Social Security Reforms in a Life Cycle Model with Human Capital Accumulation and Heterogeneous Agents

Parisa Mahboubi

PhD Candidate

University of Guelph

October 2016

Abstract

A life cycle model of human capital accumulation through learning by doing with heterogeneity in productivity and age is constructed. The model is used to evaluate the impacts of social security reforms on the welfare of individuals; and the distribution of labour supply, consumption, and physical capital accumulation over the life cycle in the long run. In the reference economy, retirement is mandatory with a Pay-As-You-Go (PAYG) social security system. The following policy reforms are considered: (i) terminating the social security system with mandatory retirement, (ii) terminating the social security system with voluntary retirement, and (iii) introducing voluntary retirement with the social security. The results from the policy experiments show that even low skilled individuals prefer an economy without a social security system. Furthermore, the impacts of voluntary retirement on an individual's decisions vary by age and productivity. In particular, shorter working life is in favor of low skilled individuals while high skilled ones choose to work longer. It also shows that an increase in the tax rate which is corresponds with higher pension benefits leads to a larger response of high skilled individuals and a decline in the working life of low skilled individuals. However, these results do not hold if human capital is assumed to be exogenous.

1 Introduction

A pay-as-you-go (PAYG) social security system has been adopted in the United States and many other developed countries, mainly because it redistributes income in the economy. In this system, income is redistributed from individuals with higher lifetime earnings to those with lower lifetime earnings. It also guarantees sufficient retirement earnings for the elderly. In contrast, there have been some concerns regarding the impacts of social security systems on saving (e.g. Feldstein, 1974; Hubbard et. al, 1995), and labour supply decisions (e.g. Diamond and Mirrless, 1978). Following Auerbuach and Kotlikoff (1987), the macroeconomic and social implications of various social security reforms have been extensively analyzed in the literature to cope with the financial challenges of an aging population on the social security[1]. However, in the existing social security literature, labour productivity is mainly assumed to be exogenous.

This study contributes to the literature by incorporating a human capital accumulation mechanism. It is important to account for human capital accumulation since it affects the labour market decisions at all ages differently when a change in the economy (e.g. a change to the social security tax rate) is studied (e.g. Shaw, 1986; Heckman et al., 1998; Imai and Keane, 2004; Hansen and Imrohoroghlu, 2009). In particular, Kean (2013) shows the assumption of endogenous human capital through learning by doing[2] increases the effects of permanent tax changes over time. Furthermore, Alvarez-Albelo (2004) studied the role of human capital accumulation through learning by doing (LBD) in a general equilibrium model of social security. She shows that the capital-labour ratio varies less, and the average hours worked changes more under the model with endogenous human capital compared to a model with exogenous efficiency units of labour. As such, she indicated that the assumption of exogenous efficiency units of labour may lead to imprecise results. However, it is not clear how social security reforms influence individuals' decisions differently with heterogeneous human capital accumulation.

The impact of social security reforms on low income individuals requires more attention, as the main purpose of a social security system is to protect disadvantaged groups such as low income elderly. Consequently, the assumption of heterogeneity may be necessary to evaluate policies in a life cycle framework. Figure 1 illustrates that non-college graduates, on average, earn less during their working

[1] Including Hubbard and Judd (1987); Imrohoroglu, Imrohoroglu and Joines (1995, 1999); Huggett and Ventura (1999); Conesa and Krueger (1999); Nishiyama and Smetters (2007); Pries (2007); Rojas and Urrutia (2008); and Chen (2010).

[2] In general, two forms of human capital accumulation have been mainly adopted in the literature: learning-by-doing (LBD) or on-the-Job training (OJT). With OJT mechanism, individuals acquire human capital by spending time to learn while with LBD, individuals accumulate human capital through past experience.

period compared to college graduates. Therefore, this study investigates how a social security reform affects non-college (low skilled) and college graduates (high skilled) differently, and particularly their labour supply decisions over their life cycle. Conesa and Krueger (1999) also confirm the importance of heterogeneity to study the political impact of social security reforms.

A life cycle model of labour supply, human capital accumulation and physical capital accumulation with heterogeneous agents has been constructed in this study. The model has been developed with the intention of explaining differences in human capital accumulation and labour supply decisions between high skilled and low skilled workers. Consequently, it explains wage differentials and life cycle wage growth in a general equilibrium setting. In this model, skill is accumulated through past work experience or LBD mechanism[3]. Heterogeneity derives from two channels in order to capture differences in life cycle decisions and behavior among individuals. First, the model allows individuals to differ within an age group as a result of differences in learning abilities and thus productivities. Second, the model allows learning abilities to vary by age. In the reference economy, it is assumed that there exists a PAYG social security system and mandatory retirement[4]. Overall, the model provides a reasonable fit to the life cycle characteristics observed in the Panel Study of Income Dynamics 1962-2011 in the US economy.

Furthermore, the quantitative effects of heterogeneous human capital on the evaluation of social security reforms are investigated in four cases: (i) terminating the social security system with mandatory retirement, (ii) terminating the social security system with voluntary retirement, (iii) allowing voluntary retirement with the social security system where pension benefits decline while the social security tax rate remains unchanged, and (iv) allowing voluntary retirement with the social security system in which the tax rate rises while pension benefit stays constant.[5] The same exercises with social security reforms have also been done with exogenous human capital to quantitatively explore how the macroeconomic outcomes are affected by this assumption. It will be demonstrated that accounting for endogenous human capital is important for the evaluation of social security reforms with voluntary retirement since it leads to different outcomes. Differences between the model with endogenous human capital and exogenous human capital derive from the differences in the price of leisure. In the model with endogenous human

[3] Hansen and Imrohoroghlu (2009) studied the effect of endogenous human capital on average hours worked by age. They argued LBD affects labour market decisions at all ages, the assumption of endogenous human capital accumulation is important for the life cycle analysis, if human capital is accumulated through LBD.

[4] In the early 1970s, about half of Americans were covered by mandatory retirement provisions and required to leave their jobs at a specified age such as 65. Congress amended the Age Discrimination in Employment Act to abolish mandatory retirement in 1986.

[5] Studies that focus on the extending the retirement age as part of social security reforms include Hviding and Merette (1998) for a number of OECD countries; De Nardi et al. (1999), and Conesa and Garriga (2003) for the U.S.; Hirte (2002) for Germany; Henin and Weitzenblum (2005) for France; Beetsma et al. (2003) for the Netherlands; Keuschnigg and Keuschnigg (2004) for Austria; and Koka and Kosempel (2014) for Canada.

capital, where work experience accumulates human capital, the opportunity cost of the leisure equals the wage plus the marginal value of work experience. Whereas, in the model with exogenous human capital, the opportunity cost of leisure equals the wage.

The welfare outcomes show that both types of agents are better off when social security tax is set to zero. By setting the tax rate to zero, the net earnings increases which leads to higher saving and consequently higher wage rate. Imrohoglue et al. (1995, 1999) and Conesa and Kruger (1999) also found similar results in which the optimal social security tax rate is zero. Furthermore, the impact of elimination of mandatory retirement with social security on individual's decisions varies by age and type of agent. It also depends on the size of social security and its distributional effect. Our main result is that the labour force participation of individuals is affected differently by type with voluntary retirement. In particular, the labour market participation of low types declines while high types work longer. In a social security system, low types prefer voluntary retirement and high types are better off in an economy with mandatory retirement due to the changes in the distributional effects. Overall, an increase in the tax rate raises the labour force participation of high skilled individuals and declines the working life of low types. Furthermore, the labour force responses by type depend on the assumption of whether human capital is exogenous or accumulated endogenously.

The remainder of this research paper is organized as follows. In section 2, the model is developed and the solution method to obtain the steady states values is described. Calibration of the model is discussed in section 3. Section 4 presents the results and discusses the findings of the model with endogenous human capital accumulation. The results of the policy reform in a model with exogenous human capital are presented in section 5. The conclusions of the paper are provided in section 5.

2 The Model

A general equilibrium model, based on the pioneering life cycle model developed by Auerbach and Kotlikoff (1987), is considered with endogenous human capital accumulation and heterogeneous agents. In the model heterogeneity derives from the initial level of human capital, ability to learn, and age. There are two types of capital in the model: physical and human. Physical capital is accumulated during life through investment. Agents begin their life with no physical capital and leave no intentional bequests at the end of their life. On the other hand, agents start their life with positive human capital and human capital is accumulated only by allocating time to work and learning by doing.

