Growth and Debt: An Endogenous Smooth Coefficient Approach^{*}

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Abstract

Is high public debt detrimental to all countries? Is the level of public debt primary reason for this concern? We employ a smooth coefficient approach that allows democracy to characterize the long-run relationship between public debt as well as other conditioning variables and economic growth, and parameter heterogeneity in the unknown functional form. We find some evidence of parameter heterogeneity in the growth effect of public debt with respect to institutional quality of countries. Our results are consistent with the previous literature that find significant negative effect of public debt on growth for the countries below a particular democracy level. However, we also find surprisingly strong evidence of adverse effect of public debt on growth for countries with high institutional quality.

Keywords: functional coefficients; local linear regression; nonparametric 2SLS estimator; series estimator; Solow economic growth convergence model

JEL classification codes: C14; C21; O47

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1 Introduction

In the aftermath of the recent global financial crisis, government debt has increased substantially across the world. For advanced economies, public debt-to-GDP ratio has risen on average from about 66% in 2007 to 105% by the end of 2015. Particularly, Greece, Ireland, Japan, Portugal, Spain, and the United Kingdom, comparable to others, have experienced a rapid increase in public debt ratio between the years 2008 and 2012. A growing concern behind these facts is that countries may not achieve debt sustainability implying higher vulnerability to economic and financial crisis (Cecchetti, Mohanty, and Zampolli, 2010). In fact, over the last two centuries there are twenty financial crisis followed by debt build-ups periods, which lasted more than a decade and are associated with lower growth than during other periods (Reinhart, Reinhart, and Rogoff, 2012). Therefore, a relevant policy question is centered on the long-term growth effects of high public debt.

The relationship between public debt and economic growth is still unresolved in both theoretical and empirical literature. Theoretically, the conventional view of public debt is that fiscal deficits in the short-run can have a positive effect on economic growth through stimulating aggregate demand and output, whereas having a potential crowding out effect on private investment in the long run (Elmendorf and Mankiw, 1999). On the other side, a large number of economic growth research papers find some evidence of nonlinearity in the effect of public debt on growth, particularly focusing on threshold levels. The idea is to detect a debt level beyond which economic growth is adversely affected implying a concave (inverted-U shape) relationship between debt and growth. Using a basic nonparametric technique, i.e., a histogram, to investigate correlation between public debt and growth, Reinhart and Rogoff (2010) find a threshold level of 90% for the 20 advanced countries over the period 1945-2009. Their findings are striking in the sense that real mean GDP growth decreases substantially (at about 4%) when public debt is beyond the 90% threshold as compared to other public debt-to-GDP ratios. Moreover, the debt-growth link disappears for the public debt ratios below 90% threshold; see Herndon, Ash, and Pollin (2014) for a criticism of Reinhart and Rogoff (2010).

In the empirical growth literature, an extensive amount of studies has tried to examine the sensitivity of Reinhart and Rogoff's 90% threshold level to model specification, alternative sets of included/excluded variables, and different data series. Table 1 in the appendix provides a summary of recent studies aimed at unveiling the nonlinear relationship between government debt and economic growth. An important observation gleaned from this table is that there is no common finding for the threshold level, except for a small number of research papers, which find a turning point for a public debt-to-GDP ratio at around 90%. As one study in the latter group of papers, Cecchetti, Mohanty, and Zampolli (2011) look at a panel of 18 OECD countries (all from advanced economies) for the period 1980-2006. Using least squares dummy variable and threshold estimation within the context of dynamic fixed-effects panel data model, they find a negative relationship between government debt and growth beyond the 85% threshold

level, after controlling for other determinants of growth including trade openness, inflation rate, and total dependency ratio (related to ageing). Their approach avoids possible feedback effect from economic growth to public debt using five-year averages of growth, so that regressors are predetermined. Their results suggest that on average, a 10 percentage points increase in public debt-to-GDP ratio is predicted to reduce economic growth by 0.13 percentage points per year. Checherita-Westpal and Rother (2012) study 12 euro area economies from 1970-2008 aiming at to investigate nonlinearity in the debt-growth link by using a quadratic equation in debt. To control for endogeneity of public debt variable, the authors use lagged value of debt and average debt of the other countries in the sample. They find a public debt threshold level in between 90%and 100%, beyond which economic growth is negatively affected. Baum, Checherita-Westpal, and Rother (2013) deal with the endogeneity problem arising from dynamic model specification in their study of 12 euro area countries from 1990-2007/2010. They find a threshold level of public debt-to-GDP ratio at 95% for the extended period. In a recent publication, Woo and Kumar (2015) look at 38 advanced and emerging economies from 1970-2008. Using several estimation strategies and subsamples, the authors examine nonlinearity in the debt-growth relationship by fitting the data to the dynamic panel regression model. They also find a 90%threshold level, beyond which public debt has a negative and significant effect on economic growth. In a last study that needs to be emphasized, Panizza and Presbitero (2014) account for the potential endogeneity of public debt using the share of foreign currency debt in total public debt as an instrument. Using the same data set and empirical approach of Cecchetti et al. (2011) as well as performing various robustness checks, they find little evidence on the adverse effect of high public debt on future growth in advanced economies.

Many other studies provide evidence of a threshold level of public debt different than 90 percent of GDP. For example, Caner, Grennes, and Koehler-Geib (2010) look at a cross-section of 101 developed and emerging market economies from 1980-2008. Using threshold estimation, they find a turning point of public debt-to-GDP ratio at 77% for the full sample, while this value is lower, at 64% of GDP, for the subsample of developing countries only, after controlling for initial GDP per capita, trade openness and inflation rate. In the Wright and Grenade (2014) study of 13 Caribbean countries from 1990-2012, the authors find a threshold level of 61% of GDP beyond which debt has a negative effect on economic growth and investment. A few other research papers closely replicate Reinhart and Rogoff's (2010) paper using econometric techniques. For example, Minea and Parent (2012) employ the panel smooth transition regression model of González, Terasvirta, and van Dijk (2015) and find a negative and gradually decreasing effect of public debt on growth below the threshold level of 115%. Their finding does, in fact, support the presence of nonlinearity in the effect of debt on growth for the debt-to-GDP ratio above 90%. On the other hand, they find a positive growth effect of debt for the debt level above 115%. Relatedly, using nonlinear threshold models for the same dataset used in Reinhart and Rogoff (2010), Égert (2015) found limited evidence for a negative nonlinear correlation between public debt and growth. The author's findings suggest that a debt threshold level can be lower than 90% of GDP depending on data coverage (in terms of country coverage and time dimension), model specification, and measure of the public debt. Eberhardt and Presbitero (2013, 2015) provide strong evidence of different nonlinearities in the debt-growth relationship across 118 countries from 1961-2012 by doing comprehensive analysis of dynamic panel time series estimation. They employ common factor framework to uncover possible heterogeneity in the effect of public debt stock on economic growth through taking into account latent factors of growth and public debt, which include a country's debt composition, macroeconomic policies related to past crises, and institutional framework. They find no evidence for the common threshold effect for all countries in their sample.

