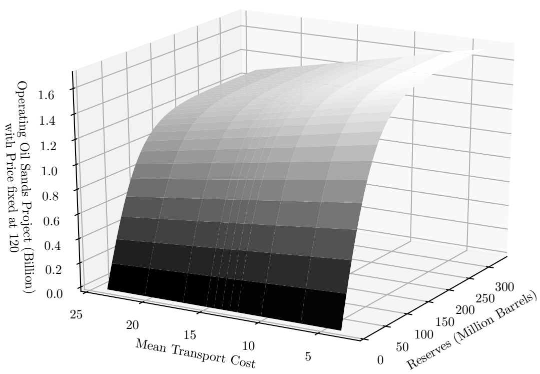
3.2 Effect of a Change in Mean Transport Cost

Consider a change in mean transport cost caused by a change in pipeline capacity. We assume that an increase in capacity will reduce mean transport costs by lowering pipeline tariffs and reducing the shadow cost of pipeline capacity. Figure 4 shows that a decrease in transport costs, resulting from an increase in pipeline capacity, will lead to an increase in the value of an oil sands project regardless of its current stage. The value of an operating project will increase, as shown in Figure 4b, because the expected present value of cash flows increase at all price levels following a decrease in mean transport costs. A similar result would be expected should average production costs decrease. The value of the option to develop an oil sands project will increase (Figure 4a) because the value of the operating

Figure 4: The impact a change in mean transport cost has on the value of an oil sands project



(a) Value of an option to develop an oils sands project with price fixed at 120



(b) Value of an operating oil sands project with an option to abandon for scrap value with price fixed at 120

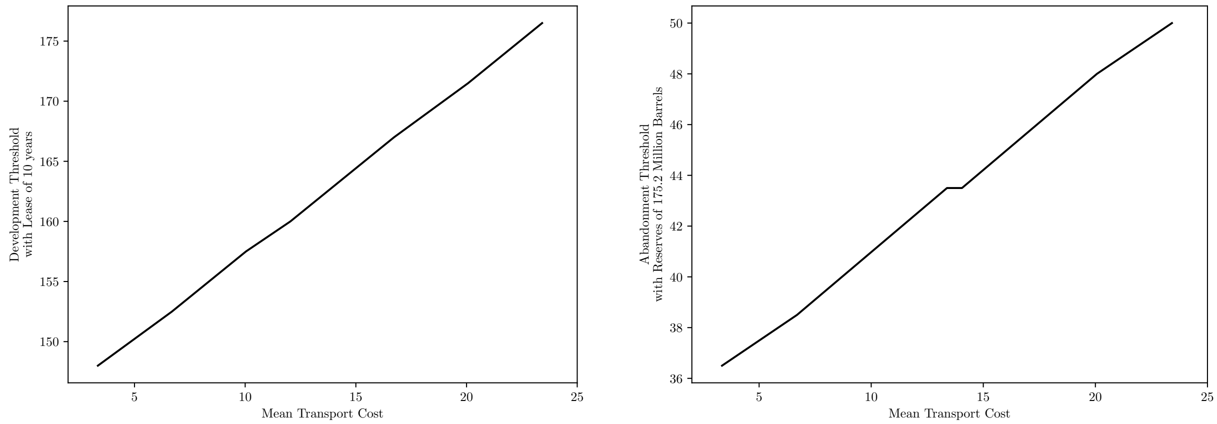
project increases. Figure 4a shows that an oil sands projects will be developed earlier as the value of the underlying asset increases and the benefits/costs from the undeveloped lease have remained unchanged. Figure 5b shows that an operating projects will be abandoned later following a decrease in transport costs resulting from an increase in pipeline capacity as cash flows from operation increase due to lower mean transport costs and the scrap value of the project remains unchanged.

3.3 Effect of a Change in Transport Cost Volatility

In the previous subsections, we approximate the value of an oil sands project and estimate the development and abandonment thresholds when transport costs are nonstochastic and evaluated how these values change when mean transport costs increase or decrease. We found that a decrease in transport costs increases the value of an oil sands project and increases the incentive to invest in new projects. In this subsection, our attention turns to the effect transport cost uncertainty has on the value of an oil sands project and the incentive to invest. The model is solved for different transport cost standard deviation levels and different transport cost starting values.

Before discussing the effect changes in transport cost volatility have on the value of an oil sands project and the development and abandonment thresholds we examine the impact stochastic transport costs have on the development and abandonment thresholds. Figure 6 shows the development and abandonment thresholds for an oil sands project that faces price and transport cost uncertainty. The long run average transport cost is equal to \$13.38 and the standard deviation is 3.53. Figure 6a shows that when transport costs deviate below the long run average oil sands projects are developed earlier as the firm takes advantage of temporarily elevated net prices, $S - C$. When transport costs rise above the long run average projects are developed later as the firm delays development to allow the net price to recover to sufficiently high levels. Deviations from the long run average transport cost has a

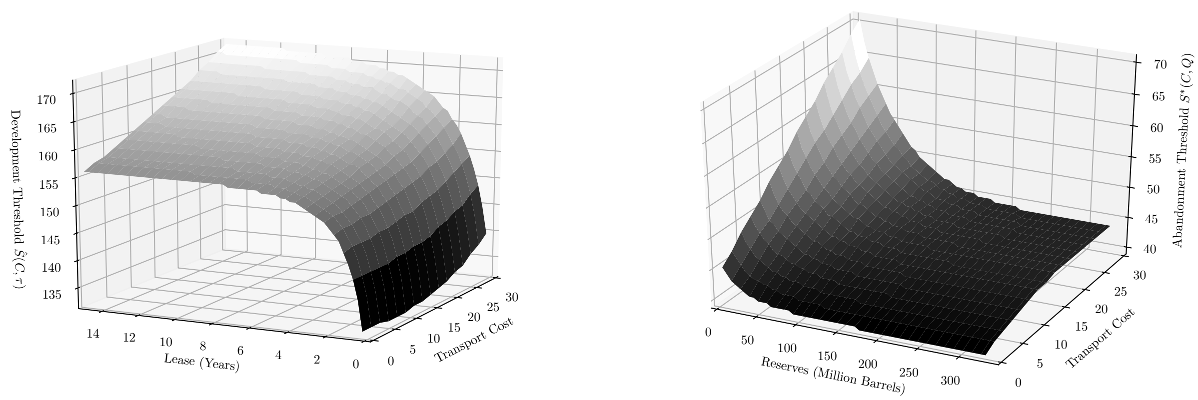
Figure 5: Impact mean transport cost has on the development and abandonment thresholds



