Incentives for Effort with Team Production

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Abstract

This article looks at the impact of uncertainty about the types of teammates and about the production process on effort decisions of agents. The model involves solving a moral hazard problem with generation of agents that overlap. The economy contains two types of agents, pyramid organizational structures, (heterogeneous or homogeneous) combination of manager-worker teams, and promotion and retirement reward structure that affects incentives. When cost of effort is low, "bad luck" in the production process is low, and proportion of low efficiency types are high, then this reduces the incentives for individuals to free ride on the efforts of the team.

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1 Introduction

This article provides a further understanding of the interactions between individuals working in a firm with joint production. The focus is on the effort decisions made by agents of differing types with uncertainty and asymmetric information in regards to the agent's teammate and production process. The results provide an explanation, based on cost of effort of agents, luck in the production process, and proportion of types, for the teamwork that can exist in work environments. The analysis also includes several important features that reflects some aspects of a professional work environment such as a pyramid organizational structure and team rewards.

An important assumption of the analysis is joint production in firms. Joint production involves a group of agents who jointly produce some observable output. Each agent's input towards the production of output is unobservable. The observable output determines the rewards of the team members.[1] Moral hazard may exist when each agent's input is unobservable. A teammate may have an incentive to contribute less input (i.e., high cost of effort) and free ride on the inputs of others.

The framework is related to the theoretical literature regarding teams in the presence of asymmetric information. Many models focus on the actions and incentive scheme of the principle to induce efficient effort from its workers in teams.[2] This article adds to the literature regarding incentives in teams in the presence of asymmetric information by including the impact of uncertainty created by the production process and unknown type of teammate on the shirking decision of the agent.

[1] Suppose a team of workers produced a single unit of output that a firm considered good quality. The firm gives the "team" a good reputation instead of reputation per individual because firms cannot observe a workers type or effort. Each member of the team, therefore, has a good reputation regardless of the effort each member exerted in producing the good quality output. Each member is given the individual reward that accompanies a good reputation. Since the reputation is given to the entire team regardless of type and effort, there exists incentives of some members to free ride on the efforts of others to achieve a good reputation.

[2] Alchian and Demsetz (1972) states that residual from production should be given to an individual whose main purpose is monitoring individual team member inputs. The incentive compatibility of the monitor is met with the residual from production while the team members' incentive compatibility constraints are met with the threat of dismissal from the team. Holmstrom (1982) demonstrate that free riding can be avoided if contracts would impose large penalties for shirking. Rasmusen (1987) introduces risk-adverse agents and shows that a budget-balancing "massacre" contract can be feasible; a contract in which one randomly selected agent benefits and all others receive zero if an out-of-equilibrium output is observed. McAfee and McMillan (1991) demonstrated that monitoring is not needed to prevent shirking by team members if the principle should offer a menu of contracts to individual workers in teams. Itoh (1991) considered a moral hazard problem in a multi-agent situation where cooperation is a problem. Agents choose an effort level for their own task and another effort level for the task of another agent to "help" them. If the wage structure involved individual wages that are a function of other agents' outcomes and own and "helping" effort is complementary, the principle finds teamwork to be optimal relative to individuals focusing on own tasks.

Most theoretical work is inadequate for many applications. The model in this article incorporates some characteristics of professional work environments to better explain the teamwork that exists. First, there is pyramid organizational structure in which a manager is responsible for multiple agents. Second, are two levels of uncertainty in the form of the type of a teammate and in the production process ("bad luck"). Lastly, firms use a promotion scheme to 1) incentivize it's workers to exert effort and 2) act as a imperfect screening mechanism to only promote high ability individuals.

It is of more interest to investigate and analyze a scenario that uses a more commonly used or observed incentive scheme to induce effort in teams (i.e., promotions). Jeon (1995) and Auriol et al. (2002) considered an incentive scheme in which wages and payoffs are dependent on the team outcome (incentives tied/linked together). By exerting effort, the team performance is better which leads to higher payoffs for the worker. Breton et al. (2000) and Breton et al. (2002) analyzed incentive schemes for teamwork if teams are composed by workers of different ages. By pairing young workers that are unestablished with old workers with an established reputation, the team output is a better signal for the young workers type. Therefore, concern for his own reputation for the future, the young worker exerts effort. Huck et al. (2001) use a common incentive scheme in bonuses for team output but assume a social norm, such as disutility of exerting less effort than his team, exists in the work environment. The bonus increases individual effort but the existence of the social norm increases the effort exerted much more. The incentive scheme this analysis uses is promotions. If a team of workers perform successfully for a firm, all low level workers in the team are offered a promotion to manager. If they do not perform well, they are not offered a promotion. As shown later, the promotion can be an incentive to exert effort and it acts as a imperfect screening mechanism for the firm to promote high ability type individuals to be managers.

This article is closest to the analysis of Bar-Isaac (2007). He introduces a framework with overlapping generations of agents with three period lifespans. Agents can start their own firms and hire junior workers that produce output with them. The junior worker can eventually have the option to purchase the firm from his manager. It proceeds to characterize an equilibrium where agents exert costly and efficient effort throughout their life by comparing the expected payoff from sticking to the equilibrium strategy to all possible deviations. He concludes that by working alone, an agent with an established reputation cannot credibly commit to exerting costly effort. However, if the agent hires a junior of unknown reputation, the uncertainty acts as an incentive for the agent to exert effort because of concerns for his own reputation. This article builds upon Bar-Isaac (2007) by incorporating another level of uncertainty in the production process and finds that the introduction of uncertainty in the production process creates the existence of an equilibrium of no shirking. Also, a different incentive scheme (promotions) is used to incentive the juniors to exert effort which provides a different result. Although the methodology is similar, this analysis provides a solution using a different incentive scheme and additional level of uncertainty that is more widely observed in reality.

This article is best suited towards illustrating work environments where production of output is by teams of workers and the individual effort is unverifiable. Examples of such work environments are in industries such as consulting, law, or manufacturing. Within these organizations, the productivity of a single individual is hard to verify when output is produced in teams. For example, consider a service based organization such as a law firm. When a team of lawyers defend a case in court, it is hard to know the effort each lawyer put into the case. If efforts of each lawyer is unobserved, incentives cannot be given according to effort. The reward may be given to the entire team with each member having an equal share (promoting all members of the team). When rewards are given in this manner, there exists the possibility of incentives for individuals to "free ride" on the efforts of others and reap the benefits of success without exerting any costly effort of their own.[3]

Consider an economy that contains many firms that produce output. Firms employ a technology that requires a team of two individuals to produce a single unit of output. One member of the team is the manager; an individual with experience in production. The other member is the worker; an individual with or without experience in production. The technology also allows the manager to manage two workers separately but simultaneously. The manager's action is applied to bot teams. This creates two teams producing two units of output for the firm.[4] The output produced can be of two qualities, good or bad, and the probability that determines the quality is dependent on the

[3]A relatable example is group assignments. A professor issues a group assignment to his class. Students hand in the assignment and the professor grades it. If the professor knows the amount of work each student put into the assignment, the professor can give a grade according to the percentage of work each member gave. A practice observed in some courses requires students to hand in evaluations of their group members and the professor gives grades to individual students according to those evaluations. If the professor does not know the work put in by each group member, each member receives the same grade (team reward).

[4]This creates a vertical hierarchy structure within firms. This is often referred to as the pyramid organizational structure where a manager supervises/manages more than one worker.

effort levels the individual team members choose to exert. The firm values good quality (success) greater than bad quality (failure).

The firm's role is to select individuals to fill the positions for production and organize them into teams for production. The firm offers a contract with wages that are dependent on the position and work history of the individual. The contract also states the amount of retirement income the individual will receive depending on the individual's work history. The wages are equal to an individual's expected worth for his age and position in the firm.

There are two types of individuals that a firm can hire to fill the positions for production (managers and workers). The types differ by their efficiency levels. A high type individual is someone that can choose to exert effort but it is costless (cost of effort is equal to zero). The other type, low types, can exert the same amount of effort as a high type but incurs a positive cost of effort. Since a high type and low type can exert the same amount of effort but low types incur a cost, a high type is considered more efficient compared to a low type.

Firms provide incentives for individuals to exert effort in the form of promotions and retirement income. If a worker's team performs successfully in production (produce good quality output) for a firm, he is promoted to manager. If a worker's team does not perform successfully, he remains a worker but is now experienced (one period older). Managers that perform successfully (produce at least one unit of good quality output) is rewarded with more in retirement. Promoting individuals with good work histories (team success) is an imperfect screening mechanism for high efficiency types. Firms would prefer to promote high type individuals to managers. Since a high type individual incurs zero cost of effort, exerting effort is a weakly dominant strategy. Also, the effort he exerts is applied to both teams. This makes hiring a high type individual as a manager a more cost effective way to increase the probability of all teams producing good quality output. Since type and effort are unobservable, promoting a worker because of his work history provides the highest probability of obtaining a high efficiency type as a manager.

Individuals live for three periods; the first two are working periods and the final is a retirement period. A period ends after production has taken place and the quality of output is observed. They first apply for a job as a worker in a firm. They are considered "young" or inexperienced. The firm pairs him with a manager in the firm for production. Types of individuals are unobservable to firms and other individuals although age is observable. The "young" worker chooses an effort level to exert in team production. After production, the quality of the output is observed. If the quality is good, the worker is offered a promotion to a manager position by a firm. As a manager, the individual is paired with two workers. He manages then separately but simultaneously. The effort he chooses to exert is applied to both teams. The production process occurs again. When the quality of team output is realized, the firm gives the manager a retirement bonus if at least one of his teams was able to produce good quality output. The individual retires after the production process as a manager. If the team produced bad quality output, a "young" worker is not offered a promotion. He remains a worker but he is "old" and experienced. He is paired with a manager and the production process occurs again. He retires after his second round of production as a worker. In neither of the manager's teams produces at least one unit of output that is good quality, then he receives no retirement income. The wages and retirement for all the positions, young and old worker and manager, were agreed upon when the firm offered a contract to the individuals.

The analysis focuses on 3 possible equilibrium strategies of an individual. It proposes a specific equilibrium strategy and finds the exogenous parameters that supports the strategy. The three equilibrium strategies that this analysis focuses on focus on the effort decisions of a low efficiency type individual. The individuals always work for the firm and the individual chooses whether to exert effort only as a "young" worker, only as a manager, or in both positions.[5] Other equilibrium strategies, such as not working for the firm or rejecting promotions if offered, are not considered in this analysis.[6]

A simulation of the model illustrates that these three alternative possible pure strategy equilibria depend on the parameters of the model; c (cost of effort), β ("bad luck" in the production process), and γ (proportion of low efficiency types in each generation). High efficiency types have a weakly dominant strategy to exert effort in all positions, worker and manager, as the effort they exert is costless. Old and low efficiency type workers never exert effort because there are no incentives to do so. The analysis focuses on the actions of the low efficiency types. Low efficiency types can choose to exert effort in three possible scenarios; in all positions (young worker and manager), only exert

[5]Only exerting effort in one of the positions or in only one period of life means the individual free rides on the efforts of the team to obtain a reward. Exerting effort in all positions or both periods means he does not free ride. An agent never exerts effort as a "old" worker because there are no incentives in the model for him to exert effort (no retirement income for "old" workers). Also, a high efficiency type has zero cost of effort and therefore, always exerts effort as a weakly dominant strategy.

[6]It is believed that these are the most interesting equilibria and trivial equilibrium strategies, such as no individuals work for the firm, are not considered.

effort as young (first period) workers, or only exert effort if promoted to manager.[7] When cost of effort, c, is sufficiently low, β is sufficiently low, and γ is sufficiently high, effort is exerted in all positions and no shirking occurs. As cost of effort increases, exerting effort becomes less attractive. The agent chooses only one position to exert effort. When β is low or γ is high, the individual only exerts effort as a manager. If β is high or γ is low, the individual only exerts effort as a young worker.