The main purpose of the above research and analysis is to reveal a nonlinear relationship between public debt and economic growth depending on the public debt level. In other words, these papers try to expose nonlinear growth effect of high public debt levels. However, this point of view ignores potential variables, either omitted from the model or included as a regressor, that may govern the debt-growth relationship. This concern raises an important question: Can negative effect of debt on growth be attributed to high public debt levels? Formally testing for several threshold variables including democracy, trade openness, fertility, life expectancy, and inflation rate, among others, Kourtellos, Stengos and Tan (2013) study 82 countries in a 10-year panel from 1980 to 2009. They employ the structural threshold regression model of Kourtellos, Stengos, and Tan (2016) to account for the endogeneity of both the threshold variable and the regressors. The authors find a strong evidence in favor of heterogeneity in the debt-growth relationship in the sense that the effect of public debt on economic growth depends on the institutional quality of a country. Particularly, they find that countries with low institutional quality experience a negative and significant effect of public debt on economic growth, holding other factors fixed, while public debt has a positive but insignificant effect on growth for countries with high institutional quality. Jalles (2011) investigate the impact of democracy and corruption on the external debt-growth relationship in a panel of 72 developing countries from 1970-2005. Using fixed effects and GMM estimation strategies under various model specifications (linear and quadratic terms in debt-to-GDP ratio), they find a negative growth effect of external debt in countries with higher levels of corruption. These findings are consistent with the new growth theories such as Azariadis and Drazen (1990) suggesting highly nonlinear cross-country growth process.

Institutional differences across countries is perceived as one of the primary factor in crosscountry income gap. In a seminal paper by Acemoglu et al. (2001), the authors document a positive relationship between democracy and per capita GDP after controlling for endogeneity of institution variable from an exogenous source of variation in it (see also Acemoglu et al. (2016) for recent work on the same subject). It is also argued that institution variable is not correctly measured as many institutional measures reflect outcome of dictatorial choices, and therefore should be seen as institutional outcome variables, not predictors of that (see, e.g., Glaeser et al., 2004; Acemoglu et al., 2005). On the other hand, Minier (1998, 2007) examine democracy as a source of heterogeneity in the relationship between growth and its determinants and provide some evidence of an indirect effect of institutions on the link between trade openness and economic growth. Our aim in this paper is, therefore, to examine whether democracy may govern the relationship between public debt and economic growth in our sample. Relatedly, we can gather a few more observations from the past literature on the empirical debt-growth nexus given in Table 1. First, the relationship appears be heterogeneous and complex. Second, there might be other factors that potentially contribute to the marginal impacts of regressors on economic growth rates, which implies that heterogeneity in the debt-growth relationship might be with respect to other variables in the model. Third, there is lack of strong evidence of the negative effect of public debt on economic growth for advanced economies. These limitations of the existing debt-growth literature, coupled with the lack of clear theoretical argument on the debt-growth link (in advanced economies), suggests that a flexible approach may be more appropriate for estimating the effect of debt on growth and letting other factors to characterize this relationship. We, therefore, present an augmented conventional Solow economic growth model with public debt-to-GDP ratio and country-specific parameters, which relax the homogeneity assumption of a standard growth regression. Specifically, we model parameters to be a function of one or more covariates including democracy, fertility, and life expectancy, among others. Our approach is also related to the empirical growth studies that use nonparametric and semiparametric models to model parameter heterogeneity in the cross-country growth process. Examples are Liu and Stengos (1999) and Ketteni et al. (2007) for an additive semiparametric partially linear model, Vaona and Schiavo (2007) for a semiparametric partial linear model, Durlauf et al. (2001), Mamuneas et al. (2006), Kourtellos (2011), and Kumbhakar and Sun (2012) for a varying coefficient model and Henderson et al. (2011) for a nonparametric model.

To ensure that our regression model captures heterogeneous effects of variables, we further assume the parameters to be unknown measurable smooth functions. This assumption enables us to use nonparametric techniques, which essentially let the data decide functional form of each parameters. In addition, the coefficient estimates avoid biasedness by the misspecification of parameter heterogeneity, which is in parametric form in existing debt-growth studies. Furthermore, economic theory does not suggest a functional form for the regression model of debt-growth relationship or even for the parameter heterogeneity in the debt-growth link. Therefore, nonparametric techniques permit unknown functions to be governed by countryspecific characteristics such as country's initial conditions, state of development variables, institutional quality, and macroeconomic policies playing an indirect role in explaining nonlinear relationship between growth and its determinants across countries and time domain.

We use a recently developed smooth coefficient instrumental variable estimator (Delgado, Ozabaci, Sun, and Kumbhakar, 2015) that assumes linearity in the regressors, but allows



Figure 1: Growth and Public Debt, 1980-2014

Log of Public Debt-to-GDP Ratio, 1980-2014

parameters vary nonparametrically with respect to a set of covariates. One advantage of this estimation method is to control for endogeneity of covariates in the functional coefficients.

In terms of our findings, we find strong evidence of heterogeneity in the effect of public debt with respect to institutional quality of countries. Our results support Kourtellos et al. (2013), which suggest an adverse effect of public debt on growth for the countries below a particular institutional quality level. However, our results also show that for countries with a democracy score above a critical level, higher public debt level leads to lower economic growth (all else equal). But, this effect is comparably less strong than for the countries with a lowest democracy score. When we control for the global factors, we find, for the period 2000-2009, an increasing negative, but insignificant, effect of public debt on growth for countries with high institutional quality above a particular level. Our findings are robust to using other measures of institutional quality, using alternative covariates in the functional form, controlling other variables in the regression model, and using different subsamples of countries. Our results from prediction exercises also suggest that our semiparametric model can better describe the underlying process that generated the data. Our paper therefore contributes to the empirical debt-growth literature from the point of view that explains parameter heterogeneity in the cross-country growth process through fundamental determinants of economic growth proposed by new growth theories.

The remainder of this paper is organized as follows. Section 2 describes our empirical methodology. Section 3 describes our data. In Section 4 we present the empirical results of the paper. In Section 5 we present robustness checks. Section 6 concludes.

2 Empirical Methodology

2.1 The augmented Solow growth model

In this section, we provide a brief description of a linear Solow growth model augmented with the debt-to-GDP ratio to investigate the impact of country's debt level on its economic growth rate. This model assumes a common regression across countries as well as constant coefficient estimates for all economic variables, which intuitively explains the average effect of the variables.

$$g_i = X_i^T \beta + u_i = \beta_0 + S_i^T \beta_s + \beta_d debt_i + u_i, i = 1, .., n,$$
(2.1)

where $X_i = [1, S_i^T, debt_i]^T$ is a $(d_s + 2) \times 1$ vector of regressors consists of a constant term, a d_s dimensional vector of standard Solow growth determinants, including $ln(yin_i)$, the logarithm of the i^{th} country's real GDP per worker in the initial year of each 10-year period; $ln(s_i)$, the logarithm of the i^{th} country's average saving rate; $ln(n_i + 0.05)$, the logarithm of the i^{th} country's average saving rate; $ln(n_i + 0.05)$, the logarithm of the i^{th} country's average years of secondary and tertiary schooling for male population over 25 years of age, and $debt_i$ which is defined as the i^{th} country's public debt-to-GDP ratio. Moreover, S_i includes a time trend. u_i is an identically independently distributed error term.

