A Unifying Theory of the Generic Competition Paradox:

Dynamic limit pricing with advertisement

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Abstract

Existing evidence shows that prescription drug monopolists tend to increase drug
prices as generic competitors penetrate the market (Generic Competition Paradox). However economic theory predicts that prices tend to fall in more competitive markets for close substitutes. This study examines the optimal pricing and advertising behavior of brand-name drug monopolists that face abolition of barriers to entry, in an optimal control framework where advertisement leads to dynamic transformations of demand and generic entry is a function of brand-name price. I argue that the existing theories behind the Generic Competition Paradox (GCP) are unified by assuming low own-price elasticity of demand for the brand-name producer post patent expiration. The model suggests that in more general environments short monopoly market lifetime explains the GCP, which is not captured in the existing literature and uniquely related to early patenting in the prescription drug industry. JEL No. D21, D42, I11, L12, C61

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1 Introduction

Manufacturers of prescription drugs receive patent protection upon the development of a novel pharmaceutical substance. For a predetermined period of time, patent protection grants immunity from market competition allowing the manufacturer to act as a monopolist and establish their brand-name in association to the new chemical substance. The expected monopoly profits encourage firms to engage in risky activities, such as research and clinical trials, which are institutionally necessary for marketing new pharmaceuticals. After patent expiration and in order to minimize the monopoly costs to society, manufacturers of generic drugs face no barriers to market entry and brand-name firms eventually confront rival competition from firms that sell virtually the same product.

Typical intuition suggests that prices tend to fall when markets become more competitive, for example, in certain microeconomic models it is optimal for the monopolist to charge below the monopoly price when the market opens up to competition in order to deter rival entry (limit pricing). If product price is the only cost to consumers, and if all products in the market are close substitutes then we expect market prices to converge over time and firms with high prices to receive a market share of zero as the market becomes perfectly competitive. However there is a substantial amount of evidence suggesting that in the prescription drug market brand-name prices are unresponsive to competition from generic drugs; in several cases, brand-name firms continue to increase their price at the same rate as prior to patent expiration and manage to retain positive market shares despite facing competition from lower priced generic drugs. This counter-intuitive result became known as the Generic Competition Paradox (hereafter GCP) and has been extensively studied by
theorists and empiricists alike but there has been no consensus among health economists about the underlying factors that give rise to this paradox.

In this paper we propose the argument that the Generics Competition Paradox is best understood in the context of model of the firm’s intertemporal profit maximization problem. Our basic model incorporates a version of the Frank and Salkever (1992) (hereafter FS) assumption about differing elasticities of demand among subsets of the patient population and allows us to consider how variations in the strength of the FS effect might affect observed pricing behavior. We work within a dynamic framework akin to that of Bhattacharya and Vogt (2002), but by importing from the dynamic industrial organization (IO) literature Gaskin’s (1971) dynamic limit pricing model we extend the modelling framework from the “one-shot” transition which B&V use to a more general IO framework.

Our starting point is the fact that research-based (or brand name) drug companies are intertemporal profit maximizers. They have two policy tools: the immediate price of their product and advertising, operating on demand through a demand-shift and rotation variable which we shall refer to as consumer attachment to the brand name product. The attachment variable, which can be thought of as being incorporated in the intercept and the slope of the demand curve which the firm faces at any instant in time, is one of the primary factors which make the firm’s problem a dynamic one. Attachment is a capital variable in the sense that the firm can accumulate it through investment activity – in our case the investment activity is spending on advertising – it depreciates over time and therefore must be maintained\(^1\) and, from the point of view of the individual firm the strength of patient attachment to that

\(^1\)The rate at which it depreciates can be varied to reflect what we might term the necessity of the drug as an input into the treatment of the condition at which it is aimed.
firm’s particular drug depends on the number of competing products in the market. This means that the firm’s decision about the optimal level of advertising expenditure at any point in time must allow for the fact that an increase in advertising today will affect demand in the future through its effect on accumulated attachment, in that it causes an outward shift in the location of the demand curve which does not vanish at the instant expenditure on advertising goes to zero, and that other factors, most notably the number of competing firms in the market, will eat away at its consumer attachment and hence cause a process of leftward shifting of the demand curve. Furthermore, the accumulation of attachment leads to a higher own-price elasticity by attracting more price-sensitive consumers which are the first to switch to consuming generic drugs once the latter enter the market, an assumption which links our model to the necessary and sufficient assumptions in FS for a GCP to occur. Generally speaking, we are, in effect, fitting the pharmaceutical industry into a dynamic version of the model of a dominant firm which faces a competitive fringe of firms and which must take account of this fringe when it sets its price and, in our case, decides on its advertising policy.

In translating the underlying IO model to the brand-name pharmaceutical sector, we incorporate some key factors of that sector and omit others. Perhaps the most important of the omitted factors is the presence of on-patent competition. It is rare, although obviously not unknown, for there to exist a single brand name drug aimed at treating a particular condition. Indeed, Lichtenberg and Philipson (2002) argue that on-patent competition is often more important to the brand name drug company than is off-patent competition. In our model there will only be off-patent (i.e. generic) competition. We also neglect the effect of differences in the design of insurance plans: the difference between reference pricing and
co-insurance in terms of their impact on the price elasticity of demand faced by the brand name firm, for example.

The key feature which we do incorporate is the fact that brand name drugs have a period of monopoly conferred on them by their patent protection. At the end of that period of patent protection, the market is opened to competition from generic drug producers who enter over time according to a dynamic limit pricing equation. Thus we extend the Bhattacharya and Vogt model, in which the end of patent protection is represented by a one-time leftward shift of the demand curve faced by the brand name firm to one in which generic firms enter gradually, at a rate which depends on their perception of the profitability of the market and which may also depend on regulatory factors – FDA\(^2\) regulation of the US market with regards to approval of production facilities, and in particular with regards to the hurdles faced by the entry into the US market of generic drugs produced in foreign manufacturing plants, has, it has been suggested, restricted entry of generic competitors for off-patent brand name drugs. While we do not focus on it, this effect can be built into our model.

We assume that the brand name drug company knows exactly when its patent will expire, and that on that known date the market will be one of free entry of generic competitors. This means that we assume away the possibility that the brand name company can extend its patent by spending on evergreening, and also that there is no possibility of its facing a successful patent challenge before the expiry date, so there is no uncertainty about the date at which generic producers enter the brand name company’s market.