2 The Model

This section presents a model to illustrate the actions of an agent throughout his lifetime while working for a firm that uses teams in the production process. It demonstrates the impact of uncertainty and asymmetric information, in regards to an agent's teammate and the production process, on an agent's contribution to the team.

There are important features that help the model reflect and provide a better understanding of professional work environments. There is a probability function that determines quality of production which obscures the actions of agents in the team. The quality of the output does not provide information on the effort choices of individual team members. A team of agents produce a single unit of output. The quality of output is dependent on the total amount of effort exerted by team members (joint production) and the public cannot tell which team member exerted effort.[8] Also, the vertical hierarchy of firms reflect an environment where an individual manages a team of workers, which is believed to be more observed in reality (i.e., pyramid organizational structure).[9]

[7] The design of the model does not offer a reward to workers in their second working period if they produced good quality output. It can certainly be included into the model and possibly several other equilibria space is created to account for the parameters that satisfy those conditions. But the qualitative results of the current equilibria should not change significantly. The addition of a retirement package for old workers to incentivize effort increases the overall expectation for a manager to produce good quality output. This only allows a higher cost to be incurred for conditions regarding the effort decision of a manager.

[8] "Two men jointly lift heavy cargo into trucks. Solely by observing the total weight loaded per day, it is impossible to determine each person's marginal productivity. With team production it is difficult, solely by observing total output, to either define or determine each individual's contribution to this output of the cooperating input." - Alchian and Demsetz (1972), pg. 779. Assuming a supervisor (in the case of this article, the firm) saw the heavy cargo on the truck, they would assume that half the weight was lifted by one man and the other half lifted by the other. Therefore, half the weight is attributed to one man and the other half to the other (assuming the supervisor was not observing the process of loading the truck).

[9] "The list of well-known companies that are using some form of a matrix is becoming long and impressive. Take, for example, a company that has annual sales of $14 billion and employs about 400,000 people in scores of diverse businesses; General Electric. For decades, despite the diversity of its businesses, GE used one basic structure throughout its organization: five functional managers reporting to one general manager... Other major corporations, in diverse activities, such as Bechtel, Citibank, Dow Chemical, Shell Oil, Texas Instruments, and TRW, to mention a

Finally, the rewards for production are given to the entire team regardless of effort because it is unobservable.

The analysis focuses on four possible equilibrium strategies of agents while working for a firm. Agents always choose to work in a firm, if possible, rather than choosing their reservation payoffs. The H type agent always chooses to exert effort while working for a firm. The L type agent can choose to exert effort as a young worker and manager, only as a young worker, only as a manager, or not exert effort in all working periods. The analysis constructs conditions for the H and L type agents for sticking to a equilibrium strategy and compares them to any deviations. The equilibrium strategy presented in this article is where the L type agent exerts effort as a young worker and manager. The construction of the conditions for other equilibriums are similar and the differences are explained in footnotes of the article. To illustrate the existence of the equilibrium, a simulation is performed and the conditions are illustrated in a parameter space. Certain combinations of values of parameters will satisfy the conditions of different equilibria creating areas with multiple equilibria. The equilibrium strategy that provides an agent with the highest lifetime expected payoff is considered the dominant strategy. Other equilibrium strategies are possible but this analysis focuses on, what is believed to be, the more interesting equilibrium strategies.

The model introduces a framework with overlapping generations of agents that live for three periods and work for a firm; the first two periods are working periods and the final period is a retirement period where agents only consume. It characterizes equilibria where agents choose to exert costly effort in one or both working periods while working for a firm. The proof of equilibria results begin by constructing the expected payoff of the agent along his decision path and then checking if there are incentives to deviate. The expected payoffs of an agent sticking to the equilibrium strategy is first constructed and then compared to the expected payoff from a deviation. If an agent behaves rationally and sticks to the equilibrium strategy, then there is a pure strategy equilibrium. A simulation is used to illustrate that any deviation from the equilibrium strategy, given certain parameters, leads to a lower expected payoff for the agent.

few, have also turned to the matrix. " - Stanley M. Davis and Paul R. Lawrence. Problems of Matrix Organizations. *Harvard Business Review.* https://hbr.org/1978/05/problems-of-matrix-organizations. May 1978.
"The management pyramid, as we know it, began to take shape around the early 1900s... This configuration worked well and why not, after all, it was borrowed from a very robust model that had proven to be successful in the most trying circumstances." - Vineet Nayar. It's Time to Invert the Management Pyramid. *Harvard Business Review.* https://hbr.org/2008/10/its-time-to-invert-the-managem. October 8, 2008.

2.1 Technology

The technology firms employ to produce a single unit of output for production requires a team of two agents. One of the agents is a manager. A manager is an agent that has worked in a firm in the previous period and was part of a team that produced good quality output. Due to his good work history (good quality output), he was offered a promotion to a manager position in a firm. The other member of the team is a worker. A worker is an agent that can be young or old. A young worker is an agent that has no experience with the production technology (an agent in their first working period). An old worker is an agent, similar to a manager, with experience in production (worked in a firm in the previous period) but did not produce good quality output and obtained a bad work history. He is not promoted and remained a worker. More details regarding how agents become managers and workers is explained in a later section.

The technology allows a manager to work with two workers simultaneously but separately. Therefore, for every manager in a firm, there are two workers hired. Since a team is composed of one manager and one worker, this creates two teams that produce a total of two units of output for the firm. The effort that a manager exerts is applied to both teams. This technology also creates a pyramid organizational structure where a higher level employee (manager) is in charge of one or more lower level employees (workers) but the lower level employees work separately.

The output produced by the team of agents can be of two qualities: good or bad. The probability that the output is of a certain quality is dependent on the amount of effort that each member of the team exerts. An explanation of the probability is provided in a later section.

2.2 Agents

Agents live for three periods. The first two periods are working periods where the agent chooses to work for the firm and the final period is a retirement period where he only consumes. A new generation of agents with a mass of 1 is born each period creating overlapping generations of agents.

An agent may be born high (H) or low (L) type which is unobservable to firms and other agents. There is a probability γ that the worker is born L type and $1-\gamma$ is born H type where $0 < \gamma < 1$. At birth, an agent knows his own type.

Agents have an effort choice while working denoted by $e \in \{0, \bar{e}\}$ where $\bar{e} > 0$. A H type agent always chooses to exert an effort level of $\bar{e}$. Exerting this level of effort is costless ($c = 0$) to the agent. Since a H type agent does not incur a cost ($c = 0$) while choosing to exert effort ($\bar{e}$), he always chooses to exert effort. Exerting effort is a weakly dominant strategy. A L type agent chooses an effort level to exert. If he chooses $\bar{e}$, he incurs a cost of $c > 0$. Choosing not to exert effort incurs no cost ($c = 0$). The difference between the two types of agents is the efficiency level. A H type agent can exert the same amount of effort as a L type agent but at a lower cost.

All agents are risk neutral, maximize their expected lifetime earnings, and have a discount factor of 1 (discount rate of 0).

2.3 Firms

There exists many identical risk neutral firms. An agent's type and effort is unobservable to the firm but age of agents is observable. Firms hire one agent that was in his first working period in his previous period and produced good quality output in a team to be a manager.[10] They also hire two agents to be workers managed by the manager.

The joint production process of the firm requires two agents; one manager and one worker. As mentioned previously, the manager manages two workers simultaneously creating two teams with a team comprising of the manager and one of the two workers. Therefore, each period, two units of output are produced by each manager for the firm.

Each unit of output can be of two qualities: Good (G) or Bad (B). The firm values good quality output at 1 and bad quality output at zero.

Firms cannot observe the type and effort of agents. Due to the nature of the joint production process, firms are unable to identify from the quality produced, which agent exerted effort. Therefore, firms arbitrarily attribute half $\left(\frac{1}{2}\right)$ the value from production to each member of the team.[11]

Reputation is earned from production in a firm. Quality of output produced in a team de-

[10] In this article, agents in their first working period are referred to as "young". Agents in their second working period are referred to as "old".

[11] The agents in the model are risk neutral. The H type agent exerts effort but it is costless. The L type agent can exert effort but incurs a cost of $c > 0$. Therefore, if the L type agent exerts effort, they obtain the same expected payoff as the H type agent minus cost of effort, c. Suppose a firm changed the share of value from production between workers and managers where it increases the expected payoff for H type agents. They will move to the new firm offering the different contract. Since the expected payoffs are the same between the two types, the new share also increases the expected payoff of L type agents. The new contract has attracted all types of workers but at a different share. Therefore, the share does not affect the qualitative results of the analysis.

termines an agent's reputation. Since effort is indistinguishable between agents in teams, the reputation earned from production by a team is given to all team members regardless of effort. This means the team member's reputation is equal to the team's reputation. If the quality of output is G, agents earn good reputations, otherwise, B quality output earns no reputations.

Rewards accompany good reputations. A young worker with good reputation is offered a promotion for the subsequent period to manager by a firm. Firms benefit from promoting H types to manager because they have a weakly dominant strategy to exert effort for zero cost. By promoting H types to manager, this increases the probability of producing good quality output of all teams at the lowest cost to induce effort. Since type and effort is unobservable, promoting workers that produced good quality output gives the highest probability of a H type manager. An old worker with a good reputation is rewarded with nothing and retires next period. A manager earns a good reputation as long as one of the teams he manages produces good quality output. A manager with good reputation is given R in retirement.[12] Agents with bad reputations are given nothing next period. In this case, a worker re-applies for a position as worker when he is old and a manager retires with nothing in the retirement period.

For services of the agents, firms pay them a wage according to their position (worker or manager) and age. They pay a wage equal to the expected value they bring to the firm. Therefore, firms make an expected profit of zero. Employment contracts are in the form of (w_{YW}, w_{OW}, w_M, R) where w_{YW} is the wage that a young worker is paid, w_{OW} is the wage an old worker is paid, w_M is the wage a manager is paid, and R is the retirement consumption given to managers.[13]

2.4 Timing

In the beginning of an agent's first working period, the agent is born and the type is revealed to him. The agent chooses to apply for a job with a firm in a worker position. A worker is considered as the low level employee in the vertical hierarchal structure. If he does not apply or is not hired by a firm, he obtains a reservation payoff of $\bar{P}$ equivalent to the expected value of producing a single

[12] It is true that providing different R's dependent on whether the outcome is one or two good units of good quality output for the manager can potentially provide a lower expected R required to induce effort. For simplicity of the results, only one R is used.

[13] Despite the possibility that the employment contract can be in the form of $(0, 0, 0, R)$ with R sufficiently large and achieve similar results, for the purpose of realism, this analysis assume a cap on R to achieve positive wage levels. The potential of including risk aversion for agents in the future provides a reason for the analysis performed here.

unit of output by himself.

If the agent is hired to be a worker in the first working period, he is paired with another agent in a manager position. The manager is considered to be a high level position on the vertical hierarchal structure. The two agents work together to produce a single unit of output for the firm. The worker decides whether or not to exert effort. If he exerts effort, the probability of producing good quality output increases. The probability of producing good quality output is dependent on the combined efforts of the worker and manager.[14] Likewise, the effort choice of the worker impacts the expected probability of producing good quality output of the manager.

In this joint production process, the effort decision of an agent can influence the outcomes of his teammate and *vice versa*. By exerting effort, an agent increases the team's probability of producing good quality output. Not exerting effort hurts both agents by not increasing the probability. Effort is unobservable and the quality of output does not provide information about which member of the team exerted effort. Firms and other agents cannot tell which agent exerted effort. This production process allows an agent to shirk on his teammate without incurring the true total cost of the team because costs are incurred by both team members.[15]

In the second working period, the quality of output from production in the first working period is observed (given he was hired by a firm in the first working period). If the team produced good quality output, the team members are all given their corresponding rewards. The worker is given a promotion to manager from a firm. The worker decides to accept the promotion and becomes a manager. The firm hires two workers for the manager to manage simultaneously. This creates two teams with a team comprising of the manager plus one of the two workers. The manager has an effort decision. The effort decision made by the manager is applied to both teams. The output produced from each team is independent from the other.