(a) Development threshold when time remaining on the lease is fixed at 10 years

(b) Abandonment threshold when reserves are fixed at 175.2 million barrels

Figure 6: Development and abandonment thresholds when transport costs are uncertain



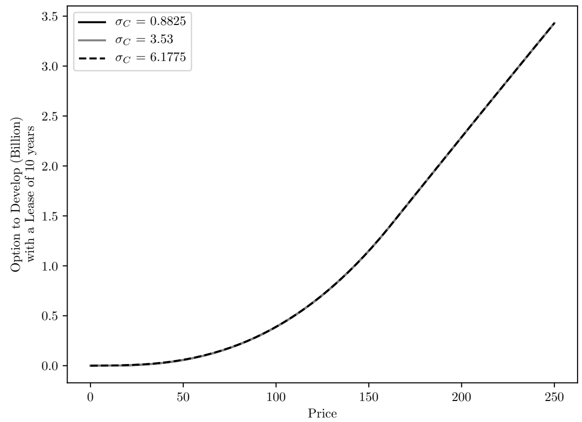
(a) Development threshold

(b) Abandonment threshold

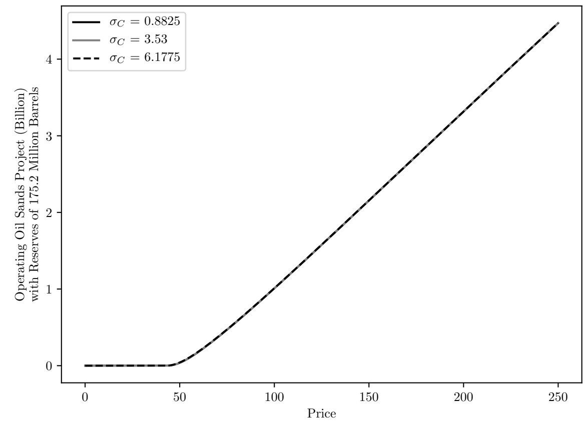
similar impact on an operating project. Figure 6b shows that a project is abandoned later if transport costs are below the long run average than if they are above the long run average. The impact of deviations in transport costs become larger as reserves are extracted. At high levels of reserves, $Q = 175.2$ million barrels, the abandonment price ranges from \$41 when transport costs are \$4 to \$44 when transport costs are \$22. As reserves are depleted, $Q = 54.75$ million the abandonment range increases to \$45 to \$51.

Figure 7 and 8 presents the results from solving the model with different transport cost standard deviation levels and different transport cost starting values. Figure 7 shows, the unexpected result, that transport cost uncertainty has no impact on the value of an oil sands project. In Figure 7a the time remaining on an oil sands lease is fixed at 10 years and in Figure 7b the amount of reserves remaining are fixed at 175.2 million barrels. The standard result in the option-pricing literature is that an increase in the volatility of the underlying asset increases the options value as upside potential increases while downside loses remain

Figure 7: The effect of σ_C on the value of an oil sands project



(a) Value of an option to develop an oil sands project when time remaining on the lease is fixed at 10 years



(b) Value of an operating oil sands project with an option to abandon for scrap value when reserves are fixed at 175.2 million barrels

unchanged. Almansour and Insley (2016) had a surprising result when they found that cost uncertainty had a significant negative impact on the value of an oil sands project. The result in this paper may be the outcome from the choice in transport cost model. Schwartz and Smith (2000) found short-term deviations from the equilibrium price level very little small impact on the value of long-lived projects. What was important in their model was long run uncertainty in the equilibrium price. In this paper, a deviation from the long run average transport cost is short lived, 0.77 years, compared to the life of the oil sands project which is 30 years. The model is re-solved with the speed of reversion set equal to zero; however, the results are unchanged. It appears what matters in this model is the expected transport cost over the life of a project. A reduction in expected transport costs, resulting from an expansion in pipeline capacity, will increase the incentive to invest in new oil sands projects as expected transport costs have been reduced. Figure 8 confirms the result. Transport cost volatility has a very small impact on the development and abandonment thresholds. The small differences likely result from numerical error.

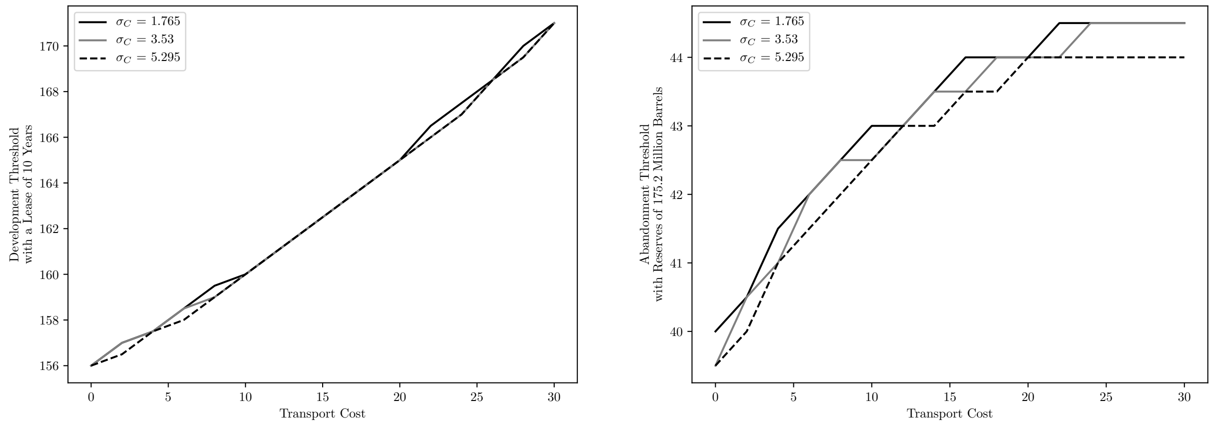
Given that transport cost volatility has no impact on the value of an oil sands project we expect development and abandonment thresholds to be unaffected by changes in transport cost volatility. Figure 8 confirms our expectations. Figure 7a plots the development threshold for three different transport cost volatility levels with lease length fixed at 10 years and Figure 7b plots the abandonment thresholds with 175.2 million reserves. The differences in the figures are on the order of \$0.50 and are likely the result of numerical error.