Time is discrete and each period corresponds to one year in reality. Let s denote an individual's age and t the time period. A new generation of equal size is born every year. Individuals face an uncertain life span and may live for a maximum of T_{max} periods. All surviving individuals retire at age T_r. Given the conditional probability, $\varphi_{s,}$ of surviving from age s to age $s + 1$, the cohort shares, $\theta_{s,}$ are obtained by

$$\theta_s = \varphi_{s-1}\theta_{s-1} \text{ for } s = 2, \dots, T^{max} \tag{1}$$

where,

$$\theta_1 = 1 - \sum_{s=2}^{T^{max}} \theta_s \tag{2}$$

2.1 Household's problem

Individuals choose optimal consumption, *c*, and leisure, *l*, to maximize their discounted life time utility:

$$\sum_{s=1}^{T_{max}} \beta^{s-1} \left[\prod_{j=0}^{s-1} \varphi_j \right] \left[\frac{(c_{s,t+s-1} l_{s,t+s-1}{}^{\gamma})^{1-\eta} - 1}{1-\eta} \right], \tag{3}$$

where γ is the disutility of non-leisure activities, η is the coefficient of relative risk aversion, and β is the discount factor.

In each period, each individual has been provided with one unit of time. During the working period, time can be allocated among leisure, *l*, and work, *n*. Workers receive income from providing labour services and from renting capital assets to the production sector. Retired agents provide no labour services and instead, receive public pensions, b, from government.

The budget constraints for working and retired agents, respectively, are as follows:

$$k_{s+1,t+s} = (1 + r_t)k_{s,t+s-1} + (1 - \tau_t)w_t n_{s,t+s-1} h_{s,t+s-1} - c_{s,t+s-1} + tr_t \tag{4}$$

$$k_{s+1,t+s} = (1 + r_t)k_{s,t+s-1} + b_t - c_{s,t+s-1} + tr_t \tag{5}$$

where k is physical capital, r is the real rental rate of physical capital, h is worker efficiency or human capital, τ is the labour income tax rate, and *tr* is a government transfer of accidental bequests to the surviving individuals.

The model accounts for two types of heterogeneity in the production of human capital. First, the levels of productivity are different for individuals within the same cohort such that high type individuals, H, can accumulate more human capital compared to low types, L, at a given age. In the model, the fractions of high types and low types are represented by ζ and 1- ζ, respectively. Second, the productivity in learning is different between cohorts and declines with age. For both types i= $\{H, L\}$, it has been assumed that human capital is accumulated through a LBD mechanism according to the following equation,

$$h^i_{s+1,t+s} = (1-\delta_h)h^i_{s,t+s-1} + \Omega^i_s h^i_{s,t+s-1} {n^i_{s,t+s-1}}^{\phi} \tag{6}$$

here, the parameter δ_h is the depreciation rate of human capital, ϕ is a parameter that affects the speed of learning by doing, and Ω^i_s is a productivity parameter which is assumed to vary by age and type. Hansen and Imrohoroglu (2009) and Alessandrini and et.al (2015) have used a similar function for human capital accumulation. However, Alessandrini and et.al (2015) assumed that human capital is acquired through education and Hansen and Imrohoroglu (2009) did not account for heterogeneity by type in their learning by doing human capital accumulation technology. In both studies, the productivity sequences decline with age but no explanation is provided for this exogenous decline. One possible explanation might be that for a given technology, there may exist diminishing returns to learning (Kosempel, 2007). Kosempel suggests that as agents age they accumulate knowledge and will approach the technology frontier. As this happens learning will become more difficult, causing a decline in learning productivity.

2.2 Production

The production sector of the model consists of competitive firms which hire efficiency units of labour, L_t, and rent physical capital, K_t, to produce output, Y_t. Letting δ_k denote the depreciation rate of physical capital, then the net-of-capital-depreciation production function for the representative firm is assumed to take the constant returns to scale Cobb-Douglas form:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} - \delta_k K_t, \tag{7}$$

where α is the capital share of output.

2.3 Government

In this economy, there is a pay as you go system in which government collects labour income taxes from workers to finance the pension payments to the retired individuals. A balanced budget is required to be maintained in every period, and the government's budget constraint is given by:

$$\tau_t w_t L_t = b_t \sum_{s=T_r}^{T_{\max}} \theta_s \tag{8}$$

Furthermore, in every period t, government equally redistributes the confiscated accidental bequests through government transfer to the survivors:

$$tr_t = \sum_{s=1}^{Tmax} (k_{s+1,t+1,h}\xi + k_{s+1,t+1,l}(1-\xi))(1-\varphi_s)\theta_s \tag{9}$$

2.4 Competitive equilibrium

For a given initial distribution of human and physical capital stocks, the stationary competitive equilibrium in the model consists of a collection of policy rules:

$$\{c^i_{s,t}, n^i_{s,t}, h^i_{s+1,t+1}, k^i_{s+1,t+1}\}_{s=1}^{T^{max}}$$

for each type i, and factor prices $\{w_t,\ r_t\}$ such that:

1- The policy rules solve the optimization problem of each household in equation (3) subject to (4) and (6) for $s < T^r$, and (5) for $s \geq T^r$.

2- Production factors are compensated by their marginal products:
$w_t = (1-\alpha)K_t^\alpha L_t^{-\alpha}$
$r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha} - \delta_k$

3- Government balances its budget constraint and transfer constraint.

4- Commodity market clears:
$K_t^\alpha L_t^{1-\alpha} = C_t + K_{t+1} - (1-\delta_k)K_t$

5- In the factor markets, individual decisions and aggregate behaviors are consistent:
$K_t = \sum_{s=1}^{Tmax} (k_{s,t,H}\xi + k_{s,t,L}(1-\xi))\theta_s$

$$L_t = \sum_{s=1}^{Tmax} (n_{s,t,H} h_{s,t,H} \xi + n_{s,t,L} h_{s,t,L} (1-\xi)) \theta_s$$

2.5 Solution Methods

The steady state of the model is solved using the computational algorithms inspired by Heer and Maussner (2005) as follows: First, we make a guess for the initial steady state values for aggregate physical capital and aggregate labour. Second, the factor prices and tax rate are computed. Third, the household optimization problem is solved separately for both types using backward induction. Then, the new aggregate values for labour and physical capital are computed. If these values do not match the initial guesses then they are updated and this procedure is repeated until convergence.

3 Model Calibration

Particular values for the parameters of the model must be assigned to obtain numerical solutions to the model. The parameters are calibrated to match averages in the US data or set to values that are commonly used in the macroeconomics literature. Table 1 summarizes the calibrated values for the parameters of the model that will be explained in this section.

3.1 Demographics

It is assumed that age 1 in the model corresponds to the start of one's working life, that is, age 18 in reality. Individuals may live up to T^{max}=60 years in order to match the life expectancy at age 18 of males born in 1960 estimated by Bell and Miller (2002). The survival probabilities are also obtained from Bell and Miller (2002). Although human capital is accumulated in the model via LBD, it is well known that age-learning profiles differ by education level (see Figure 1). Therefore, the fraction of high types, ζ, is taken to be 0.46 to be consistent with the fraction of individuals who pursued an education beyond high school from the Panel Study of Income Dynamics (PSID) for the period of 1969-2011.

3.2 Policy Parameters

In the benchmark economy, it is assumed that individuals are not allowed to continue to work when they start collecting their pension benefits. Mandatory retirement age, T^r, is set to 48 to target the retired to active population ratio of 21.6%. It also matches the normal (or full) retirement age of 66 at which individuals are eligible to collect their full benefits in the US. Furthermore, the replacement ratio of pension benefits is chosen to be 53.3% to be consistent with the social security payroll tax rate of 10.1 % in 1978[6]. The pension benefit, b, associated with this tax rate is 0.1935 for the benchmark model.

3.3 Technology and preferences

The capital share of 0.36 is taken from Kydland and Prescott (1982, 19877) to match the US time series data. By setting the depreciation rate of physical capital to 10% per year as in Hansen (1985), the discount factor, β, and disutility from work, ɣ, are chosen to be 0.9747 and 1.819, respectively, to target the return to physical capital of 6% and the average time spent working of 0.3325 in the steady state. The rate of return to physical capital is taken from Alessandrini et al. (2015) and the average time spent working is generated from the Panel Study of Income Dynamics 1969-2011 for individuals aged 18 to 65. In the literature, the coefficient of the risk aversion is commonly assumed to be between 1 and 2 (e.g. Imrohoroglu and et. al, 1998). Gandelman and Hernández-Murillo (2014) estimate a constant relative risk aversion (CRRA) function with GMM to obtain the coefficient of risk aversion for 75 countries using data from Gallup World Poll. The value of 1.384 for the risk aversion parameter, η, is taken from their estimate for the US.

