The Impact of Spatial Price Differences on Oil Sands Investments

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Abstract

In this article, a two-factor real options model is developed to examine the impact spatial price differences have on the value of an oil sands project and the incentive to invest. Large, volatile price differences between locations can emerge when demand to ship exceeds capacity limits. This may have a significant impact on policy, production and investment in exporting regions. We assume the price difference between two locations follows a stationary process implying prices in different locations move together. The investment decision is formulated as a linear complementarity problem that is solved numerically using a fully implicit finite difference method. Results show the value of an oil sands project and the incentive to invest in a new project will increase when price differences decrease. Surprisingly, the standard deviation of the price difference has very little impact on project value or the incentive to invest.

1 Introduction

The feasibility of natural resource investments depends critically on access to markets. The decision of whether to build additional pipeline capacity to export crude bitumen and its derivatives from Alberta has been a contentious policy issue. Proponents argue the large price difference between Western Canadian Select (WCS) and international benchmarks is mostly attributed to inadequate pipeline infrastructure[1] and claim that both firms and governments would benefit from expanding pipeline capacity. Firms would gain access to international markets, higher world prices, and lower transportation costs. Governments would receive more tax revenue from higher royalties. The claims are supported, theoretically, by spatial arbitrage models which show that significant variation in price differences can emerge as a result of capacity constraints (Coleman (2009)).

Figure (1) plots monthly spot price data for Brent blend, West Texas Intermediate (WTI), WCS, and Mexican Maya as well as the price difference between Mexican Maya and WCS from January 2005 to December 2015. Mexican Maya is a heavy crude oil similar in quality to WCS located in the Gulf Coast. Prior to 2011, Mexican Maya received a small location premium over WCS and large price differences were short lived. Beginning in 2011 Mexican Maya and WCS diverged. The persistence of the large price difference suggests there are substantial costs required to move heavy crude oil from Northern Alberta to the Gulf Coast, if not, there is a opportunity to arbitrage between the two markets[2].

In this paper we incorporate transportation costs into a real options model to study the impact spatial price differences have on the value of an oil sands project and the incentive to invest. The value of an oil sands project is contingent upon uncertain output prices and transportation costs. We assume the output price follows a geometric Brownian motion (GBM) and transportation costs follow a Ornstein-Uhlenbeck (OU) mean-reverting process. This assumption implies the world oil market is 'one great pool' (Adelman (1984)) as crude oil prices in different markets move together. The decisions of when to invest in

[1]WCS is the benchmark for heavy crude oils in Canada. It is a blend of heavy crude oil, crude bitumen and diluents with an API gravity of 20.5°.

[2]The spatial equilibrium model of Samuelson (1952) and Takayama and Judge (1971) show the price of a good in one market is equal to the price of the good in another market minus the cost of transporting it between the two markets.

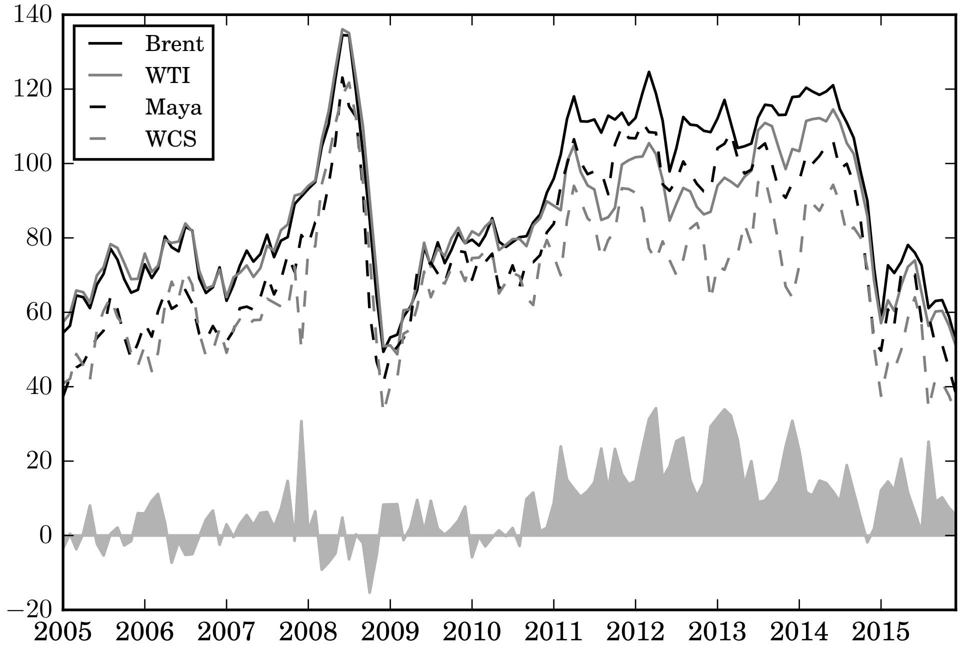
a new project and when to abandon a project for scrap value are evaluated under different scenarios. Optimal stopping is used identify the threshold prices when it is optimal to invest in a new project and abandon an operating project. The optimal stopping problems result in free boundary problems that do not have known analytical solutions. Following Wilmott et al. (1993) and Insley and Rollins (2005), the free boundary problems are redefined as linear complementarity problems and we approximate the solutions numerically using the fully implicit finite difference method (IFDM). The model parameters are chosen to approximate a typical in situ oil sands project in Northern Alberta.

To preview the results, we find that a decrease in transportation costs increases the value of the oil sands project, investments in new projects happen earlier, and operating projects are abandoned later. These results are consistent with the claims made by supporters of the policy to expand pipeline capacity. Surprisingly, we also find that an increase in transportation cost uncertainty has little effect on the value of the oil sands project or on the decision of when to invest and when to abandon. Typically, the value of an option increases as uncertainty increases as upside potential increases while the option limits downside loses.

1.1 Literature Review

Evaluating natural resource investments using real options analysis is a standard approach in the literature. Brennan and Schwartz (1985) apply option pricing theory to the problem of valuing uncertain investments. They determine the combined value of the options to shut down and restart a copper mine when spot prices are uncertain and the convenience yield is constant. Paddock et al. (1988) combine option-pricing techniques with a model of equilibrium in the market for the underlying asset to value offshore petroleum leases. Bjerksund and Ekern (1990) value a Norwegian oil field with options to defer and abandon. Clarke and Reed (1990) consider the option to abandon a currently producing oil-well when oil prices and extraction rates are uncertain. Conrad and Kotani (2005) determine the trigger prices to initiate investment in the Arctic National Wildlife Refuge under different assumptions about the evolution of crude oil prices. Morck et al. (1989) value forestry resources under stochastic inventories and prices. Insley (2002) and Insley

Figure 1: Monthly crude oil spot prices and Mexican Maya-WCS price difference in Canadian dollars from January 2005 to December 2015. Brent and WTI data was collected from the EIA, Mexican Maya data was collected from Bloomberg, and WCS data was collected from Natural Resources Canada.



and Rollins (2005) consider the optimal tree harvest problem when tree harvesting can be delayed and output prices follow known stochastic processes. Conrad (2000) determines the order and timing of wilderness preservation, resource extraction, and development when amenity value, the value of the resource, and return from development all follow geometric Brownian motions.

Recently, a number of papers have analyzed the management of oil sands projects and the rate of oil sands development using real options analysis. Almansour and Insley (2016) extend the Brennan and Schwartz (1985) model to include cost uncertainty and study the optimal management of an oil sands project. In situ oil sands projects face high levels of cost uncertainty from fluctuations in natural gas prices, natural gas is an important input in the extraction process. Commodity prices follow a non-stationary stochastic process made up of three factors: a long-run factor (non-stationary process), a short-run factor (stationary process), and a deterministic function that represents seasonality in the prices[3]. They find

[3]Almansour and Insley (2016) extend the Schwartz and Smith (2000) two factor commodity price model by incorporating a deterministic seasonality component.

the value of the project is significantly negatively affected by stochastic costs and the value of the project decreases as cost volatility increases.