2.2 An endogenous smooth coefficient model

We consider the following semiparametric varying coefficient model of Delgado, Ozabaci, Sun, and Kumbhakar (2015) for the augmented Solow growth model:

$$\begin{cases} g_i = \theta_0(Z_i) + \sum_{j=1}^{d_s} \theta_{sj}(Z_i) S_{ji} + \theta_d(Z_i) debt_i + \epsilon_i \\ Z_i = \mu_Z + a_1(E_{i,1}) + a_2(E_{i,2}) + \dots + a_p(E_{i,p}) + u_i, i = 1, \dots, n, \end{cases}$$
(2.2)

$$(i)E[u_i|\mathbf{E_i}] = 0$$
$$(ii)E[\epsilon_i|\mathbf{E_i}, u_i] = E[\epsilon_i|u_i], i = 1, ..., n,$$

where Z_i is an endogenous variable defined as an additive nonparametric functions of E_{ij} , j = 1, ..., p, where $E_i = [E_{i,1}, E_{i,2}, ..., E_{i,p}] = [S_i^T, debt_i, W_i^T]^T$ is a $p \times 1$ vector of continuous variables including a d_w dimensional vector of instrumental variables, W_i^T . $a_t(\cdot), t = 1, ..., p$, $\theta_0(\cdot), \theta_s(\cdot)$, and $\theta_d(\cdot)$ are all unknown smooth measurable functions and u_i is zero-mean error term.

In Equation (2.2), the object of estimation is structural model that necessitates different identification strategy than standard nonparametric regression, which is used to estimate conditional expectations. Additive separability of Z and conditional mean of ϵ and u given in (i) and (ii) in Equation (2.2) are nonparametric restrictions for identification in this model¹.

¹In another paper (Newey and Powell, 2003) conditional mean of disturbances given instruments are assumed to be zero without imposing an additive structure for the endogeneous variables.

After setting $E[\epsilon_i|u_i] \equiv b(u_i)$ and denoting $v_i \equiv \epsilon_i - b(u_i)$ that satisfies $E[v_i|\mathbf{E}_i, u_i] = 0$, we can rewrite Model (2.2) as

$$g_i = \theta_0(Z_i) + \sum_{j=1}^{d_s} \theta_{sj}(Z_i) S_{ji} + \theta_d(Z_i) debt_i + b(u_i) + v_i, i = 1, ..., n,$$
(2.3)

provided that $b(\cdot)$ is an unknown smooth function. Equation (2.3) consists of two additive components, $\theta_0(Z_i)$ and $b(u_i)$, together with the functional coefficient terms, $\sum_{j=1}^{d_s} \theta_{sj}(Z_i)S_{ji}$ and $\theta_d(Z_i)debt_i$. According to Newey, Powell, and Vella (1999), identification of unknown functions in Equation (2.3) is the same as identification in Equation (2.2), as the additive structure of Equation (2.3) is equivalent to conditional mean restriction (assumption (*ii*)) in Equation (2.2). The sufficient condition for identification of unknown functions in Equation (2.3) is, therefore, assuming no additive functional relationship between Z_i and u_i (see Newey et al. (1999), Theorem 2.1 and 2.2 on page 567-568).

If we assume that Z and all conditioning variables are exogeneous, then the first equation in (2.2) is a pure varying coefficient model that can be consistently estimated using the nonparametric kernel estimator of Li et al. (2002); otherwise, this estimator yields a bias in estimation of unknown functional coefficients. Assuming exogeneity of covariates seems to be strong in the present growth application; we, therefore, allow variables representing Z to be endogeneous. It is this endogeneity assumption that growth regression in this paper is formulated as in structural form of Model (2.2) called as a triangular nonparametric simultaneous equations model.

Nonparametric estimators for regression models that include endogeneity problem have been proposed in the context of varying coefficient models, for example, Das (2005), Cai et al. (2006), and Cai and Li (2008). However, these papers allow for endogeneous variables in the parametric part of a regression. The estimator proposed by Delgado et al. (2015), on the other hand, deals with endogenous variables that appear in the nonparametric part of a smooth coefficient model. This estimator is applicable to the economic studies, where endogeneous variable has a potential interaction effect with the other regressors on response variable. For example, child care use may have a potential indirect effect on students' test scores that can be modeled as in the functional coefficient form that vary with respect to mother's education, age, and experience, among other regressors (see Bernal and Keane (2011) for a parametric estimation and full description of the regressors and Ozabaci, Henderson, and Su (2014) for an additive nonparametric regression estimation).

To circumvent the endogeneity problem, Delgado et al. (2015) use the control function approach in the estimation of structural function of interest. Since u enters Equation (2.3) as a conditioning variable and it is generally unobserved, Delgado et al. (2015), first, calculate \hat{u} from the regression of Z on E_i using second equation of Model (2.2). Then, they estimate $\theta(Z_i)$ and $b(\hat{u})$ via sieve approximation approach by an ordinary least squares method. In the third step, they use a local linear regression method to estimate $\theta(Z_i)$ and $\theta'(Z_i)$. They show that their estimator is oracle efficient in the sense that large sample distribution of the estimator is the same regardless of whether the function $b(\cdot)$ is known. It is also noted that third-step estimator is not affected from the errors in the first two steps of estimation. The estimation procedure is given in detail as follows.

In the first step, Delgado et al. (2015) approximate unknown functions $a_1(\cdot),...,a_p(\cdot)$ by series expansions²

$$a_m^*(e) = \sum_{l=1}^{L_n} \alpha_{ml} \phi_l(e), \qquad (2.4)$$

for m = 1, ..., p, where $\alpha_m = (\alpha_{m1}, \alpha_{m2}, ..., \alpha_{mL_n})^T$ is $L_n \times 1$ vector of unknown coefficients, $\{\phi_j(\cdot)\}_{j=1}^{L_n}$ is a sequence of square integrable orthonormal basis functions over the interval $[0, \infty)$, and L_n denotes the number of basis functions. It is noteworthy that Laguerre polynomial series is used to approximate the unknown functions as it is one of the common choices for series expansions when a function has a domain over $[0, \infty)$ (see, e.g., Assumption 1(ii) in Delgado et al. (2015) and Chen (2007, p.5574) for further details).

The coefficients α_m , m = 1, ..., p in (2.4) can be consistently estimated from the ordinary least squares (or OLS) regression of Z_i on $a_1^*(E_{i,1}), a_2^*(E_{i,2}), ..., a_p^*(E_{i,p})$. Then, the OLS estimator of the unknown function is given by $\hat{a}_m(e) = \sum_{l=1}^{L_n} \hat{\alpha}_{ml} \phi_l(e), m = 1, ..., p$. Fitted values and the residuals from the OLS regression can be calculated as $\hat{Z}_i = \hat{\mu} + \hat{a}_1(E_{i,1}) + \hat{a}_2(E_{i,2}) + ... + \hat{a}_p(E_{i,p})$ and $\hat{\epsilon}_i = Z_i - \hat{Z}_i$ for all i = 1, ..., n, respectively.