3.4 Alternative Price Process

In this subsection the value of an oil sands project will be calculated under the alternative assumption that crude oil prices follow an OU mean-reverting process,

$$dS = \mu(\bar{S} - S)dt + \sigma_S dW_S. \tag{23}$$

Figure 8: The effect of σ_C on the development and abandonment thresholds



(a) Development threshold when time remaining on the lease is fixed at 10 years

(b) Abandonment threshold when reserves are fixed at 175.2 million barrels

Crude oil prices revert to the long run average price, $\bar{S}$, at a speed determined by μ. Crude oil prices have a constant standard deviation of σ_S.[17] If prices are mean-reverting the PDE for an oil sands project is

$$\delta_G G = \mathrm{M} + \mu(\bar{S} - S)G_S + \kappa(\bar{C} - C)G_C - G_\tau + \frac{1}{2}(\sigma_S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S\sigma_C\rho_{S,C}G_{SC}), \quad (24)$$

when the project is in the development stage and

$$\delta_F F = \pi(q^*) + \mu(\bar{S} - S)F_S + \kappa(\bar{C} - C)F_C - q^* F_Q + \frac{1}{2}(\sigma_S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C}F_{SC}), \quad (25)$$

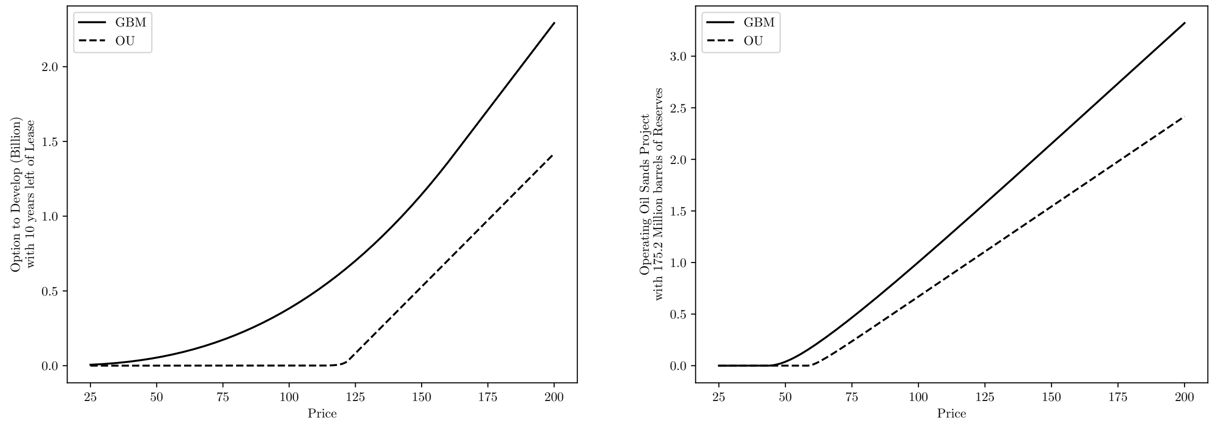
when the project is in the operating stage. The parameters of equation (23) are estimated using WTI data following the same procedure as described above. The long run average price, $\bar{S}$, is 70.27, the speed of reversion to the long run average, $\hat{\mu}$, is 0.058, and the standard deviation is $\hat{\sigma}_S$, is 3.783. Deviations from the long run average have a half life of 5.2 years. The resulting LCPs are solved using a method similar to that described in Appendix A.1.

There is no consensus regarding stationarity versus non-stationarity in crude oil prices. Pindyck (1999) found evidence of mean-reversion in oil prices over a period of 127 years, however, the speed of reversion was slow. Alternatively, a number of papers have examined the co-movement of crude oil prices using various cointegration approaches; testing for cointegration requires crude oil prices to be non-stationary. Insley (2002) shows that the choice of stochastic process for the underlying asset is important as the value of a real option is sensitive to this decision. Conrad and Kotani (2005) found the expected present net revenue from Arctic National Wildlife Refuge fields is higher when oil prices follow GBM than when they follow an OU mean-reverting process. Their results also show that investment is initiated earlier when oil prices follow GBM than OU mean-reverting process.

Figures 9 and 10 compare the results when oil prices follow a GBM and an OU mean-reverting process. Figure 9 plots the value of an oil sands project when time remaining on

[17] Alternative mean-reverting processes considered include $dS = \mu(\bar{S} - S)dt + \sigma_S S dW_S$ and $dS = \mu(\bar{S} - S)Sdt + \sigma_S S dW_S$.