Kobari et al. (2014) evaluate the rate of oil sands expansion under different environmental cost scenarios in a dynamic, game-theoretic model. Their model considers a multi-plant/multi-agent setting with price and cost uncertainty. Like Almansour and Insley (2016), cost uncertainty is driven by uncertainty in natural gas prices. The price of oil follows a mean-reverting process with an increasing long-run average price. The cost of natural gas depends on a deterministic seasonality component and a mean-reverting stochastic component. They consider two environmental cost scenarios: an increasing environmental cost scenario and a decreasing environmental cost scenario. Their results show that decreasing environmental costs cause new investments to be delayed compared to increasing environmental costs but decreasing environmental costs have little effect on projects that have already been constructed.

Almansour and Insley (2016) and Kobari et al. (2014) both assume that the price of crude oil and natural gas in Northern Alberta follows the same dynamics as international crude oil and natural gas benchmarks[4]. These assumptions ignore the crude oil price differential and factors that affect the differential such as the availability of pipeline capacity, weather and the cost of diluent[5]. The Bank of Canada (2013) expects Canadian crude oil prices to remain depressed and more volatile than international crude oil benchmarks until sufficient capacity is in place. They believe this is an important issue facing Canadas energy sector and a major factor restraining business investment. This paper hopes to contribute to this literature by focusing on the effect transportation costs have on a firm's investment decision. Due to the cost of investing in new pipeline projects, understanding how oil sands producers will respond to a decrease in transportation costs is important for oil transportation firms proposing new pipeline projects and for policymakers weighing the cost and benefit of these new pipeline projects.

[4]Almansour and Insley (2016) use weekly WTI futures and Henry Hub (HH) natural gas futures data from January 1995 to August 2010 to calibrate their model and Kobari et al. (2014) use daily WTI futures and HH natural gas futures data from February 2, 2009 to May 10, 2012 to calibrate their model.

[5]Diluent is any lighter hydrocarbon added to heavy crude oil or bitumen in order to facilitate its transportation in crude oil pipelines, National Energy Board (2013, p. 80).

The rest of the paper is organized as follows. Section 2 presents the general valuation model when price and cost are uncertain. Section 3 presents the results for each transportation cost scenario. Section 4 summarizes the results and concludes the paper.

2 General Model

This section presents a model for the valuation of a nonrenewable resource asset with nested real options subject to uncertain prices and transportation costs. Here, an oil sands project has two stages. The development stage where a firm holds an option to develop an oil sands project and the operating stage where a firm operates an oil sands project and has an option to abandon for scrap value. Similar to Paddock et al. (1988), the value of the oil sands project in the development stage is contingent on the value of the oil sands project in the operating stage and is therefore a compound option, option on an option.

The motivation for this model is a Canadian oil sands project located in Northern Alberta that must transport its output from remote production areas to consuming markets thousands of kilometers away,[6] but we believe it can be applied to any nonrenewable resource project that faces price and transportation cost uncertainty.

2.1 Option to Develop an Oil Sands Project

Consider a firm that holds a lease to a previously undeveloped parcel of land that contains a known quantity of crude oil. We assume all expenditures relating to exploration have been made. The lease gives the firm the proprietary right to extract and sell the crude oil for a specified period of time. If, by the end of the lease[7], minimum production requirements[8] as specified in the lease agreement have not been met the lease expires and the land is returned to the leasee[9]. If minimum production requirements have been met the

[6]The distance between Hardisty, Alberta and Cushing, Oklahoma, two major transportation hubs, is over 2500 kilometers.

[7]The standard term of a primary lease is 15 years.

[8]The required minimum level of production per term year is 2400 m^3 of bitumen per section (40 barrels per section per day), on average, over the lease term.

[9]Generally, the leasee refers to the provincial Crown as it own 97 percent of oil sands mineral rights.

lease is extended indefinitely, meaning the lease is extended until reserves are exhausted or the project is abandoned.

The lease is viewed as an option to develop an oil sands project. The underlying asset is an operating oil sands project whose value is contingent on the price of the crude oil, the cost of transporting crude oil to market, and the amount of reserves in place. The exercise price is the cost of building the required production facilities and transportation infrastructure. The firm's problem is to determine the value of the option to develop and decide at what point in time they will exercise the option to develop given price and transportation cost follow known stochastic processes.

Assume the stochastic process for price, $S(t)$, and transportation cost, $C(t)$, are

$$dS = \mu S dt + \sigma_S S dW_S, \tag{1}$$

$$dC = \kappa(\bar{C} - C)dt + \sigma_C dW_C. \tag{2}$$

Where μ is the drift and σ_S is the standard deviation in price, κ is the speed of reversion, $\bar{C}$ is the long-run average transportation cost, and σ_C is the standard deviation in transportation cost. dW_S and dW_C are increments of a correlated Brownian motion with correlation coefficient of $\rho_{S,C}$.

Geometric Brownian motion in commodity prices is standard in the real options literature (Brennan and Schwartz (1985) copper prices, Paddock et al. (1988) for the value of developed reserves, Clarke and Reed (1990) and Conrad and Kotani (2005) for crude oil prices). Schwartz and Smith (2000) consider a two-factor model for commodity prices that incorporates short-term deviations from the equilibrium price and long-term random fluctuations in the equilibrium price. They show that for long-term investments, short-term deviations from the equilibrium price have little effect on the value of the investment. Therefore, they argue, to simplify analysis a single-factor model that considers uncertainty in the equilibrium price can be used to value long-term investments.

Assuming transportation costs follow the Ornstein-Uhlenbeck mean-reversion process is consistent with the literature on crude oil price differentials. A number of authors have examined the comovement of crude oil prices (Gülen (1997 and 1999), Hammoudeh et al.

(2008), and Fattouh (2010)) and have found that crude oil prices differences are stationary. Recently, Wilmot (2013) found that secondary crude oil blends of similar and differing qualities are cointegrated with a structural break.

Let $G(S, C, \tau)$ be the value of the option to develop an oil sands project at the current price, S, current transportation cost, C, and with τ time remaining on the lease. Where $\tau = \bar{T} - t$ and $\bar{T}$ is the expiration date of the lease. If $F(S, C, \bar{Q})$ is the value of an operating oil sands project with initial reserves $\bar{Q}$ and IC is the required investment cost then the firm's payoff from exercising the option to develop is $F(S, C, \bar{Q}) - \mathrm{IC}$. If the firm decides not to exercise the option, they receive a payoff of $\mathrm{M}(t)$ per unit of time from the undeveloped parcel of land[10], and the option to develop an oil sands project in the next period.

The firm's problem of valuing the option to develop an oil sands project and determining the optimal development threshold can be formulated as an optimal stopping problem

$$G(S, C, \tau) = \max \left\{ F(S, C, \bar{Q}) - \mathrm{IC},\ \mathrm{M}dt + \frac{\mathrm{E}_t[G(S + dS, C + dC, \tau + d\tau)]}{1 + \delta_G dt} \right\}. \tag{3}$$

Where E_t is the conditional expectations operator and δ_G is the risk-adjusted constant discount rate.

The optimal development threshold defines a surface that divides the (S, C, τ)-space into two regions: the continuation region and the development region. Let $\hat{S}(C, \tau)$ be the optimal development threshold. The optimal development threshold specifies a price for a given τ and C where the payoff from exercising the option is equal to the payoff from waiting. The continuation region lies below the surface, $S < \hat{S}(C, \tau)$. In this area it is optimal to continue holding the option to develop. The development region lies above the surface, $S > \hat{S}(C, \tau)$. In this area it is optimal to exercise the option to develop. When $S = \hat{S}(C, \tau)$, the continuation payoff equals the exercise payoff.

In the continuation region, $S \leq \hat{S}$, the value of the option to develop an oil sands project satisfies the following Bellman equation

$$\delta_G G = \mathrm{M} + (1/dt)\mathrm{E}_t[dG]. \tag{4}$$

[10]The payoff from the undeveloped parcel of land can be either positive or negative.

The Bellman equation requires the firm's payoff from waiting to exercise the option to develop, the right hand side of (4), to equal the required return from holding the option to develop.

Insley and Wirjanto (2010) compare the dynamic programming and contingent claims approaches for valuing risky investments using real options analysis. They find that the contingent claims approach is preferred when data exists that allows for the estimation of the market price of risk or the convenience yield. Contingent claims assumes that project risk can be eliminated with a perfect hedged position with another risky asset. In this setting, it might not be possible to create the perfect hedge as transportation cost risks (i.e. crude oil price spreads) may not be traded in the markets.