If he rejects the promotion, he chooses to work alone and earns a reservation payoff of P equivalent to the expected value of producing a single unit of output by himself given that he produced good quality output in a firm in the previous period.[16]

[14] Each team of workers produce one unit of output that can be of two qualities: good or bad. The firm values each quality differently. The section on firms, presented later in the article, provides details on the role of the firm and its valuation on quality.

[15] If efforts are observable, it is possible to only penalize the agent that chose not to exert effort. Therefore, the agent incurs the full cost of shirking. If effort is unobservable, the team is punished and therefore some costs are incurred by the agent that chooses to exert effort.

[16] An individual that does not have a history (bad) of production from a firm has a reservation payoff of $\bar{P}$. This

If bad quality was produced in the first working period, the worker is not rewarded with a promotion to manager. The agent applies for a worker position in his second working period. The reputation earned from producing bad quality is unobservable to the public.

Similarly, if the agent was not hired in the first working period as a worker, he applies for a job in his second working period. Producing good quality output while working alone does not get any rewards from a firm.

The output quality from an individual's second working period is observed. If, at least one of the teams, good quality output is produced, then the manager is given a retirement consumption of $R > 0$ in the retirement period. If neither of the teams produced good quality output, the manager is given 0. Workers in their second working period also retire but receive no retirement benefits. Therefore, the only way to obtain R is to work for a firm as a manager and produce at least one unit of good quality output in each working period.

2.5 Probability of Quality

In general, when members of a team all choose to exert effort, the probability of producing high quality output is usually greater compared to if they choose not to exert effort. Even though the team members exert effort, sometimes people make mistakes and therefore the quality of work suffers or the task was completed not to standard. There are no guarantees in the outcome of a task unless all team members choose to not exert effort.

This article uses a probability function to determine the quality of output produced by a team in a firm that incorporates characteristics of teamwork explained above. The function adopted, for N team members, is the following,

$$p(e_1, ..., e_N) = \frac{\sum_{i=1}^{N} e_i}{\sum_{i=N}^{N} e_i + \beta} \qquad \text{where} \qquad \beta > 0 \quad \text{and} \quad e_i \in \{0, \bar{e}\} \tag{1}$$

The equation is a modification of the popular Tullock contest success function. Instead of the players exerting effort to defeat the other, they exert effort to help the team succeed.[17]

is given to young or old agents that are not hired by firms. An individual with a (good) history of production from a firm has a different reservation payoff given by P. The reservation payoff for an agent with a production history is updated to reflect the "good reputation" he earned from a firm. (i.e., Using a successful history at a firm to advertise his own business.)

[17] Amegashie (2006) proposed a similar contest success function in which he examined the degree to which luck as

First, $p(e_1, ..., e_N)$ is increasing and concave in all e_i's. This equation satisfies the property that the probability of good quality is increasing in each individual's effort.

Second, the function includes a measurement of error involved in production. The error can arise from team member accidents, mistakes, bad luck, or any other uncontrollable/unobservable factors. The magnitude of these factors' effect on the production process is captured in β. The β captures the extent to which luck as opposed to effort determines the success in a team environment. As β increases, the probability of good quality decreases (i.e., the likelihood of a good quality product depends more on luck).

When β is strictly positive, the probability of good quality is never 1. If the entire team exerts effort, that is the highest probability of producing good quality output the team can achieve. Therefore, if they produced a bad quality output while all team members exert effort, it is caused strictly by uncontrollable factors and not by the efforts of team members. On the other hand, if all members of the team exert zero effort, then the team produces bad quality output with certainty.

In this article, a team working for a firm is composed of two agents; a worker and a manager. Since types are unobservable, agents form expectations of the probability that good quality output will be produced. Therefore, the expected probability conditional on the worker's type for a worker and manager position are denoted by $p^W(e_W; e_M^H, e_M^L)$ and $p^M(e_M; e_W^H, e_W^L)$, respectively. The worker's expected probability is a function of the effort choice of the worker, e_W, and the effort choice of the manager he is paired with in equilibrium depending on whether he is H, e_M^H, or L type, e_M^L. The manager's expected probability is a function of the effort choice of the manager, e_M, and the effort choice of the workers in equilibrium depending on whether they are H, e_W^H, or L types, e_W^L. The expected probabilities, $p^W(e_W; e_M^H, e_M^L)$ and $p^M(e_M; e_W^H, e_W^L)$, must all be consistent with the on and off equilibrium actions of the agents.

2.6 Wages

As mentioned in a previous section, firms compete for agents and pay a wage equal to the expected value they bring to the firm.

opposed to effort affects behaviour in different contest settings. His paper presents and discusses the properties of the contest success function which is very similar to the probability function used in this article.

The wage given to a young worker is,

$$w_{YW} = \frac{1}{2}\left[p(H^{YW})p_H^W(e_W; e_M^H, e_M^L) + p(L^{YW})p_L^W(e_W; e_M^H, e_M^L)\right] \tag{2}$$

The wage is constructed as follows. If the team produces good quality output, the firm values it at one and attributes half $\left(\frac{1}{2}\right)$ the value produced to the young worker. To the firm, the worker can be of two types; H and L. If the young worker is H type, $p(H^{YW})$, he is expected to produce good quality with probability $p_H^W(e_W; e_M^H, e_M^L)$. If he is L type, $p(L^{YW})$, he is expected to produce good quality output with probability $p_L^W(e_W; e_M^H, e_M^L)$. If bad quality output is produced, the firm values it at zero.

The wage given to an old worker is,

$$w_{OW} = \frac{1}{2}\left(p(U) + p(F)\right)\left[p(H^{OW})p^W(\bar{e}; e_M^H, e_M^L) + p(L^{OW})p^W(0; e_M^H, e_M^L)\right] \tag{3}$$

The wage is constructed as follows. If the team produces good quality output, the firm values it at one and attributes half $\left(\frac{1}{2}\right)$ the value produced to the old worker. For an agent to be an old worker, he must have been unemployed in the previous period, $p(U)$, or part of a team that produced bad quality output in the previous period, $p(F)$. Similarly to the young worker's wage, the probability of an old worker producing good quality output is given by the probability the worker is of either type, $p(H^{OW})$ and $p(L^{OW})$, multiplied by the probability of producing good quality output, $p^W(\bar{e}; e_M^H, e_M^L)$ and $p^W(0; e_M^H, e_M^L)$, depending on whether he is H or L type respectively.[18]

The wage given to a manager is,

$$\begin{aligned} w_M = & \left[2(p(H^M)p^{M2}(e_M^H; e_W^H, e_W^L) + p(L^M)p^{M2}(e_M^L; e_W^H, e_W^L))\right. \\ & \left. + (p(H^M)p^{M1}(e_M^H; e_W^H, e_W^L) + p(L^M)p^{M1}(e_M^L; e_W^H, e_W^L))\right] \\ & - 2(p(N)w_{YW} + (p(U) + p(F))w_{OW}) - Rp^M(e_M; e_W^H, e_W^L) \end{aligned} \tag{4}$$

The wage given to a manager is constructed as follows. The manager is paid the residual of the expected value earned in production. A manager manages two teams simultaneously If

[18] A L type old worker chooses to never exert effort. He gets paid an old workers wage for being hired but there are no incentives for him to exert effort because there are no rewards in his retirement period.

both teams produce good quality output, the firm values the output at 2. The probability of both teams producing good quality output is given by the probability the manager is H or L type, $p(H^M)$ and $p(L^M)$, respectively, multiplied by the corresponding probability both his teams produce good quality output given the managers type, $p^{M2}(e_M^H; e_W^H, e_W^L)$ and $p^{M2}(e_M^L; e_W^H, e_W^L)$. Similarly, if only one of the manager's teams produces good quality output, the firm receives a value of 1 and is multiplied by the probability of only one team producing good quality output, $p(H^M)p^{M1}(e_M^H; e_W^H, e_W^L) + p(L^M)p^{M1}(e_M^L; e_W^H, e_W^L)$. If no teams produce good quality output, the value is zero. The expected value from production for a firm is the first two terms in the managerial wage. The firm must pay wages to the two workers hired for production. The workers hired by a firm can be young or old. Finally, the firm pays the manager a retirement consumption of R next period if at least one of his teams produces good quality output, $p^M(e_M; e_W^H, e_W^L)$. Subtracting the expected workers wages and retirement consumption of the manager from the expected value from production is the manager's wage.

If an agent does not work for a firm, he receives a reservation payoff equal to the expected value from production if he was working alone instead of in a team. There are two potential reservation payoffs for the worker; a reservation payoff if he does not have a history of production from a firm or if he produced bad quality output with a firm in the previous working period (P) or a reservation payoff if he produced good quality output in a firm in the previous working period ($\bar{P}$).

Suppose an agent was in their first working period and was not hired by a firm or in their second period but produced bad quality output in the previous period, he has no production history. Therefore, his reservation payoff is given by,

$$P = p(H)\frac{\bar{e}}{\bar{e} + \beta} \tag{5}$$

The reservation payoff is the expected payoff from producing output by himself. Good quality output is valued at 1 and bad quality valued at zero. The expected payoff is the probability the agent is H type provided his age is observable multiplied by the probability of producing good quality output while working alone. L type agents working alone do not exert effort and produce bad quality with certainty because their are no rewards next period for producing output alone. Since there are no incentives next period, an agent chooses no effort.

If an agent produced good quality output in a firm the previous period, the agent accepts a promotion to manager in equilibrium. But if he rejects the promotion, the agent produces alone. Given a history of good quality output last period in a firm, the agent has a different reservation payoff.

$$\bar{P} = p(H|G)\frac{\bar{e}}{\bar{e}+\beta} \tag{6}$$

Similar to P, the reservation payoff is the probability the agent is H type given he produced good quality output in a firm in the previous period multiplied by the probability of producing good quality output while working by himself.

2.7 Probability Agents Hired as Workers

In each working period, there are agents, young and old, applying for jobs as a worker. Applying for a job with a firm as a worker does not guarantee the agent a job. The agent has an expected probability of being hired by the firm in each working period. The expected probability that an agent is hired as a worker in a working period is constructed as follows.

The number of workers hired by firms is directly related to the number of managers hired by firms. For each manager hired by a firm, two workers are hired. The expected number of managers in a period is given by N,

$$N = \alpha p^W(e_W; e_M^H, e_M^L) \tag{7}$$

The above equation is constructed as follows. To become a manager in the second working period, the agent must have been a worker in the first working period. Therefore, the agent was a young worker. Each period, a new generation of agents is born with a size of 1. Of the agents born, a proportion of them apply to be workers and are hired. This is represented by the variable α. To be a manager, the worker must have been part of a team that produced good quality output. The probability that a worker produces good quality output is given by $p^W(e_W; e_M^H, e_M^L)$. Therefore, the expected number of managers in a period is N.

Each firm hires two workers for each manager hired. Therefore, the number of worker vacancies (V) each period is,

$$V = 2\alpha p^W(e_W; e_M^H, e_M^L) \tag{8}$$

The number of applicants (A) for worker positions in a period is given by,

$$A = 1 + (1-\alpha) + \alpha\left(1 - p^W(e_W; e_M^H, e_M^L)\right) \tag{9}$$

The above equation is constructed as follows. There are three groups of agents that apply for worker positions in a given period. Each period, there is a new generation of agents born with a size 1. There is a proportion of young agents last period that were not hired by firms last period $(1-\alpha)$. Lastly, there are agents that were hired in their first working periods as workers, α, that were part of a team that produced bad quality output, $\left(1 - p^W(e_W; e_M^H, e_M^L)\right)$. They re-apply for worker position in their second working period.