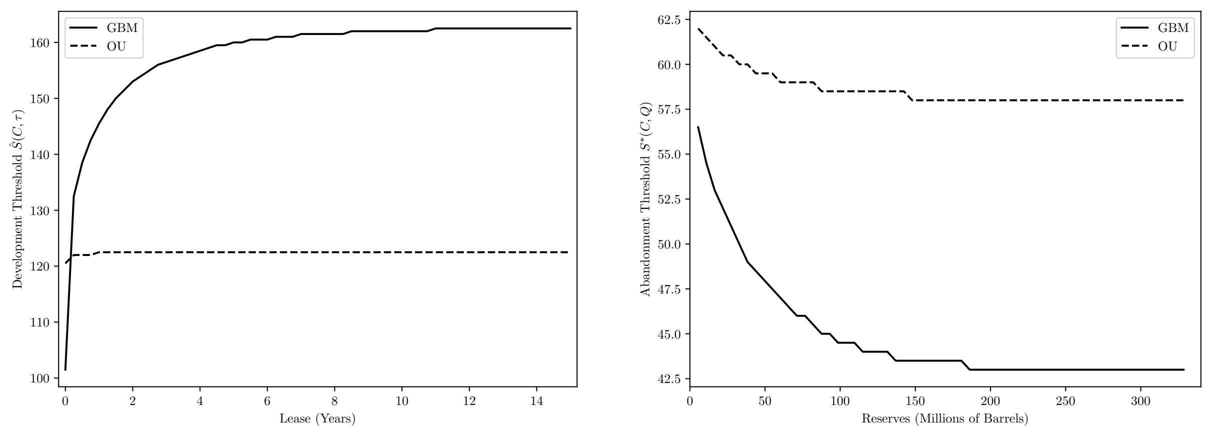
Figure 9: Value of an oil sands project when price follows a mean-reverting process



(a) Option to develop an oil sands project

(b) Operating oil sands project with the option to abandon for scrap value

Figure 10: Development and abandonment thresholds when price follows a mean-reverting process



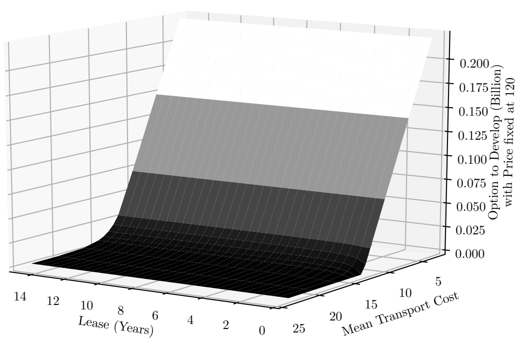
(a) Development threshold

(b) Abandonment threshold

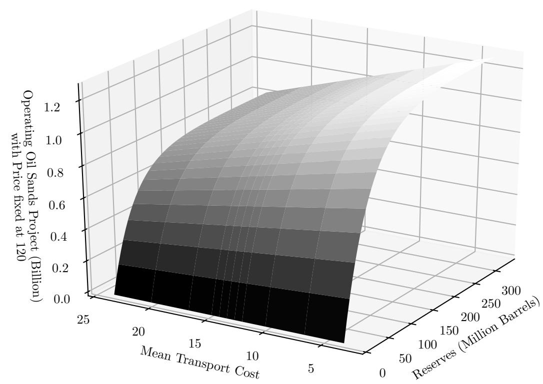
the lease is fixed at 10 years and the amount of reserves are fixed at 175.2 million barrels. Similar to the results found by Conrad and Kotani (2005), Figure 9 shows the value of an oil sands project is higher when oil prices follow a GBM than when they follow an OU mean-reverting process. When oil prices follow an OU mean-reverting process with a long run average equal to \$70.27, the option to develop will have very little value until oil prices rise above roughly \$120, whereas, when oil prices follow a GBM the option to develop will have similar value at very low oil prices. This difference in value is because it is very unlikely an oil sands project will be developed when oil prices follow an OU mean-reverting process. Figure 10a plots the development threshold, it ranges from \$120.5 when the lease expires to \$122.5 when there is 15 years remaining on the lease. This threshold lies well above the long run average oil price, roughly 13 standard deviations.

Figures 11 and 12 show the effect a change in mean transport cost has on the value of an oil sands project and development and abandonment thresholds. Figure 11 plots the value

Figure 11: The impact a change in mean transport cost has on the value of an oil sands project when price follows a mean-reverting process



(a) Value of an option to develop an oil sands project with price fixed at 120



(b) Value of an operating oil sands project with an option to abandon for scrap value with price fixed at 120

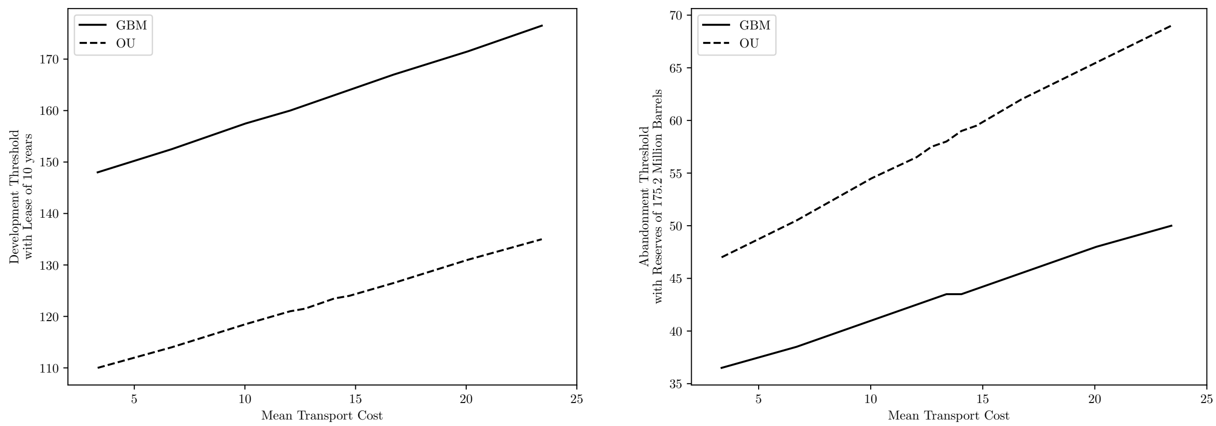
of an oil sands project at different mean transport costs. The figure shows similar results to those found in Figure 4. We find that regardless of the assumed price process, a decrease in mean transport cost increases the value of an oil sands project as expected cash flows increase at all prices levels. Figure 12a plots the development threshold when time remaining on the lease is fixed at 10 years and Figure 12b plots the abandonment threshold when reserves are fixed at 175.2 million barrels. The optimal thresholds have very similar shapes under both assumptions about oil price dynamics. As mean transport costs decrease so to does the development and abandonment thresholds. If oil prices follow a mean-reverting process lower average transport costs resulting from an increase in pipeline capacity will increase the value of an oil sands project and will lower the development and abandonment thresholds so that project will be developed sooner and abandoned later.