Our model, then, is set up as a two-stage optimal control problem in which the brand

\(^2\)U.S. Food and Drug Administration.
name firm’s objective is to maximize the present value of its lifetime profits. The first
stage is the on-patent stage during which the brand name firm has a monopoly over the
market and the second stage is the off-patent stage during which competitors enter according
to a dynamic limit pricing equation, which relates the rate of competitive entry to the
profitability of the market. In the terminology of optimal control, the first stage is a fixed,
finite horizon problem since the date of patent expiry is known with certainty. The second
stage is, at least potentially, an infinite horizon problem from the perspective of the brand
name firm although that firm could make it a finite horizon problem by choosing to exit
the market. This is consistent with the literature which suggests that in many cases brand
name firms simply abandon the market to the generic entrants. In the optimal control
framework this means that the second stage problem can be set up as a free horizon problem
with a terminal transversality condition which determines whether the firm chooses to stay
permanently in the market or to exit it at a finite point in time. The choice of horizon
for the second stage of the problem is part of the brand name firm’s overall optimization
problem.

The key to understanding a two-stage optimal control problem is the fact that the two
stages are linked by a transversality condition. In essence this means that every decision
the firm makes about the choice of its control variables, in the following model price and
advertising, during the first stage must take account of the impact of those decisions on its
profitability in the second stage. We argue that it is in the formal transversality condition
linking the two stages that we find the essence of the generics competition paradox, and
that understanding the nature of the transversality condition linking the two stages is key
to understanding under what conditions the paradox arises.
We have said that the first stage of our optimal control problem is a fixed, finite horizon problem. This does not mean that we neglect one key aspect of the brand name drug market, the fact that the period of on-market patent protection experienced by particular firms may vary. A drug patent is issued for a fixed period, usually 20 years, from date of filing of the patent. Research-based drug companies usually patent very early in the research process, before clinical trials are underway and well before marketing approval has been granted, assuming the drug is one of the relatively small number which survive the clinical trial process. This means that the actual length of the on-patent market period varies from drug to drug. The longer it takes a drug to get through the testing and approval process the shorter the first stage of the control problem. Our first stage should be seen as beginning not at the time of patenting but at the time of market entry by the brand name company. Thus its decisions about pricing and advertising spending will depend on the length of the on-patent market period it faces. We will see that this plays a role in whether the pricing decisions made about a particular drug display the characteristics of the generics pricing paradox.

2 Background and related literature

Before generic drugs can enter the market they must be scientifically proven bioequivalent to the brand-name drug which, in economic terms, implies a *de facto* high substitutability between any two goods in the market for a given chemical substance. In actuality generic and brand name drugs differ only in their inactive ingredients which are a small part of each products composition, and in the absence of consumer adverse effects to the inactive
ingredients, e.g. allergies, pharmaceutical products are perfectly substitutable; a rather uncommon phenomenon in modern industries. This makes the GCP peculiar to economists since its explanation is not straightforward with the use of traditional microeconomic models. However, price differences between similar products can be theoretically justified if consumers perceive the products as different. This paper shows that under the assumption that product differentiation for the brand-name drug can be built through promotional activities, the GCP is not all that paradoxical but rather a result of the standard optimization problem of a monopolist that expects rival entry at a given point in time.

Frank and Salkever using principal microeconomic foundations provide the first explanation of the GCP in which they assume that demand for brand-name drugs is exogenously segmented to one part that is loyal to the firm in the sense that some consumers always purchase the brand-name drug regardless of the price of generic drugs, and another in which consumers are sensitive to price differences between generic and brand-name drugs. The proposed justification for this assumption is sticky physician prescription habits, that is, doctors are reluctant to prescribe generic drugs and continue to prescribe brand-name drugs after patent expiration. However, not only does empirical evidence suggest to the contrary, it is also often the case that pharmacists, rather than doctors, decide which product to dispense to the consumer, and given that generic drugs are bioequivalent to the respective brand-name products the suggestion that physicians habitually do not prescribe generic drugs is doubtful, unless some consumers are allergic to the inactive ingredients in all generic versions of a given brand name drug - an unlikely case in the presence of mul-

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3Hellerstein (1994).
4E.g. most insurance drug plans in Canada require mandatory substitution by generic drugs unless explicitly prohibited by the physician
tiple generic drugs in the given market. Regardless of these concerns, the basic economic assumption behind this theory is an elasticity assumption; demand for brand-name drugs is inherently infinitely inelastic for a segment of the population with respect to generic drug price in the FS model. In the approach that follows, rather than assuming an exogenously fixed proportion of consumers which are loyal to the firm to the extent that their consumption choice is unaffected by the price differential, it is assumed that consumer loyalty is endogenous and can be gradually and partly controlled by the brand-name firm through the amount of promotional activities over time and the consumer attachment stock variable. Frank and Salkever extend the basic stackelberg game by assuming that advertisement is a choice variable for the brand-name firm, in addition to price. In this scenario, in order to observe a GCP it is necessary that first, increased advertising leads to an increase in the own-price elasticity of demand and, second, advertising has a negative effect on the demand response to entry, the latter assumed negative. We incorporate these assumptions in our model and show that, in a general dynamic framework, they are not sufficient to produce a GCP.

Other explanations for the GCP have focused on the role of insurance. Ferrara and Kong (2008) and Kong (2009) show that it may be optimal for the firm to increase its price after rival entry, under certain conditions, when demand consists of population groups with different insurance coverage. In many countries the majority of prescribed drug expenditure is covered by public or private insurance plans and only a minor percentage is paid out-of-pocket by end drug consumers. Once again, this fact can be expressed in elasticity terms; demand stemming from the population of consumers with generous insurance plans is more inelastic to changes in brand-name prices relatively to consumers that pay larger
out-of-pocket amounts of drug expenditure. In the model that we propose we assume that through promotional activities the brand-name firm attracts some of the more price-sensitive consumers, perhaps the ones with not so generous insurance plans, which results in increased demand in the future and a more price elastic demand.

The above theories are developed in the context of a two-period stackelberg game. In essence, these problems boil down to maximizing the leaders profit function. That is, given the best response from the generic market as a function of or leadership price, the stackelberg equilibrium is determined by solving the brand-name firms profit function. More generally, given the assumptions on player’s beliefs, the solution to the two-period problem is a subgame perfect Nash equilibrium in which, under certain conditions, it is optimal for the firm to increase their price as rival manufacturers penetrate the market. In the FS model with advertisement, for example, one of the additional conditions assumes that rival entry leads to a decline in the own-price elasticity of brand-name demand. The following work unifies the assumptions and the results behind the theories that have been proposed in explanation to the GCP with the traditional dynamic literature on limit pricing.