The expected probability of being hired as a worker in equilibrium (α) is the number of vacancies (V) divided by the number of applicants (A).

$$\alpha = \frac{V}{A} = \frac{2\alpha p^W(e_W; e_M^H, e_M^L)}{1 + (1-\alpha) + \alpha\left(1 - p^W(e_W; e_M^H, e_M^L)\right)} \implies \alpha = \frac{2(1 - p^W(e_W; e_M^H, e_M^L))}{p^W(e_W; e_M^H, e_M^L)} \tag{10}$$

3 Equilibrium Strategies

This article focuses on the three possible equilibrium strategies that are believed to be the most interesting. Other equilibrium strategies and even multiple equilibria are possible but are not analyzed in this article but is considered in future iterations. The analysis focuses on the four possible equilibrium strategies of effort for an L type agent; exerting effort as a young worker and manager, only as a young worker, or only as a manager. All agents choose to work for a firm if possible. Other possible equilibrium strategies such as deviations in their career decisions (i.e., not working for the firm or rejecting promotions to manager) are not looked at.

The analysis begins by hypothesizing the existence of one of the equilibrium strategies of the L type individual and finding the exogenous parameter values that support the its existence. The proof of the results starts by outlining the equilibrium strategy of agents in each period of life and checking possible deviations from the equilibrium strategy to verify no agent has an incentive to deviate. The expected payoff of each action in the equilibrium strategy is constructed and compared to the expected payoff of a deviation. Conditions are formed to show that, given a set of exogenous parameters, any deviations from the equilibrium strategy leads to a lower expected payoff. The analysis starts with the last decision an agent makes and works backwards in his decision path towards the first decision of his life.

The conditions constructed in this model are similar to incentive compatibility and participation constraints. They illustrate that, if the expected payoff is greater when performing an action compared to another, the action with the higher expected payoff is taken. Assuming agents in this model are rational, the following inequalities can be considered as rationality conditions as agents choose the action that provides the higher expected payoff.[19]

The equilibrium strategy of both types is to apply for jobs when possible and accept any employment contract offered by a firm so long as such contract offers him in equilibrium at least as high an expected payoff as his reservation payoff of working alone.

The conditions constructed in this section illustrate an equilibrium strategy where the L type agent chooses to exert effort as a young worker and manager.[20] Other equilibrium strategies with the L type agent exerting effort in a different combination of positions (i.e., only young worker, only manager, or never exert effort) is constructed in a similar approach. The expected payoff of sticking to the equilibrium strategy is constructed and compared to the expected payoff of a deviation. The changes in wages and α reflect the beliefs and on and off equilibrium actions of the agents and firms.

The H type agent always choose to exert effort and it is costless ($c = 0$). As long as the expected payoff from working for a firm is greater than his reservation payoff, H type agents always work for a firm. Suppose a H type agent is offered a promotion in his second working period because he produced good quality output in the previous period in his team, the H type worker accepts the promotion if,

$$w_M + p^M(\bar{e}; e_W^H, e_W^L)R \geq \bar{P} \tag{11}$$

The above inequality is constructed as follows. The left hand side (LHS) of the inequality is

[19] "In the standard view, rational choice is defined to mean the process of determining what options are available and then choosing the most preferred one according to some consistent criterion. In a certain sense, this rational choice model is already an optimization-based approach." - Jonathan Levin and Paul Milgrom. Introduction to Choice Theory. *Jonathan Levin: Teaching and Lecture Notes.* http://web.stanford.edu/ jdlevin/Econ%20202/Choice%20Theory.pdf. September 2004.

[20] The L type agent never exerts effort as a old worker because they retire next period and is guaranteed zero regardless of the effort exerted. Also, the equilibrium strategy is described in terms of the L types actions because the H types do not have conditions to determine their effort choice. Since all types' equilibrium strategy is to always work for the firm, the focus of the analysis is on the actions of the L type agent while working for a firm.

the expected payoff of an H type agent accepting the promotion. The manager is given a wage of w_M. If one of the teams that he is part of produces good quality output, $p^M(e_M; e_W^H, e_W^L)$, the firm pays him a retirement package of R. The right hand side (RHS) of the inequality is the expected payoff of a worker that rejects the promotion and chooses to work alone. He makes $\bar{P}$, the income he obtains by selling output produced alone given he produced good quality output in a firm last period.

Inequality (11) can be re-written as,

$$w_M + p^M(\bar{e}; e_W^H, e_W^L)R - \bar{P} \geq 0 \tag{11*}$$

If a H type agent produces bad quality output in the previous period in a team, he re-applies for a worker position with a firm in his second working period.

$$\alpha w_{OW} + (1-\alpha)P \geq P \implies w_{OW} \geq P \implies w_{OW} - P \geq 0 \tag{12}$$

The above inequality is constructed as follows. The LHS of the equation is the expected payoff of a H type agent if he re-applies for a worker position in a firm in his second working period. If he is hired as a worker, α, he is given a wage of w_{OW}. If the agent is not hired by the firm, $1-\alpha$, the agent works alone and earns P, a reservation payoff for an agent with no history of production from a firm. If he chooses not to apply for a job, he obtains P with certainty (RHS). Since cost of applying for a job is zero, if the wage earned in a firm is greater than the reservation payoff, P, the H type agent applies.

Moving backwards on the decision path, the H type agent applies for a worker position in a firm in his first working period.

$$\begin{aligned} & w_{YW} + p^W(\bar{e}; e_M^H, e_M^L)\left(w_M + p^M(\bar{e}; e_W^H, e_W^L)R\right) + (1 - p^W(\bar{e}; e_M^H, e_M^L))\left(\alpha w_{OW} + (1-\alpha)P\right) \\ & \geq \\ & P + \alpha w_{OW} + (1-\alpha)P \end{aligned} \tag{13}$$

Again, since cost of applying is zero, if the expected payoff from working in a firm is greater than his expected reservation payoff, the agent applies. If the H type agent is hired, he earns the young worker's wage, w_{YW}. In production, if his team produces good quality output, $p^W(\bar{e}; e^H_M, e^L_M)$, the agent has an expected payoff of accepting a promotion (Inequality (11)). If the team produces bad quality output, $1 - p^W(\bar{e}; e^H_M, e^L_M)$, the agent re-applies for a job in the next period and has an expected payoff of re-applying for a worker position (Inequality (12)). If the H type agent does not apply for a job as a worker (RHS), the agent receives P with certainty in the first working period for producing alone and next period applies for a job as a worker and has an expected payoff of applying in the second working period (Inequality (12)).[21]

Equation (13) can be re-written as,

$$w_{YW} - P - p^W(\bar{e}; e^H_M, e^L_M)\left[\alpha w_{OW} + (1-\alpha)P - w_M - p^M(\bar{e}; e^H_W, e^L_W)R\right] \geq 0 \qquad (13^*)$$

The results of the analysis focus on the L type agent because of their effort decisions. Similar to the methodology for the H type agent, the analysis begins with the final decision an L type agent makes in his lifetime and works backwards towards the first decision. The equilibrium strategy of a L type agent is to exert effort as a young worker and manager and the conditions presented compare the expected payoffs of sticking to the equilibrium strategy to a deviation. The last decision an L type agent makes is an effort choice as a manager.[22]

$$-c + p^M(\bar{e}; e^H_W, e^L_W)R \geq p^M(0; e^H_W, e^L_W)R \qquad (14)$$

The above inequality is constructed as follows. The LHS of the inequality is the expected payoff of a L type manager that chooses to exert effort. If he exerts effort, he incurs a cost of c. If the manager produces at least one unit of good quality output with his teams, $p^M(\bar{e}; e^H_W, e^L_W)$, he receives a retirement package of R. The RHS is the expected payoff of a manager that chooses not to exert effort. The manager does not incur the cost of c because he does not exert effort. The manager receives a retirement package of R if at least one of his teams produces good quality

[21] If an agent was unemployed in the first working period, they are equivalent to an agent that produced bad quality output in a firm. In the second working period, the condition for an agent to re-apply as a worker in his second working period after producing bad quality output in the previous period is the same for an agent that was working alone in the previous period.

[22] This assumes the L type manager produced good quality output last period, a firm offered a promotion, and the agent accepted the promotion.

output, $p^M(0; e_W^H, e_W^L)$. The H type agent exerts effort as a manager if the LHS is greater than or equal to the RHS.[23]

Inequality (14) can be re-written as,

$$[p^M(\bar{e}; e_W^H, e_W^L) - p^M(0; e_W^H, e_W^L)]R \geq c \tag{14*}$$

The action prior to exerting effort as a manager is accepting a promotion offered by a firm. If a L type agent exerts effort as a manager in equilibrium, the condition for the agent to accept a promotion from a firm is,

$$w_M - c + p^M(\bar{e}; e_W^H, e_W^L)R \geq \bar{P} \tag{15}$$

The above inequality is constructed as follows. The LHS is the expected payoff of a L type agent accepting the promotion to manager. By accepting, the manager receives a wage w_M. If the manager exerts effort in equilibrium, the expected payoff from production is given by the LHS of inequality (14). The RHS is the reservation payoff from rejecting the promotion and choosing to work alone given he produced good quality output in the previous period in a firm, $\bar{P}$.[24]

Inequality (15) can be re-written as,

$$w_M - \bar{P} + p^M(\bar{e}; e_W^H, e_W^L)R \geq c \tag{15*}$$

The decisions made by a L type agent so far relate if the agent produced good quality output in the previous period. If the agent produced bad quality output, the agent does not receive an offer of promotion. He re-applies for a position with a firm as a worker. This condition was explained previously and is the same for a H type worker. This condition is inequality (12).

Moving backwards, the L type agent makes an effort decision in his first working period while

[23]If the equilibrium strategy is for the L type agent to not exert effort as a manager, inequality (14) is reversed with the RHS greater than or equal to the LHS with consistent beliefs.