Apply Ito's Lemma to $G(S, C, \tau)$ and substitute equations (1) and (2) and rearrange to get

$$\begin{aligned} dG =& (\mu G_S + \kappa(\bar{C} - C)G_C - G_\tau + \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S G_{SC}))dt \\ &+ \sigma_S S G_S dW_S + \sigma_{TC} G_C dW_{TC}. \end{aligned} \tag{5}$$

Equation (5) is the stochastic differential equation for the option to develop an oil sands project. Substitute (5) into the Bellman equation (4) and pass it through the expectations operator to obtain the partial differential equation for the value of the option to develop an oil sands project in the continuation region,

$$\delta_G G = \mathrm{M} + \mu S G_S + \kappa(\bar{C} - C)G_C - G_\tau + \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S G_{SC}). \tag{6}$$

The partial differential equation is subject to the following boundary condition,

$$G(S, C, 0) = \max\{F(S, C, Q) - \mathrm{IC}, 0\}. \tag{7}$$

If the lease reaches the expiration date and the oil sands project has not yet been developed, the option to develop an oil sands project is exercised if the value of the operating oil sands project exceeds the investment cost otherwise the option to develop expires unused.

The optimal development threshold is determined by the following value-matching condition

$$G(\hat{S}(C,\tau),C,\tau) = F(\hat{S}(C,\tau),C,Q) - \mathrm{IC}, \tag{8}$$

and smooth-pasting conditions

$$G_S(\hat{S}(C,\tau),C,\tau) = F_S(\hat{S}(C,\tau),C,Q), \tag{9.1}$$

$$G_C(\hat{S}(C,\tau),C,\tau) = F_C(\hat{S}(C,\tau),C,Q). \tag{9.2}$$

The value-matching condition matches the value of the option to develop to the value of the operating oil sands project minus the investment cost on the optimal stopping boundary. The smooth-pasting conditions are required to jointly solve for the unknown function G and the unknown development threshold $\hat{S}$. On the boundary the functions, G and F, must meet tangentially for $\hat{S}$ to be the optimal stopping boundary[11].

2.1.1 Option to Develop as a Linear Complementarity Problem

Equation (6) and conditions (7), (8), and (9) define a free boundary problem, the solution to the problem determines the value of the option to develop an oil sands project as well as the optimal development threshold. We follow Wilmott et al. (1993) and Insley and Rollins (2005) and redefine the free boundary problem as a linear complementarity problem (LCP)[12]. A solution to the LCP is a solution of the free-boundary problem and *vice versa*[13]. A benefit of redefining the free boundary problem as a LCP is that the complications caused by the free-boundary are eliminated and the free boundary can be recovered after the LCP has been solved.

The free boundary problem for the option to develop can be redefined as the following

[11]See Dixit and Pindyck (1994) for a detailed discussion on value-matching and smooth-pasting conditions.

[12]A LCP has the following form

$$\begin{aligned} x, F(x) &\geq 0, \\ x^T F(x) &= 0. \end{aligned} \tag{10}$$

Where x is a vector and $F(x)$ is a linear vector valued function.

[13]See Elliot and Ockendon (1982), Friedman (1988), and Kinderlehrer and Stampacchia (1980) for proofs of the existence and uniqueness of the solutions.

LCP

$$
\begin{aligned}
&\delta_G G - \mathrm{M} - \mu S G_S - \kappa(\bar{C} - C) G_C + G_\tau \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S G_{SC}) \geq 0, && (11.1) \\
&G - F + \mathrm{IC} \geq 0, && (11.2) \\
&\big(\delta_G G - \mathrm{M} - \mu S G_S - \kappa(\bar{C} - C) G_C + G_\tau \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 G_{SS} + \sigma_C^2 G_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S G_{SC})\big) \times \big(G - F + \mathrm{IC}\big) = 0. && (11.3)
\end{aligned}
$$

The option to develop, like all American-type options, defined as LCPs has the intuitive interpretation of a rational individual's strategy with regard to holding versus killing the option. For the option to develop, equation (11.1) holds with an equality when it is optimal to hold the option to develop and equation (11.2) is a weak inequality. Equation (11.1) holds with a weak inequality and equation (11.2) holds with an equality when it optimal to exercise the option to develop. Equation (11.1) can be interpreted as the difference between the required return for holding the option to develop and the actual return from holding the option. When the required return equals the actual return it is optimal to hold the option to develop. When the required return exceeds the actual return it is optimal to exercise the option to develop. Equation (11.1) is nonnegative as realized returns cannot be consistently greater than required returns in equilibrium. Equation (11.2) is nonnegative, if negative it is optimal to exercise the option.

2.2 Operating Oil Sands Project

In subsection (2.1) we determine free boundary problem for the value of the option to develop and oil sands project and the optimal development threshold for a given value function for the operating oil sands project, $F(S, C, \bar{Q})$. Now we turn to the problem of valuing an operating oil sands project with the option to abandon for scrap value.

After exercising the option to develop, the firm receives an operating oil sands project with the option to abandon for scrap value. While it is operating, crude oil is extracted, transported, and then sold in a perfectly competitive market. The after-tax cash flows from operations, $\pi(q;S,C,Q)$, are affected by the amount of output sold, q, the current price and transportation cost, and the amount of reserves remaining. The payoff to the firm from the operating oil sands project is the cash flows from operations and the future value of the operating oil sands project. If the firm decides to exercise the option to abandon the firm gets the scrap value of the oil sands project, $\Omega(S,C,Q)$. Here, scrap value represents all the costs associated with abandoning the project and restoring the land to its previous state and is likely to be negative[14].

The firm's problem of valuing the operating oil sands project with the option to abandon for scrap value can be represented by the following optimal stopping problem

$$F(S,C,Q) = \max\left\{\Omega(S,C,Q),\right.$$
$$\left.\max_{q\in[\underline{q},\bar{q}]}\pi(q;S,C,Q)dt + \frac{\mathrm{E}_t[F(S+dS,C+dC,Q+dQ)]}{1+\delta_F dt}\right\}. \tag{12}$$

The value of an operating oil sands project is the larger of either exercising the option to abandon immediately or continuing to operate the project. Where δ_F is the risk-adjusted constant discount rate for the operating oil sands project. The firm chooses the flow of output overtime to maximize the expected discounted value of the operating oil sands project. Due to technological and capacity constraints management cannot produce output below $\underline{q}$ or above $\bar{q}$.

The optimal abandonment threshold defines a surface that divides the (S,C,Q)-space into two regions: the continuation region and the abandonment region. Let $S^*(C,Q)$ be the optimal abandonment threshold. The threshold specifies a price for a given Q and C where the payoff from abandonment equals the payoff from continuing operations. The continuation region lies above the surface, $S > S^*(C,Q)$. In this area it is optimal to continue operating the project. The abandonment region lies below the surface, $S < S^*(C,Q)$. In this area it is optimal to abandon the project for scrap value. When $S = S^*(C,Q)$, the continuation payoff is equal to the abandonment payoff.

[14]Scrap Value may be positive if the option to abandon is exercised before reserves are exhausted and the restored land has some value to other oil producers or another purposes.

In the continuation region the value of an operating oil sands project satisfies the following Bellman equation

$$\delta_F F = \max_{q \in [\underline{q}, \bar{q}]} \pi(q) + (1/dt)\mathrm{E}_t[dF]. \tag{13}$$

Similar to equation (4), the Bellman equation here requires the firm's payoff from operations to be equal to the required return from operations.