4 Conclusion

This paper examines the impact spatial price differences have on the value of an oil sands project and the incentive to invest in new projects. A real options model for the valuation of an oil sands project located in Northern Alberta is developed that incorporates price and transport cost uncertainty. The free-boundary problems that determines the value of the oil sands and the investment thresholds are defined as linear complementarity problems and numerically solved using a fully implicit finite difference method.

Results for a typical *in situ* oil sands project show that average transport costs are an important factor in the decision whether to start a new project or not while transport cost uncertainty has virtually no impact on the investment decision. Results indicate that the price difference faced by oil sands producers is an important factor restraining new investments. The result suggests new pipeline projects that would reduce the price difference would increase the value of existing oil sands projects and would increase the incentive to

Figure 12: The impact a change in mean transport cost has on the development and abandonment thresholds when price follows a mean-reverting process



(a) Development threshold when time remaining on the lease is fixed at 10 years

(b) Abandonment threshold when reserves are fixed at 175.2 million barrels

invest in new projects.

We have seen that transport cost uncertainty has very little impact on the value of an oil sands project and the incentive to invest. What matters here is the average transport cost over the life of the oil sands project. In this paper we considered changes in the average transport cost but ignored uncertainty in average transport costs. Future research would incorporate this uncertainty into the value of an oil sands project by modeling average transport costs as a Poisson process. At some future date transport costs might jump up or down as a result of a decrease or an increase in pipeline capacity.

References

Adelman, M. (1984). International oil agreements. *The Energy Journal*, 5(3):1–9.

Almansour, A. and Insley, M. (2016). The impact of stochastic extraction cost on the value of an exhaustible resource: An application to the Alberta oil sands. *The Energy Journal*, 37(2):61–88.

Bjerksund, P. and Ekern, S. (1990). Managing investment opportunities under price uncertainty: From "last chance" to "wait and see" strategies. *Financial Management*, 19(3):65–83.

Brennan, M. J. and Schwartz, E. S. (1985). Evaluating natural resource investments. *The Journal of Business*, 58(2):135–157.

Canadian Association of Petroleum Producers (2015). Crude oil forecast, markets, and transportation. Technical report, Canadian Association of Petroleum Producers.

Carney, M., Macklem, T., Murray, J., Lane, T., Cote, A., and Schembri, L. (2013). Monetary policy report. Technical report, Bank of Canada.

Clarke, H. R. and Reed, W. J. (1990). Oil-well valuation and abandonment with price and extraction rate uncertain. *Resources and Energy*, 12:361–382.

Conrad, J. M. (2000). Wilderness: Options to preserve, extract, or develop. *Resource and Energy Economics*, 22:205–219.

Conrad, J. M. and Kotani, K. (2005). When to drill? trigger prices for the arctic national wldlife refuge. *Resource and Energy Economics*, 27:273–286.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under Uncertainty*. Princeton University Press.

Elliot, C. M. and Ockendon, J. R. (1982). *Weak and Variational Methods for Free and Moving Boundary Problems*. Pitman.

Fattouh, B. (2010). The dynamics of crude oil price differentials. *Energy Economics*, 32:334–342.

Friedman, A. (1988). *variational Principles and Free Boundary Problems*. Robert Krieger Publishing.

Gülen, S. G. (1997). Regionalization in the world crude oil market. *The Energy Journal*, 18(2):109–126.

Gülen, S. G. (1999). Regionalization in the world crude oil market: Further evidence. *The Engergy Journal*, 20(1):125–139.

Hammoudeh, S. M., Ewing, B. T., and Thompson, M. A. (2008). Threshold cointegration analysis of crude oil benchmarks. *The Energy Journal*, 29(4):79–95.

Insley, M. (2002). A real options approach to the valuation of a forestry investment. *Journal of Environmental Economics and Management*, 44:471–492.

Insley, M. and Rollins, K. (2005). On solving the multirotational timber harvesting problem with stochastic prices: A linear complementarity formulation. *American Journal of Agricultural Economics*, 87(3):735–755.

Insley, M. and Wirjanto, T. (2010). Contrasting two approaches in real options valuation: Contingent claims versus dynamic programming. *Journal of Forest Economics*, 12:157–176.

Kinderlehrer, D. and Stampacchia, G. (1980). *An Introduction to Variational Inequalities and Their Applications*. Academic Press.

Kobari, L., Jaimungal, S., and Lawryshyn, Y. (2014). A real options modol to evaluate the effect of environmental policies on the oil sands rate of expansion. *Energy Economics*, 45:155–165.

Millington, D., Murillo, C. A., and McWhinney, R. (2014). Canadian oil sands supply costs and development projects (2014-2046). Technical report, Canadian Energy Research Institute.

Morck, R., Schwartz, E., and Stangeland, D. (1989). The valuation of forestry resources under stochastic prices and inventories. *The Journal of Financial and Quantitative Analysis*, 24(4):473–487.

Paddock, J. L., Siegel, D. R., and Smith, J. L. (1988). Option valuation of claims on real assets: The case of offshore petroleum leases. *The Quarterly Journal of Economics*, pages 479–508.

Pindyck, R. S. (1999). The Long-Run Evolution of Energy Prices. *The Energy Journal*, 20(2):1–27.

Samuelson, P. A. (1952). Spatial price equilibrium and linear programming. *The American Economic Review*, 42(3):283–303.

Schwartz, E. and Smith, J. E. (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science*, 46(7):893–911.

Takayama, T. and Judge, G. G. (1971). *Spatial and Temporal Price and Allocation Models*. North-Holland.

Wilmot, N. A. (2013). Cointegration in the oil market among regional blends. *International Journal of Energy Economics and Policy*, 3(4):424–433.

Wilmott, P., Dewynne, J., and Howison, S. (1993). *Option Pricing: Mathematical Models and Computation*. Oxford Financial Press.

Zhu, Y., Wu, X., and Chern, I.-L. (2004). *Derivative Securitites and Difference Methods*. Springer.