3.4 Human Capital

The parameters associated with equation (6) are calibrated as follows: By rewriting the human capital accumulation function, the sequence of age-specific learning abilities, $\left\{\Omega^i\right\}_{s=1}^{T^r-1}$, for each type is estimated as follows:

$$\Omega_{s,i} = \frac{h^*_{s+1,i} - (1-\delta_h) h^*_{s,i}}{h^*_{s,i} n^{*\phi}_{s,i}} \tag{11}$$

[6] It equals to the US Old Age and Survivors Insurance (OASI) tax rate in 1978

The average time spent working, $n^*_{s,i}$ and efficiency weights or exogenous human capital, $h^*_{s,i}$, for each age s and ability type are obtained using data on hourly earnings and annual work hours from the Panel Study of Income Dynamics (PSID) 1969-2011[7]. To do this, first, male head of households are divided into two groups based on their level of educational attainment and six age groups (18-24, 25-34, 35-44, 45-54, 55-64 and 65-77) are defined. In particular, low type individuals are those who achieved at most a high school diploma while high types obtained further education above high school. Second, the average hours worked, $\bar{n}_{s,i}$ and the average hourly earnings[8], $\bar{E}_{s,i}$, for each age group and type is obtained. By using the average hourly earnings, the efficiency weights for each type and for each age group are computed following the methodology put forward by Hansen (1993). Then, polynomial interpolation method is used to interpolate the values of average time spent at work and efficiency weights for each age and type. The parameter ϕ is set to be 0.02[9] to target the ratio of average efficiency weights between high type and low type from PSID[10]. In the model, the initial levels of human capital for low and high types are chosen to be 0.5773 and 0.5855, respectively, to match the calibrated efficiency weights, $h^*_{1,L}$ and $h^*_{1,H}$, from the data.

Furthermore, the depreciation rate of human capital, δ_h, is assumed to be the same across all individuals regardless of their type. In the literature, there is no consensus on the value of the depreciation rate of human capital particularly with LBD skill accumulation. In order to select a value for this parameter, the age-specific learning abilities series are generated with different values for the depreciation rate of human capital within an acceptable range (e.g. 1%, 5%, 7.5% and 10%). The wage profile from the data has a humped shape indicating that the wage declines toward the end of the working period due to human capital deccumulation. In order to obtain similar profiles in the model, it requires either the learning ability series take negative values or, more plausibly, a depreciation rate of human capital that is sufficiently high. By setting $\delta_h = 0.1$, the model is able to replicate the declines in wage profiles late in life.

Panels (a) and (b) of Figure 2 illustrate the calibrated exogenous human capital (h^*_i) and age-specific abilities (Ω_i), respectively, over the working years by type and age. The efficiency weights have a hump shape and are increasing at the beginning of the life and declining as individuals get older. At any given

[7] Data for years 1998, 2002, 2004 and 2008 were missing.
[8] Hourly earnings are converted into 1969 constant dollars.
[9] Starting with an initial guess for ϕ, the sequence of age-specific learning abilities is generated to run a simulation to obtain a profile of human capital for each type of agent. If the ratio of average human capital between high type and low type does not match the ratio of average efficiency weights between high type and low type from the data, the guess for ϕ is updated until convergence.
[10] For example, $\frac{\bar{h}_H}{\bar{h}_L} = 1.5079$.

age, the values of the efficiency weights are higher for high type agents compared to low type individuals. The calibrated age-specific abilities decline by age for both types and are higher for high types.

4 Results and discussions

This section presents the simulation results of the developed model with learning by doing at the steady state. The simulated life cycle profiles are compared with the observed data from the PSID for the period of 1969 to 2011. The life cycle profiles of labour supply are illustrated for each type of agent to evaluate the ability of these models to replicate the life cycle profiles observed in data. Then, a number of social security policy experiments are conducted and the impacts of each policy on the variables of interest in the steady states are discussed.

4.1 Life cycle analysis

Panels (a) to (d) of Figure 3 present the corresponding steady state values of physical capital, consumption, human capital and hours worked for the benchmark model by age and type. Results indicate that each type of agent makes different decisions to maximize their utility over the life cycle. High types devote less time to work and more to leisure when they are young, compared to low types. This is an optimal response by the high types given the rapid increase in wages they anticipate over their life cycle. For low types, wages are more constant over the life cycle. The simulated physical capital-age profiles illustrate that the high types start their life with borrowing, and hold fewer assets compared to low types in the model when they are young. However, the rate of accumulating physical capital is higher for high types so they accumulate more physical capital at any age above 37 in the model. Capital accumulation and labour earnings are correlated. For example, high types start their life by spending less time to work and borrowing to cover their expenses. However, when high types start allocating more time to work, their earnings accelerate at a higher rate compared to low types due to their higher level of human capital accumulation and consequently they are able to accumulate more physical capital too.

Optimal choices by both types result in increasing consumption profiles during the working periods but consumption declines sharply at the time of retirement which consistent with the literature (Bernheim, Skinner, and Weinberg, 2001; Hurd and Rohwedder, 2006). Nevertheless, the reduction in consumption upon retirement is higher for high types than low type agents. Human capital and hours worked have hump shapes which are consistent with the corresponding life-cycle profiles from the actual data. During the working period, high type agents accumulate a higher level of human capital at any age and allocate more time to work with the exception of the first few years compared to low types. This is because high types are more productive in learning compared to low types and they also start their life with a slightly higher level of human capital in the model.

4.2 Model Evaluation

Figure 4 presents the ability of the model to match the empirical life-cycle profiles for both types for hours worked, wages and human capital. Data on wages and hours worked are from PSID (1969-2011). Wages in both the data and model have been normalized to one at age 20 since they have different units. The overall performance of the model is satisfying. The simulated human capital and hours worked are able to replicate the main characteristics of data from PISD. In particular, human capital increases quickly and labour supply slowly at the beginning of the life cycle. On the other hand, the human capital profile is flat later in life while labour supply declines towards the end of the working period. The calibrated productivity series play important roles in the success of the model to imitate the data. Thus, the developed model is capable of strongly replicating the human capital series and wages across all age ranges. The model is not fully able to match the life-cycle profile of time spent working, although there is an agreement between the observed data and model simulations for the shape of the labour supply profile of high type individuals. The model slightly underestimates the labour supply at the end of working periods for both types and overestimates it for low types at the beginning of life cycle. These inconsistencies can be explained by the fact that the model does not account for some features of the data that may impact the labour supply (e.g. indivisibilities in hours worked). In reality, there is more heterogeneity among individuals compared to what can be considered in the model developed here. Other sources of heterogeneity that have not been modeled may also cause discrepancies between the data and model for life cycle labour supply. That said, the average hours worked is 31.8% for low types and 34.5% for high types in the data. In comparison, in the model, low type and high type agents allocate, on average, 31.4% and 35.4% of their time to work, respectively. Overall, the model provides a reasonable fit to the life cycle characteristics observed in the data.

4.3 Social Security reforms

This section provides the quantitative evaluation of the impact of social security reforms on the economy and individuals' decisions in the long run with an emphasis on heterogeneity among agents by age and type. Four reforms are considered to study the economic effects and welfare implications associated with eliminating the PAYG social security system ($\tau = 0$) and/or introducing voluntary retirement:

- Model 1 represents an economy with mandatory retirement and no PAYG social security system.
- Model 2 characterizes an economy with voluntary retirement and no PAYG social security system.

In the next two experiments, we introduce voluntary retirement. However, the reform affects the government revenue. When voluntary retirement is studied, it is necessary to either raise contribution tax rate or lower the benefit level:

- Model 3 corresponds to an economy with a social security system and voluntary retirement wherein the payroll tax matches the social security tax rate in the benchmark (BM) economy but the pension benefit is lower.
- Model 4 illustrates an economy with a social security and voluntary retirement in such a way that the pension benefit is the same as it is in the BM economy while the tax rate is higher.

Note that for all model economies, we assume that individuals can not start collecting their pension benefits until they reach the full retirement age and the pension benefits are tax exempted. We study the social security reforms by first examining the life cycle and macroeconomic effects. Then, we examine the welfare effects.