In the second step, using series expansions they approximate unknown functions $\theta(z)$ and $b(\hat{\epsilon}_i)$, respectively, by

$$\theta_k^*(z) = \sum_{l=1}^{L_n} \beta_{kl} \phi_l(z), \quad \text{and} \quad b^*(\hat{\epsilon}) = \sum_{l=1}^{L_n} \gamma_l \phi_l(\hat{\epsilon}), \quad (2.5)$$

where $\beta_k = (\beta_{k1}, \beta_{k2}, ..., \beta_{kL_n})^T$ for $k = 0, ..., d_s + 1$, and $\gamma = (\gamma_1, \gamma_2, ..., \gamma_{L_n})^T$ are all $L_n \times 1$ vectors of unknown coefficients. Model (2.3) can be, now, approximated by substituting equalities in (2.5) for $\theta_k(z)$, $k = 0, ..., d_s + 1$, and $b(\hat{\epsilon})$ in Model (2.3).

$$g_i \approx \sum_{k=0}^{d_s+1} \sum_{l=1}^{L_n} \beta_{kl} \phi_l(z) X_{ki} + \sum_{l=1}^{L_n} \gamma_l \phi_l(\hat{\epsilon}_i) + v_i, i = 1, .., n,$$
(2.6)

where residuals $\hat{\epsilon}_i$ is calculated from the first step. The least squares problem is, then, defined as follows:

$$[\hat{\beta}^{T}, \hat{\gamma}^{T}]^{T} = \arg\min_{(\beta, \gamma)} \sum_{i=1}^{n} \left\{ g_{i} - \sum_{k=0}^{d_{s}+1} \sum_{l=1}^{L_{n}} \beta_{kl} \phi_{l}(z) X_{ki} + \sum_{l=1}^{L_{n}} \gamma_{l} \phi_{l}(\hat{\epsilon}_{i}) \right\}^{2}.$$
 (2.7)

In the third step, Delgado et al. (2015) use the local linear regression approach to estimate the functional coefficients, $\theta(\cdot)$, and its first-order derivatives, $\theta'(\cdot)$. Following Delgado et al.

 $^{^{2}}$ The authors use B-spline smoothing in the first two steps assuming domain of the basis functions over the closed interval.

(2015), we assume that unknown function, $\theta(Z)$ is continuously differentiable up to second order so that we can apply a first order Taylor series approximation of $\theta(Z)$ around a given point z, technically by $\theta(Z) \approx \theta(z) + \theta'(z)(Z-z)$. We, further, assume that $K(\cdot)$ to be a kernel weight function assigning more weights to the observations closer to point z, satisfying: (i) $\int K(a)da = 1$, (ii) K(a)=K(-a), and (iii) $\int a^2 K(a)da > 0$. In case of higher dimensional covariate vector, Z, that includes continuous and discrete covariates, the kernel function is the product kernel, $K = WL(Z^d, z^d, \lambda)$, where $W = W((Z^c - z^c)/h)$, Z^c is the continuous covariate, L_{λ} is the kernel function for the discrete variable, Z^d is the discrete variable, and λ is the smoothing parameter for the discrete covariate; see Racine and Li (2004) for further details kernel functions for the categorical variables. The kernel function given in (2.8) is for single continuous covariate.

Replacing $b(\epsilon_i)$ in Equation (2.3) by $\hat{b}(\hat{\epsilon}_i)$ calculated from the second step estimation and treating $\hat{g}_i = g_i - \hat{b}(\hat{\epsilon}_i)$ as a dependent variable, Delgado et al. (2015) show that a consistent estimate of $(\theta(\cdot), \theta'(\cdot))$ can be obtained from a minimization of a kernel-weighted objective function:

$$\min_{\theta(z),\theta'(z)} \sum_{i=1}^{n} [\hat{g}_i - X_i^T \theta(z) - X_i^T \theta'(z) (Z_i - z)]^2 K((Z_i - z)/h),$$
(2.8)

where $\theta'(z)$ reflects the partial effects $\partial \theta(z)/\partial z$ and h is the bandwidth controlling the size of the local neighborhood around an interior point z.

Letting $\delta(z) = [\theta(z), \theta'(z)]$, the solution of problem (2.8) is given by

$$\tilde{\delta}(z) = (\mathbf{X}^T \mathbf{K} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{K} \hat{g}, \qquad (2.9)$$

where **X** is a $n \times 2(d_s + 2)$ matrix having $(X_i^T, X_i^T(Z_i - z))$ as its *i*th row and **K** is a $n \times n$ diagonal matrix with the *i*th diagonal element being $K((Z_i - z)/h)$.

The bandwidth parameter has a particular importance in estimation of non-/semiparametric models as it determines the degree of smoothing. We use a cross-validation method, a data-driven approach, to choose the bandwidth parameter so that the bias-variance trade-off in the estimation is optimized by using the data itself. We also provide wild-bootstrap standard errors, which are robust to heteroscedasticity, using 399 bootstrap replications (Härdle and Marron, 1991, p.782).

We use three goodness-of-fit measures including in-sample R^2 , out-of-sample R^2 , and average squared predicted error (ASPE). The out-of-sample measures are robust to over-fitting of the model, which, therefore, implies that the model of interest may better describe the underlying process that generated the data. The predictive exercises are based on 1000 bootstrap replications. We use 80 percent of the data to estimate the model parameters and evaluate on the hold-out data; see Henderson and Parmeter (2015, p.141).

3 Data

We employ the same data set as used in Kourtellos et al. (2013) to investigate long-run growth effect of public debt. We provide the source and definition of each variable in Table 3 in the Appendix. We have a balanced 10-year period panel dataset covering 82 countries in 1980-1989, 1990-1999, and 2000-2009. An advantage of working with 10-year averages is to avoid any short-run fluctuations in macroeconomic variables. We also obtain an extended dataset and construct 10-year and 5-year averages for a sample 78 countries using the latest version of Penn World Table (PWT 9.0)³.

We use the per capita real GDP growth rate as a measure of economic growth. We include traditional Solow regressors as control variables in our model. These variables are initial level of income at the beginning of each ten-year period, which is expected to be negatively related to economic growth rates, the population growth rate and the rate of physical capital investment, which are used as proxies for the growth rate of input factors in the aggregate production function. Additional regressors are the following: public debt, the logarithm of percent of public debt to GDP, is the primary variable that we are interested in this paper, which comes from the International Monetary Fund historical public debt database. Inflation rate is included as a finance related variable that is expected to be positively related to public debt, which therefore may help to partly explain causal effect of debt on growth.

The main covariate, or auxiliary variable, in this study is democracy, for which we use democracy index as a proxy for institutions constructed by the Center for Systemic Peace as in the Polity IV project. The democracy index ranges from 0 to 10, and higher scores indicate a greater extent of institutionalized democracy that incorporates "the presence of institutions and procedures through which citizens can express effective preferences about alternative policies and leaders", "the existence of institutionalized constraints on the exercise of power by the executive", and "the guarantee of civil liberties to all citizens in their daily lives and in acts of political participation" (Polity IV Project: Dataset Users' Manual, 2016, pp.14-15).

It is believed that there are many determinants of economic growth that may be correlated with institutions, but omitted from the regression model. Moreover, the democracy indicators are viewed as noisy measures of "true" institutional quality and subject to considerable measurement error, which therefore potentially result in attenuation bias in the estimate. Acemoglu et al. (2001) use the mortality rates of European settlers in the colonial countries as an instrument for the institutions and eliminate these two potential bias sources simultaneously. In a recent study by Acemoglu et al. (2016), the authors use regional waves of democratization after 2011 as an instrument for democracy variable. They also construct a new measure of democracy variable to circumvent measurement error problem in the standard dynamic panel

³Excluded countries are Guyana, Nicaragua, Papua New Guinea, and Syria. Guyana and Papua New Guinea are not reported in PWT 9.0. We exclude Nicaragua as the outlier along with Guyana. Public debt for Syria after 2010 is missing.

regression estimation. In our paper, we rely on lagged values of democracy, which may still lead to underestimation of the impact, but can eliminate omitted variable bias.