Appendices

A Numerical Methods

A.1 Fully Implicit Finite Difference Method

The fully implicit finite difference method (IFDM) is an established technique for numerically solving option pricing problems (Wilmott et al. (1993) and Zhu et al. (2004)) that involves discretizing the domain and replacing partial derivatives with backward difference and symmetric central difference approximations. A benefit of the IFDM is that it does not require step lengths in one direction on the domain to be proportionate to step lengths in another direction for stability or convergence. In this appendix we numerically approximate the value of an oil sands project using the IFDM. The following linear complementarity problems determine the value of the oil sands project. The option to develop an oil sands project is the solution to equation (11) and the value of an operating oil sands project with the option to abandon is the solution to equation (21).

The value functions $G(S,C,\tau)$ and $F(S,C,Q)$ depend on three state variables. To simplify the numerical scheme and reduce the dimensionality of the domain, let $P = S - C$ be the net price, so that $g(P,\tau) = G(S,C,\tau)$, and $f(P,Q) = F(S,C,Q)$. The partial derivatives in equation (11) can be replaced with,

$$\begin{array}{ll} G_S = g_P, & G_{SS} = g_{PP}, \\ G_C = -g_P, & G_{CC} = g_{PP}, \\ G_\tau = g_\tau, & G_{SC} = -g_{PP}. \end{array}$$

Similarly for equation (21)

$$\begin{array}{ll} F_S = f_P, & F_{SS} = f_{PP}, \\ F_C = -f_P, & F_{CC} = f_{PP}, \\ F_Q = f_Q, & F_{SC} = -f_{PP}. \end{array}$$

Substitution and rearrange to get simplified LCPs for the option to develop

$$\begin{aligned} &\delta_G g - \mathrm{M} - (\mu S - \kappa(\bar{C} - C))g_P + g_\tau \\ &\qquad - \frac{1}{2}(\sigma_S^2 S^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S)g_{PP} \geq 0, &(26.1)\\ &g - f + \mathrm{IC} \geq 0, &(26.2)\\ &\big(\delta_G g - \mathrm{M} - (\mu S - \kappa(\bar{C} - C))g_P + g_\tau \\ &\qquad - \frac{1}{2}(\sigma_S^2 S^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S)g_{PP}\big) \times \big(g - f + \mathrm{IC}\big) = 0. &(26.3) \end{aligned}$$

and the simplified LCP for the operating project

$$\begin{aligned} &\delta_F f - \pi(q^*) - (\mu S - \kappa(\bar{C} - C))f_P + q^* f_Q \\ &\qquad - \frac{1}{2}(\sigma_S^2 S^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S)f_{PP} \geq 0, &(27.1)\\ &f - \Omega \geq 0, &(27.2)\\ &\big(\delta_F f - \pi(q^*) - (\mu S - \kappa(\bar{C} - C))f_P + q^* f_Q \\ &\qquad - \frac{1}{2}(\sigma_S^2 S^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S)f_{PP}\big) \times \big(f - \Omega\big) = 0. &(27.3) \end{aligned}$$

Define on the axes for S, τ, and Q by

$$\begin{aligned} &\{0, S_1, \ldots, S_i, \ldots, S_M\}, \\ &\{0, \tau_1, \ldots, \tau_n, \ldots, \tau_N\}, \\ &\{0, Q_1, \ldots, Q_j, \ldots, Q_K\}. \end{aligned} \qquad (28)$$

For a given value of C, a typical grid point $(S_i - C, \tau_n)$ on the discretized $(S - C) \times \tau$ mesh, the value of the option to develop is $g(S_i - C, \tau_n) = g_i^n$. For a typical grid point $(S_i - C, Q_j)$ on the discretized $(S - C) \times Q$ mesh, the value of the operating project is $f(S_i - C, Q_j) = f_i^j$.

The IFDM involves using backward difference approximation for g_τ and f_Q and symmetric central difference approximation for the terms g_P, g_{PP}, f_P and f_{PP}. The backward difference and symmetric central difference equations can be written

$$\begin{array}{ll} g_\tau = \frac{g_i^{n+1}-g_i^n}{\Delta\tau} + O(\Delta\tau) & f_Q = \frac{f_i^{j+1}-f_i^j}{\Delta Q} + O(\Delta Q) \\ g_P = \frac{g_{i+1}^{n+1}-g_{i-1}^{n+1}}{2\Delta P} + O(\Delta P^2) & f_P = \frac{f_{i+1}^{j+1}-f_{i-1}^{j+1}}{2\Delta P} + O(\Delta P^2) \\ g_{PP} = \frac{g_{i+1}^{n+1}-2g_i^{n+1}+g_{i-1}^{n+1}}{\Delta P^2} + O(\Delta P^2) & f_{SS} = \frac{f_{i+1}^{j+1}-2f_i^{j+1}+f_{i-1}^{j+1}}{\Delta P^2} + O(\Delta P^2) \end{array} \tag{29}$$

where ΔP is the constant step length in the P direction,[18] $\Delta\tau$ is the constant step length in the τ direction, and ΔQ is the constant step length in the Q direction.

Assume the flow of benefits (costs) from an undeveloped oil sands lease is

$$\mathrm{M} - \lambda_P g(P,\tau) \tag{30}$$

and the cash flow from operations is

$$\begin{aligned} \pi(q^*; S-C, Q) =& \big((1-\lambda_R)(S-C) - AC\big)q^* + \max\{\lambda_I[\big((1-\lambda_R)(S-C) - AC\big)q^*], 0\} \\ & - \lambda_P f((S-C), Q). \end{aligned} \tag{31}$$

Regardless of whether the project has been developed on not, property tax rates, λ_P, are applied to the value of the oil sands project. When the project has been developed, royalty rates, λ_R, are applied to net revenue and income tax rates, λ_I, are applied to profits net royalty payments. The output flow, q^* and the average cost of producing a barrel of oil, AC, are assumed to be constant over the life of the project.