Let $q(t)$ represent the quantity of reserves extracted at a particular point in time so that changes is reserves are

$$dQ = -qdt. \tag{14}$$

Apply Ito's Lemma to $F(S, C, Q)$ and make the appropriate substitutions to get

$$\begin{aligned} dF =& (\mu S F_S + \kappa(\bar{C} - C)F_C - qF_Q + \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S F_{SC}))dt \\ &+ \sigma_S S F_S dW_S + \sigma_C F_C dW_C. \end{aligned} \tag{15}$$

Equation (15) is the stochastic differential equation for the operating oil sands project. Substitute (15) in to the Bellman equation (13) and pass through the expectations operator to obtain the following partial differential equation for the value of an operating oil sands project with the option to abandon in the continuation region,

$$\delta_F F = \max_{q \in [\underline{q}, \bar{q}]} \pi + \mu S F_S + \kappa(\bar{C} - C)F_C - qF_Q + \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S F_{SC}) \tag{16}$$

The optimal flow of output is determined by differentiating the right hand side of equation (17) with respect to q

$$\pi_q - F_Q.$$

π_q is the marginal cash flow and F_Q is the shadow price of producing an extra unit of output. Let $q^*(S, C, Q)$ be the optimal flow rate

$$q^* = \begin{cases} \underline{q} & \text{if } \pi_q(\underline{q}) < F_Q \\ q' & \text{if } \pi_q(q') = F_Q \\ \bar{q} & \text{if } \pi_q(\bar{q}) > F_Q \end{cases}$$

The firm will produce at the lower boundary if the shadow price exceeds the marginal cash flow at $\underline{q}$, the firm will produce at the upper boundary if the marginal cash flow exceeds the shadow price at $\bar{q}$, and the firm will produce at an interior point if marginal cash flow equals the shadow price at q'. At the optimal flow of output the partial differential equation becomes

$$\delta_F F = \pi(q^*) + \mu S F_S + \kappa(\bar{C} - C)F_C - q^* F_Q + \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S \sigma_C \rho_{S,C} S F_{SC}) \quad (17)$$

The partial differential equation is subject to the following boundary condition,

$$F(S, C, 0) = \Omega(S, C, 0). \quad (18)$$

When reserves are exhausted the value of an operating oil sands project is equal to the remaining scrap value of the project.

The optimal abandonment threshold is determined by the value-matching

$$F(S^*(C, Q), C, Q) = \Omega(S^*(C, Q), C, Q), \quad (19)$$

and smooth-pasting conditions

$$F_S(S^*(C, Q), C, Q) = \Omega_S(S^*(C, Q), C, Q), \quad (20.1)$$

$$F_C(S^*(C, Q), C, Q) = \Omega_C(S^*(C, Q), C, Q). \quad (20.2)$$

2.2.1 Operating Oil Sands Project as a Linear Complementarity Problem

Equation (17) and conditions (18), (19), and (20) define a free boundary problem that determines the value of an operating oil sands project and the optimal abandonment threshold. The free boundary problem for the operating oil sands project can be redefined as the following LCP

$$\begin{aligned}
&\delta_F F - \pi(q^*) - \mu S F_S - \kappa(\bar{C} - C)F_C + q^* F_Q \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC}) \geq 0, && (21.1)\\
&F - \Omega \geq 0, && (21.2)\\
&\big(\delta_F F - \pi(q^*) - \mu S F_S - \kappa(\bar{C} - C)F_C + q^* F_Q \\
&\qquad - \frac{1}{2}(\sigma_S^2 S^2 F_{SS} + \sigma_C^2 F_{CC} + 2\sigma_S\sigma_C\rho_{S,C} S F_{SC})\big) \times \big(F - \Omega\big) = 0. && (21.3)
\end{aligned}$$

Equation (21) has the same intuitive interpretation as equation (11).

3 Results

In this section we use the fully implicit finite difference method (IFDM) to approximate the value of an typical in situ oil sands project in Northern Alberta and determine the optimal development and abandonment thresholds. A detailed explanation of the IFDM can be found in appendix A.1. There are two main methods for recovering crude bitumen from the oil sands mixture, the choice of extraction method depends on the depth of the oil sands deposits. Open-pit mining is used to recover crude bitumen from shallow deposits while in situ methods are used to extract it from deep deposits[15]. We focus on a project that uses in situ methods in this paper for two reasons. First, production from in situ projects has exceed the production from mining projects since 2014 (CAPP, 2015). Second, approximately 80 percent of oil sands deposits are too deep to be recovered from open-pit mining and must be extracted using in situ methods.

[15]In situ methods involves drilling several wells into deep oil sands deposits then injecting steam to heat the bitumen so that it flows and can be pumped to the surface. The primary in situ methods used today are the thermal techniques of Cyclic Steam Stimulation (CSS) and Steam Assisted Gravity Drainage (SAGD).

Table 1: In situ Oil Sands Project Design Parameters

Option to Develop	
Length of Lease, years (T)	15
Investment Costs, millions of dollars (IC)	\$1050
Benefits(Costs) from lease (M)	0
Discount Rate (δ_G)	10%
Operating Project	
Production life, years	30
Initial Reserves, millions of barrels ($\bar{Q}$)	328.5
Annual Production, millions of barrels (q)	10.95
Average cost, per barrel (AC)	\$35.00
Scrap Value, (Ω)	0
Royalty Rate (λ_R)	30%
Income Tax Rate (λ_I)	40%
Property Tax Rate (λ_P)	10%
Discount Rate (δ_F)	10%

Table 1 summarizes the assumptions we make about a typical in situ oil sands project. The investment cost, initial reserves, and annual production are from Millington et al. (2014) who estimate the supply costs[16] for various oil sand projects based on their type. In 2014, a number of firms operating in situ projects reported average production costs ranging from \$25-\$49. Millington et al. (2014) estimate supply costs of \$50.89 (excluding transportation and blending costs). We assume the deflated average cost of producing a barrel of oil is constant and equal to \$35. In Alberta, the royalty rates applied to gross revenue and net revenue depend on the price of WTI. To simplify the analysis we assume a constant royalty rate of 30% applied to net revenue. Income tax rate includes both provincial and federal taxes. We assume that the discount rate for the option to develop and the operating project are both 10 per cent.

To estimate the parameters in equations (1) and (2) we collect monthly spot price data for WTI and WCS for the period January 2005 to December 2015. WTI data was collected from the EIA and WCS data was collected from Natural Resources Canada. The WTI price series were converted to Canadian dollars using Canada/U.S. exchange rates from the U.S. Federal Reserve and both price series were deflated using the CPI from Statistics

[16]Supply cost is the constant dollar price needed to recover all capital expenditures, operating costs, royalties, and taxes and earn a specified return on investment.

Canada. Transportation costs estimates were generated by subtracting WCS from WTI.

Table 2: Summary Statistics

	WTI	WCS	TC
Mean	71.94	58.52	13.42
St Dev.	14.32	14.65	5.64
Min	40.44	25.88	3.55
Max	117.03	104.32	37.28
Obs	132	132	132

Crude oil prices are log-normally distributed with a mean of $\mu - \sigma_S^2/2$ and a variance of σ_S^2. Following Wilmott et al. (1993), the mean and variance are estimated with the following equations

$$\hat{m} = \frac{1}{ndt} \sum_{t=1}^{n} \log(\mathrm{WTI}_t/\mathrm{WTI}_{t-1}),$$
$$\hat{\sigma}_S^2 = \frac{1}{(n-1)dt} \sum_{t=1}^{n} (\log(\mathrm{WTI}_t/\mathrm{WTI}_{t-1}) - \hat{m})^2.$$

The drift, $\hat{\mu}$, is recovered by adding $\hat{\sigma}_S^2/2$ to $\hat{m}$. For the selected data period, the average growth rate in WTI is 1 percent with a standard deviation of 28 percent. The parameters for equation (2) are estimated by running the regression