We also use other set of covariates that are used as the threshold variables that resulted in a rejection of the null hypothesis of global linearity in the model of Kourtellos et al. (2013). These covariates include fertility, the logarithm of the average total fertility rate; life expectancy, the logarithm of the average life expectancy at birth; government consumption, the logarithm of average ratios of government consumption to real GDP per capita; and trade openness, the average ratio for each period of exports plus imports to GDP.

4 Estimation Results

4.1 Homogeneous Models and Mean Parameter Estimates

We present estimates from various model specifications for the augmented Solow growth model and an endogeneous semiparametric smooth coefficient model in Table 1. We first aim to compare mean parameter estimates from the semiparametric specifications with those from parametric model regression estimation. Columns 1-7 show estimates for four homogeneous model specifications from ordinary least squares and three model specifications from two-stage least squares estimation. Since semiparametric models take democracy into account through the functional coefficients, we include democracy as an additional conditioning variable in the standard growth model specifications. Year indicator is another factor that is controlled for in the parametric regression models in columns 1-7. Columns 1-4 show that the OLS estimates for the coefficient of public debt are negative and significant at the 5% and 10% levels with their values ranging from -0.0058 to -0.0080. The OLS regression in column 3 suggests that a 10 percentage points increase in the debt-to-GDP ratio is, on average, associated with a 0.060% decrease in subsequent 10-year period real per capita GDP growth rate.

The 2SLS estimates for public debt variable in columns 5-7 are also significant at the 10% level within the same magnitude level as the OLS estimates. The 2SLS estimate of the impact of democracy on economic growth, 0.0022, is highly significant with a standard error of 0.0007. This estimate is larger than the OLS estimates in columns 2-4. This suggests that there is a downward bias in the OLS estimates of democracy, which may be because of measurement error in the democracy index that creates attenuation bias (an estimate biased toward zero) or caused by endogeneity⁴.

Columns 8-10 reports average of semiparametric smooth coefficient instrumental variable

⁴Acemoglu et al. (2001) evaluate the difference between OLS and 2SLS estimates of democracy variable in their paper by using different measure of institutions variable, executive constraints, as an instrument. It is expected that using this variable as an instrument would not solve endogeneity problem, but correctly address the measurement error assuming that it is properly measured. The estimated effect of institutions variable from 2SLS method is 0.87 with highly significant. They conclude that measurement error in the institutions variable could be the primary reason in the difference between the OLS and 2SLS estimates.

		SPSCM-IV			2SLS				OLS	Variable
(10)	(9)	(8)	(7)	(6)	(5)	(4)	(3)	(2)	(1)	
-0.0171	0.0100	0.0148	-0.0068	-0.0068	0.0236^{c}	-0.0126	-0.0203	0.0258^{c}	0.0355^{b}	Intercept
0.0346	0.0355	0.0103	0.0457	0.0444	0.0144	0.0450	0.0437	0.0143	0.0143	
-0.0041^{c}	-0.0056^{b}	-0.0045^{c}	-0.0055^{c}	-0.0058^{c}	-0.0064^{c}	-0.0058^{c}	-0.0060^{c}	-0.0067^{b}	-0.0080^{b}	Public Debt
0.0023	0.0025	0.0025	0.0033	0.0033	0.0033	0.0033	0.0033	0.0033	0.0034	
			0.0021^{a}	0.0022^{a}	0.0015^{b}	0.0014^{b}	0.0014^{b}	0.0012^{a}		Democracy
			0.0007	0.0007	0.0004	0.0006	0.0006	0.0004		
-0.0086^{a}	-0.0098^{a}		-0.0061^{c}	-0.0060^{a}		-0.0051	-0.0049			Initial Income
0.0024	0.0023		0.0034	0.0021		0.0035	0.0035			
0.0080^{b}	0.0081^{b}		0.0181^{a}	0.0183^{a}		0.0176^{a}	0.0178^{a}			Investment Rate
0.0039	0.0039		0.0053	0.0053		0.0053	0.0053			
-0.0314^{b}	-0.0317^{b}		-0.0069	-0.0073		-0.0102	-0.0111			Population Growth Rate
0.0131	0.0133		0.0248	0.0247		0.0248	0.0248			
0.0093^{a}	0.0093^{a}		0.0048	0.0047		0.0051	0.0050			Schooling
0.0028	0.0027		0.0040	0.0039		0.0040	0.0039			
-0.0024^{b}			-0.0017			-0.0015				Inflation Rate
0.0011			0.0012			0.0012				
			0.0013	0.0019	0.0038^{c}	0.0017	0.0023	0.0041^{b}	0.0054^{a}	Trend
			0.0018	0.0017	0.0020	0.0018	0.0023	0.0019	0.0018	
0.4150	0.3827	0.1803	0.2094	0.2025	0.1191	0.2154	0.2093	0.1211	0.0832	In-Sample R^2
0.3411	0.3099	0.1187	0.2698	0.2600	0.1379	0.2767	0.2684	0.1399	0.0982	Out-of-Sample R^2
0.00040	0.00041	0.00049	0.00074	0.00073	0.00047	0.00044	0.00044	0.00046	0.00048	ASPE

Table 1: Summary of the results

1. Semiparametric model specifications allow coefficients to vary with respect to democracy.

2. We use Gaussian kernel function for all semiparametric estimation. The cross-validated bandwidth in column 9 is 1.62. Moreover, L_n is equal to 2.

3. Statistically significant parameter estimates: a, significance at 1%; b, significance at 5%; c, significance at 10%.

4. Column 8-10 reports the mean coefficient estimates and their respective standard errors.

5. Out-of-sample \mathbb{R}^2 and ASPE report mean of 1000 bootstrap replications.

estimates and its standard error. Column 8 and 9 show that the coefficient estimates of public debt are negative and statistically significant at 5% and 1% level with having the values around -0.0071 and -0.0072, respectively. The estimated effect suggests that a 10 percentage points increase in the debt-to-GDP ratio is associated with a 0.072% decrease in subsequent ten-year period real GDP growth rate, on average. Comparing column 9 and 3, we observe that mean value of public debt coefficient estimates from the semiparametric model estimation is almost in agreement with that of from the ordinary least squares estimation.