4.3.1 Life cycle and macroeconomic effects

Table 2 compares the long run economic outcomes after each policy implementation with the steady state outcomes of the benchmark (BM) economy in which retirement is mandatory and there exists a social security system as described in section 2. Although macroeconomic outcomes of social security reforms are well known due to the vast existing literature, the life cycle analysis of a reform needs more attention to understand how social security reforms impact individuals differently by age and particularly type. Figures 5 shows the life cycle profiles for physical capital, consumption, human capital and hours worked for low type agents in panels (a) to (d), respectively. Figure 6 illustrates the same life cycle profiles for high type agents. Furthermore, Table 3 presents the average individual outcomes of policy reforms for separate age groups and by type. The life cycle analysis shows that the impact of social security reforms on labour supply, physical capital, and consumption depend on an individual's age and type.

In Model 1, the elimination of the social security system with mandatory retirement results in 14.7% and 6.9% increases in aggregate physical capital and labour, correspondingly, compared to the benchmark economy. Consequently, the capital labour ratio is higher. The increase in the capital-labour ratio causes a 11% reduction in the rental rate of physical capital and a 2.4% increase in the wage rate. In an economy with no social security in which the sole source of income during retirement is capital earnings, individuals have an incentive to save more. Consequently, they accumulate more physical capital to acquire sufficient income to finance their expenses during the retirement period. Furthermore, elimination of the social security tax rate coupled with a higher wage rate has a substitution effect and an income effect on labour supply. Overall, the results indicate that the substitution effect dominates the income effect in which the average hours worked, $\bar{n}$, increases when $\tau = 0$. The increase in hours worked in each period not only raises the current earnings but it also boosts up human capital and consequently the future earnings of individuals. However, the impact of the social security tax on labour-leisure decisions is greater for low types compared to high types in Model 1. This is because low types are less productive at work compared to high types. Therefore, without a social security system, low type individuals need to work more to increase their earnings so that they have sufficient savings for retirement. Due to a lower interest rate in this economy, both types accumulate less capital when they are young (high types borrow more). However, middle-aged and old workers spend more time at work and accumulate more physical capital to guarantee a sufficient retirement income upon the retirement. Although individuals consume more during their working period, consumption drops sharply upon retirement and stays below the consumption in the BM during the retirement period due to lower retirement incomes.

Model 2 expands on the policy experiment in model 1 by allowing for voluntary retirement in addition to the elimination of social security. The macroeconomic effects are qualitatively similar to Model 1 in such a way that the aggregate physical capital, labour, consumption and human capital are higher compared to the benchmark economy. Nevertheless, the elimination of social security tax accompanying with higher wage rate has different impacts on each type and age when retirement is voluntary. Although both types choose to retire later in their life compared to Model 1, high types' working life duration (or labour force participation) increases considerably by 9 periods compared to low types by 2 periods. The explanation for this is that high types are relatively more productive later in life, and they will work more if given the opportunity. However, average labour supply during the working period declines for high types compared to the benchmark economy and Model 1 since they have less incentive to accumulate more physical capital for their retirement due to being able to work longer. Comparing Model 1 and Model 2 reveals that a change in the labour force participation has different impacts on physical capital at the individual level. The accumulation of physical capital is larger under voluntary retirement for middle-age low types as a result of a relatively higher interest rate in Model 2 than Model 1. In contrast, middle-age and senior high types accumulate less physical capital under voluntary retirement since they receive labour earnings for longer periods and consequently, they have less incentive to accumulate more physical capital for the retirement compared to Model 1. The results in Table 3 also show a reduction in physical capital for middle age high types compared to the correspondingly age group in the BM economy.

In Model 3, voluntary retirement is introduced into an economy with a social security system. In this economy, the social security tax rate matches the benchmark economy. It results in a 1.5% and 2.4% increase in aggregate physical capital and labour, correspondingly, compared to the benchmark economy. Subsequently, a relatively lower capital-labour ratio results in a 2% increase in the rental rate of physical capital and slightly decrease in the wage rate. Given the same tax rate as benchmark economy, the pension benefits must also decrease by 6% since the removal of mandatory retirement is associated with early retirement of low types. Furthermore, the average time spent working for high types declines although they work longer compared to the benchmark economy. Low types prefer to retire earlier since they are less productive at work and their productivities decline faster as they age compared to high types. Thus, a reduction in the wage rate makes working less attractive and discourages low types to work longer. Instead, their average labour supply increases by 3.5% during their working period so they can save more due to a higher rate of return compared to the benchmark economy. The decline in low type's labour force participation has some impacts on their consumption profile in which young and middle-age groups consume less compared to the benchmark economy. The consumption during the retirement is higher for both types in Model 3 since the pension benefits and interest rate are higher.

Model 4 is similar to Model 3 in that there exits voluntary retirement and social security. However, in model 4 the benefit level is increased to match its value in the BM economy. The social security tax rate must increase by about 1.1% with voluntary retirement if the pension benefit remains unchanged compared to the benchmark economy. This is because low types leave the labour market earlier than the time they collect pension benefits and average hours worked over the working period of high types decline by 7% even though they retire later. In this economy, a larger decline in aggregate physical capital than in aggregate labour causes a lower capital-labour ratio compared to the benchmark economy. As a result, the rental rate of capital rises by 2.7% while the wage rate drops by 0.6%. Young low types accumulate more physical capital and young high types borrow less due to a higher interest rate. In addition, the changes in individuals' labour force participation coupled with an increase in the tax rate are associated with a negative response of middle-aged individuals on their savings. In particular, low types accumulate less physical capital due to early retirement and high types need to save less for their retirement as they work longer given a higher return on capital.

Comparing models 3 and 4, the results show that individuals respond differently in these economies by type. In particular, an increase in the pension benefits and consequently the tax rate does not promote an early retirement for high types while encourages low type individuals to retire earlier. However, higher tax rate is associated with lower aggregate human capital in the economy. These findings may have some policy complications for policy makers who seek solutions to alleviate the negative impacts of population aging on the social security and labour market.

4.3.2 Welfare Effects

Following Koka and Kosempel (2014), the welfare benefits are measured as a fixed percentage of consumption, Δ, that is required to make individuals indifferent between living in the benchmark economy without compensation, and the alternative economy under a policy reform with compensation at each age:

$$v\left\{\left(c_s^i, l_s^i\right)|B\right\} = E_0 \sum_{s=1}^{T_{max}} \beta^{s-1} \left[\prod_{j=0}^{s-1} \varphi_j\right] u\left\{\left(1+\Delta^i\right)c_s^i, l_s^i\right)|PR\right\} \tag{12}$$

where the expected discounted life time utilities for each type of agent born in the benchmark economy and the alternative economy under a given policy reform are denoted by $v\left\{\left(c_s^i, l_s^i\right)|B\right\}$ and $v\left\{\left(c_s^i, l_s^i\right)|PR\right\}$, respectively. Therefore, the percentage consumption compensation for each type of agent is computed using:

$$\Delta^i = \left(\frac{v\left\{\left(c_s^i, l_s^i\right)|B\right\}}{v\left\{\left(c_s^i, l_s^i\right)|PR\right\}}\right)^{\frac{1}{1-\eta}} - 1 \tag{13}$$

Furthermore, the constant percentage of consumption compensation across all types of individuals in the economy is obtained from the weighted expected discounted life time utilities in the economy as it is utilized in Imrohoroglu et al. (1995) to measure the average utility in each economy, $v\{(c_s, l_s)|.\}$:

$$v\{(c_s, l_s)|B\} = E_0 \sum_{s=1}^{T_{max}} \beta^{s-1} \left[\prod_{j=0}^{s-1} \varphi_j \right] (\zeta u\{(1+\Delta)c_s^i, l_s^i)|PR\} + (1-\zeta) u\{(1+\Delta)c_s^i, l_s^i)|PR\}) \quad (14)$$

The computed percentage consumption compensations at equilibrium for each type of agent and for aggregate individuals are provided in Table 4. At the aggregate level, the results show that individuals prefer the proposed policy changes to the benchmark. For example, individuals born in Model 1 where social security is eliminated with mandatory retirement require 0.18% reduction in per period consumption to be indifferent with the outcomes of benchmark economy. However, the outcomes are interesting when the compensation variation is computed for each type of agent.

First, the results alter at the individual level by type when voluntary retirement is introduced with a social security system as in Model 3 and 4. In particular, high types born in the benchmark economy are better off than high types born in an economy with social security and voluntary retirement while low types prefer voluntary retirement. In Model 3 and 4, high types require 0.008% and 0.015% increases in per period consumption to be as well of as high types in mandatory retirement while low type require to give up 0.011% and 0.032% of their per period consumption correspondingly. The reason for these differences is that the redistributional effects of social security are larger in Model 4 than in the benchmark economy since benefits are not proportional to taxes paid. In particular, high types bear a larger tax burden compared to the benchmark economy due to an increase in tax rate. Low types benefit from an increase in the amount of redistribution since they work for a shorter period of time but they receive the same pension benefits.