[24]If the equilibrium strategy for L type agents is not to exert effort as managers, the inequality sign in condition (14) is reversed with the RHS greater than or equal to the LHS. The expected payoff from production on the LHS of inequality (15), $-c + p^M(\bar{e}; e_W^H, e_W^L)R$, becomes the RHS of inequality (14) because not exerting effort is the rational decision, $w_M + p^M(0; e_W^H, e_W^L)R$.

working for a firm as a worker.

$$
\begin{aligned}
& -c + p^W(\bar{e}; e_M^H, e_M^L)(w_M - c + p^M(\bar{e}; e_W^H, e_W^L)R) + (1 - p^W(\bar{e}; e_M^H, e_M^L))(\alpha w_{OW} + (1-\alpha)P) \\
& \geq \qquad (16) \\
& p^W(0, e_M^H, e_M^L)(w_M - c + p^M(\bar{e}; e_W^H, e_W^L)R) + (1 - p^W(0, e_M^H, e_M^L))(\alpha w_{OW} + (1-\alpha)P)
\end{aligned}
$$

Inequality (16) is constructed as follows. The LHS is the expected payoff of a L type worker choosing to exert effort in the first working period while working for a firm. He incurs a cost of c for exerting effort. If his team produces good quality output for the firm, $p^W(\bar{e}; e_M^H, e_M^L)$, he has an expected payoff given by accepting a promotion and exerting effort as a manager (LHS of inequality (15)). If the worker's team produces bad quality output, $1 - p^W(\bar{e}; e_M^H, e_M^L)$, the worker has an expected payoff given by re-applying for a worker position with a firm next period (LHS of inequality (12)). The RHS is the expected payoff of a L type worker choosing not to exert effort in the first working period. The worker does not incur a cost of c. If his team produces good quality output, $p^W(0, e_M^H, e_M^L)$, he obtains an expected payoff given by accepting a promotion to manager and exerting effort. If the team produces bad quality output, his expected payoff is given by re-applying for a worker position next period. The expected payoffs for the second working period are the same if he exerts effort or not in the first working period. A L type agent sticks to the equilibrium strategy of exerting effort as a young worker if the LHS is greater than or equal to the RHS.[25]

Inequality (16) can be re-written as,

$$
\frac{p^W(\bar{e}; e_M^H, e_M^L) - p^W(0, e_M^H, e_M^L)}{1 + p^W(\bar{e}; e_M^H, e_M^L) - p^W(0, e_M^H, e_M^L)} \big(w_M + p^M(\bar{e}; e_W^H, e_W^L)R - \alpha w_{OW} - (1-\alpha)P\big) \geq c \qquad (16^*)
$$

The first decision the L type agent makes is to apply for a job as a worker in a firm in his first working period. Since cost of applying for a job is zero, if expected payoff of being hired is greater than the reservation payoff of producing alone, the L type agents apply for jobs in the first working

[25] If the equilibrium strategy for L type agents is not to exert effort as young workers, the RHS is greater than or equal to the LHS. Furthermore, if the equilibrium strategy is for L type agents to never exert effort as young workers and managers, first, the RHS is greater than or equal to the LHS and, in addition, the second working period expected payoff for producing good quality output changes to the expected payoff for not exerting effort as a manager.

period.

$$w_{YW} - c + p^W(\bar{e}, e_M^H, e_M^L)(w_M - c + p^M(\bar{e}; e_W^H, e_W^L)R) + (1 - p^W(\bar{e}, e_M^H, e_M^L))(\alpha w_{OW} + (1-\alpha)P))$$
$$\geq \tag{17}$$
$$P + \alpha w_{OW} + (1-\alpha)P$$

The above inequality is constructed as follows beginning with the LHS. If the agent is hired, he obtains a wage of w_{YW} and an expected payoff given by exerting effort as a young worker and manager and accepting a promotion if offered one by a firm (LHS of inequality (16)). For an agent to apply for a job in his first working period, this must be greater than the expected payoff of the RHS. If the agent chooses to not apply, he receives P for working alone and an expected payoff in the second working period given by re-applying for a worker position.[26]

Inequality (17) can be re-written as,

$$\frac{1}{1 + p^W(\bar{e}, e_M^H, e_M^L)} \left(w_{YW} - P + p^W(\bar{e}, e_M^H, e_M^L)(w_M + p^M(\bar{e}; e_W^H, e_W^L)R - \alpha w_{OW} - (1-\alpha)P)\right) \geq c \tag{17*}$$

If the equilibrium strategy for the L type agent is to exert effort as a young worker and manager, the sufficient conditions are (11*) to (17*). The cost of effort must be sufficiently low for a L type agent to choose to exert effort in all working periods.

4 Simulation

To provide a comprehensible solution to the problem presented in the previous section, a simulation is performed. The exogenous variables in the model are maximum effort levels ($\bar{e}$), cost fo effort (c), luck (β), and proportion of types (γ). Values are selected for certain parameters to show the impact of a change in other exogenous variable on effort choices of agents in teams.

The simulation begins by presenting the conditions for each equilibrium strategy. Values pre-

[26]Similar changes occur to inequality (17) if the equilibrium strategy for L type agents were to not exert effort in either position of young worker or manager as inequality (16). If the equilibrium strategy is L type agents do not exert effort as young workers, the LHS does not include a c and the probability for production changes. If no effort is exert as a manager, the expected payoff given good quality output is produced changes. A combination of the two changes can also occur depending on the equilibrium strategy.

viously selected are substituted into the conditions and plotted in a parameter space of exogenous variables (i.e., values are selected for $\bar{e}$ and γ and the conditions are illustrated in a (c,β) parameter space). The area in which the parameter values satisfy all conditions is the equilibrium space where the equilibrium strategy exists given selected parameter values. After all equilibrium strategies are shown, the parameter spaces are combined into a single graph. Any overlapping areas, where parameter values satisfy conditions for multiple equilibrium strategies, are resolved by determining the strategy that provides the agent with the highest lifetime expected payoff. The strategy that provides the highest lifetime expected payoff is considered the preferred strategy.

The maximum value for effort is set to 1 ($\bar{e} = 1$). There are parameter values of c (cost of effort), β ("bad luck"), and γ (probability agent is born L type) that exist to satisfy all conditions that characterize an equilibrium. The following graphs illustrate areas in a (β, c) and (γ, c) space where the conditions are satisfied for each equilibrium strategy (exerting effort in one working period, both, or none).

The graphs presented in this section are only of the L type agents. If the parameter values satisfy a rationality condition for a certain equilibrium strategy for L types, by transitivity, H types' conditions are satisfied. Agents are risk neutral. A H type agent always chooses to exert effort and exerting effort is costless ($c = 0$). Therefore, a H type agent has a weakly dominant strategy to exert effort. The contract only needs to satisfy a H type agent's participation constraints to work for a firm. Suppose a L type agent chooses to work for the firm and exerts effort. The L type agent receives the same expected payoffs as the H type agent and has the same probability of receiving a reward from production. The difference is the L type agent incurs a cost of effort lowering his expected payoff. If the contract satisfies the L type agent to work for a firm and exert effort, a H type agent also chooses to work for a firm. On the other hand, an equilibrium strategy of a L type agent can be to work for a firm but exert no effort. The similarity between the two types of agents is they do not incur a cost of effort but the H type has a higher probability of obtaining a reward, increasing his expected payoff. Therefore, if a contract satisfies the conditions for a L type to work for a firm, a H type also works for the firm.

Figure 1 illustrates the parameter values that satisfy the conditions for the equilibrium strategy where L type agents choose to exert effort in all working periods as a young worker and manager.[27]

[27]The scale of the axes are chosen to highlight and clearly illustrate the area where the parameter values satisfy the

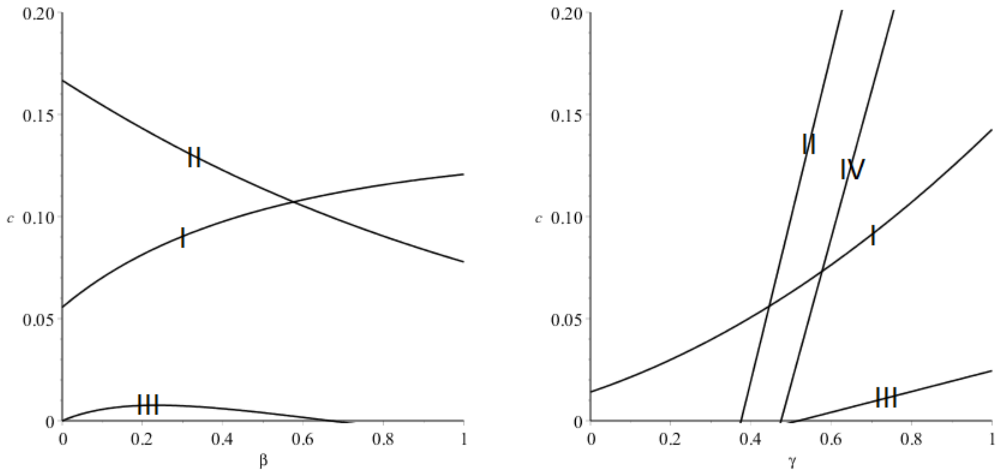

Figure 1: The graphs illustrate the combination of parameters that satisfy the conditions for the equilibrium strategy where the L type agent exerts effort in both working periods as a young worker and manager. The graph on the left illustrates the combinations of β and c given that $\gamma = \frac{2}{3}$. The graph on the right illustrates the combinations of γ and c given that $\beta = \frac{1}{4}$.

The graph on the left illustrates the combinations of β and c for the case that $\gamma = \frac{2}{3}$. For all conditions to be satisfied for this equilibrium, parameter values of β and c must lie below all the lines; cost of effort and β must be sufficiently low.

Line (I) is the condition for the L type to exert effort as a manager. The parameter values that satisfy the condition for the L type agent to exert effort as a manager must lie below the line. As the level of bad luck increases in the production process, the decrease in expected payoff is greater when less team effort is exerted. Therefore, a higher cost can be incurred because exerting effort decreases the agent's expected payoff by a smaller amount. Line (II) is the condition for the L type agent to apply for a job with a firm as a worker in the first working period. Again, all parameter values that satisfy the condition is below the curve. As bad luck increases in the production process, the probability of obtaining a reward decreases. Also, wages fall due to lower expectations of producing good quality output. Therefore, the maximum cost of effort must fall as β increases for a L type agent to work for a firm as a worker. Line (III) is the condition for a L type agent to exert effort as a worker in the first working period. Cost of effort and β must be sufficiently low for the L type agent to exert effort. As β increases past a sufficient level, the

equilibrium conditions. Some lines representing actions in the equilibrium strategy lie beyond the maximum values of the axes. Each line represents the parameter values where the agent is indifferent between sticking to the equilibrium strategy and deviating. Therefore, parameter values that satisfy a specific equilibrium strategy must lie on, below, or above a line depending on the equilibrium strategy considered.

expected payoff of producing bad quality output becomes greater than the expected payoff from producing good quality. Therefore, to increase the probability of obtaining the higher expected payoff, no effort is exerted even if effort is costless.

The graph on the right illustrates the combinations of γ and c for the case that $\beta = \frac{1}{4}$. All conditions are satisfied for the equilibrium strategy below the lines; the cost of effort must be sufficiently low and γ must be sufficiently high.

Line (I) is the condition for a L type agent to exert effort as a manager. The parameter values that satisfy the agent's condition to exert effort fall beneath the line. As the proportion of L type agents increase (as γ increases), a manager is more likely to be paired with an old, L type worker that does not exert effort lowering his chances of receiving a reward. As γ increases, the decrease in the probability of obtaining a reward is greater when less total team effort is exerted (similar to increasing β). Therefore, a higher cost can be incurred because exerting effort decreases the agent's expected payoff by a smaller amount. Line (II) is the condition where the L type agent accepts a promotion. Parameter values must fall beneath the line for the condition to be satisfied. As more H types are born into each generation, γ decreases, the expected payoff of working alone (reservation payoff) increases making working alone more attractive below a sufficient level of γ. Line (III) is the condition where the L type agent exerts effort as a young worker. Parameter values must fall beneath the line for the condition to be satisfied. As γ decreases, the reservation payoff increases which increases the expected payoff of producing bad quality output. Similar to β increasing, when γ decreases beyond a sufficient value, the agent prefers to not exert effort to increase his probability of obtaining the higher expected payoff. Line (IV) is the condition where the L type agent chooses to apply for a job with a firm as a worker in the first working period. When γ is sufficiently low, the reservation payoff becomes more attractive than working for a firm.

Figure 2 illustrates the parameter values that satisfy the conditions for the equilibrium strategy where L type agents choose to exert effort only as a young worker while working for a firm. The graph on the left illustrates the combinations of β and c for the case that $\gamma = \frac{2}{3}$. The parameter values that satisfy all the conditions for this equilibrium strategy for the L type agent falls between the downward sloping line and the β-axis.