Nevertheless, the in-sample goodness of fit of the semiparametric model (38%) is higher than for the parametric model (20%). This comparison holds for all specifications between semiparametric and parametric regression models. We further investigate the model's outof-sample performance to decide on whether this improvement reflects over-fitting. In each semiparametric models in columns 8-10, the out-of-sample R^2 (ASPE) is in general higher (lower) than in the corresponding parametric models. These results indicate that semiparametric smooth coefficient model in column 9 is 7.3% more efficient than the parametric linear model in column 3 in terms of out-of-sample predictive ability, which, therefore, implies that the semiparametric model may better describe the underlying process that generated the data than the parametric model does. One may have a concern that higher-order polynomial terms in the homogeneous model may be sufficient to capture the parameter heterogeneity. This concern can be highlighted by the bias-variance trade-off in both parametric and nonparametric model estimation. Adding polynomial terms in a parametric regression model reduces the bias of the estimates (since more information is used in the estimation), but the parameters is less accurately estimated (i.e., standard errors are larger). Therefore, nonlinearity in the parametric model may be captured at the cost of efficiency. Nonparametric regression model, on the other hand, allows to control the bias-variance trade-off through the selection of bandwidth parameter, which essentially determines the local sample size for the estimation of each point of interest. Furthermore, the bandwidth can be chosen using the data via the cross-validation method. In other words, the nonparametric modeling approach allows a researcher to use the data to optimize the bias-variance trade-off. It might be also asked that a linear interaction term in a parametric model might explain the idea that public debt might have a different effect for countries which have different institutional quality. Since the estimate for public debt reflects on average effect on growth rate for all countries and adding interaction term for each variable in the model can result in loss of efficiency, parametric model may not fully explain the parameter heterogeneity. However, the smooth coefficient approach models the interaction effect between regressors and some covariates in a flexible way as opposed to a predetermined structure considered in the parametric specifications. It should emphasized that as both parametric model and non-/semiparametric model approximate the unknown true relationships in their own capacity, however, the non-/semiparametric model imposes less restrictions than the parametric model and thus is believed to bring a better fit to the data and more reliable inference.

The coefficients on other explanatory variables (initial per capita income, investment rate, population growth rate and average years of schooling) in columns 9-11 are of the predicted sign and significant at conventional levels. Column 11 reports the mean estimates for the semiparametric regression model, which controls for inflation rate as well. All variables have statistically significant coefficient estimates at conventional levels, but the magnitude of the coefficient estimate of public debt decreases more than half as the inflation rate accounts for part of its negative effect on economic growth. This result is, in fact, consistent with the theoretical literature on inflation and economic growth (Barro, 1995). Homogeneous model specifications in column 4 and 8, on the other hand, do not estimate economically significant drop in the growth effect of public debt when inflation rate is included as additional conditioning variable.

4.2 Parameter Heterogeneity

Figure 1 displays country-specific coefficient estimates for public debt variable from the semiparametric regression model in column 10 of Table 1 along with 95% bootstrap percentile



Figure 2: Estimated coefficient curve for public debt variable from the model in column 9 of Table 1. The figure corresponds to the functional coefficient $\theta_d(\cdot)$, graphing the semiparametric smooth coefficient instrumental variable estimate (solid line with small circles) with 95% bootstrap percentile confidence intervals (solid lines), and the standard varying coefficient estimate (dashed line).

confidence intervals⁵. We first observe that more public debt leads to lower economic growth for countries with democracy score less than 1 and greater than 7.6, holding other factors fixed. This result is consistent with the existing literature that suggest that countries with weak institutional quality are the only ones that tend to have an adverse effect of more public debt on growth. However, our results also show that more public debt can be detrimental to growth for countries with strong institutional quality (all else equal). On the other hand, for countries with a democracy score in between 1 and 7.6, public debt has no significant effect on growth. Particularly, the impact of public debt on growth for countries with a median level of democracy score reduces to values around zero, which are therefore economically insignificant as well. This result can be justified by the fact that countries in transition to institutionalized democracy may obey their debt payments, which therefore allows them to borrow again so that public debt becomes growth neutral for these countries.

The mean coefficient estimates of the countries in the low and high democracy groups are -0.0114 and -0.0069, respectively, while average public debt-to-GDP ratio for the two country groups are 52.46 and 71.52, respectively. This result contradicts traditional parametric regression models, which suggest high public debt and low economic growth relationship beyond

⁵Henderson, Kumbhakar, and Parmeter (2012) suggest to plot gradient estimates in a 45° plot to expose parameter heterogeneity that exists in the estimates. Their suggestion is useful especially when covariate vector is more than one dimension. Since in our model estimation the coefficients vary with respect to only one variable, from the graphical point of view it is better to plot coefficient estimates on a Cartesian coordinate system.

various threshold public debt levels. In particular, this result implies that public debt adversely affects economic growth in countries with weak institutional quality at lower level than countries with strong institutional quality. Our results also show some evidence of parameter heterogeneity for countries with democracy score less than 1. However, since we use pooled sample that includes three ten-year periods in the estimation, our results in Figure 1 does not reflect country characteristics as well as global factors. In fact, the semiparametric regression model in column 10 estimates the same value, -0.0071, for advanced countries, which have a democracy score of 10. Moreover, thirty seven countries, which have a democracy score of zero, are predicted to have the same growth effect of public debt, i.e., -0.0118.

The dataset includes two countries that have outliers for the public debt-to-GDP ratio. Guyana has a public debt of 560% of GDP in 1980 and 503% in 1990. Nicaragua has a public debt-to-GDP ratio of 484 percent in 1990. Average democracy score of these countries are 3.6 and 5.3, respectively. From Figure 3(a), we observe very little support for the parameter heterogeneity for the public debt variable after excluding the outliers. Similar to the results of full coverage data set, only high democracy score countries have a significant negative effect of public debt on growth.

4.2.1 Including the period 2010-2014

We re-estimate the semiparametric model including the recent years 2010-2014. Solow variables including schooling for the period 1980-2014 are obtained from the latest version of the Penn World Table (PWT 9.0). Democracy index is obtained from the latest release of the Polity IV dataset. Since there is no data available for Guyana and Papua New Guinea in the PWT 9.0 and for Syria for the public debt variable in the IMF public debt database beyond 2010, we didn't include these countries in the extended dataset together with Nicaragua as an outlier. Figure 3(b) displays estimated functional coefficient for public debt variable. We find significant negative effect of public debt on growth for the countries with a democracy score higher than 5.4. On average, public debt has a stronger effect on growth having -0.010 (almost two-fold increase compared to the results from original data) after including the recent years. We also observe that the coefficient estimates sharply decrease by three folds from 0.0043 to -0.0141, in magnitude, as the democracy index increases from zero to 2. It is interesting that all three figures (Figure 2 and 3(a)-(b)) have a common path that public debt has an increasing (even though slightly) negative effect on growth for the countries with a democracy score greater than 9.

Overall, our results give some support to the recently shifted focus in the debt-growth nexus that institutions may be one of the main factor that determines the debt-growth link. Our contribution in this paper also puzzles over the complexity of debt-growth relationship for advanced countries. We would like to mention first that we are not able to expose heterogeneity in the effect of public debt on growth within the advanced country group in Figure 1 as there



Figure 3: Functional Coefficient estimates for other regressors

is no variability in the *democracy* score of these countries. This result essentially highlights the need of other potential candidate covariates such as sovereign debt credit ratings that might have a variability within the advanced country group. We defer this research for future study. The literature on moral hazard and public debt, on the other hand, gives one of the possible explanation for significant negative effect of public debt on growth for advanced countries in Figure 1. Particularly for some of the euro area countries including Greece, Italy, Portugal, and Spain, the root of the public debt crises is at an excessive risk taking behavior of economies that results from widespread support to the financial system (Allen, Carletti, Goldstein, and Leonello, 2015). Relatedly, in a recent study of Yener, Stengos, and Yazgan (2017), the authors find that Greece is the only country among others including Portugal, Spain, and Iceland, which have also external debt problem, having unsustainable debt projections, which, therefore, indicates a debt crises looming for Greece. The authors emphasize the main reason behind this result as the weak fundamentals with an increasing risk taking behavior of the politicians.