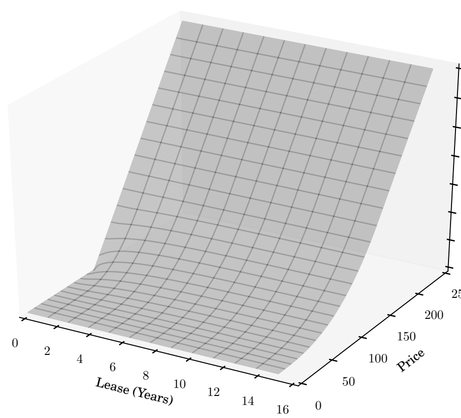
$$TC_t - TC_{t-1} = a + bTC_{t-1} + \epsilon_t$$

and then calculating

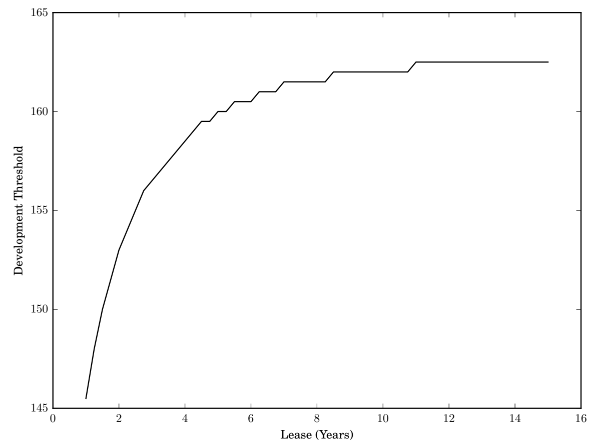
$$\hat{\kappa} = \frac{-\hat{a}}{\hat{b}},$$
$$\bar{C} = -\log(1 + \hat{b}),$$
$$\hat{\sigma}_C = \sigma_\epsilon \sqrt{\frac{\log(1 + \hat{b})}{(1 + \hat{b})^2 - 1}},$$

where $\hat{\sigma_\epsilon}$ is the standard error of the regression. Over this period the long run average transportation cost, $\bar{C}$, is \$13.38, the speed of reversion to the long run average, $\hat{\kappa}$, is 0.39, and the standard deviation, $\hat{\sigma_C}$, is \$3.53. The estimated correlation between oil prices and transportation costs, $\hat{\rho}_{S,C}$, is 0.14.

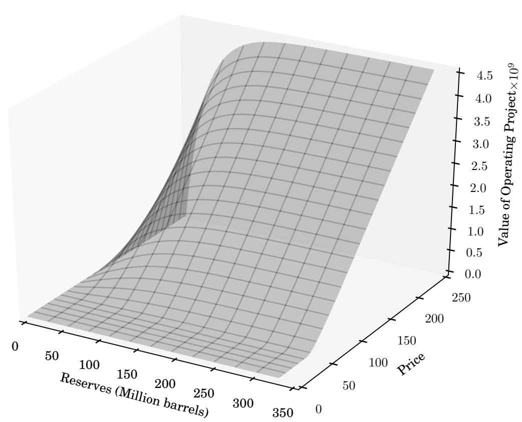
Figure 2: Value of an oil sands project that faces fixed transportation costs of \$13.38



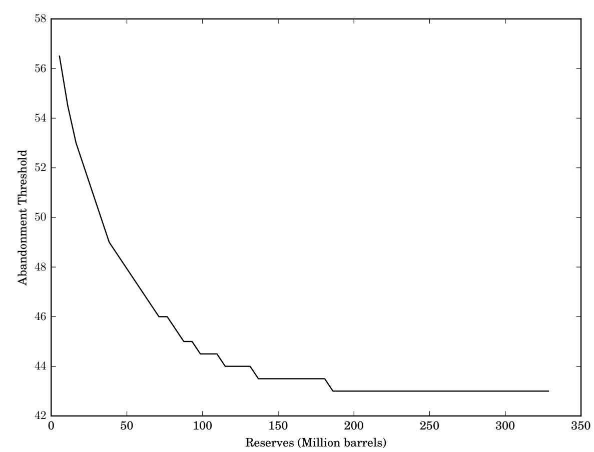
(a) Option to Develop an Oil sands Project



(b) Optimal Development Threshold



(c) Operating Oil sands Project with the Option to Abandon for Scrap Value



(d) Optimal Abandonment Threshold

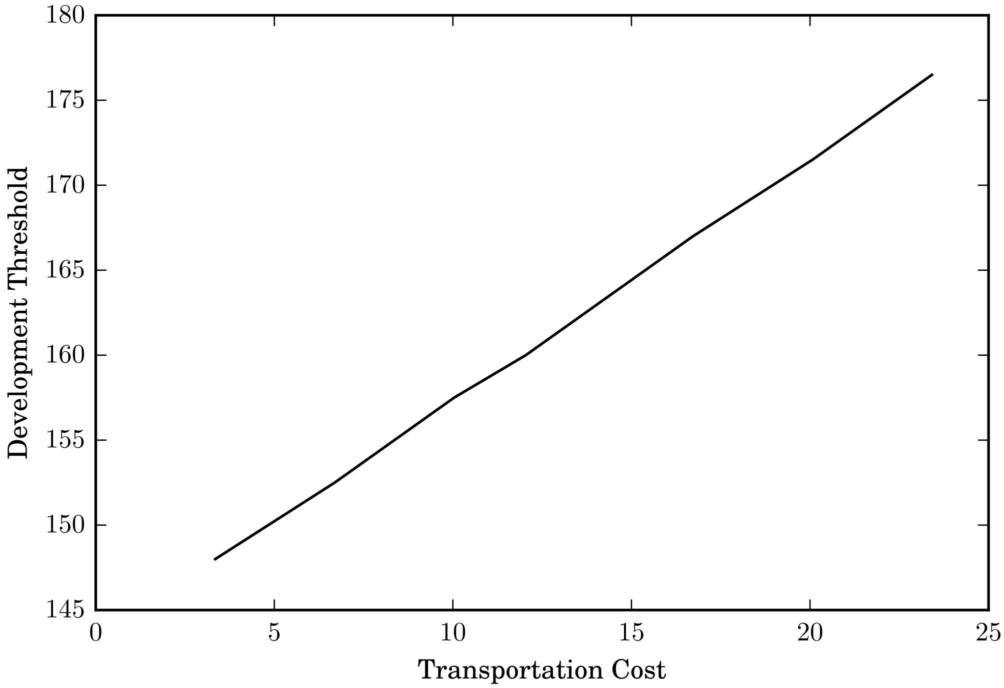
To understand the effect changes in transportation costs have on the value of an oil sands project and the incentive to invest we consider changes in the mean and variance of transportation costs. To focus on the first-order effects of a change in mean we set transportation cost variance equal to zero then solve the model for different mean values. To focus on the second-order effects of transportation cost uncertainty we set the current transportation cost equal to its mean value then solve the model for different standard deviation values.

Figure 2 plots the value of an oil sands project that faces fixed transportation costs of \$13.38 per barrel. In the development stage, the value of the option to develop (figure 2a) is increasing in both price and lease. When the lease expires if the price of oil is above \$101.50 the option is exercised and the project is developed, if not, the lease expires unused and the value of the project is zero. The optimal development threshold is shown in figure 2b. It exceeds the supply costs estimated by Millington et al. (2014) but is comparable to some of the results found by Kobari et al. (2014). Millington et al. (2014) estimate supply costs of US\$84.99 per barrel when adjusting for blending and transportation for a steam-assisted gravity drainage project[17]. In their increasing environmental cost scenario, Kobari et al. (2014) find critical thresholds ranging from \$50 to \$150 per barrel. In their decreasing environmental cost scenario, they find critical thresholds ranging from \$150 to \$300 per barrel.

In the operating stage, the value of the operating project (figure 2c) is increasing in both price and reserves. When reserves are exhausted the value of the project equals the project's scrap value, in this case zero. The optimal abandonment threshold is shown in figure 2d. Cash flows from operations range from -\$5 to -\$16 on the optimal abandonment threshold. In this example the project will have a negative net present value before it is abandoned because of the positive value of managerial flexibility. The abandonment threshold found in this example is similar to abandonment threshold found by Almansour and Insley (2016). In their paper, an oil sands project is closed when the price of bitumen is between US\$20 and US\$35 per barrel and a project is abandoned when the price of bitumen is between US\$10 and US\$20 per barrel. They assume the difference between

[17] They assume a fixed exchange rate of US\$0.98.