Line (I) is the condition where the L type agent chooses to exert effort as a young worker. As β increases, the probability of obtaining a reward decreases. Therefore, cost of effort must decrease

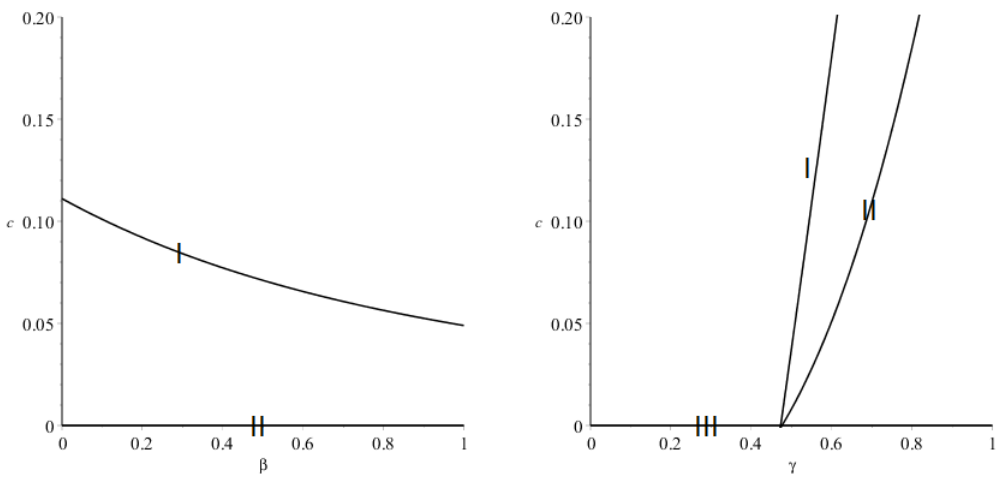

Figure 2: The graphs illustrate the combination of parameters that satisfy the conditions for the equilibrium strategy where the L type agent exerts effort only as a young worker. The graph on the left illustrates the combinations of β and c given that $\gamma = \frac{2}{3}$. The graph on the right illustrates the combinations of γ and c given that $\beta = \frac{1}{4}$.

to maintain the equilibrium strategy. Parameter values that satisfy this equilibrium condition falls below the curve. Line (II), which lies on top of the β-axis, is the condition where the L type agent exerts effort as a manager. In this equilibrium strategy, the manager does not exert effort. Therefore, $R = 0$ because there is no need for an incentive to exert effort. Any c greater than zero, the agent chooses not to exert effort.

The graph on the right illustrates the combination of parameters for γ and c that satisfy the conditions for this equilibrium strategy if $\beta = \frac{1}{4}$. The combination of parameters must fall below the two curves and above the γ-axis.

Line (I) is the condition where the L type agent chooses apply for a job as a worker in the first working period. As γ decreases, the expected payoff from producing good quality output falls and the reservation payoff increases. Below a sufficient level, the outside option of the agent is more attractive. Parameter values to satisfy this condition lie below the curve. Line (II) is the condition where the L type agent chooses to exert effort as a young worker. Decreasing γ increases the expected payoff from producing bad quality output above the expected payoff of producing good quality output. To increase the probability of producing bad quality output and obtain the higher expected payoff, the agent chooses to not exert effort. Line (III), lies on top of the γ-axis, is the condition where the L type agent chooses to exert effort as a manager. Since $R = 0$, if c is greater than zero, he chooses not to exert effort.

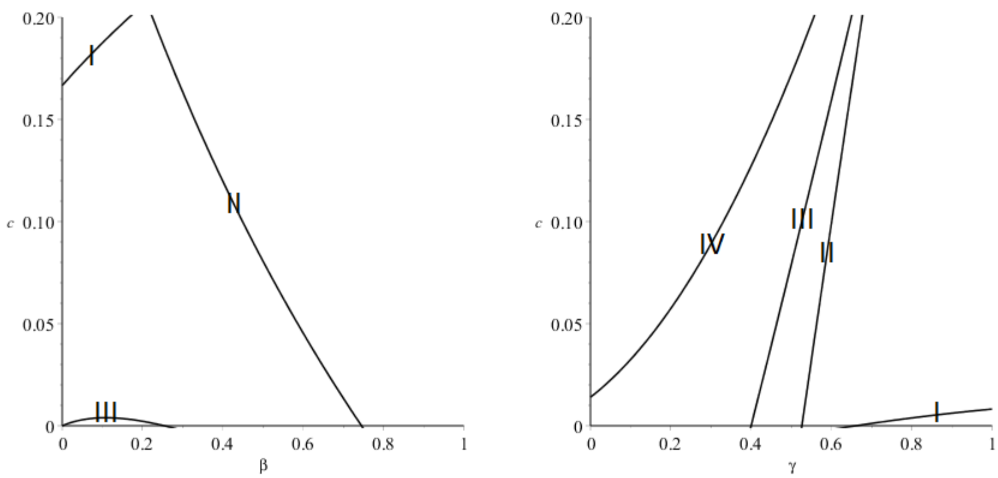

Figure 3: The graphs illustrate the combination of parameters that satisfy the conditions for the equilibrium strategy where the L type agent exerts effort only as a manager. The graph on the left illustrates the combinations of β and c given that $\gamma = \frac{2}{3}$. The graph on the right illustrates the combinations of γ and c given that $\beta = \frac{1}{4}$.

Figure 3 illustrates the combination of parameters that satisfy the conditions for the equilibrium strategy where L type agents choose to exert effort only as managers while working for a firm. The graph on the left illustrates the combination of parameter values of β and c for $\gamma = \frac{2}{3}$ that satisfy all the conditions. The parameter values that satisfy all the conditions for this equilibrium strategy fall below the two top curves and above the bottom curve. The value of β must be sufficiently low and the value of c must be sufficiently low but above a certain value.

Line (I) is the condition where the L type agent chooses to accept a promotion to manager. As β increases, the expected payoff from working alone decreases. From equation (4) for the managerial wage, the expected payoff of a manager is a constant. Therefore, if the expected payoff from accepting a promotion is constant and the expected payoff of rejecting is decreasing as β increases, the maximum value of c can increase and the agent sticks to the equilibrium strategy. Line (II) is the condition for the L type agent to apply for a job as a worker in the first working period. As β increases, the wages and the probability of receiving rewards fall. Therefore, cost of effort must fall for the agent to work for the firm. If β is sufficiently high, even if $c = 0$, the outside option provides a higher expected payoff. Line (III) is the condition where the agent chooses to exert effort as a young worker. Similar to the curves from the other equilibriums, the increase in β increases the expected payoff from producing bad quality output beyond the expected payoff of producing good quality output. Exerting no effort increases the probability of obtaining a higher expected payoff.

The graph on the right illustrates the combination of parameter values of γ and c that satisfy all the conditions for this equilibrium strategy where $\beta = \frac{1}{4}$. The combination of parameter values for γ and c that satisfy all conditions for this equilibrium strategy falls in the area on the right above the small triangular area on the bottom right. The values of γ must be sufficiently high and c must be high above a certain value.

Line (I) is the condition where the L type agent chooses to exert effort as a young worker. Parameter values must be above the curve for the agent to stick to the equilibrium strategy. As γ increases, the expected payoff from producing bad quality output increases making not exerting effort more attractive. Line (II) is the condition for a L type agent to apply for a job as a worker in the first working period. Line (III) is the condition for a L type agent to accept the promotion to manager. Parameter values that satisfy the conditions for an agent to always work for a firm must fall below both curves. As γ decreases, the reservation payoff increases making working alone more attractive than working for a firm. Line (IV) is the condition for a L type agent to exert effort as a manager. As γ increases, the probability of a manager being paired with an agent that does not exert effort increases. By exerting effort, his expected payoff increases greatly allowing for higher levels of cost ot be incurred.

The final equilibrium strategy to consider is L type agents choosing to never exert effort. This equilibrium is shown to not exist because L type agents always have a beneficial deviation to not applying for a position as a worker in the first working period for all values of γ. Condition (17) is violated for all values of γ. The equilibrium strategy of the L type agent is to not exert effort while working for a firm. Therefore, the expected probability of producing good quality output in a team as a worker is low. If the probability of producing good quality output is low, the likelihood of being promoted to manager is low which lowers an agent's expectations of being hired, α falls. Also, wages reflect the beliefs of the firm and since L types are expected to never exert effort, wages are low. The low expected probability of obtaining a reward plus the low wages makes the reservation payoff of producing alone more profitable. Therefore, there are only three possible equilibriums; the L type agent chooses to exert effort as a young worker and manager, only as a young worker, or only as a manager.

Overlaying the graphs from Figures 1 to 3 illustrates the parameter values that satisfy conditions of each equilibrium strategy. From combining the graphs from Figures 1 to 3, there exists

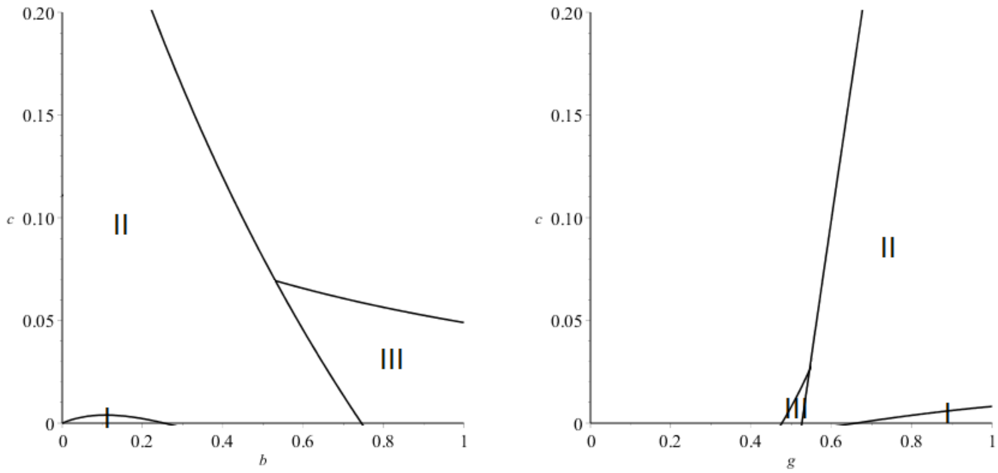

Figure 4: The graphs illustrate the combination of parameters that satisfy the conditions for each equilibrium strategy for the L type agent. The graph on the left illustrates the combinations of β and c given that $\gamma = \frac{2}{3}$. The graph on the right illustrates the combinations of γ and c given that $\beta = \frac{1}{4}$.

areas of multiple equilibria. These are areas with parameter values that satisfy conditions from multiple equilibria. For instance, some parameter values of β and c that satisfy sufficient equilibrium conditions for a L type agent to exert effort only as a young worker also satisfy the conditions to only exert effort as a manager. To determine the dominant equilibrium in this overlapping area, the expected lifetime payoff of the L type worker is compared. The dominant equilibrium is the one that provides the L type agent the higher lifetime expected payoff. Figure 5 in the Appendix shows that, for all values of β and γ, the equilibrium strategy where the L type agent only exerts effort as a manager provides the highest lifetime expected payoff, followed by exerting effort as a young worker and manager, and followed lastly by only exerting effort as a young worker.[28]

Applying the criteria to determine the dominant equilibrium, the resulting graphs are illustrated in Figure 4. The graph on the left highlights the parameter values of β and c that satisfy the conditions for each equilibrium strategy of the L type agent ($\gamma = \frac{2}{3}$). When cost of effort and β are sufficiently low, effort is exerted in all periods. This is illustrated by area (I). As β and c increase,

[28]The reason that only exerting effort as a manager provides the highest lifetime expected payoff lies in the wage for old workers. Exerting effort in all periods incurs the highest cost. Therefore, potentially, it could not be the most profitable. Between exerting effort as a young worker or manager, the probability of success in a single team is the same because there is only one person exerting effort. Therefore, other variables such as α and probability of obtaining a reward as a worker is relatively the same. If the equilibrium strategy was to only exert effort as a young worker, there exist more teams that will not produce good quality output because there are teams that exert no effort (i.e., managers with old workers). Therefore, the old worker wage is much lower compared to the equilibrium strategy where agents only exert effort as managers.