4.2.2 Parameter heterogeneity in the relationship between growth and other regressors

The curves in Figure 2 show how democracy affects the coefficients of other conditioning variables. In Figure 2a, we find that countries with an institutionalized democracy, a score greater than 4.7, have an increasing significant negative effect of initial income on economic growth, which confirms the conditional β -convergence hypothesis. The curve in Figure 2b exhibits a significant positive and an inverse U-shape relationship between the real investment rate and the real GDP per capita growth rate for the countries with a democracy score in between 1.2 and 7. It is observed in Figure 2c that higher population growth rate is associated with a slowdown in an economic growth for the countries with a democracy score greater than 6.6. Lastly, schooling in Figure 2d has a significant positive effect on growth rate for the countries with a democracy score greater than 6.3. For a better exposition of parameter heterogeneity for these variables, we plot the estimated curves in a larger scale in Figure 3 in the appendix. It is clearly seen that for each regressors, except for the investment rate, there is a heterogeneous relationship in the effect of the variable on economic growth rate for the mid- and high-level democracy score countries.

5 Robustness Checks

We aim to see the main results obtained from the semiparametric model in the previous section are robust to additional model specifications.

5.1 Alternative measure for democracy

We examine whether our main results are sensitive to different measures of institutional quality such as executive constraints obtained from the same data source, Polity IV. We find that countries with an executive constraint score less than 2.8 and greater than 5.8 have a significant negative coefficient estimates for public debt variable. This result does not alter the conclusions drawn from our main results; that is, institutional quality is an important factor that governs the effect of public debt on growth.

5.2 Additional control variables

We estimate semiparametric endogeneous smooth coefficient model that excludes three Solow variables (investment rate, population growth rate and average years of schooling) and adds inflation rate and trade openness. The results show that only the coefficients of public debt and inflation rate are significant and negative at conventional levels. Our main results are the same in this model estimation. We also do the estimation of the regression model that includes government consumption. We continue to find a negative and significant relationship between public debt and per capita GDP growth rates for the countries with low and high democracy scores. Moreover, estimated coefficient curve for public debt retains the same path as in the model in column 10 of Table 1.

6 Conclusion

We employ a semiparametric smooth coefficient model with an endogenous variable in the nonparametric part to analyze heterogeneous relationship between debt and growth with a sample of 82 countries over the three 10-year periods. Most of the papers in the growth literature have aimed at finding a nonlinearity in the debt-growth relationship depending on the level of public debt. As it is emphasized with evidences in Kourtellos et al. (2013), nonlinear effect of debt on growth depends on a country's institutional quality. In the same vein, our results show some evidence of heterogeneity in the effect of debt on growth for the countries with a democracy score below 1. Our results also show some evidences for the adverse effect of public debt on growth in advanced countries.

7 Appendix

Variable	Mean	Std. Dev.	Min	Max
Growth	0.013737	0.022960	-0.099459	0.083383
Initial income	8.423263	1.266566	5.86825	10.71059
Lag of initial income	8.335907	1.232223	5.77992	10.5477
Investment rate	3.046038	0.351779	1.873230	3.891546
Lag of investment rate	3.055552	0.394585	1.743324	4.312730
Population growth rate	-2.711415	0.160957	-3.2289	-2.384709
Lag of population growth rate	-2.690982	0.165420	-3.083584	-2.276809
Schooling	0.598071	0.768791	-2.183513	1.970172
Lag of schooling	0.320655	0.901583	-2.662667	1.901029
Public debt	4.080373	0.610824	2.173561	6.327447
Lag of public debt	3.924271	0.730973	1.116990	6.462404
Fertility	3.616907	1.729945	1.2080	7.7777
Lag of fertility	4.064228	1.888114	1.1660	7.8244
Life expectancy	4.170482	0.175197	3.634336	4.406866
Lag of life expectancy	4.139680	0.176947	3.636147	4.385104
Trade openness	66.5114	36.4878	9.7683	199.8575
Lag of trade openness	61.0066	35.8041	9.6979	180.0895
Democracy	5.742649	3.834012	0.00	10.00
Lag of democracy	5.021545	4.167344	0.00	10.00
Executive constraints	4.958977	2.047979	1.00	7.00
Lag of executive constraints	4.512398	2.332962	1.00	7.00
Government consumption	2.195023	0.439004	1.056177	3.560925
Lag of government consumption	2.192095	0.477742	1.014359	3.694487
Inflation rate	2.298081	1.167341	-1.951826	7.571372
Lag of inflation rate	2.338690	1.193889	-1.459525	8.258299

 Table 2: Summary statistics of the variables in the dataset



Figure 4: Coefficient estimates for other regressors

Variable	Source	Definition
Growth	PWT 7.0.	Growth rate of real per capita GDP in chain series for the periods 1980-1989, 1990-1999, and 2000-2009.
Initial income	PWT 7.0.	Logarithm of real per capita GDP in chain series at 1980, 1990, and 2000. Lagged values correspond to 1975, 1985, and 1995.
Investment rate	PWT 7.0.	Logarithm of average ratios over each period of investment to real GDP per capita for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Population growth rate	PWT 7.0.	Logarithm of average population growth rates plus 0.05 for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Schooling	Barro and Lee (2000)	Logarithm of average years of male secondary and tertiary school attainment for ages above 25 in 1980, 1990, and 1999. Lagged values correspond to 1975, 1985, and 1995.
Public debt	IMF, Debt Database Fall 2011 Vintage	Logarithm of average percentages over each period of public debt to GDP for the periods 1980- 1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995- 1999.
Fertility	World Bank	Logarithm of average total fertility rate in 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Life expectancy	World Bank	Logarithm of average average life expectancy at birth for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Trade openness	PWT 7.0.	Average ratios for each period of exports plus imports to real GDP per capita for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Government consumption	PWT 7.0.	Logarithm of average ratios for each period of government consumption to real GDP per capita for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Inflation rate	World Bank	$\label{eq:logarithm} \begin{tabular}{lllllllllllllllllllllllllllllllllll$
Democracy	Polity IV	An index ranges from 0 to 10 where higher values equals a greater extent of institutionalized democracy. Average for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.
Executive constraints	Polity IV	An index ranges from 1 to 7 where higher values equals a greater extent of institutionalized constraints on the power of chief executives. Average for the periods 1980-1989, 1990-1999, and 2000-2009. Lagged values correspond to 1975-1979, 1985-1989, and 1995-1999.