Figure 3: Optimal Development Threshold

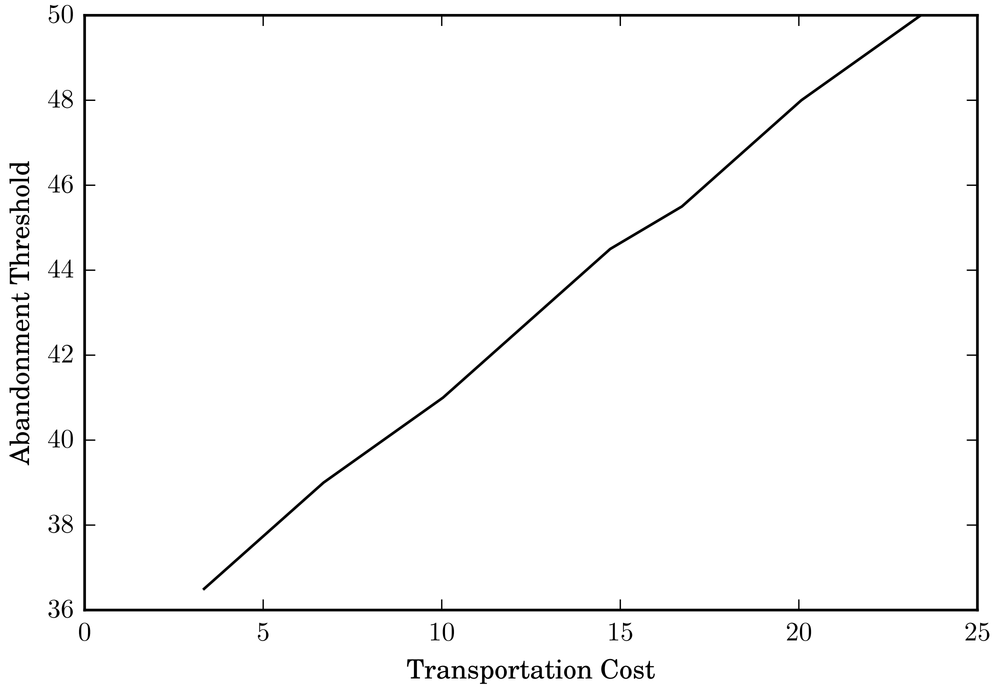


crude oil prices and crude bitumen prices is about US$30 per barrel. Adding the US$30 to their closure and abandonment results and they look similar to the abandonment threshold found here.

3.1 Effect of a Change in Transportation Cost Mean

Consider a change in transportation costs caused by a change in pipeline capacity. A decrease in transportation costs, resulting from an increase in pipeline capacity, will lead to an increase in the value of an oil sands project. The value of an operating project will increase as the expected present value of cash flows increase for all price levels. The value of the option to develop an oil sands project increases because the value of the operating project increases. Oil sands projects will be developed earlier (figure 3) because the value of the underlying asset has increased and the benefits from the undeveloped lease have remained the same. Operating projects will be abandoned later following an increase in pipeline capacity (figure 4) as cash flows increase and the scrap value of the project remains unchanged.

Figure 4: Optimal Abandonment Threshold



3.2 Effect of a Change in Transportation Cost Volatility

So far we have estimated the value of an oil sands project and the optimal development and abandonment thresholds when transportation costs are fixed and evaluated how these values change when transportation costs increase or decrease. We have seen that a decrease in transportation costs increases the value of an oil sands project and increases the incentive to invest in new projects. Now we turn our attention to the effect of transportation cost uncertainty on the value of a project. We solve the model for different transportation cost standard deviation levels while holding current transportation cost equal to its long-run average.

Table 3 presents results from solving the model with different transportation cost standard deviation levels. We can see that transportation cost uncertainty has a small positive impact on the value of the option to develop an oil sands project, a 10 percent increase in transportation cost standard deviation increases the value of the option to develop by 2 percent. The positive relationship between volatility and option value is consistent with option-pricing literature, an increase in volatility increases the upside potential of an option. The effect of transportation cost uncertainty on the value of the operating project is more complicated. Increasing the standard deviation from 0 to 3.53 reduces the value of the operating project but an increase in the standard deviation, when it is already positive, increases the value of the operating project. Although transportation cost uncertainty has a small effect on the value of an oil sands project, surprisingly, it has virtually no effect on the optimal development and abandonment thresholds. Increasing transportation cost standard deviation from zero to a positive value decreases the development boundary by $0.50 but large deviations from the estimated standard deviation has no effect on either of the optimal thresholds.

Table 3: Changes in Transportation Cost Volatility

	Transportation Cost Volatility, σ_C					
	0	2.65	3.18	3.53	3.88	4.41
Option to Develop	-0.07	-0.04	-0.02	0	0.02	0.04
Development threshold	162	161.50	161.50	161.50	161.50	161.50
Operating Project	0.003	-0.004	-0.002	0	0.002	0.005
Abandonment threshold	43	43	43	43	43	43

Option to Develop and Operating Project shows the average percent difference from the value of an oil sands project with transportation cost standard deviation of 3.53.
Lease is fixed at $\tau = 10$ and reserves are fixed at $Q = 219000000$.

The results presented here are similar to those from Schwartz and Smith (2000). The risks of short-term deviations in transportation costs has very little effect on the value of a oil sands project that has a operating life of 30 years. What matters here is the expected transportation cost over the life of a project. A reduction in expected transportation costs, resulting from an expansion in pipeline capacity, will increase the incentive to invest in new oil sands projects.

4 Conclusion

This paper examines the impact transportation costs have on the value of an oil sands project and the incentive to invest in new projects. A real options model for the valuation of an oil sands project located in Northern Alberta is developed that incorporates price and transportation cost uncertainty. The free-boundary problems that determines the value of the oil sands and the investment thresholds are defined as linear complementarity problems and numerically solved using the fully implicit finite difference method.

Results for the typical in situ oil sands project show that average transportation costs are an important factor in the decision whether to start a new project or not while transportation cost uncertainty has virtually no impact on the investment decision. Results indicate that the price differential faced by oil sands producers is an important factor restraining new investments. New pipeline projects that would reduce the price differential would increase the value of existing oil sands projects and would increase the incentive to invest in new projects.

We have seen that transportation cost uncertainty has very little impact on the value of an oil sands project and the incentive to invest. What matters here is the average transportation cost over the life of the oil sands project. In this paper we considered changes in the average transportation cost but ignored uncertainty in average transportation costs. Future research would incorporate this uncertainty into the value of an oil sands project by modeling average transportation costs as a Poisson process. At some future date transportation costs might jump up or down as a result of an decrease or increase in pipeline capacity.

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Appendices

A Numerical Methods

A.1 Fully Implicit Finite Difference Method

The fully implicit finite difference method (IFDM) is an established technique for numerically solving option pricing problems (Wilmott et al. (1993) and Zhu et al. (2004)) that involves discretizing the domain and replacing partial derivatives with backward difference and symmetric central difference approximations. A benefit of the IFDM is that it does not require step lengths in one direction on the domain to be proportionate to step lengths in another direction for stability or convergence. In this appendix we numerically approximate the value of an oil sands project using the IFDM. The following linear complementarity problems determine the value of the oil sands project. The option to develop an oil sands project is the solution equation (11) and the value of an operating oil sands project with the option to abandon is the solution to equation (21).

The value functions $G(S, C, \tau)$ and $F(S, C, Q)$ depend on three state variables. To simplify the numerical scheme and reduce the dimensionality of the domain, let $P = S - C$ so that $g(P, \tau) = G(S, C, \tau)$, and $f(P, Q) = F(S, C, Q)$. The partial derivatives in equation (11) can be replaced with,

$$\begin{aligned} G_S &= g_P, & G_{SS} &= g_{PP}, \\ G_C &= -g_P, & G_{CC} &= g_{PP}, \\ G_\tau &= g_\tau, & G_{SC} &= -g_{PP}. \end{aligned}$$

Similarly for equation (21)

$$\begin{aligned} F_S &= f_P, & F_{SS} &= f_{PP}, \\ F_C &= -f_P, & F_{CC} &= f_{PP}, \\ F_Q &= f_Q, & F_{SC} &= -f_{PP}. \end{aligned}$$