The problem that the brand-name firm solves in the succeeding model is founded upon the same rationale behind the two-period stackelberg game theoretic model in a more general framework where the brand-name firm chooses what price to set in order to maximize profits at every instant rather than at a single point in time under an exogenously given best
response function from the generic fringe with respect to brand-name price.\textsuperscript{5} By doing so, we link the assumptions in the above theories to the standard dynamic limit price model developed by Gaskins. Generic manufacturers decide whether to enter a drug market or not based on the current brand-name drug price, and given this optimal response function, brand-name producers maximize their own profits by controlling price and advertisement thereby affecting the volume and the elasticity of demand for the brand-name drug over time.

As in the FS stackelberg game with several entrants, the brand-name firm implicitly controls the rate of rival entry through the choice of price. This is also the main assumption behind limit price models; if monopolists set their price below a certain level then rival producers find it too costly to produce and decide not to enter the market, therefore monopolists choose market entry, to some extent, when setting their own-price and the non-myopic firm will consider this fact in its maximization problem.

It turns out that in a dynamic framework it is straightforward to express all of the assumptions in the advertisement FS model that are necessary for a GCP to occur in the two-period stackelberg game, if we model demand as a function of a stock variable in which advertisement and rival entry have opposite first and second order effects on demand.

Throughout what follows, we refer to this stock variable as consumer attachment and denote it by $F(t)$ where $t$ represents time, for lack of a better term. The reader should always keep in mind that the value of $F(t)$ captures the lagged effects of past values of real advertisement

\textsuperscript{5}Since firms are allowed to abandon the market altogether the time horizon may be finite, or when the firms plan to stay in the market forever they consider their problem under an infinite time horizon. One of the difficulties associated with this extension are conceptual problems around the optimal choice of finite vs infinite horizon values, and the optimal time to exit the market.
exposure and the rate of rival entry on brand-name demand.

Grabowski and Vernon (1992) observe that in most cases out of 18 drug products which lost patent protection in the U.S. prescription drug market between 1983 and 1985, brand-name firms continued to increase their price after patent expiration despite the fact that their generic competitors entered the market quoting much lower prices. They find that it takes about two years after patent expiration for the typical product in the sample to lose half of the market to competition and that the number of competitors in each market is statistically related to market profitability. Caves et al. find no evidence of limit pricing and estimate that generic drugs capture roughly 25% of the market when the ratio of generic to branded price is 0.45. The contradiction between the empirical evidence and economists’ typical intuition became known as the ‘Generics Competition Paradox’ (GCP) (Scherer, 1993). More recent evidence documenting this paradox in the U.S. markets include, e.g., Saha et al. (2006) and Regan (2007). A similar phenomenon has been observed in the Canadian and E.U. drug prescription market (e.g. Vandoros and Kanavos (2012), Kanavos et al. (2008)).

Bhattacharya and Vogt (2002) propose a dynamic model where they assume a one-time effect of patent expiration on brand-name demand. In their framework, the profit maximizing firm charges a higher price for its product after patent expiration under the assumption that rival entry makes demand more elastic which is in contradiction to the assumption of Frank and Salkever. In our approach, we assume that rival entry causes brand-name demand to become less elastic. The intuition behind this assumption is that as generic competitors enter the market, it is the more price sensitive consumers that switch to buying generics. We expand upon the aforementioned literature by assuming that the
effect of rival entry on brand-name demand is continuous in time.

3 Model

Throughout this section we assume that the brand-name firm intends to stay in the market for an infinite amount of time after patent expires. In the next section we consider the case of a finite competition horizon.

3.1 Assumptions

The firm faces a demand of the form $Q(p(t), F(t))$ where $t$, $p$ and $F$ take values in the non-negative real line and denote time, product price and the capital of consumer attachment respectively.

We assume that $Q$ is twice continuously differentiable in $(p, F)$, with finite partial derivatives and satisfies (i) through (vi):

(i). $\frac{\partial Q(p,F)}{\partial p} < 0$, (ii). $\frac{\partial^2 Q(p,F)}{\partial p^2} = 0$, (iii). $\frac{\partial Q(p,F)}{\partial F} > 0$, (iv). $\frac{\partial^2 Q(p,F)}{\partial F^2} < 0$,

(v). $\frac{\partial^2 Q(p,F)}{\partial p \partial F} < 0$ and (vi). $Q(p, 0) > 0$ for some $p > 0$.

Assumptions (i) and (ii) imply that quantity demanded decreases at a constant rate as price increases, while assumptions (iii) and (iv) imply that quantity demanded increases at a decreasing rate as consumer attachment increases.

Through assumption (v) an increase in $F$ causes quantity demanded to fall more, for a given increase in price. That is, an increase in $F$ rotates the linear demand curve in $(Q, p)$ making it more price elastic.
For $t \in [0, T]$, which we shall call the first phase, the firm acts as a monopolist. During this phase familiarity builds up according to the equation of motion:

$$\dot{F}(t) = g(A(t)) - \delta F(t)$$

where the variable $A$ represents the real level of advertising exposure, the parameter $\delta \in (0, 1)$ is the rate of consumer attachment depreciation assumed constant over time, and $g$ has the standard properties of a production function: $g_A > 0$, $g_{AA} < 0$ and $g(0) = 0$. Thus, the firm can not directly control the level of $F(t)$ but by choosing the level of $A(t)$ it affects the future values of the former.

We assume that the firm can obtain units of $A$ at a constant price per unit $v$. Since $g$ is an increasing function an additional unit of $A(t)$ causes $\dot{F}(t)$ to increase, as a result future quantity demanded rises (assumption (iii)) and demand becomes more price elastic (assumption (v)).