Using the finite difference equations defined in (29) and equation (30), the discretized LCP for the option to develop at an interior node is

$$-\Delta\tau a_i g_{i-1}^{n+1} + (1+\Delta\tau(\delta_G + \lambda_P + a_i + b_i))g_i^{n+1} - \Delta\tau b_i g_{i+1}^{n+1} - g_i^n - \Delta\tau \mathrm{M} \geq 0 \tag{32.1}$$

$$g_i^{n+1} - f_i^N + \mathrm{IC} \geq 0 \tag{32.2}$$

$$\begin{aligned} &\big(-\Delta\tau a_i g_{i-1}^{n+1} + (1+\Delta\tau(\delta_G + \lambda_P + a_i + b_i))g_i^{n+1} - \Delta\tau b_i g_{i+1}^{n+1} - g_i^n - \Delta\tau \mathrm{M}\big) \\ &\qquad \times (g_i^{n+1} - f_i^N + \mathrm{IC}) = 0. \end{aligned} \tag{32.3}$$

With equation (31), the discretized LCP for the operating project at an interior node is

$$\begin{aligned} &-\Delta Q a_i f_{i-1}^{j+1} + (q + \Delta Q(\delta_F + \lambda_P + a_i + b_i))f_i^{j+1} - \Delta Q b_i f_{i+1}^{j+1} - q f_i^j \\ &\qquad - \Delta Q(((1-\lambda_R)P_i - AC)q - \max\{\lambda_I[((1-\lambda_R)P_i - AC)q], 0\}) \geq 0 \end{aligned} \tag{33.1}$$

$$f_i^{j+1} - \Omega \geq 0 \tag{33.2}$$

$$\begin{aligned} &\big(-\Delta Q a_i f_{i-1}^{j+1} + (q + \Delta Q(\delta_F + \lambda_P + a_i + b_i))f_i^{j+1} - \Delta Q b_i f_{i+1}^{j+1} - q f_i^j \\ &\qquad - \Delta Q(((1-\lambda_R)P_i - AC)q - \max\{\lambda_I[((1-\lambda_R)P_i - AC)q], 0\})\big) \\ &\qquad \times \big(f_i^{j+1} - \Omega\big) = 0 \end{aligned} \tag{33.3}$$

[18] Here the step length $\Delta P = \Delta S$ because $P = S - C$.

Where

$$a_i = \frac{\sigma_S^2 S_i^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S_i}{2\Delta P^2} - \frac{\mu S_i - \kappa(\bar{C} - C)}{2\Delta P}, \tag{34.1}$$

$$b_i = \frac{\sigma_S^2 S_i^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S_i}{2\Delta P^2} + \frac{\mu S_i - \kappa(\bar{C} - C)}{2\Delta P}. \tag{34.2}$$

To implement the IFDM we need to impose the following boundary conditions on the value of the option to develop an oil sands project,

$$g(-C, \tau) = \mathrm{M}\Delta t + \frac{\mathrm{E}_t[g(-C, \tau + d\tau)]}{1 + \delta_G dt} \tag{35.1}$$

$$\lim_{S\to\infty} g(S - C, \tau) = \lim_{S\to\infty} f(S - C, Q) - \mathrm{IC}, \tag{35.2}$$

When price goes to zero, The likelihood of development gets very small and the value of the option to develop approaches the present discounted value of benefits (costs) from the undeveloped land. When the price gets very large, the option to develop will be exercised immediately as the benefits from immediate development outweigh the costs. From these assumptions we get the following boundary conditions for the discrete LCP

$$g_0^n = \frac{(1+\delta_G)(1+\lambda_P) - [(1+\delta_G)(1+\lambda_P)]^{n-1}}{(1+\delta_G)(1+\lambda_P) - 1}\frac{\mathrm{M}}{1+\lambda_P} + \frac{g_0^0}{[(1+\lambda_P)(1+\delta_G)]^n}, \tag{36.1}$$

$$g_M^n = f_M^K - \mathrm{IC}. \tag{36.2}$$

The boundary conditions for the operating oil sands project are

$$f(-C, Q) = \Omega \tag{37.1}$$

$$\lim_{S\to\infty} f(S - C, Q) = \lim_{S\to\infty} \pi(\bar{q}; S - C, Q)dt + \frac{\mathrm{E}_t[F(S - C, Q + dQ)]}{1 + \delta_F \Delta t}. \tag{37.2}$$

When the prices goes to zero, the option to abandon will be exercised immediately. When the price gets very large, the value of an operating project approaches the present discounted value of cash flows from operation and the value of the option to abandon goes to zero. This happens because the likelihood of exercising the abandonment option is very small when the price is very large. From these assumptions we get the following boundary conditions for the discrete LCP

$$f_0^j = \Omega \tag{38.1}$$

$$\begin{aligned} f_M^j =& \frac{(1+\lambda_P)(1+\delta_F) - [(1+\lambda_P)(1+\delta_F)]^{n-1}}{(1+\lambda_P)(1+\delta_F) - 1}\frac{\big((1-\lambda_I)(1-\lambda_R)(S_M - C) - AC\big)q}{1+\lambda_P} \\ &+ \frac{f_M^0}{[(1+\lambda_P)(1+\delta_F)]^n}. \end{aligned} \tag{38.2}$$

For a given value of τ_n, equation (26) can be arranged from S_1 to S_{M-1} to form the following system of equations

$$Ag^{n+1} - g^n - \Delta\tau\mathrm{M}^n \geq 0 \tag{39.1}$$

$$g^{n+1} - (f^K - \mathrm{IC}) \geq 0 \tag{39.2}$$

$$\langle Ag^{n+1} - g^n - \Delta\tau\mathrm{M}^n, g^{n+1} - (f^K - \mathrm{IC})\rangle = 0 \tag{39.3}$$

Where A is a $M-1\times M-1$ tridiagonal positive semi-definite matrix,[19] with diagonal terms $A_{i,i}=1+\Delta\tau(\delta_G+\lambda_P+a_i+b_i)$ and off diagonal terms $A_{i,i-1}=-\Delta\tau a_i$ and $A_{i,i+1}=-\Delta\tau b_i$. g^{n+1} is an $M-1$ vector of unknown values, g^n and M^n are $M-1$ vectors of known values.