Substitution and rearrange to get simplified LCPs for the option to develop

$$\begin{aligned} &\delta_G g - \mathrm{M} - (\mu(P+C) - \kappa(\bar{C} - C))g_P + g_\tau \\ &\qquad - \frac{1}{2}(\sigma_S^2 (P+C)^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}(P+C)g_{PP} \geq 0, \end{aligned} \tag{22.1}$$

$$g - f + \mathrm{IC} \geq 0, \tag{22.2}$$

$$\begin{aligned} &\big(\delta_G g - \mathrm{M} - (\mu(P+C) - \kappa(\bar{C} - C))g_P + g_\tau \\ &\qquad - \frac{1}{2}(\sigma_S^2 (P+C)^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}S)g_{PP}\big) \times \big(g - f + \mathrm{IC}\big) = 0. \end{aligned} \tag{22.3}$$

and the simplified LCP for the operating project

$$\delta_F f - \pi(q^*) - (\mu(P+C) - \kappa(\bar{C}-C))f_P + q^* f_Q \\ \quad - \frac{1}{2}(\sigma_S^2(P+C)^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}(P+C))f_{PP} \geq 0, \tag{23.1}$$

$$f - \Omega \geq 0, \tag{23.2}$$

$$\big(\delta_F f - \pi(q^*) - (\mu S - \kappa(\bar{C}-C))f_P + q^* f_Q \\ \quad - \frac{1}{2}(\sigma_S^2(P+C)^2 + \sigma_C^2 - 2\sigma_S\sigma_C\rho_{S,C}(P+C))f_{PP}\big) \times \big(f - \Omega\big) = 0. \tag{23.3}$$

Define on the axes for S, τ, and Q by

$$\begin{aligned} &\{0, S_1, \ldots, S_i, \ldots, S_M\}, \\ &\{0, \tau_1, \ldots, \tau_n, \ldots, \tau_N\}, \\ &\{0, Q_1, \ldots, Q_j, \ldots, Q_K\}. \end{aligned} \tag{24}$$

For a given value of C, a typical grid point $(S_i - C, \tau_n)$ on the discretized $(S-C) \times \tau$ mesh, the value of the option to develop is $g(S_i - C, \tau_n) = g_i^n$. For a typical grid point $(S_i - C, Q_j)$ on the discretized $(S-C) \times Q$ mesh, the value of the operating project is $f(S_i - C, Q_j) = f_i^j$.

The IFDM involves using backward difference approximation for g_τ and f_Q and symmetric central difference approximation for the terms g_P, g_{PP}, f_P and f_{PP}. The backward difference and symmetric central difference equations can be written

$$\begin{array}{ll} g_\tau = \frac{g_i^{n+1} - g_i^n}{\Delta\tau} + O(\Delta\tau) & f_Q = \frac{f_i^{j+1} - f_i^j}{\Delta Q} + O(\Delta Q) \\ g_P = \frac{g_{i+1}^{n+1} - g_{i-1}^{n+1}}{2\Delta P} + O(\Delta P^2) & f_P = \frac{f_{i+1}^{j+1} - f_{i-1}^{j+1}}{2\Delta P} + O(\Delta P^2) \\ g_{PP} = \frac{g_{i+1}^{n+1} - 2g_i^{n+1} + g_{i-1}^{n+1}}{\Delta P^2} + O(\Delta P^2) & f_{SS} = \frac{f_{i+1}^{j+1} - 2f_i^{j+1} + f_{i-1}^{j+1}}{\Delta P^2} + O(\Delta P^2) \end{array} \tag{25}$$

where ΔP is the constant step length in the P direction[18], $\Delta\tau$ is the constant step length in the τ direction, and ΔQ is the constant step length in the Q direction.

[18]Here the step length $\Delta P = \Delta S$ because $P = S - C$.

Assume the flow of benefits (costs) from an undeveloped oil sands lease is

$$\mathrm{M} - \lambda_P g(P, \tau) \tag{26}$$

and the cash flow from operations are

$$\begin{aligned}\pi(q^*; S-C, Q) = &\big((1-\lambda_R)(S-C) - AC\big)q^* + \max\{\lambda_I[\big((1-\lambda_R)(S-C) - AC\big)q^*], 0\}\\ &- \lambda_P f((S-C), Q).\end{aligned} \tag{27}$$

Regardless of whether the project has been developed on not, property tax rates, λ_P, are applied to the value of the oil sands project. When the project has been developed, royalty rates, λ_R, are applied to net revenue and income tax rates, λ_I, are applied to profits net royalty payments. The output flow, q^* and the average cost of producing a barrel of oil, AC, are assumed to be constant over the life of the project.

Using the finite difference equations defined in (25) and equation (26), the discretized LCP for the option to develop at an interior node is

$$-\Delta\tau a_i g_{i-1}^{n+1} + (1 + \Delta\tau(\delta_G + \lambda_P + a_i + b_i))g_i^{n+1} - \Delta\tau b_i g_{i+1}^{n+1} - g_i^n - \Delta\tau \mathrm{M} \geq 0 \tag{28.1}$$

$$g_i^{n+1} - f_i^N + \mathrm{IC} \geq 0 \tag{28.2}$$

$$\begin{aligned}&\big(-\Delta\tau a_i g_{i-1}^{n+1} + (1 + \Delta\tau(\delta_G + \lambda_P + a_i + b_i))g_i^{n+1} - \Delta\tau b_i g_{i+1}^{n+1} - g_i^n - \Delta\tau \mathrm{M}\big)\\ &\quad \times (g_i^{n+1} - f_i^N + \mathrm{IC}) = 0.\end{aligned} \tag{28.3}$$

With equation (27), the discretized LCP for the operating project at an interior node is

$$
\begin{aligned}
& -\Delta Q a_i f_{i-1}^{j+1} + (q + \Delta Q(\delta_F + \lambda_P + a_i + b_i)) f_i^{j+1} - \Delta Q b_i f_{i+1}^{j+1} - q f_i^j \\
& \qquad - \Delta Q(((1-\lambda_R)P_i - AC)q - \max\{\lambda_I[((1-\lambda_R)P_i - AC)q], 0\}) \geq 0 && (29.1) \\
& f_i^{j+1} - \Omega \geq 0 && (29.2) \\
& \big(-\Delta Q a_i f_{i-1}^{j+1} + (q + \Delta Q(\delta_F + \lambda_P + a_i + b_i)) f_i^{j+1} - \Delta Q b_i f_{i+1}^{j+1} - q f_i^j \\
& \qquad - \Delta Q(((1-\lambda_R)P_i - AC)q - \max\{\lambda_I[((1-\lambda_R)P_i - AC)q], 0\}) \big) \\
& \qquad \times \big(f_i^{j+1} - \Omega \big) = 0 && (29.3)
\end{aligned}
$$

Where

$$
\begin{aligned}
a_i &= \frac{\sigma_S^2 S_i^2 + \sigma_C^2 - 2\sigma_S \sigma_C \rho_{S,C} S_i}{2\Delta P^2} - \frac{\mu S_i - \kappa(\bar{C} - C)}{2\Delta P}, && (30.1) \\
b_i &= \frac{\sigma_S^2 S_i^2 + \sigma_C^2 - 2\sigma_S \sigma_C \rho_{S,C} S_i}{2\Delta P^2} + \frac{\mu S_i - \kappa(\bar{C} - C)}{2\Delta P}. && (30.2)
\end{aligned}
$$

To implement the IFDM we need to impose the following boundary conditions on the value of the option to develop an oil sands project,

$$
\begin{aligned}
g(-C, \tau) &= \mathrm{M}\Delta t + \frac{\mathrm{E}_t[g(-C, \tau + d\tau)]}{1 + \delta_G dt} && (31.1) \\
\lim_{S \to \infty} g(S - C, \tau) &= \lim_{S \to \infty} f(S - C, Q) - \mathrm{IC}, && (31.2)
\end{aligned}
$$

When price goes to zero, The likelihood of development gets very small and the value of the option to develop approaches the present discounted value of benefits (costs) from the undeveloped land. When the price gets very large, the option to develop will be exercised immediately as the benefits from immediate development outweigh the costs. From these assumptions we get the following boundary conditions for the discrete LCP

$$
\begin{aligned}
g_0^n &= \frac{(1+\delta_G)(1+\lambda_P) - [(1+\delta_G)(1+\lambda_P)]^{n-1}}{(1+\delta_G)(1+\lambda_P) - 1} \frac{\mathrm{M}}{1+\lambda_P} + \frac{g_0^0}{[(1+\lambda_P)(1+\delta_G)]^n}, && (32.1) \\
g_M^n &= f_M^K - \mathrm{IC}. && (32.2)
\end{aligned}
$$