Intuitively, we are looking at a population of potential consumers of a fixed size - the population with the condition that the drug is aimed at. With no advertisement, $F = 0$ and for some $p > 0$ where $Q > 0$ there exist some consumers that choose not to buy the drug, presumably having done the calculation and decided that the price was not matched by the marginal health benefit. An increase in advertisement exposure will attract some of the consumers who are more price sensitive, or have a lower willingness to pay, than the ones that already consume the product. This increases market size by bringing in a more price elastic segment of the market and as a result demand becomes more elastic overall. That is, through direct or indirect methods of advertisement, the brand-name firm has the ability to implicitly control the stock of consumer loyalty which attaches part
of the consumer population to the specific brand-name drug and as a result consumer attachment accumulation the standard demand function is shifted and rotated in the two-dimensional price and quantity space. In other words, despite the fact that products in the given market are close substitutes, promotional activities may lead to consumers having increasingly different perceptions about product effectiveness and lead to the establishment of a certain a degree of product differentiation between brand-name and generic drugs.

After time $T$ there are no barriers to entry and the law of motion for $F$ becomes$^6$:

$$F(t) = g(A(t)) - \delta F(t) - \gamma(p(t) - \overline{p})$$

The term $\gamma(p(t) - \overline{p})$ represents the rate of entry of rival producers. If the number of competitors in each market is related with market profitability and if rival competitors view current product price as a proxy for market profitability then the rate of entry of rival producers must be a function of current product price. As in Gaskins’ model we assume that rival entry is a linear function of current product price and define $\overline{p}$ as the price at which net entry is equal to zero with $\gamma$ being a positive response coefficient. Pricing above the limit price $\overline{p}$ causes positive rival entry at time $t$: the increase in the term $\gamma(p(t) - \overline{p})$ has no effect on $Q(p(t), F(t))$ at time $t$, but by decreasing $F(t)$ it reduces the dominant firms’ demand in the future through (iii), *ceteris paribus*. Moreover, the reduction in $F$ makes demand more price inelastic through assumption (v). As rival competitors enter the market and $F$ is reduced it is the more price sensitive individuals who leave, making demand less elastic and shifting it towards the origin.

$^6$We will refer to this time period as the second phase, or simply phase 2.
3.2 Necessary conditions

The problem of the firm is to choose the path for price and advertisement exposure that will maximize the present value of its lifetime profits. We further assume that future income is discounted at a constant rate $r$ and that $c$, the average total cost of production, is also constant and that $c < \bar{p}$.

Formally, the problem can be written as

$$\max_{(p(t), A(t))} \int_0^\infty [(p(t) - c)Q(p(t), F(t)) - \nu A(t)]e^{-rt}dt$$

subject to

$$F(t) = g(A(t)) - \delta F(t) - \gamma(p(t) - \bar{p})$$

(2)

$$\gamma = 0 \text{ if } t \in [0, T], \text{ otherwise } \gamma > 0$$

(3)

$$F(0) = 0$$

(4)

$$F(t) \geq 0 \forall t \in [0, \infty)$$

(5)

The current value Hamiltonian associated with problem (1) can be expressed as

\footnote{We will work on the case of an infinite second period horizon and examine how the solution changes if the second period is finite.}
\[ H(p, F, A, \psi) = (p - c)Q(p, F) - vA + \psi[g(A) - \gamma(p - \bar{p}) - \delta F] \]

where the time argument is omitted for notational simplicity. In the language of optimal control, \( p \) and \( A \) are control variables, \( F \) is the state variable and \( \psi \) is the co-state variable. In economic terms, \( \psi \) is the value associated with a marginal increase in \( F \) or the current value shadow price of \( F \).

The necessary conditions are:

\[ 0 = Q(p^*, F) + p^*Q_p(p^*, F) - cQ_p(p^*, F) - \gamma\psi \tag{6} \]

\[ 0 = -v + \psi g_A(A^*) \tag{7} \]

\[ \dot{\psi} = (r + \delta)\psi - (p - c)Q_F \tag{8} \]

\[ \dot{F} = g(A) - \gamma(p - \bar{p}) - \delta F \tag{9} \]

and must hold for all \( t \).

The transversality condition linking the two-phases is given by

\[ e^{-rt}\psi^* = \frac{\partial \varphi}{\partial F^*} \tag{10} \]

which must be satisfied at \( t = T \), where \( \varphi = \max_{(p(t), A(t))} \int_T^\infty [(p(t) - c)Q(p(t), F(t)) - vA(t)]e^{-rt}dt \) subject to (2) with \( \gamma > 0 \) and \( F(T) \) free. Condition (10) is a necessary terminal condition.
for phase one and prevents the optimal value of the co-state from changing discontinuously at time \( T \).

Condition (6) can be rewritten as

\[
cQ_p(p^*, F) + \gamma \psi = Q(p^*, F) + p^* Q_p(p^*, F)
\]

and during the first phase simplifies to

\[
cQ_p(p^*, F) = Q(p^*, F) + p^* Q_p(p^*, F)
\]

The above conditions implicitly define the optimal price \( p^*(F, \psi) \) as a function of the state and co-state variables. In the first period \( \gamma = 0 \) and the optimal price is a function of the state variable only since the term \( \gamma \psi \) vanishes. The economic intuition behind these conditions is that the optimal pricing policy \( p^*(t) \) must equate marginal cost to marginal revenue at every instant. During the second period, the term \( \gamma \psi \) reflects the additional costs that stem from increased competition: pricing above the limit price will result in positive rival entry which will decrease \( F \) in the future. This additional cost to the firm is equal to the change in \( F \) times the shadow price of \( F \).

Similarly condition (7) implicitly defines the optimal amount of advertisement exposure \( A^*(\psi) \) as a function of the co-state variable and has the same economic interpretation: the marginal cost of advertisement must be equal to the marginal benefit. In addition, (7) implies that the current value shadow price of \( F \) is always positive.

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8The price induced change in the growth of \( F \) is equal to \(-\gamma \) when price is increased marginally. Therefore at the next instant the change in \( F \) is exactly \(-\gamma \).

9The present value of the co-state however is zero when \( t \to \infty \).
Substituting conditions (6) and (7) into the law of motion for $F$ and $\psi$ yields a pair of autonomous differential equations:

\[ \dot{\psi} = (r + \delta)\psi - [p^*(F, \psi) - c]Q_F(p^*, F) \tag{11} \]

\[ \dot{F} = g(A^*(\psi)) - \gamma[p^*(F, \psi) - p] - \delta F \tag{12} \]

From (6), we can calculate

\[ p_F^* = \frac{Q_F + (p - c)Q_{pF}}{-2Q_p} \]

and

\[ p_{\psi}^* = \frac{\gamma}{2Q_p} < 0 \]

We will assume that $Q_F > -Q_{pF}$ through most of $F$ so that additional units of attachment will increase the equilibrium price. Intuitively, a decrease in consumer attachment during the first phase causes demand to fall and also become more price inelastic. If the former effect dominates the latter then the firm will decrease its price in order to recapture demand that was lost due to a decrease in $F$.