L type agents choose to only exert effort as managers. This is represented by area (II). When β gets sufficiently high, area (III), while costs remain sufficiently low, the agent only exerts effort as a young worker.[29]

When β is equal to zero, there is no uncertainty in the production process. If one member of the team exerts effort, good quality output is produced with certainty. Therefore, at $\beta = 0$, the equilibrium strategy for the L type agent is to only exert effort as a manager as it provides a higher lifetime expected payoff. As β increases and uncertainty is introduced into the production process, the existence of an equilibria where more effort is exerted appears. If cost of effort is sufficiently low, increasing β, uncertainty in the production process, a L type agent chooses to exert more effort by also exerting effort as a young worker. Uncertainty in the production process acts as a potential incentive for L type agents to exert effort under the right conditions.

The graph on the right of Figure 4 highlights the equilibrium strategies, after applying the criteria to determine the dominant equilibrium, of the L type agent given parameter values for γ and c where $\beta = \frac{1}{4}$. Area (I), where γ is sufficiently high and c is sufficiently low, the parameter values satisfy the conditions for the equilibrium strategy of the L type agent exerting effort as a young worker and a manager. The area above, area (II), contains the combination of parameter values that satisfy the condition for the equilibrium strategy where the L type agent exerts effort only as a manager. Area (III), below the line are the parameter values of γ and c that satisfy the conditions for the equilibrium strategy where the L type agent exerts effort only as a young worker.

As a L type agent is more certain to be paired with a H type agent that always exerts effort, a L type agent deviates from his equilibrium strategy of exerting effort (or working for a firm). If γ increases and the L type agent is more likely paired with someone that might not exert effort, introducing uncertainty into the production process by way of less effort, equilibrium strategies with more effort appear. Again, uncertainty in the production process introduced by the increase in the probability of being paired with an agent that might not exert effort acts as an incentive for L type agents to exert effort. Similar to increasing β, uncertainty acts as a potential incentive for effort under the right conditions.[30]

[29]The area above all lines might contain parameter values that satisfy conditions for mixed strategy equilibria. The idea of mixed strategy equilibria may have non-symmetric actions of agents of the same type. For example, a proportion of L type agents may exert effort while the other does not. This is not solved in this article but might be considered in future iteration.

[30]Figure 6 to 8 in the Appendix further illustrates the impact of the parameter changes on the parameter areas of

5 Conclusion

This article illustrates the possible impact of uncertainty and asymmetric information in regards to the agent's teammate and production process on teamwork. It included several important features of professional work environments such as promotions as an incentive, a pyramid organizational structure, team rewards, and heterogeneous agents in efficiency to better model firms seen in reality.

The model suggests, under the assumptions and framework of the model, agents tend to shirk on their teammates when cost of effort is sufficiently high. This is a obvious result. As cost of effort rises, an agent chooses to exert less effort. Since effort is binary, for an agent to exert less effort, he must choose to exert effort only in one of two periods. When the bad luck in the production process is sufficiently low or the proportion of low efficiency types are high, agents free ride as young workers to get a promotion. As bad luck increases or the proportion of low efficiency types decrease sufficiently, agents free ride as managers. The first objective behind this article is to provide firms and organization with an explanation of the teamwork that exists in their workplace. If they observe free riding among teams of workers, it could be due to the flaws of the production process or proportion of types in the workforce.

Another result is the potential positive impact of uncertainty in the production process on teamwork. When there is certainty in the production process and effort from agents provide guaranteed outcomes, agents tend to shirk. If a little uncertainty in the production process is introduced, in the form of bad luck or uncertainty in type of teammates, agents begin to worry about the impact their teammate might create to his own probability of obtaining rewards. Therefore, agents are incentivized to exert effort to prevent major negative effects under the right conditions.

each equilibrium strategy.

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6 Appendix

6.1 Expected Probabilities

The equation for $p^W(e_W; e_M^H, e_M^L)$ is,

$$p^W(e_W; e_M^H, e_M^L) = p(H^M)\left(\frac{e_W + e_M^H}{e_J + e_M^H + \beta}\right) + p(L^M)\left(\frac{e_J + e_M^L}{e_J + e_M^L + \beta}\right) \tag{18}$$

The equation for $p^M(e_M; e_W^H, e_W^L)$ is,

$$\begin{aligned}
p^M(e_W^H, e_W^L, e_M) = p(H^W)^2 & \left[\left(\frac{e_W^H + e_M}{e_W^H + e_M + \beta}\right)^2 + (2)\left(\frac{e_W^H + e_M}{e_W^H + e_M + \beta}\right)\left(\frac{\beta}{e_W^H + e_M + \beta}\right)\right] \\
& + (2)(p(H^W))(p(L^W))\left[\left(\frac{e_W^H + e_M}{e_W^H + e_M + \beta}\right)\left(\left(\frac{N}{N+F+U}\right)\left(\frac{e_W^L + e_M}{e_W^L + e_M + \beta}\right)\right.\right. \\
& + \left.\left(\frac{F+U}{N+F+U}\right)\left(\frac{e_M}{e_M + \beta}\right)\right) + \left(\frac{\beta}{e_W^H + e_M + \beta}\right)\left(\left(\frac{N}{N+F+U}\right)\left(\frac{e_W^L + e_M}{e_W^L + e_M + \beta}\right)\right. \\
& + \left.\left(\frac{F+U}{N+F+U}\right)\left(\frac{e_M}{e_M + \beta}\right)\right) + \left(\frac{e_W^H + e_M}{e_W^H + e_M + \beta}\right)\left(\left(\frac{N}{N+F+U}\right)\left(\frac{\beta}{e_W^L + e_M + \beta}\right)\right. \\
& + \left.\left.\left(\frac{F+U}{N+F+U}\right)\left(\frac{\beta}{e_M + \beta}\right)\right)\right] \\
& + p(L^W)^2\left[\left(\left(\frac{N}{N+F+U}\right)\left(\frac{e_W^L + e_M}{e_W^L + e_M + \beta}\right) + \left(\frac{F+U}{N+F+U}\right)\left(\frac{e_M}{e_M + \beta}\right)\right)^2\right. \\
& + (2)\left(\left(\frac{N}{N+F+U}\right)\left(\frac{e_W^L + e_M}{e_W^L + e_M + \beta}\right) + \left(\frac{F+U}{N+F+U}\right)\left(\frac{e_M}{e_M + \beta}\right)\right) \\
& \left.\left(\left(\frac{N}{N+F+U}\right)\left(\frac{\beta}{e_W^L + e_M + \beta}\right) + \left(\frac{F+U}{N+F+U}\right)\left(\frac{\beta}{e_M + \beta}\right)\right)\right]
\end{aligned} \tag{19}$$

The equation for $p^{M2}(e_W^H, e_W^L, e_M)$ is,

$$\begin{aligned}
p^{M2}(e_W^H, e_W^L, e_M) = p(H^W)^2 & \left[\left(\frac{e_W^H + e_M}{e_W^H + e_M + \beta}\right)^2\right] \\
& + (2)(p(H^W))(p(L^W))\left[\left(\frac{e_W^H + e_M}{e_W^H + e_M + \beta}\right)\left(\left(\frac{N}{N+F+U}\right)\left(\frac{e_W^L + e_M}{e_W^L + e_M + \beta}\right)\right.\right. \\
& + \left.\left.\left(\frac{F+U}{N+F+U}\right)\left(\frac{e_M}{e_M + \beta}\right)\right)\right] \\
& + p(L^W)^2\left[\left(\left(\frac{N}{N+F+U}\right)\left(\frac{e_W^L + e_M}{e_W^L + e_M + \beta}\right) + \left(\frac{F+U}{N+F+U}\right)\left(\frac{e_M}{e_M + \beta}\right)\right)^2\right]
\end{aligned} \tag{20}$$

The equation for $p^{M1}(e_W^H, e_W^L, e_M)$ is,

$$p^{M1}(e_W^H, e_W^L, e_M) = p^M(e_W^H, e_W^L, e_M) - p^{M2}(e_W^H, e_W^L, e_M) \tag{21}$$

6.2 H Type Agent Equilibrium Strategy Conditions

The H type agent's conditions for the equilibrium strategy where L type agents choose to exert effort as a young worker and manager are inequalities (11*) to (13*). The LHS of inequalities (11*) to (13*) are greater than or equal to zero for all values of β. Substituting the values for $\gamma = \frac{2}{3}$ into the equations, the LHS of the inequalities (11*) to (13*) become, respectively,

$$\frac{16 + 56\beta + 76\beta^2 + 50\beta^3 + 16\beta^4 + 2\beta^5}{4(\beta+2)^3(\beta+1)^2} \geq 0$$
$$\frac{3\beta + 4}{3(\beta+2)(\beta+1)} \geq 0$$
$$\frac{5.3 + 21.3\beta + 32\beta^2 + 21.3\beta^3 + 5\beta^4 - 0.33\beta^6 - 0.66\beta^5}{(\beta+1)^3(\beta+2)^4} \geq 0$$

Referring back to inequalities (11*) to (12*), this means for all values of β, the H type agent accepts a promotion and re-applies for a job as a worker in the second working period. The last one is downward sloping in β. The equation is equal to zero when $\beta \approx 5$. A H type agent does not apply for a worker position in a firm in the first working period if $\beta \gtrsim 5$, which is well outside the value of β for the equilibrium strategy of L type agents ($\beta \lesssim 0.7$).

Similarly, the equation on the LHS for the H type agent's conditions (11*) to (13*) for $\beta = \frac{1}{4}$ are, respectively,

$$0.5 \geq 0$$
$$\frac{4 + 32g}{45} \geq 0$$
$$-0.64 + 1.35g \geq 0$$

The first two are positive for all values of $0 < \gamma < 1$. The H type agent accepts a promotion and re-applies for a job as a worker in the second working period. The last equation is greater than zero if $\gamma \gtrsim 0.47$ which is consistent with the conditions for L type agents ($g \gtrsim 0.52$).

For the equilibrium strategy where L type agents choose to exert effort only as young workers, conditions for H type agents, (11*) to (13*), when $\gamma = \frac{2}{3}$ are, respectively,

$$\frac{162 + 918\beta + 2254.5\beta^2 + 3135\beta^3 + 2700.5\beta^4 + 1476\beta^5 + 500\beta^6 + 96\beta^7 + 8\beta^8}{(2\beta+3)^2(\beta+1)^2(\beta+2)^2(4\beta^2+12\beta+9)} \geq 0$$

$$\frac{3\beta+4}{9(\beta+2)(\beta+1)} \geq 0$$

$$\frac{498\beta^7 + 1812.4\beta^6 + 4161.8\beta^5 + 6261.5\beta^4 + 6177\beta^3 + 3854\beta^2 + 216.0 + 78.2\beta^8 + 5.3\beta^9 + 1380\beta}{(2\beta+3)^2\left(4\beta^2+12\beta+9\right)(1+\beta)^3(2+\beta)^3} \geq 0$$

For all values of β, the conditions are true. The H type agent always chooses to work for the firm for all values of β.

For the equilibrium strategy when L type agents choose to exert effort only as young workers when $\beta = \frac{1}{4}$, the conditions for H type agents(11*) to (12*), are, respectively,

$$0.5 \geq 0$$

$$\frac{4(1-\gamma)(8\gamma+1)}{45} \geq 0$$

For all values of $0 < \gamma < 1$, the equation on the LHS are positive. The final condition for H type agents where he chooses to apply for a position as a worker in his first working period is also satisfied for all values of $\gamma \gtrsim 0.473$. The equation is very large and is not presented here. For the conditions of a L type agent to hold for the equilibrium strategy where he exerts effort only as a young worker, $\gamma \gtrsim 0.476$.