Second, low types prefer to live in an economy without social security and pension benefits during retirement. Although the amount of consumption in each period that low types are required to sacrifice in order to be indifferent with the outcomes of benchmark economy is 0.05% lower than high types, the magnitude of compensation variation is still significant for low types. This is because the social security tax generates a distortion against labour supply, and the cost associated with this distortion is larger than benefits of income redistribution through the social security.

Third, both types prefer mandatory retirement when social security is eliminated since the absolute compensation variation as a percent of consumption in each period is higher for both types in Model 1 compared to Model 2. Koka and Kosempel (2014) explain that a coordination problem in the unconstraint economy (the voluntary retirement economy) results in a lower utility compared to an economy with mandatory retirement. When all high types are forced to retire early, aggregate employment falls and consequently the capital-labour ratio declines. This change positively affects the wage rate. The higher wage rate benefits all individuals and offsets any costs associated with a shorter working life.

Finally, in an economy with social security and voluntary retirement, low types prefer an economy with higher benefits even though it derives a higher tax rate while high types prefer an economy with a lower tax rate. This happens because a higher tax rate leads to a shorter working life for low type individuals and increases their life time utility due to more leisure they obtain, while the working periods increases for high types. Furthermore, the redistributional effects of social security are larger in Model 4 compared to Model 3, since benefits are not proportional to tax paid. In particular, high types bear a larger tax burden due to an increase in the tax rate since they retire later.

5 Exogenous Human Capital

In order to show why it is important to account for endogenous human capital when social security policies are studied, the impact of social security reforms on the economy and individuals' decisions are investigated in an alternative model with exogenous human capital and heterogeneous agents. Then, the outcomes from the alternative economy are compared with the benchmark model which is described in section 2. In this economy, the given age specific human capital for each type is taken from the calibrated exogenous human capital, $h^{*}_{s,i}$, in section 3.4. Consequently, equation (6) is eliminated in this model. All parameters are taken from the benchmark model to maintain the same characteristics of individuals for comparison purposes. Similar to section 4.3, four policy reforms are considered.

Table 5 compares the steady states outcomes of this alternative economy with the outcomes of the benchmark economy in section 4 and the outcomes of all policy models (e.g. Model A1, Model A2, Model A3 and Model4) with exogenous human capital. As it is expected, the aggregate physical capital and aggregate labour are higher in the benchmark economy with endogenous human capital compared to the alternative economy. However, a larger increase in the aggregate physical capital compared to an increase in the aggregate labour pushes the wage rate up by 0.4% while the interest rate declines by 1.9 %. The reason is that in the alternative economy, the wage rate is taken as given and individuals labour supply decisions are affected only through the opportunity cost of leisure and it is independence of work experience. Thus, the opportunity cost of leisure is higher when the assumption of human capital is relaxed. In particular, the increase in the labour supply not only raises the current earnings of individuals but it also increases their future earnings. Consequently, the average time spent working is higher for both types in the benchmark economy compared to the alternative economy.

Comparing Table 2 and Table 5, the results show that the assumption of exogenous human capital not only affects the key variables of the economy, but it predicts different outcomes for the labour market participation of individuals in some cases when retirement is voluntary. For example, after the elimination of social security and mandatory retirement in Model 2, both high types and low types postpone their retirement by one more period when human capital is endogenous compared to the corresponding model with endogenous human capital, Model A2. This is because first, an increase in the wage rate is slightly lower in the model with exogenous human capital compared to the model with endogenous human capital. Second, a larger increase in the average labour supply of low types or a lesser decline in the average labour supply of high types is not associated with higher human capital accumulation to enhance the future earnings in the alternative economy after the reform. Thus, individuals have less incentive to provide labour services for one more period.

Furthermore, comparing Tables 5 and 4 demonstrates how the assumption of exogenous human capital leads to contradictory outcomes in a model with a social security system and voluntary retirement. In particular, low types prefer Model 4 to Model 3 with endogenous human capital due to a higher level of pension benefits and lower labour market participation while the results alter when human capital is assumed to be exogenous. Low types prefer Model A3 to Model A4. This is because low types' labour force participation declines more in this setting so they have higher ability with extra leisure. Furthermore, a larger response among high types for their labour market participation declines the pension benefits at a lower rate and increases the distributional effects of social security in Model 3 with the exogenous human capital.

6 Conclusion

In this study, a life cycle model of labour supply and human capital with heterogeneous agents has been constructed. In the baseline model, human capital is accumulated through learning by doing. Agents differ in productivity and initial level of human capital. In addition, productivity declines by age. Then, two types of social security policies have been considered in constructing various policy reforms. In one experiment, the PAYG social security system is eliminated and the social security payroll tax rate and pension benefits are set to be zero with mandatory retirement. In the second experiment, the PAYG social security system and mandatory retirement are eliminated. The third and forth experiments represent voluntary retirement policy with a social security system where either tax rate or pension benefits, respectively, matches the baseline economy.

The results of these experiments demonstrate that individuals respond to a social security policy differently by age and type. In particular, the variation in labour force participation with voluntary retirement is diverse by type. Therefore, it is necessary to account for heterogeneity in the model to study the social security systems. In general, both types prefer an economy with no social security and welfare effect is stronger with mandatory retirement although retirement consumption is lower than the reference economy. In a social security system, low types prefer voluntary retirement and high types are better off in an economy with mandatory retirement due to the changes in the distributional effects. Overall, an increase in the tax rate raises the labour force participation of high skilled individuals and declines the working life of low types.

Furthermore, the same exercises with social security reforms have been done with exogenous human capital to explore how quantitatively macroeconomic outcome are affected by this assumption. The results imply that in addition to aggregate economy, the behavior of individuals impacted differently when retirement is voluntary. In particular, individuals prefer to retire earlier due to lower opportunity cost of leisure with exogenous human capital. However, a change in the tax rate does not have any impact on high types' decision on when to retire. In conclusion, the assumption of endogenous human capital is important for evaluation of social security reforms with voluntary retirement.

7 References

Alessandrini, D., Kosempel, S. and Stengos, T., 2015. The Business Cycle Human Capital Accumulation Nexus and Its Effect on Hours Worked Volatility. Journal of Economic Dynamics and Control: 356–77.

Alvarez-Albelo C. D., 2004. Endogenous versus Exogenous Efficiency Units of Labour for the Quantitative Study of Social Security: Two Examples,. Applied Economics Letters 11: 693–97.

Auerbach, A. J. and Kotlikoff, L. J., 1987. Dynamic Fiscal Policy,. Cambridge University Press, Cambridge.

Beetsma, R., Bettendorf, L., Broer, P., 2003. The budgeting and economic consequences of aging in the Netherlands,. Econic Modelling 20(5): 987–1013.

Bell F. C. And Miller, M. L., 2002. Life tables for the United States social security area. Actuarial Study, Social Security Administration, 116.

Chen, K., 2010. A Life-Cycle Analysis of Social Security with Housing,. Review of Economic Dynamics 13: 597-615.

Conesa, J. and Krueger, D., 1999. Social Security Reform with Heterogeneous Agents,. Review of Economic Dynamics 2: 757-795.

Conesa, J. and Krueger, D., 1999. Social Security Reform with Heterogeneous Agents,. Review of Economic Dynamics 2: 757-795.

Conesa, J.C., Garriga, C., 2003. Status quo problem in social security reforms,. Macroeconnomic Dynamics. 7: 691–710.

De Nardi, M., Imrohoroglu, S., Sargent, T.J., 1999. Projected U.S. demographics and social security,. Review of Economic Dynamics 2: 575–615.

Diamond, P., and Mirrlees, J., 1978. A model of social insurance with variable retirement,. Journal of Public Economics: 295-336.

Feldstein, ed., 1978. The effects of taxes on capital accumulation,. Chicago: University of Chicago Press: 7-48.

Gandelman, N. and Hernández-Murillo, R., 2014. Risk Aversion at the Country Level, doi:Federal Reserve Bank of St. Louis, Working Paper 2014-005B.

Haley, W. J., 1976. Estimation of the earnings profile from optimal human capital accumulation,. Econometrica, 44(6): 1223–1238.

Hansen, G. D., and Imrohoroglu, D. **(**2009), Business Cycle Fluctuations and the Life Cycle: How Important Is On-the-Job Skill Accumulation?, Journal of Economic Theory 144 (6): 2293–309.

Heckman, J. (1976) A life-cycle model of earnings, learning, and consumption, Journal of Political Economy, 84(4): S11–S44.