From (7) we obtain

\[ A^*_\psi = -\frac{g_A(A)}{\psi g_{AA}(A)} > 0 \]

As expected the optimal amount of real advertisement exposure increases as $F$ becomes more valuable.

The stationary locus for $\psi(p^*, F(A^*))$ is defined as
and it follows by the implicit function theorem that

\[
\frac{d\psi}{dF} \bigg|_{\psi=0} = \frac{p^*_F Q_F + (p^* - c)(Q_{FF} + Q_{Fp^*} p^*_F)}{(r + \delta) - [(p^* - c)Q_{Fp^*} + Q_F] p^*_\psi} \tag{14}
\]

In phase 1, the right hand side of the above expression is positive for small values of \(F\) and becomes negative as \(F\) increases. Thus, \(\dot{\psi} = 0\) is increasing in \(F\) for small values of \(F\) and decreasing for large values and the locus must be concave in \((\psi, F)\) (see appendix for a proof).

In phase 2, the additional term in the denominator \([(p^* - c)Q_{Fp^*} - Q_F] p^*_\psi\) is positive and causes \(\psi\) to change less rapidly along \(\dot{\psi} = 0\) with respect to \(F\). Assuming that \((r + \delta) - p^*_F Q_F > (p^* - c)Q_{Fp^*}\) the locus is first increasing and then decreasing in \(F\), as in phase 1, otherwise the first order properties in \(F\) are reversed.

In addition, during phase one, equation (13) requires the value of \(\psi\) to be positive at \(F = 0\), say \(\bar{\psi}\). In phase 2, the value of \(\psi\) is positive and less than \(\bar{\psi}\) if some consumer attachment has accumulated up to time \(T\), since \(Q_F\) will be smaller in magnitude through assumption (iv).

We now turn our attention to the slope of the stationary locus for the state variable:

\[
0 = g(A^*(\psi)) - \gamma[p^*(F, \psi) - \bar{p}] - \delta F \tag{15}
\]

For this locus we have
\[
\frac{d\psi}{dF} \bigg|_{F=0} = \frac{[\gamma p_F^* + \delta]}{[g_A A_{\psi}^* - \gamma p_F^*]}
\]

so \( \hat{F} = 0 \) is increasing in \( F \) during the first period, also during the second under the assumption that \( g_A A_{\psi}^* > \gamma p_F^* \).

If the value of \( \psi \) at \( F = 0 \) is less than \( \tilde{\psi} \) then, by continuity, the two loci intersect at \((F^*, \psi^*)\) where \( \psi(F^*, \psi^*) \) and \( F(F^*, \psi^*) \) are both equal to zero.

In addition

\[
\frac{d^2\psi}{dF^2} \bigg|_{F=0} = \frac{\gamma p_F^* (g_A A_{\psi}^* - \gamma p_{\psi}^*) - (-\gamma p_{\psi}^*) (\gamma p_F^* + \delta)}{\gamma (g_A A_{\psi}^*)^2 - 2 < 0}
\]

which implies that the stationary locus for \( F \) is concave in \((F, \psi)\).

Since \( p_{\psi} = 0 \) in phase 1 this expression becomes

\[
\frac{d^2\psi}{dF^2} = \frac{\gamma p_F^* g_A A_{\psi}^*}{(g_A A_{\psi}^*)^2} = 0
\]

However, with \( F \) on the horizontal axis the formula for the curvature of this locus during phase one is given by

\[
\frac{d^2F}{d\psi^2} = \frac{g_A A_{\psi}^* + A_{\psi\psi}^* g_A}{\delta}
\]

which is negative since \( A_{\psi\psi}^* < 0 \).

Therefore the stationary locus for \( F \) is concave in \((F, \psi)\) during both phases.
Figure 1 summarizes our findings on the stationary loci. The two loci divide the $(\psi, F)$ space into four regions with different dynamics, represented by the phase arrows, that determine the time path of the system in each region. There is a unique trajectory that reaches the steady state $E$ where both $F$ and $\psi$ are not growing over time, represented by the stable path $\Omega$. That is for all initial pairs $(\psi, F) \in \Omega$ the system converges to $E$. Any other trajectory going through $(\psi, F) \notin \Omega$ eventually diverges from $E$. After time $T$ the loci shift but have the same first and second order properties in $(\psi, F)$ under our assumptions. The dynamics of the space change accordingly so that the phase two dynamics are the same as in figure 1 with respect to the position of the phase two stationary loci. We will now examine the direction of the shift of the loci after patent expiration.
3.3 The effect of patent expiration

 Totally differentiating (13) yields

\[
\frac{d\psi}{d\gamma} |_{\psi=0} = \frac{p^*_\psi [Q_{Fp}(p^* - c) + Q_F]}{(r + \delta) - [(p^* - c) Q_{Fp} + Q_F]} p^*_\psi
\]  

(17)

The above expression is positive if

\[
\frac{(r + \delta)}{p^*_\psi} - Q_F < (p^* - c)Q_{Fp} < -Q_F
\]

and negative if

\[
(p^* - c)Q_{Fp} > -Q_F
\]

or

\[
(p^* - c)Q_{Fp} > \frac{(r + \delta)}{p^*_\psi} - Q_F
\]

From the first order condition for \( p \) we can calculate

\[
\frac{dp^*}{d\gamma} = \frac{\psi}{2Q_p} < 0
\]

Recall that we have assumed that \( Q_F > -(p - c)Q_{pF} \) through most of \( F \) in order for \( \frac{dp^*}{dF} > 0 \).

Thus, \((p^* - c)Q_{Fp} > -Q_F\) which implies that the post entry stationary locus for \( \psi \) will be below the respective pre entry stationary locus.
The stationary locus for $F$ is given by $0 = g(A^*(\psi)) - \gamma [p^*(F, \psi) - \bar{p}] - \delta F$.