$$g^{n+1}=\begin{pmatrix} g_1^{n+1}\\ \vdots \\ g_{M-1}^{n+1}\end{pmatrix},\quad g^n=\begin{pmatrix} g_1^{n}\\ \vdots \\ g_{M-1}^{n}\end{pmatrix},\quad \mathrm{M}^{n+1}=\begin{pmatrix}(1+a_1)\mathrm{M}+a_1\frac{1-\lambda_P-\lambda_P\delta_G}{1+\delta_G}g_0^n\\ \mathrm{M}\\ \vdots\\ \mathrm{M}+b_{M-1}(f_M^K-IC)\end{pmatrix}.$$

The terminal condition specifies the value of the option to delay development of an oil sands project when the lease has expired. Using this and the information given by equations (35.1) and (35.2) the value of the option to develop an oil sands project can be approximated at all other nodes in the domain. The optimal stopping boundary $\hat{S}_i(C,\tau_n)$ is recovered using equation (39.2). For a given τ_n, the smallest indexed price S_i where $g_i^n=f_i^K-\mathrm{IC}$ is the price where it is optimal to exercise the option to develop.

For a given value of Q_j, equation (27) can be arranged to form the following system of equations

$$Bf^{j+1}-f^j-\Delta Q\Pi^j\geq 0 \tag{40.1}$$
$$f^{j+1}-\Omega\geq 0 \tag{40.2}$$
$$\langle Bf^{j+1}-f^j-\Delta Q\Pi^j, f^{j+1}-\Omega)\rangle=0 \tag{40.3}$$

B is a $M-1\times M-1$ tridiagonal positive semi-definite matrix, with diagonal elements $B_{i,i}=q+\Delta Q(\delta_F+\lambda_P+a_i+b_i)$ and off diagonal elements $B_{i,i-1}=-\Delta Qa_i$ and $B_{i,i+1}=-\Delta Qb_i$. f^{j+1} is a $M-1$ vector of unknown values, f^j and Π^j are $M-1$ vectors of known values. With

$$f^{j+1}=\begin{pmatrix} f_1^{j+1}\\ \vdots \\ f_{M-1}^{j+1}\end{pmatrix},\quad f^j=\begin{pmatrix} f_1^{j}\\ \vdots \\ f_{M-1}^{j}\end{pmatrix},$$

$$\Pi^j=\begin{pmatrix}((1-\lambda_R)((S_1-C)-AC)q+a_1\Omega\\ ((1-\lambda_R)(S_2-C)-AC)q-\max\{\lambda_I[(1-\lambda_R)(S_2-C)-AC]q,0\}\\ \vdots\\ (1-\lambda_I)((1-\lambda_R)(S_{M-1}-C)-AC)q+b_{M-1}f_M^{j+1}\end{pmatrix},$$

and

$$f_M^{j+1}=\frac{(1-\lambda_I)((1-\lambda_R)(S_M-C)-AC)q}{1+\lambda_P}+\frac{f_M^j}{(1+\delta_F)(1+\lambda_P)}.$$

The terminal condition specifies the value of the operating oil sands project when reserves are exhausted. Using this condition and the conditions given by equations (37.1) and (37.2) the value of the of an operating oil sands project with an option to abandon can be approximated on all nodes in the domain. The optimal stopping boundary $S_i^*(C,Q_j)$ is recovered using equation (40.2). For any Q_j, the highest indexed price S_i where $f_i^j=\Omega$ is the price where it is optimal to exercise the option to abandon for scrap value.

[19] A is a strictly diagonally dominant matrix.

A.2 Pseudo Code

We use the python package OpenOpt to numerically solve equations (39) and (40). OpenOpt is a package designed to numerically solve complementarity problems. To employ OpenOpt, LCPs must be written in the following form

$$\begin{aligned} w &= Mz + q, \\ w \geq 0, z \geq 0, &\text{ and } w^T z = 0, \\ &\text{with } M \text{ and } q \text{ given.} \end{aligned}$$

For the option to develop, let $w_g \equiv Ag^{n+1} - g^n - \Delta\tau M^n$ and $z_g \equiv g^{n+1} - f^K + \text{IC}$. Then equation (26.3) can be written

$$w_g = Az_g + A(f^K - IC) - (g^n + \Delta\tau \mathrm{M}^{n+1}),$$

with the conditions $w_g \geq 0$, $z_g \geq 0$, and $w_g^T z_g = 0$ where A and $A(f^K - IC) - (g^n + \Delta\tau\mathrm{M}^{n+1})$ are given. Similarly for the operating project we get

$$w_f = Bz_f + B\Omega - (f^j + \Delta Q \Pi^{j+1})$$

with $w_f \geq 0$, $z_f \geq 0$, and $w_f^T z_f = 0$ where B and $B\Omega - (f^j + \Delta Q\Pi^{j+1})$ are given.

When an element of w_{gi} is equal to zero the option to develop is in the continuation region and it is optimal to continue to hold the option and delay development. The value of the option to develop is

$$g_i^{n+1} = [A^{-1}(g^n + \Delta\tau\mathrm{M}^{n+1})]_i.$$

When an element of z_{gi} is equal to zero the option to develop is in the development region and it is optimal to exercise the option to develop. The value of the option to develop is

$$g_i^{n+1} = f_i^K - \text{IC}.$$

Similarly for the operating project, if w_{fi} is equal to zero the value of the project is

$$f_i^{j+1} = [B^{-1}(f^j + \Delta Q\Pi^{j+1})]_i.$$

When z_{fi} is equal to zero the value of the project is

$$f_i^{j+1} = \Omega.$$

The option to develop depends on the value of the operating project. We start by solving for the value of the operating project with the option to abandon for scrap value. Iterating over reserves from reserve exhaustion, $j = 0$, to initial reserves, $j = K - 1$. Then we solve the option to develop an oil sands project using the solution to the value of the operating project at initial reserves. Iterate over time remaining on lease from expiration, $\tau = 0$, to initial day of lease, $\tau = N - 1$.