Table	3:	Data	descrip	tion

Table 4: List of countries grouped into coefficient estimates from SPSCM-IV and democracy score

	Negative and Significant		Insignificant
≤ 1	$\geq 7.6~\& < 9$	≥ 9	
Algeria (1980, 1990)	Argentina (2000)	Australia (1980, 1990, 2000)	Argentina (1980, 1990)
Bangladesh (1980)	Bolivia (1990, 2000)	Austria (1980,1990,2000)	Benin (1990, 2000)
Benin (1980)	Botswana (2000)	Belgium (1980, 1990, 2000)	Bangladesh (1990, 2000)
Burundi (1980, 1990)	Brazil (1990, 2000)	Canada (1980, 1990, 2000)	Bolivia (1980)
Cameroon (1980, 1990, 2000)	Chile (1990, 2000)	Costa Rica (1980, 1990, 2000)	Botswana (1980, 1990)
Central African Republic (1980)	Colombia (1980, 1990)	Cyprus (1980, 1990, 2000)	Brazil (1980)
Chile (1980)	Dominican Republic (2000)	Denmark (1980, 1990, 2000)	Burundi (2000)
Cote'd Ivoire (1980, 1990)	Ecuador (1980, 1990)	Finland (1980, 1990, 2000)	Central African Republic (1990, 2000)
Egypt (1980, 1990, 2000)	France (1980)	France (1990, 2000)	Congo Republic (1990)
Gabon (1980, 1990, 2000)	Greece (1980)	Greece (1990, 2000)	Cote'd Ivoire (2000)
Gambia (2000)	Guatemala (2000)	Ireland (1980, 1990, 2000)	Colombia (2000)
Ghana (1980)	India (1980, 1990, 2000)	Italy (1980, 1990, 2000)	Dominican Republic (1980, 1990)
Guyana (1980)	Republic of Korea (2000)	Israel (1980, 1990, 2000)	Ecuador (2000)
Indonesia (1980, 1990)	Lesotho (2000)	Jamaica (1980, 1990, 2000)	Gambia (1980, 1990)
Iran (1980)	Mexico (2000)	Japan (1980, 1990, 2000)	Ghana (1990, 2000)
Kenya (1980, 1900)	Panama (1990, 2000)	Netherlands (1980, 1990, 2000)	Guatemala (1990)
Lesotho (1980)	Paraguay (2000)	New Zealand (1980, 1990, 2000)	Guyana (1990, 2000)
Malawi (1980)	Peru (2000)	Norway (1980, 1990, 2000)	Honduras (1980, 1990, 2000)
Mali (1980)	Philippines (2000)	Portugal (1980, 1990, 2000)	Kenya (2000)
Mauritania (1980, 2000)	Senegal (2000)	Spain (1980, 1990, 2000)	Lesotho (1990)
Morocco (1980, 1990, 2000)	South Africa (1990, 2000)	Sweden (1980, 1990, 2000)	Malaysia (1980, 2000)
Nicaragua (1980, 1990)	Thailand (1990)	United Kingdom (1980, 1990, 2000)	Malawi (1990, 2000)
Niger (1980)	Trinidad & Tobago (1990, 2000)	United States (1980, 1990, 2000)	Mali (1990, 2000)
Panama (1980)	Turkey (1990, 2000)	Uruguay (1990, 2000)	Mexico (1980, 1990)
Paraguay (1980)	Venezuela (1980, 1990)		Nepal (1980, 1990, 2000)
Sierre Leone (1980)			Nicaragua (1990)
Swaziland (1980, 1990, 2000)			Niger (1990, 2000)
Syria (1980, 1990, 2000)			Pakistan (1980, 1990, 2000)
Togo (1980, 1990, 2000)			Papua New Guinea (1980, 1990, 2000
Tunisia (1980, 1990, 2000)			Paraguay (1990)
Zambia (1980)			Republic of Korea (1980, 1990)
Zimbabwe (1990)			Peru (1980, 1990)
			Sierre Leone (1980, 2000)
			South Africa (1980)
			Sri Lanka (1980, 1990, 2000)
			Thailand (2000)
			Turkey (1980)
			Venezuela (2000)
			Zambia (1990, 2000)
			Zimbabwe (1980 2000)

Paper	Sample	Empirical methodology	Debt measure	Instrumental variable	Findings
Caner et al. (2010)	101 developing and developed countries (1980- 2008)	Cross-section; Threshold Least Squares	General govern- ment gross debt (% GDP) from IMF	No instru- ments	Significant negative effect; debt thresh- old is 77% for all countries; 64% for the sample of devel- oping countries only
Cecchetti et al. (2011) Checherita- Westpal and Rother (2012)	 18 OECD countries (1980-2010) 12 Euro area countries (1970-2008) 	Panel data; FE; panel threshold; LSDV Panel data; FE; 2SLS; GMM	General govern- ment debt from IMF Gross govern- ment debt (% GDP) from AMECO	No instru- ments Lagged debt- to-GDP ratio up to the 5th lag; average of the debt levels of the other countries in the sample	Significant negative effect; threshold level is 85% Significant negative effect; debt turning point is in between 90% and 100%
Minea and Parent (2012)	20 advanced countries as in Reinhart and Rogoff (2010) (1945-2009)	Panel data; panel smooth threshold re- gression	Public debt from IMF	No instru- ments	Negative effect be- low the threshold level of 115%; pos- itive effect beyond this level of debt
Baum et al. (2013)	12 Euro area countries (EMU) (1990- 2007/2010)	Panel data (yearly); non- /dynamic panel threshold model; OLS; GMM	Public debt from AMECO	No instrument for debt vari- able	Significant positive effect below the threshold level of 67% for the period 1990-2007; insignif- icant effect beyond that threshold; significant negative effect beyond the threshold level of 95% for the period 1990-2010
Kourtellos et al. (2013)	82 countries (1980-2009)	Panel data (10- year averages); structural threshold re- gression; 2SLS; GMM	Public debt (% of GDP) from IMF	Lag of public debt	Threshold vari- able is democracy; significant neg- ative effect for low-democracy regime countries; insignificant effect for countries in high-democracy regime

Table 5: List of literature on the relationship between public debt and economic growth

Paper	Sample	Empirical methodology	Debt measure	Instrumental variable	Findings
Wright and	13 Caribbean	Panel data;	$\mathrm{Debt}/\mathrm{GDP}$	No instru-	61% is the threshold
Grenade	countries	PDOLS	from IMF	ments	level
(2014)	(EMU) (1990-				
	2012)				
Eberhardt	118 countries	Unbalanced	Gross general	No instru-	No common thresh-
and Pres-	(1961-2012)	panel data;	government	ments	old level of public
bitero (2015)		panel time	debt from WDI		debt for all coun-
		series approach;	and IMF		tries; evidence for
		ECM			differences in debt-
					growth relationship
					across countries
Égert (2015)	20 advanced	Panel data;	Central govern-	No instru-	Little evidence on
	and 21 emerg-	threshold re-	ment debt from	ments	90% threshold level;
	ing economies	gression	the same source		some evidence for
	(1946-2009)		in Reinhart and		lower threshold
			Rogoff (2011)		level
Woo and Ku-	38 advanced	Panel data; BE;	Gross govern-	5th lag of debt	Significant negative
$\max(2015)$	and emerg-	pooled OLS;	ment debt ($\%$	variable	effect; threshold
	ing economies	FE; SGMM	of GDP) from		level of 90% , be-
	(1970-2008)		IMF		yond which debt
					has a negative effect

Table 5: List of literature on the relationship between public debt and economic growth (Cont'd.)

1. European Commission AMECO database

3. Woo and Kumar (2015) found the threshold level by adding interaction terms into the model.

4. Égert's (2015) dataset for advanced countries excludes Ireland and includes Switzerland.

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