Therefore

$$\frac{d\psi}{d\gamma} \bigg|_{F=0} = \frac{(p^* - \bar{p}) + \gamma p^*_\gamma}{g_A A^*_\psi - \gamma P^*_\psi}$$

The denominator is positive so the sign of the above expression will depend on the sign of the numerator, specifically if $(p^* - \bar{p}) > -\gamma p^*_\gamma$ (strictly less than), then the stationary locus for $F$ post entry is above (below) the respective locus in the pre entry phase. So there are two cases to consider\(^{10}\)

**Case 1:** $(p^* - \bar{p}) > -\gamma p^*_\gamma$ for all $t$.

\(^{10}\)We ignore the case in which the direction of the inequality changes at some $t$.\n
Figure 2: Optimal trajectories: Case 1
This case is depicted in figure 2 with a phase diagram where we have drawn the stationary loci for both phases. During phase one, the loci intersect at $E_1$ while during phase two at $E_2$. Working backwards we are able to sketch the optimal trajectory of the system. Since the second phase has an infinite horizon, when the system is controlled optimally it approaches the steady state $E_2$ asymptotically. There is a unique locus of points in $(\psi, F)$ along which the system can reach $E_2$, denoted by $\omega_2$. Thus $(\psi^*, F^*) \in \omega_2$ for all $t > T$. The transversality condition (10) for our two phase problem prevents discontinuous changes in $\psi^*$ at time $T$, that is, the value of $\psi^*$ at the beginning of the second phase is $\psi^*(T)$. For this condition to hold, the optimal trajectory must approach the locus $\omega_2$ for some time up to time $T$ and reach a point in $\omega_2$ exactly when patent expires. Now consider the first phase of the problem. At $t = 0$ the value of familiarity is zero so the planner picks a point on the vertical axis of figure 2. The optimal trajectory must reach a point in $\omega_2$ at exactly time $T$ without violating the direction of the phase arrows. Thus, the set of optimal trajectory candidates are all the trajectories that satisfy $F(0) = 0$ and reach a point in $\omega_2$ at time $T$. The length of the first phase (i.e. the value of $T$) determines the optimal trajectory uniquely. In figure 2, $\omega'$ and $\omega''$ are both optimal trajectories in the sense that they satisfy the optimality conditions (1) – (4) given $F(0) = 0$. If $T$ is relatively small then the unique optimal trajectory is $\omega'$ and if $T$ is large then the unique optimal trajectory is $\omega''$. Any other trajectory fails to reach a point in $\omega_2$ at time $T$ and satisfy the transversality condition. As $T \to \infty$ the optimal trajectory reaches a neighborhood close to $E_1$ where the growth rates of the state and co-state variables become infinitesimal.

Assume that $T$ is such that $\omega''$ is the optimal trajectory. Then time can be decomposed
into three different time periods:

Time period I: From $T = 0$ and until the system crosses the phase one stationary locus for $F$, $\psi$ decreases while $F$ increases over time. During this time period $A^*$ is falling while $p^*$ is rising and $\dot{F} \geq 0$, the equality holds for the instant that the trajectory crosses the $\dot{F} = 0$ locus, say $t'$.

Time period II: After the system crosses the stationary locus for $F$ in phase 1, $\psi$ decreases over time but now $F$ is also falling over time. Throughout the duration of this period, $A^*$ is falling and $p^*$ may either fall or rise over time depending on the magnitude of $p^*_F$ relatively to that of $p^*_\psi$.

Time Period III: After the end of Phase 1 the system asymptotically approaches $E_2$ and $(\psi^*, F^*) \in \omega_2$ for the whole duration of phase 2. During this period, $\psi$ is rising and $F$ is falling. This causes $p^*$ to fall and $A^*$ to rise over time.

Therefore in Case 1 the optimal solution dictates brand-name price to fall after rival competitors enter the market. This result is the standard result that prices tend to decrease when markets become more competitive.

However, when $T$ is small then the optimal trajectory is $\omega'$ since it is the unique trajectory that reaches $\omega_2$ at exactly time $T$. In this scenario the system approaches $E_2$ from the left during phase two and the optimal value of price increases over time while advertisement is decreases, for the whole duration of the problem.

Case 2: $(p^* - \bar{p}) < -\gamma p^*_T$ for all $t$.

This case is depicted in figure 3. In this case the optimal path of price is rising post-patent expiration for all optimal trajectories with finite $T$, a result in contradiction to most
models of competition. If the duration of the first phase is long enough then a trajectory such as \( \omega'' \) or \( \omega''' \) is optimal. For small values of \( T \) trajectories such as \( \omega' \) are optimal. Any trajectory that begins above the phase one stable arm, such as \( \omega_c \) cannot be optimal since it diverges from \( E_2 \) as \( T \to \infty \).

4 Summary of results

In both cases 1 and 2, the optimal price is increasing and \( F \) is growing for \( t \in [0, t') \). This is also the case for optimal trajectories that begin above the stationary locus for \( \psi \) such as \( \omega''' \) in figure 3. After time \( t' \) and up to \( T \) the system is approaching \( \omega_2 \) and \( A^* \) continues to fall while the optimal price is increasing if \( p^*_F > p^*_\psi \) and decreasing otherwise. However if the duration of the first phase is short then time period \( II \) does not occur because there
is not enough time for the optimal trajectory to cross the stationary locus for $F$ during the monopoly phase. This implies that at the beginning of the problem, when $F = 0$ it is optimal for the firm to start with a high value of real promotional exposure\textsuperscript{11} and then gradually reduce it up to time $t'$. As consumer attachment accumulates, the firm finds it profitable to increase its price. The continuous decrease in $A^*$ causes $F$ to grow negatively after $t'$ and since $\frac{dA^*}{dt}$ remains negative throughout $t \in (t', T]$, consumer attachment will decrease at a faster rate as the time of patent expiration approaches. After the time that the growth rate of consumer attachment becomes negative\textsuperscript{12}, it is optimal for the firm to charge a higher price for its product if the total effect of $\psi$ is greater than the total effect of $F$ on $p^*$. Intuitively, the firm would like to settle at $E_1$ if the monopoly phase would last forever. As patent expiration approaches and advertisement is falling, consumer attachment starts to grow negatively. This has two results, it reduces quantity demanded and it makes demand more price inelastic. The firm is still a monopolist in the market therefore the decrease in the price elasticity of demand would have a positive effect on the optimal price but since the firm now faces imminent rival entry, any increases in price would result in a higher rate of rival entry after patent expiration and therefore greater reductions in quantity demanded and own-price elasticity. Depending on the relative magnitude of our exogenous parameters, the firm may choose to lose some of the more price sensitive consumers by increasing its price as patent expiration approaches.