The boundary conditions for the operating oil sands project are

$$f(-C, Q) = \Omega \tag{33.1}$$
$$\lim_{S\to\infty} f(S-C, Q) = \lim_{S\to\infty} \pi(\bar{q}; S-C, Q)dt + \frac{\mathrm{E}_t[F(S-C, Q+dQ)]}{1+\delta_F \Delta t}. \tag{33.2}$$

When the prices goes to zero, the option to abandon will be exercised immediately. When the price gets very large, the value of an operating project approaches the present discounted value of cash flows from operation and the value of the option to abandon goes to zero. This happens because the likelihood of exercising the abandonment option is very small when the price is very large. From these assumptions we get the following boundary conditions for the discrete LCP

$$f_0^j = \Omega \tag{34.1}$$
$$\begin{aligned} f_M^j =& \frac{(1+\lambda_P)(1+\delta_F) - [(1+\lambda_P)(1+\delta_F)]^{n-1}}{(1+\lambda_P)(1+\delta_F)-1} \frac{\big((1-\lambda_I)(1-\lambda_R)(S_M - C) - AC\big)q}{1+\lambda_P} \\ &+ \frac{f_M^0}{[(1+\lambda_P)(1+\delta_F)]^n}. \end{aligned} \tag{34.2}$$

For a given value of τ_n, equation (22) can be arranged from S_1 to S_{M-1} to form the following system of equations

$$Ag^{n+1} - g^n - \Delta\tau \mathrm{M}^n \geq 0 \tag{35.1}$$
$$g^{n+1} - (f^K - \mathrm{IC}) \geq 0 \tag{35.2}$$
$$\langle Ag^{n+1} - g^n - \Delta\tau \mathrm{M}^n, g^{n+1} - (f^K - \mathrm{IC})\rangle = 0 \tag{35.3}$$

Where A is a $M-1 \times M-1$ tridiagonal positive semi-definite matrix[19], with diagonal terms $A_{i,i} = 1 + \Delta\tau(\delta_G + \lambda_P + a_i + b_i)$ and off diagonal terms $A_{i,i-1} = -\Delta\tau a_i$ and $A_{i,i+1} = -\Delta\tau b_i$.

[19] A is a strictly diagonally dominant matrix.

g^{n+1} is an $M-1$ vector of unknown values, g^n and M^n are $M-1$ vectors of known values.

$$g^{n+1} = \begin{pmatrix} g_1^{n+1} \\ \vdots \\ g_{M-1}^{n+1} \end{pmatrix}, \quad g^n = \begin{pmatrix} g_1^n \\ \vdots \\ g_{M-1}^n \end{pmatrix}, \quad \mathrm{M}^{n+1} = \begin{pmatrix} (1+a_1)\mathrm{M} + a_1 \frac{1-\lambda_P-\lambda_P\delta_G}{1+\delta_G} g_0^n \\ \mathrm{M} \\ \vdots \\ \mathrm{M} + b_{M-1}(f_M^K - IC) \end{pmatrix}.$$

The terminal condition specifies the value of the option to delay development of an oil sands project when the lease has expired. Using this and the information given by equations (31.1) and (31.2) the value of the option to develop an oil sands project can be approximated at all other nodes in the domain. The optimal stopping boundary $\hat{S}_i(C,\tau_n)$ is recovered using equation (35.2). For a given τ_n, the smallest indexed price S_i where $g_i^n = f_i^K - \mathrm{IC}$ is the price where it is optimal to exercise the option to develop.

For a given value of Q_j, equation (23) can be arranged to form the following system of equations

$$Bf^{j+1} - f^j - \Delta Q \Pi^j \geq 0 \tag{36.1}$$

$$f^{j+1} - \Omega \geq 0 \tag{36.2}$$

$$\langle Bf^{j+1} - f^j - \Delta Q \Pi^j, f^{j+1} - \Omega) \rangle = 0 \tag{36.3}$$

B is a $M-1 \times M-1$ tridiagonal positive semi-definite matrix, with diagonal elements $B_{i,i} = q + \Delta Q(\delta_F + \lambda_P + a_i + b_i)$ and off diagonal elements $B_{i,i-1} = -\Delta Q a_i$ and $B_{i,i+1} = -\Delta Q b_i$. f^{j+1} is a $M-1$ vector of unknown values, f^j and Π^j are $M-1$ vectors of known values. With

$$f^{j+1} = \begin{pmatrix} f_1^{j+1} \\ \vdots \\ f_{M-1}^{j+1} \end{pmatrix}, \quad f^j = \begin{pmatrix} f_1^j \\ \vdots \\ f_{M-1}^j \end{pmatrix},$$

$$\Pi^j = \begin{pmatrix} ((1-\lambda_R)((S_1 - C) - AC)q + a_1\Omega \\ ((1-\lambda_R)(S_2 - C) - AC)q - \max\{\lambda_I[(1-\lambda_R)(S_2 - C) - AC]q, 0\} \\ \vdots \\ (1-\lambda_I)((1-\lambda_R)(S_{M-1} - C) - AC)q + b_{M-1}f_M^{j+1} \end{pmatrix},$$

and

$$f_M^{j+1} = \frac{(1-\lambda_I)((1-\lambda_R)(S_M - C) - AC)q}{1+\lambda_P} + \frac{f_M^j}{(1+\delta_F)(1+\lambda_P)}.$$

The terminal condition specifies the value of the operating oil sands project when reserves are exhausted. Using this condition and the conditions given by equations (33.1) and (33.2) the value of the of an operating oil sands project with an option to abandon can be approximated on all nodes in the domain. The optimal stopping boundary $S_i^*(C, Q_j)$ is recovered using equation (36.2). For any Q_j, the highest indexed price S_i where $f_i^j = \Omega$ is the price where it is optimal to exercise the option to abandon for scrap value.

A.2 Pseudo Code

We use the python package OpenOpt to numerically solve equations (35) and (36). OpenOpt is a package designed to numerically solve complementarity problems. To employ OpenOpt, LCPs must be written in the following form

$$w = Mz + q,$$
$$w \geq 0, z \geq 0, \text{ and } w^T z = 0,$$
$$\text{with } M \text{ and } q \text{ given.}$$

For the option to develop, let $w_g \equiv Ag^{n+1} - g^n - \Delta\tau M^n$ and $z_g \equiv g^{n+1} - f^K + \text{IC}$. Then equation (22.3) can be written

$$w_g = Az_g + A(f^K - IC) - (g^n + \Delta\tau \mathrm{M}^{n+1}),$$

with the conditions $w_g \geq 0$, $z_g \geq 0$, and $w_g^T z_g = 0$ where A and $A(f^K - IC) - (g^n + \Delta\tau \mathrm{M}^{n+1})$ are given. Similarly for the operating project we get

$$w_f = Bz_f + B\Omega - (f^j + \Delta Q \Pi^{j+1})$$

with $w_f \geq 0$, $z_f \geq 0$, and $w_f^T z_f = 0$ where B and $B\Omega - (f^j + \Delta Q \Pi^{j+1})$ are given.

When an element of w_{gi} is equal to zero the option to develop is in the continuation region and it is optimal to continue to hold the option and delay development. The value of the option to develop is

$$g_i^{n+1} = [A^{-1}(g^n + \Delta\tau \mathrm{M}^{n+1})]_i.$$

When an element of z_{gi} is equal to zero the option to develop is in the development region and it is optimal to exercise the option to develop. The value of the option to develop is

$$g_i^{n+1} = f_i^K - \mathrm{IC}.$$

Similarly for the operating project, if w_{fi} is equal to zero the value of the project is

$$f_i^{j+1} = [B^{-1}(f^j + \Delta Q \Pi^{j+1})]_i.$$

When z_{fi} is equal to zero the value of the project is

$$f_i^{j+1} = \Omega.$$

The option to develop depends on the value of the operating project. We start by solving for the value of the operating project with the option to abandon for scrap value. Iterating over reserves from reserve exhaustion, $j = 0$, to initial reserves, $j = K - 1$. Then we solve the option to develop an oil sands project using the solution to the value of the operating project at initial reserves. Iterate over time remaining on lease from expiration, $\tau = 0$, to initial day of lease, $\tau = N - 1$.