Heckman, J., Lochner, L. and Taber, C., 1998. Explaining rising wage inequality: explanations with a dynamic general equilibrium model of labor earnings with heterogeneous agents,. Review of Economic Dynamics: 1–58.

Heer, B. and Maussner, A., 2009. Dynamic general equilibrium modeling,. Springer, 2ed. Berlin.

Henin, P.Y., Weitzenblum, Th., 2005. Welfare effects of alternative pension reforms: assessing the transition costs for French socio-occupational groups,. Journal of Pension Economics and Finance 4(3), 249 - 271

Heylen, F. and Pozzi, L., 2007. Crises and human capital accumulation,. Canadian Journal of Economics, 40(4): 1261–1285.

Hirte, G., 2002. Welfare and macroeconomic effects of the German pension acts of 1992 and 1999: a dynamic CGE study,. German Economic Review 3 (1): 81–106.

Hubbard, R. G. and Judd, K., 1987. Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax,. American Economic Review 77, 630-646.

Hubbard, R. G., Skinner, J., and Zeldes, S.P., 1995. Precautionary Saving and Social Insurance,. Journal of Political Economy: 360-99.

Huggett, M. and Ventura, G., 1999. On the distributional Effects of Social Security Reform,. Review of Economic Dynamics 2: 498-531.

Hviding, K., Merette, M., 1998. Macroeconomic effects of pension reforms in the context of aging populations: OLG simulations for seven OECD countries,. OECD Working Paper, No. 201, ECO/WKP(98)14.

Imai, S. and Keane, M. P., 2004. Intertemporal labor supply and human capital accumulation,. International Economic Review, 45(2): 601–41.

Imrohoroglu, A., Imrohoroglu, S., Joines, D., 1995. A Life Cycle Analysis of Social Security. Economic Theory 6: 83-114.

Imrohoroglu, A., Imrohoroglu, S., Joines, D., 1999. Social Security in an Overlapping Generations Economy with Land. Review of Economic Dynamics: 638-665.

Johnson, T. and Hebein, F. J., 1974. Investments in human capital and growth in personal income,. The American Economic Review, 64(4):604–615.

Keuschnigg, C., Keuschnigg,M., 2004. Aging, labormarkets, and pension reformin Austria,. FinanzArchiv: Public Finance Analysis 60 (3): 359–392.

Koka, K. and Kosempel, S., 2014. Life-cycle analysis of ending mandatory retirement. Economic Modelling, 38, 57–66.

Nishiyama, S. and Smetters, K., 2007. Does Social Security Privatization Produce Effciency Gains?,. The Quarterly Journal of Economics 122(4): 1677-1719.

Pries, M. J., 2007. Social Security Reform and Intertemporal Smoothing,. Journal of Economic Dynamics and Control 31(1): 25-54.

Rojas, J. A. and Urrutia, C., 2008. Social Security with Uninsurable Income Risk and Endogenous Borrowing Constraints,. Review of Economic Dynamics 11(1): 83-103.

Shaw, K., 1986. Life-cycle labor supply with human capital accumulation,. International Economic Review, 30: 431–56.

Table 1- Calibrated Baseline Model Parameters with Heterogenous Agents

Parameters	set to target	Value from US data	Value from Model
$T^{max} = 60$	Life expectancy at age 18	58.99	
$T^{r} = 48$	Retired to active population ratio	21.60%	21.1%
$\beta = 0.9705$	Average annual interest rate	6%	6%
$\gamma = 1.92$	Average time spent working of workers ($\bar{n}^*$)	0.3325	0.3325
$\varphi = 0.04$	$\bar{h}_h^*/\bar{h}_l^*$	1.5079	1.5079
$\tau = 0.101$	Social security payroll tax rate in 1978	10.1%	
$\delta = 0.1$			
$\eta = 1.384$			
$\alpha = 0.36$			

Table 2- Steady States Outcomes of Policy Reforms with Endogenous Human Capital Accumulation

	BM	Model 1	%Δ from BM	Model 2	%Δ from BM	Model 3	%Δ from BM	Model 4	%Δ from BM
K	1.0011	1.1485	14.7226	1.1614	16.0094	1.0163	1.5174	0.9814	-1.9624
L	0.2829	0.3023	6.8933	0.3161	11.7471	0.2895	2.3518	0.2806	-0.8047
H[a]	0.9525	0.9546	0.2138	0.9901	3.9488	0.953	0.0519	0.9360	-1.7328
r	0.0599	0.0532	-11.2039	0.0565	-5.794	0.0612	2.1149	0.0616	2.7391
w	1.0101	1.0348	2.4424	1.0226	1.2426	1.0056	-0.4431	1.0043	-0.5729
b	0.1651	0	-100	0	-100	0.1551	-6.0731	0.1651	0
τ	0.101	0	-100	0	-100	0.101	0	0.1021	1.0505
K/Y	2.2455	2.3494	4.6278	2.2999	2.4246	2.2337	-0.5225	2.2287	-0.7485
K/L	3.5393	3.7985	7.3244	3.6743	3.8142	3.5104	-0.8152	3.4980	-1.1670
C	0.3462	0.374	8.0302	0.3844	11.0226	0.3493	0.8931	0.3488	0.7500
$\bar{c}_l^b$	0.2951	0.3144	6.5457	0.3211	8.813	0.2956	0.1908	0.2938	-0.4398
$\bar{c}_h$	0.4179	0.4498	7.6309	0.4693	12.3082	0.4256	1.8362	0.4272	2.2313
$\bar{n}^c$	0.3325	0.3537	6.3945	0.3291	-1.028	0.3308	-0.4967	0.3328	0.0907
$\bar{n}_l$	0.3141	0.3397	8.1497	0.3226	2.6996	0.3251	3.4861	0.3361	6.9966
$\bar{n}_h$	0.354	0.3702	4.5663	0.3366	-4.911	0.3376	-4.6454	0.3289	-7.1027
t_l^d	47	47		49		43		39	
t_h	47	47		56		51		53	

a. Aggregate level of human capital in the economy; b. The dash over the variables indicates the average; c. The average labour supply during the working period; d. Duration of working period.

Table 3- Average Individual Outcomes of Policy Reforms with Heterogeneous Agents for Separate Age Groups

Age Groups*	Physical Capital		Consumption		Hours Worked	
	Low type	High type	Low type	High type	Low type	High type
Benchmark Economy						
18-35	0.3040	-0.3432	0.2250	0.3062	0.3782	0.3603
36-64	1.7185	1.4597	0.3175	0.4635	0.2743	0.3501
64+	0.8948	1.5822	0.3421	0.4709	0	0
Model 1						
18-35	0.1911	-0.6243	0.2608	0.3590	0.3732	0.3514
36-64	1.9771	1.4748	0.3428	0.5041	0.3190	0.3818
64+	1.6984	2.4014	0.3252	0.4542	0	0
Model 2						
18-35	0.2887	-0.5362	0.2550	0.3519	0.3774	0.3542
36-64	2.1893	1.2266	0.3472	0.5131	0.3009	0.3669
64+	1.7969	2.0433	0.3542	0.5341	0.0221	0.1411
Model 3						
18-35	0.3315	-0.3140	0.2231	0.3040	0.3791	0.3615
36-64	1.7453	1.3826	0.3153	0.4669	0.2467	0.3442
64+	1.0011	1.6384	0.3522	0.5016	0	0.0558
Model 4						
18-35	0.3267	-0.3032	0.2223	0.3028	0.3782	0.3623
36-64	1.6481	1.3863	0.3106	0.4672	0.2173	0.3428
64+	0.9657	1.5508	0.3553	0.5103	0	0.0744

* Age groups represent the actual age groups in reality.