For the equilibrium strategy where L type agents choose to only exert effort as a manager, conditions for H type agents, (11*) to (13*), when $\gamma = \frac{2}{3}$ are, respectively,

$$\frac{162 + 1476\beta^5 + 2700.5\beta^4 + 3135\beta^3 + 2254.5\beta^2 + 500\beta^6 + 8\beta^8 + 96\beta^7 + 918\beta}{\left(4\beta^2+12\beta+9\right)^2(1+\beta)^2(2+\beta)^2} \geq 0$$

$$\frac{3\beta+4}{3(\beta+2)(\beta+1)} \geq 0$$

$$\frac{648 - 11395.3\beta^5 - 9324\beta^4 - 2040\beta^3 + 3006\beta^2 - 53.3\beta^9 - 7482.7\beta^6 - 597.3\beta^8 - 2853.3\beta^7 + 2592\beta}{\left(4\beta^2+12\beta+9\right)^2(2\beta+3)(1+\beta)^2(2+\beta)^3} \geq 0$$

The first two conditions state illustrate that they are true for all values of β. This means that a H type agent chooses to accept a promotion and re-apply as a worker in the second working period for all values of β. The last condition is true for all values of $\beta \lesssim 0.8$. Therefore, for all values of β less than approximately 0.8, H type agents choose to apply for a position in a firm as a worker in their first working period. Any $\beta \gtrsim 0.75$, the conditions for this equilibrium strategy do not hold for L type agents.

For the equilibrium strategy when L type agents choose to exert effort only as managers when $\beta = \frac{1}{4}$, the conditions for H type agents(11*) to (12*), are, respectively,

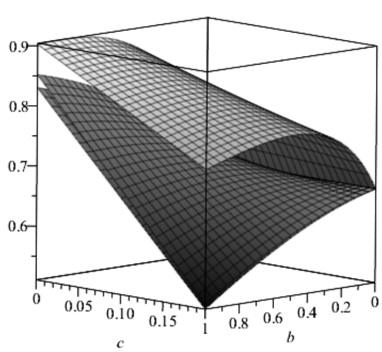
$$\frac{0.5\gamma^2 - 10\gamma + 50^2}{\gamma - 10} \geq 0$$

$$\frac{32\gamma + 4}{45} \geq 0$$

$$\frac{550.5\gamma^2 - 1483.5\gamma + 2.7\gamma^4 - 68.2\gamma^3 + 642}{(-10 + \gamma)(\gamma^2 - 20\gamma + 100)} \geq 0$$

The first two conditions are true for all values of $0 < \gamma < 1$. H type agents choose to accept a promotion or re-apply for a worker position in a firm in their second working period. The last condition is true for all values of $\gamma \gtrsim 0.53$. The minimum parameter value of γ for all L type conditions to hold in this equilibrium is $\gamma \gtrsim 0.66$.

6.3 Lifetime Expected Payoffs of L Type Agent by Equilibrium Strategy



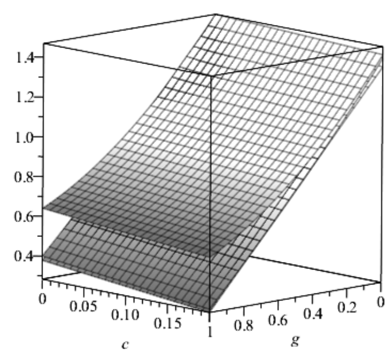


Figure 5: The graph on the left illustrates the expected lifetime payoff of a L type agent given parameter values of β and c with $\gamma = \frac{2}{3}$. The highest curve is the expected payoff of the equilibrium strategy of only exerting effort as a manager. The second highest is the curve for the equilibrium strategy of exerting effort as a young worker and manager. The lowest curve is for the equilibrium strategy of exerting effort only as a young worker. The graph on the right is for the parameter values of γ and c with $\beta = \frac{1}{4}$. The ordering of the curves are the same as the graph on the right.

6.4 Equilibrium Strategy Area

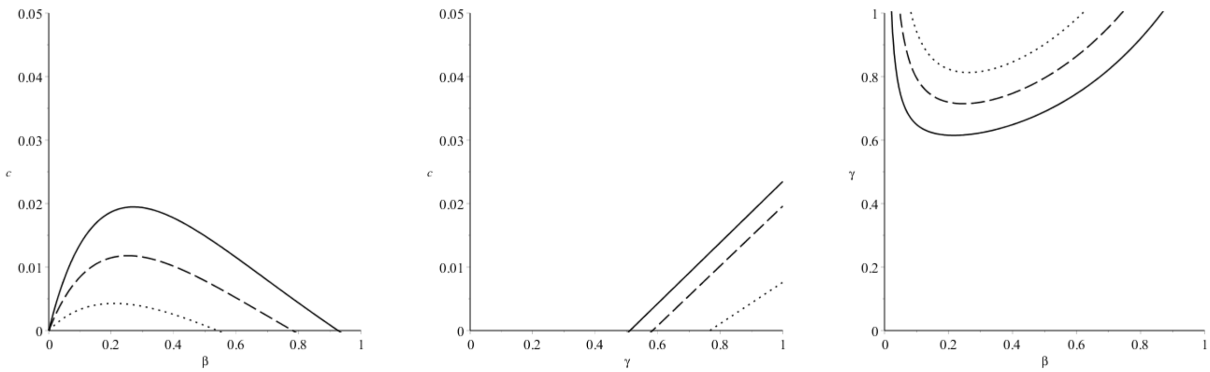


Figure 6: The graphs illustrate the parameter values that satisfy the conditions for the equilibrium strategy where L type agents exert effort as a young worker and manager. The graph on the left illustrates the parameter values of β and c. The solid line is where $\gamma = 0.6$, dash line is $\gamma = 0.75$, and dot line is $\gamma = 0.9$. The equilibrium area shrinks as γ increases. The graph in the middle illustrates the parameter values of γ and c. The solid line is where $\beta = 0.2$, dash line is $\beta = 0.5$, and dot line is $\beta = 0.8$. The equilibrium area shrinks as β increases. The graph on the right illustrates the parameter values of β and γ. The sold line is where $c = 0.005$, dash line is $c = 0.01$, and dot line is $c = 0.015$. As cost of effort increases, the equilibrium area shrinks. The condition shown here is for the L type agent to exert effort as a young worker. If this condition is satisfied for the equilibrium strategy, all other conditions are satisfied. As γ increases, a manager is more likely to be paired with a old worker that does not exert effort. This decreases the value of a reward making exerting effort less attractive unless cost of effort or β decreases. If β increases, this decreases the likelihood of obtaining a reward. To stick to the equilibrium strategy, cost of effort must fall or the reservation payoff must be less attractive (γ increases) for the agent to exert effort. If cost increases, the expected payoff from exerting effort decreases. If β falls, obtaining a reward is more likely. If γ increases, the reservation payoff is less attractive. Both incentivizes the agent to exert effort.

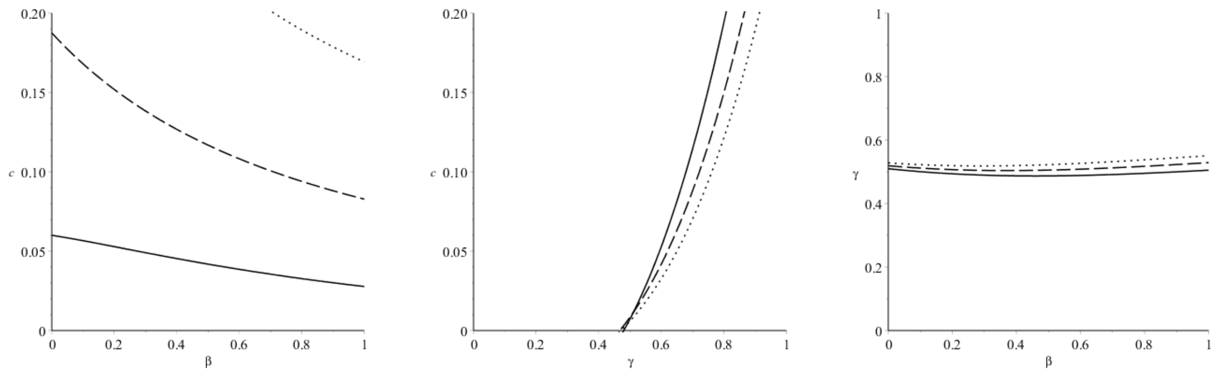


Figure 7: The graphs illustrate the parameter values that satisfy the conditions for the equilibrium strategy where L type agents exert effort as a young workers. The graph on the left illustrates the parameter values of β and c. The solid line is where $\gamma = 0.6$, dash line is $\gamma = 0.75$, and dot line is $\gamma = 0.9$. The equilibrium area expands as γ increases. The graph in the middle illustrates the parameter values of γ and c. The solid line is where $\beta = 0.2$, dash line is $\beta = 0.5$, and dot line is $\beta = 0.8$. The equilibrium area shrinks as β increases. The graph on the right illustrates the parameter values of β and γ. The sold line is where $c = 0.005$, dash line is $c = 0.01$, and dot line is $c = 0.015$. As cost of effort increases, the equilibrium area shrinks. Similar to the graphs previously, the condition illustrated is the one where an L type agent chooses to exert effort as a young worker. Increasing γ greatly decreases the value of the reservation payoff and an old worker's wage. Therefore, a worker can incur a higher cost for exerting effort. The explanations for the other graphs are the same as the previous graphs.

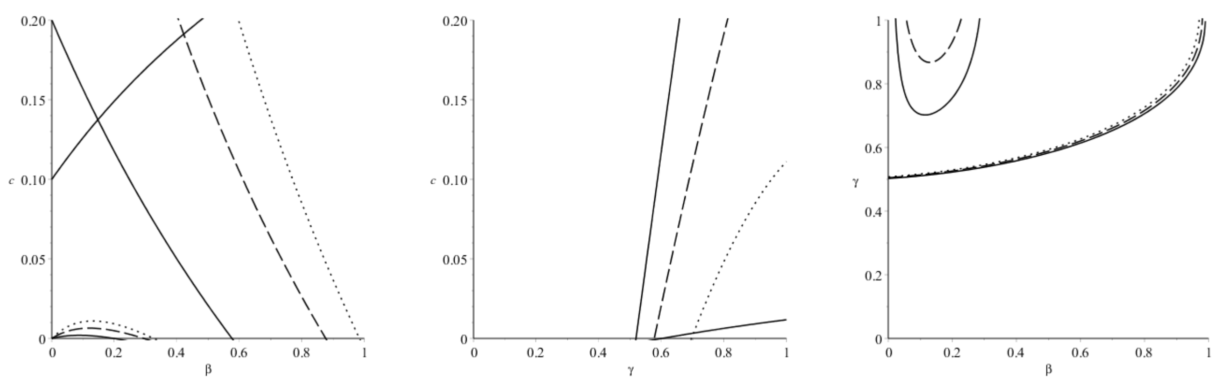


Figure 8: The graphs illustrate the parameter values that satisfy the conditions for the equilibrium strategy where L type agents exert effort as a manager. The graph on the left illustrates the parameter values of β and c. The solid lines are where $\gamma = 0.6$, dash line is $\gamma = 0.75$, and dot line is $\gamma = 0.9$. The equilibrium area expands as γ increases. The graph in the middle illustrates the parameter values of γ and c. The solid line is where $\beta = 0.2$, dash line is $\beta = 0.5$, and dot line is $\beta = 0.8$. The equilibrium area shrinks as β increases. The graph on the right illustrates the parameter values of β and γ. The sold line is where $c = 0.005$, dash line is $c = 0.01$, and dot line is $c = 0.015$. As cost of effort increases, the equilibrium area shrinks. As γ increases, the reservation payoff greatly decreases and the likelihood of being paired with agents that do not exert effort increases. This greatly decreases the expected payoff of not exerting effort. Therefore, the agent can incur a higher cost of effort or larger β. The explanation for the other graphs are the same as previous graphs.