When the horizon of post-patent competition is infinite our model predicts that the

\textsuperscript{11} If the optimal trajectory begins above the stationary locus for $\psi$ then $A^*$ is increasing for some time after $t = 0$ but eventually crosses $\dot{\psi} = 0$ in the first phase and $A^*$ is decreasing up to $T$.

\textsuperscript{12} If the duration of the first phase is not long enough then this may not happen, see for example $\omega'$ in Fig.1 and Fig.2.
optimal price may either rise or fall depending on the magnitude of \((p^* - \bar{p})\) relatively to \(-\gamma p_t^*\). If the limit price and the response parameter \(\gamma\) is high and \(Q_p\) is relatively small then 
\[(p^* - \bar{p}) < -\gamma \frac{\psi}{Q_p}\]
and the optimal price is increasing post patent expiration for all finite values of \(T\). On the other hand, if the limit price is relatively low and \(Q_p\) is large, then 
\[(p^* - \bar{p}) > -\gamma \frac{\psi}{Q_p}\]
and the optimal price is decreasing post patent expiration. If the first period is short then the optimal trajectory may approach \(\omega_2\) without crossing the stationary 
locus for \(F\), so time period II may not occur; in this case if \((p^* - \bar{p}) > -\gamma \frac{\psi}{Q_p}\) then \(p^*\) may be decreasing or increasing after \(T\), whereas if \((p^* - \bar{p}) < -\gamma \frac{\psi}{Q_p}\) then \(\frac{dp^*}{dt} < 0\) for all \(t > T\). Finally, as \(T \to 0\), \((\psi^*, F^*) \in \omega_2\) at \(t = 0\) and \(\frac{dp^*}{dt} > 0\) for all \(t > T\).

The difference between cases 1 and 2 highlight the importance of assumptions about the demand price elasticity. We argue that the literature that explains the GCP is unified under case 2. Under the assumptions that there exists a segment of demand in which consumers do not change their consumption pattern based on the price of brand-name drugs, either because of insurance plans or sticky physician prescription habits, for those consumers \(Q_p\) is relatively low and demand is not greatly affected by changes in the brand-name price. That is, in the static models, after patent expiration the brand-name firm expects to lose all of the price sensitive consumers to generic drugs and that \(Q_p\) for the remaining demand is relatively low. When this is the case, \((p^* - \bar{p}) < -\gamma \frac{\psi}{Q_p}\) and the brand-name firm always increases its own price after patent expiration in order to capitalize on the low price elasticity of the remaining "loyal" consumers or the consumers with generous insurance plans. Therefore the firms are able to maintain monopoly power over the loyal consumers and price increases are a result of monopolistic behavior. However, in our dynamic framework we show that price increases may also occur when the market is closer to one that is competitive, that is when
$Q_p$ is very high after patent expires and $(p^* - \bar{p}) > -\gamma \frac{\psi}{\partial p}$. This is case 1, where observing a GCP or not depends on the duration of monopoly lifetime. In general, short monopoly periods always lead to a GCP, regardless of the level of competition and, consequently, regardless of demand responsiveness with respect to changes in brand-name prices.

5 Finite second phase horizon

So far we have assumed that the profit maximizing firm views the post-patent phase as an infinite horizon problem. In practice however brand-name firms may decide to abandon the market altogether after patent expiration. If this is the case then the optimal time of market exit is endogenous to the firms’ problem

$$\max_{(p(t), A(t), F(\tau_2), \tau_2)} \int_0^{\tau_2} [(p(t) - c)Q(p(t), F(t)) - vA(t)]e^{-rt}dt$$

(18)

subject to (2), (3), (4) and (5), where $\tau_2$ is the (finite) terminal time of the second phase.

The necessary conditions are

$$e^{-rt}\psi^*(\tau_2) = 0$$

(19)

$$e^{-rt}H(p(t), F(t), A(t), \psi(t)) = 0 : t = \tau_2^*$$

(20)

in addition to conditions (6) through (10). Condition (19) states that a necessary condition for choosing $F(\tau_2)$ optimally is that its present value shadow price must be driven to zero at the time that the firm exits the market and condition (20) that the present value of the hamiltonian is driven to zero at time $\tau_2$. Intuitively, (19) is automatically satisfied in an infinite horizon, i.e. as $\tau_2^* \to \infty$ and for $\tau_2^*$ to be finite, it is necessary that $\psi^*(\tau_2)$, the current value shadow price of $F$ at time $\tau_2$ is exactly zero. Since $\psi^*$ captures the future effect of the
current rate of change of $F$ on lifetime profits when the system is controlled optimally, if $\psi^*(\tau_2) > 0$ then the firm could increase the maximized value of lifetime profits by increasing the value of $\tau_2$ and if $\psi^*(\tau_2) < 0$ then it would be optimal for the firm to abandon the market at an earlier point in time. This is the necessary condition for choosing the value of the co-state variable as time approaches a finite value in which the firm abandons the market. If $\tau^*_2$ is predetermined at the beginning of the optimization problem, for example, if the firm knows that it will bring a new drug in the market after patent expiration and wishes to use all of the production facilities towards manufacturing the new drug so that $\tau^*_2$ is exogenously given by previous and ongoing research, adding condition (19) is the only additional condition that must be satisfied in the optimum. However if $\tau^*_2$ is a choice variable then (20) must also hold. This implies that not only $\psi^*$ but also $H(p, F, A, \psi)$ must be zero at a finite point in time. For a heuristic derivation of the necessity of condition (20) the reader is referred to Leonard and Long (1992).