Table 4- Welfare Comparison with the Benchmark Economy

	Model 1	Model 2	Model 3	Model 4
% Consumption Compensation:				
$\boldsymbol{\Delta}$	-0.1776	-0.1688	-0.0035	-0.0129
$\boldsymbol{\Delta_l}$	-0.1568	-0.1509	-0.0113	-0.0320
$\boldsymbol{\Delta_h}$	-0.2073	-0.1944	0.0079	0.0154

Table 5- Steady States and Welfare Outcomes of Policy Reforms with Exogenous Human Capital

Outcomes	AL	% Δ from Alternative Economy (AL)				
		BM	Model A1	Model A2	Model A3	Model A4
K	0.957	4.627	12.938	17.031	1.648	-1.721
L	0.272	3.882	6.229	12.289	-1.555	-0.673
r	0.061	-1.906	-10.804	-5.609	3.157	2.625
w	1.006	0.409	2.382	1.217	-0.667	-0.556
b	0.159	3.957	-100	-100	-1.696	0
τ	0.101	0	-100	-100	0	1.187
K/Y	2.235	0.458	3.997	2.683	2.070	-0.676
K/L	3.514	0.717	6.315	4.223	3.254	-1.055
C	0.332	4.158	7.432	10.372	1.177	0.983
$\bar{c}_l^*$	0.286	3.189	5.974	8.255	0.137	-0.036
$\bar{c}_h$	0.400	4.568	7.063	11.581	2.625	2.334
$\bar{n}$	0.321	3.482	5.603	-0.335	0.505	0.216
$\bar{n}_l$	0.305	3.095	7.320	3.378	7.835	7.027
$\bar{n}_h$	0.341	3.888	3.800	-4.233	-7.190	-6.932
Working Period Duration:						
t_l	47	47	47	48	38	39
t_h	47	47	47	55	53	53

* The dash over the variables indicates the average.

Figure 1- Hourly Wages Profiles of College and Non College Graduates and all Individuals

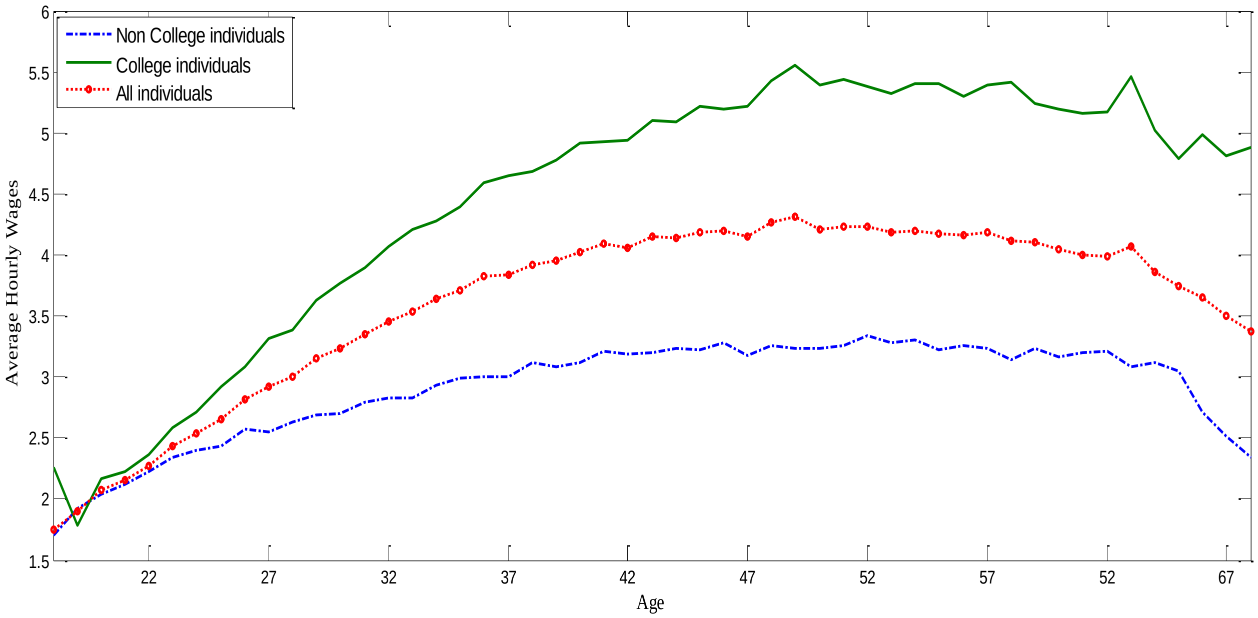

Figure 2- Calibrated Efficiency Weights and Productivity Sequence by Type and Age

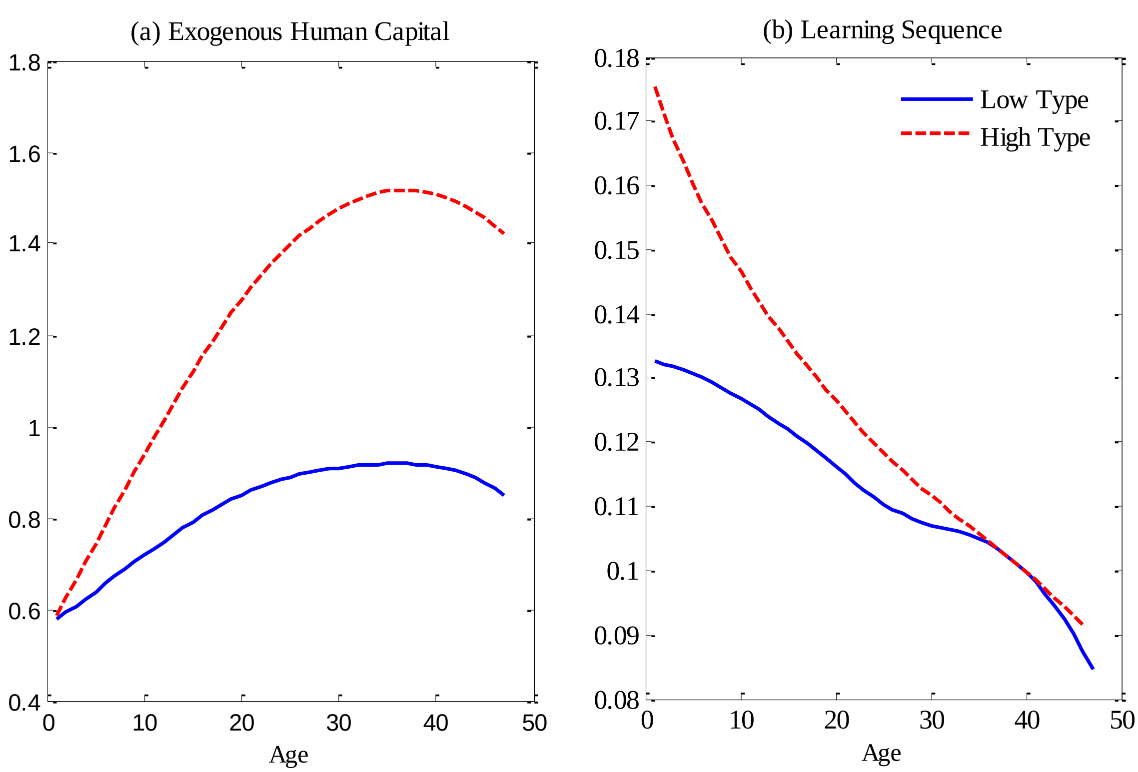

Figure 3- Steady States Profiles for High Type and Low Type Individuals for Baseline Model