Assuming that (20) can be satisfied for finite values of $\tau_2$, we can use a phase diagram to examine optimal solutions. Under the restriction imposed in the previous section, we have two extreme cases, in which the stationary locus for $\psi$ in phase 2 is always above, or always below, the respective phase 1 stationary locus. These cases are respectively presented in figures 4 and 5 where $\tau_1$ denotes the exogenously given time of patent expiration and $s(\tau_1)$ is the position of the system at $\tau_1$ in the $(F, \psi)$ space. In case 1 (figure 4) we have drawn two solution candidates, $\omega'$ and $\omega''$ that satisfy all of the necessary conditions for different values of $\tau_1$. Short monopoly market life corresponds to trajectory $\omega''$ and the position of the system in the phase diagram space is $s''(\tau_1)$ while large monopoly lifetime corresponds to trajectory $\omega'$ where $s'(\tau_1)$ is the position of the system at patent expiration. The same
notation is used in figure 5 which depicts the second case. We observe that in all cases, as the system approaches a point on the horizontal axis the movement in price is ambiguous since it depends on the relative magnitudes of $p_F^*$ and $p_\psi^*$. In this scenario the length of patent protection affects the immediate post-patent dynamics but as the system approaches the equilibrium both $\psi$ and $F$ change in the same direction in all cases. The differences are best highlighted in case 2 (figure 5) where price is increasing post patent expiration for a small value of $\tau_1$ and decreasing for a large value of $\tau_1$. This is an example of how small patent lifetimes may cause a GCP when firms plan to abandon the market even when $Q_p$ is high after patent expires.

Figure 4: Finite second phase horizon: Case 1
Figure 5: Finite second phase horizon: Case 2

6 Discussion

Our proposed dynamic limit pricing model incorporates the effects of demand transformations as a result of promotional activities that lead to rotations of the standard demand curve. We are able show that in some cases it is optimal for the firm to increase its price after patent expiration and that the previous GCP explanations can be united through their common assumptions on the own-price elasticity of brand-name demand. Existing theories assume that for exogenous reasons a group of consumers exists such that the post-patent responsiveness of quantities demanded for the brand-name firm with respect to price changes is low. In our approach, consumer attachment to the brand-name drug is endogenous to the firms’ profit maximization problem, and the assumption that the price elasticity of demand is sufficiently low after patent expiration guarantees that a GCP will occur. However, if the
firm plans to abandon the market then it may a low value of own-price elasticity does not
guarantee prices to increase after patent expiration. In this scenario, a GCP will occur, at
least briefly, after patent expiration whenever monopoly lifetime is brief.

When the post-patent price elasticity of demand is high the length of monopoly lifetime
plays a critical role in observing a GCP suggesting that an alternative explanation to the
GCP is patenting in the early trial stages resulting in brief monopoly market lifetime. In the
presence of advertisement effects on demand, if demand is sensitive to changes in the brand-
name price, and if the duration of monopoly period is above a certain threshold, patent
expiration induces the brand-name firm to reduce their price. However, if the duration of
monopoly lifetime is too great then after a point the path of price is ambiguous, unless
further restrictions are imposed. In general, if we relax some of our assumptions we may
obtain different optimal paths for the firms’ choice variables. Unfortunately, firm-level
time series data for the drug industry are not publicly accessible and we are not able to
empirically verify the validity of our assumptions.

Our approach focuses on markets for prescribed medicines, motivated by the general
belief that pharmaceutical firms allocate a significant amount of resources on promotional
expenditures. However, it is our belief that with the appropriate modifications in the as-
sumptions, the model is applicable to any industry in which firms receive market exclusivity
for a period of time, face rival entry in the future and set the path of price and advertisement
exposure in order to maximize the present value of their lifetime income stream.

The model contributes to the dynamic literature about the optimal pricing and ad-
vertisement behavior of monopolists given the abolition of barriers to market entry at a
deterministic point in time. It also provides a novel explanation to the GCP, one that
is uniquely related to the research and development industry for new drugs, where early patenting, that is, patenting before the product enters the market, is common practice. Although patent protection is typically 20 years, it is often the case that brand-name firms patent new chemical substances early in the trial stages and by the time the product is approved for the market a monopoly life of 8 years remains. In the presence of advertisement effects, a possible explanation for the GCP is early patenting.

From a policy point of view, the main tool that a central planner controls in the context of our model is monopoly lifetime. Much of the early economic literature on innovation incentives has focused on the determining the optimal monopoly lifetime, for example, Kamien et al. (1974). Since then, significant progress has been made on the design of incentive schemes, however, health economists have paid little attention on alternative innovation incentive mechanisms in the prescription drug industry so that the problems are free of the underlying dynamic issues related to the optimal choice of patent lifetime for innovation and the standard dynamic tradeoff between static and dynamic efficiency innovation. The following chapter is partly motivated by this lack of general static models of alternative designs of innovation incentives.

7 Appendix

The curvature of $\psi = 0$ can be determined from
During phase one, $\gamma = 0$ and assuming that the third derivatives of $Q$ are zero the above expression becomes

$$\frac{d^2 \psi}{dF^2} \bigg|_{\psi=0} = \{ [p_{FF}^* Q_F + 2(Q_{FF} + Q_{FP})p_F^*] + (p^* - c)(Q_{FFF} + Q_{FP}p_F^* + Q_{FPP}p_F^2 + p_{FF}^2 Q_{FP}) + p_F^*(Q_{FF} + Q_{FP}p_F^*]) \\
* [(r + \delta) - [(p^* - c)Q_{FP} + Q_F]p_F^*] \\
+ [p_{FP}^*[p_{PP}^*(p^* - c)Q_F + Q_F] + [(p^* - c)(Q_{PP}^2 + Q_{PP}p_F) + p_F^* Q_{FP} + (Q_{FF} + Q_{FP}p_F^*)]p_F^*] \\
* [p_F^*(Q_{FP} + (p^* - c)(Q_{FF} + Q_{FP}p_F^*)]) \\
* ((r + \delta) - [(p^* - c)Q_{PP} + Q_F]p_F^*)^{-2}$$

Differentiating the first order condition for price:

$$p_{FF}^* = \{ [Q_{FF} + (p - c)Q_{PP}^2](-2Q_{PP}^2) + 2Q_{PP}^2 [Q_F + (p - c)Q_{PP}^*] \} \ast (-2Q_{PP}^*)^{-2}$$

$$= [Q_{FF} + (p - c)Q_{PP}^2](-2Q_{PP}^2) + 2Q_{PP}^2 Q_F + 2Q_{PP}^2 (p - c) \ast (-2Q_{PP}^*)^{-2}$$

thus $p_{FF}^* < 0$ if $[Q_{FF} + (p - c)Q_{PP}^2](-2Q_{PP}^2) + 2Q_{PP}^2 Q_F > 2Q_{PP}^2 (p - c)$

and

$$p_{FP}^* = -\frac{\gamma}{2(Q_{PP}^2)^2} < 0$$
Therefore, $\frac{d^2\psi}{dT^2} < 0$ during the first phase.

8 References

References