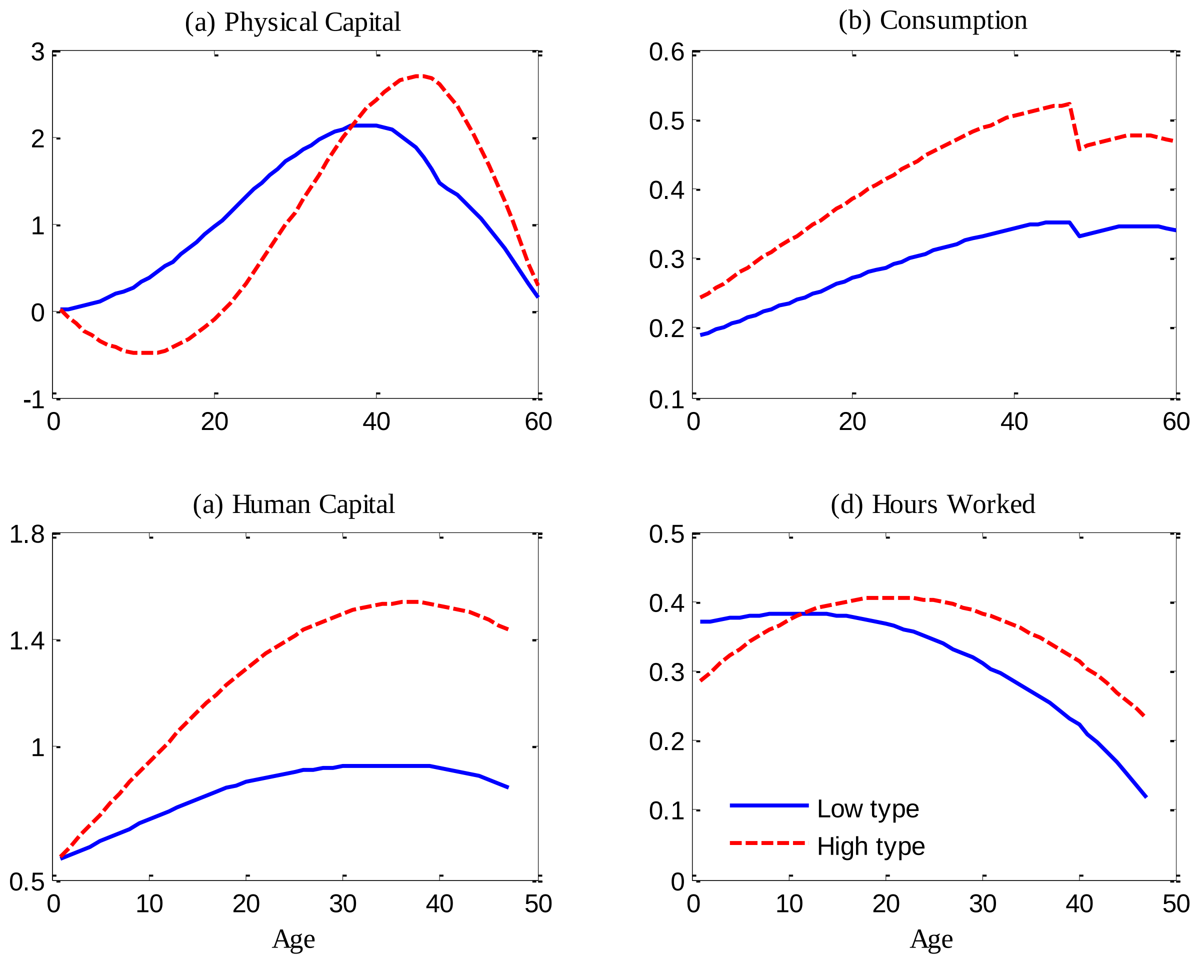

Figure 4- Ability of the Baseline Model (LBD) to Replicate the Wages, Human Capital and Hours Worked from the US Data for High Type and Low Type Individuals

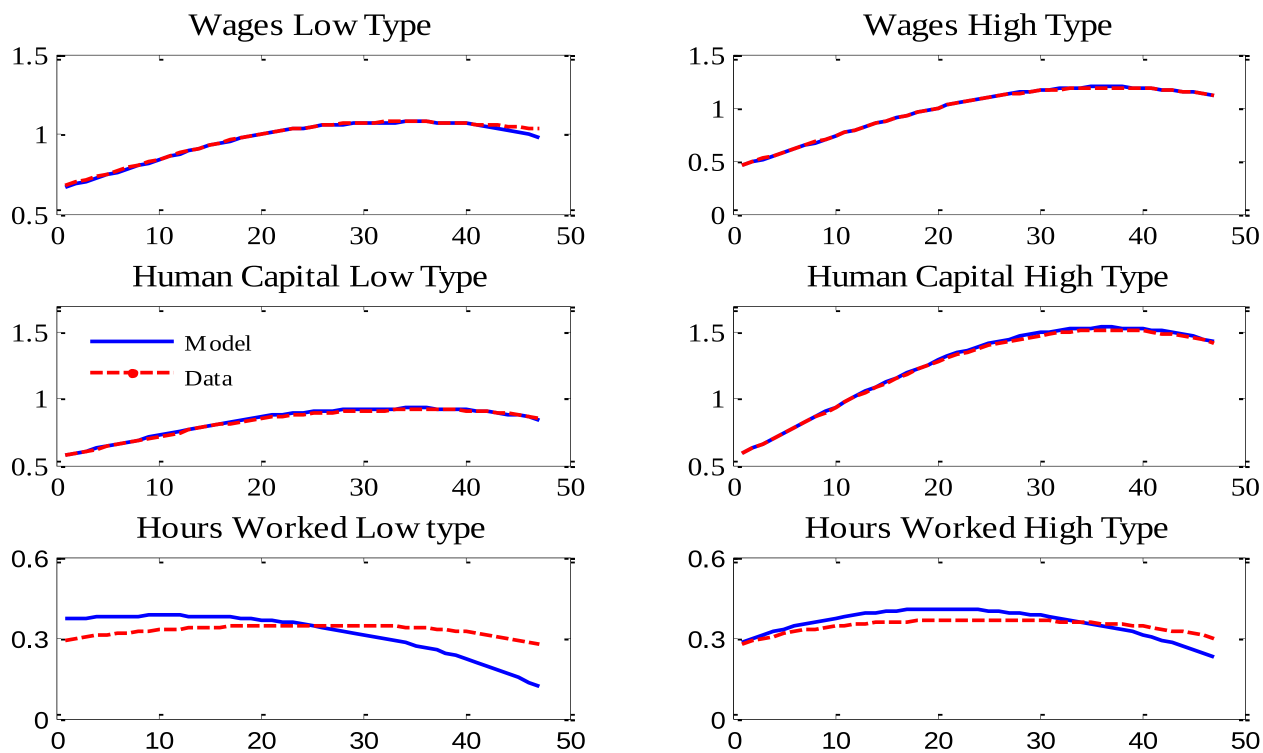

Figure 5- Impacts of Social Security Reforms on Life Cycle Profiles of Low Type Individuals

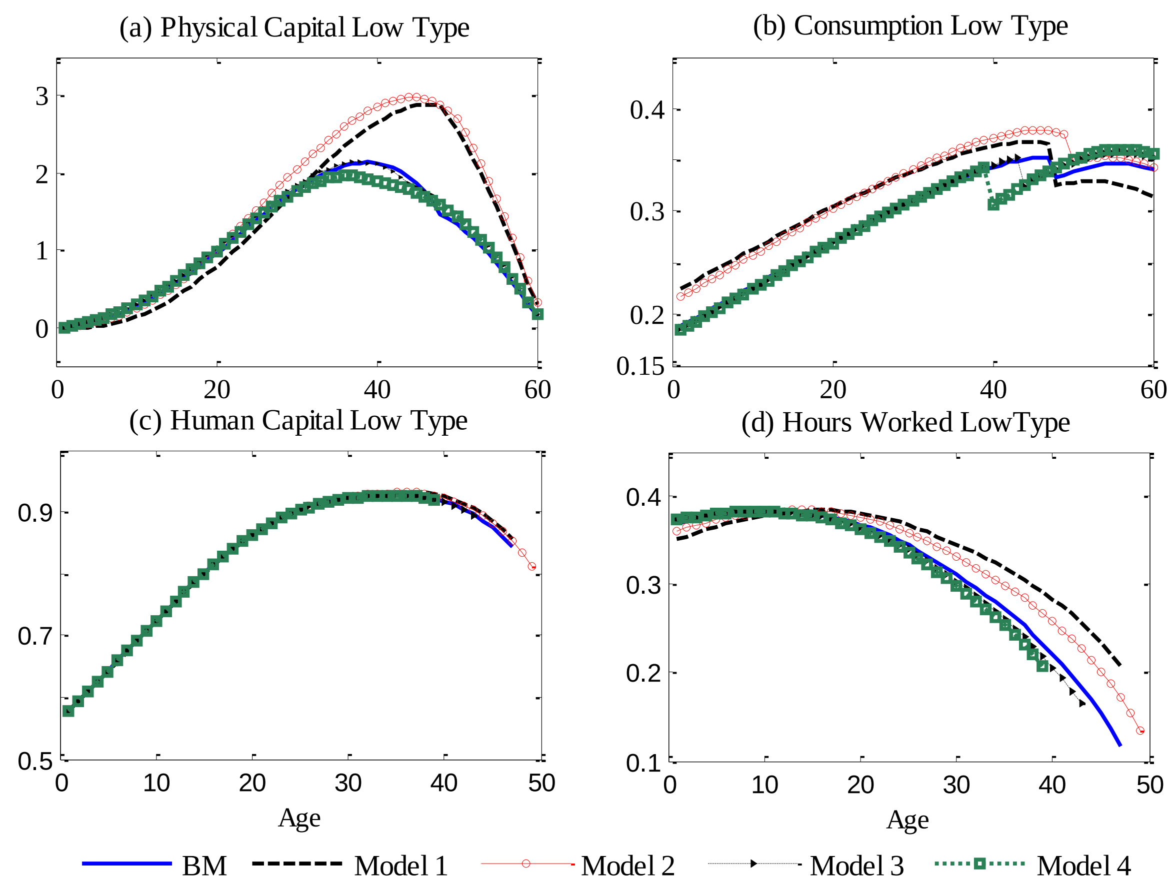

Figure 6- Impacts of Social Security Reforms on Life Cycle Profiles of High Type Individuals