Dynamic Competition in Electricity Markets under Uncertainty

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Abstract

We study discrete-time infinite-horizon imperfect competition between asymmetric firms producing from different technologies. Specifically, one firm produces from hydroelectric units and the others operate thermal generators. This type of structure is common in some electricity markets. What makes this research interesting is that firms have different types of constraints, face different kinds of uncertainties, need to allocate their resources over time, and yet produce strategically. For the renewable energy holder, the key issue is how to allocate water between current and future electricity generation given the thermal firms' strategic actions along with demand and/or water inflow uncertainties. We analyze equilibrium outcomes (e.g., the price distribution) and market inefficiencies stemming from both production constraints and imperfect competition. We show that equilibrium price volatility and skewness are generally lower than optimal, although average price is higher than optimal. The hydro producer under-utilizes the available water, which leads to more water being available to smooth price fluctuations. However, in the extreme case of water inflows so plentiful that the hydro firm is never constrained, prices can be more volatile than optimal. We also demonstrate that the lack of social optimality of the market outcome is tempered by the capacity constraints: the welfare loss under the oligopoly market structure is much less than would occur in the absence of water and capacity constraints. These results are demonstrated using numerical simulations of the infinite horizon game, one of which is calibrated to match the characteristics of the Ontario wholesale electricity market.

Keywords: Electricity markets; Price volatility; Hydroelectric generation; Imperfect competition; Dynamic Games; Chebyshev polynomials.

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1 Introduction

A common feature of many electricity markets is the coexistence of a variety of generation technologies, such as hydro, nuclear and thermal (coal, oil, gas) generation. In some jurisdictions, hydroelectric power generation is the dominant source of electricity. It accounts for 80% of generation in New Zealand, 77% in Brazil, 90% in Quebec, and 98% in Norway (IEA, 2013). In other jurisdictions, such as Ontario and the Western United States, it is a significant source of electricity, but not as dominant. It is not uncommon to observe large hydro producers competing with thermal generators. For example, in Honduras, large state-owned hydro generation facilities coexist with privately owned thermal generators.[1] Colombia has a similar structure, a large hydro operator with 64% of the installed capacity coexists with a thermal production sector with the rest of capacity.[2]

We investigate the implications of the coexistence of hydro and thermal generation technologies for price and water dynamics over a long horizon when the producers are imperfectly competitive. One characteristic of hydroelectric power generation that makes this situation particularly interesting is that it is constrained by the dynamics of water availability. For hydro generators that use a reservoir to store water, generating more electricity in one period reduces the available water in the next period, and hence constrains generation at that time. In addition, these alternative generation technologies have rather different cost structures. Hydroelectric generation can be characterized by low marginal cost when operating, but subject to the availability of water to drive the turbines. In contrast, thermal generation units have more flexibility in the sense that their inputs (gas, coal, etc.) are not subject to the same constraints as water in a reservoir, however the marginal cost of generation is higher as generators need to purchase the fuel inputs.

The dynamic management of hydroelectric facilities takes on added importance when we consider that both the flow of water and the state of demand can fluctuate randomly. A common feature of restructured electricity markets is price volatility, due in large part to the difficulty in storing electricity for the purpose of smoothing price fluctuations. However, the ability to store water behind a hydro dam does allow for some degree of price smoothing. A hydro operator may benefit from withholding water in periods with low prices in order to have more available for use in periods with high prices. In a perfectly competitive market, it is likely that the hydro operators would choose their

[1] Installed generation capacities are approximately two-thirds hydro and one-third thermal in Honduras (see ENEE at www.enee.hn).

[2] See www.creg.gov.co.

water release in a socially optimal way.[3] However, in most jurisdictions, hydroelectric generators tend to be rather large producers, in which case there is no guarantee that water will be released optimally in an unregulated environment. We investigate this issue by analyzing a dynamic game between a hydro generator and a thermal generator in a fully unregulated environment. Comparing the equilibrium of this game to the socially optimal outcome allows us to determine the potential for sub-optimal water use in an imperfectly competitive market for electricity. We find that if water inflows are not too low, the hydro generator uses less water than optimal due to its exercise of market power. However, this means that more water is generally available to the hydro generator to use in reaction to demand uncertainty. Consequently, prices are both less volatile and less skewed in the market equilibrium than is efficient. Prices are higher due to market power, but less volatile and less skewed due to increased water availability under imperfect competition compared to social optimum.

The use of hydro generation to improve the performance of electricity markets is an increasingly important topic as the share of intermittent supply from renewable generation technologies has increased. Many countries have issued and implemented Renewable Energy Laws or Green Energy Acts so as to produce clean electricity, reduce air emissions, and diversify generation portfolios.[4] In many jurisdictions, pumped-storage is used to move electricity generation from off-peak to peak periods. Crampes and Moreaux (2010) determine the conditions for optimal pumped storage use when demand varies over two periods: thermal generation is diverted in the off-peak period to pump water from a low reservoir to a higher one to be used during the peak period. Steffen and Weber (2013) extend this analysis to incorporate renewable generation as well. In contrast to this work, rather than focusing on optimal or efficient use of hydro generation, we examine the equilibrium use of hydro generation in a decentralized setting and contrast the outcome with the optimal solution.[5]

The fact that hydro producers have relatively large market shares in many jurisdictions has led to work that examines the issue of the use of market power by hydro producers. Imperfect competition among hydro producers only has been examined by Scott and Read (1996), Garcia et al (2001), Ambec and Doucet (2003) and Hansen (2009). Scott and Read (1996) examines the behavior of

[3]See Evans et al (2013) for an analysis of a perfectly competitive mixed thermal/hydro electricity market.

[4]Genc and Reynolds (2019) examine implications of wind generation expansion and of Green Energy Act in Ontario market. Ambec and Crampes (2012) examine the interaction between reliable and intermittent generators.

[5]Another difference with our approach is the treatment of natural water inflows. Since pumped storage uses no exogenous inflow of water, the question in that literature is essentially how to move thermal generation across periods. In contrast, by allowing natural inflows, hydro generation does not solely rely on previous thermal generation in our model.

imperfectly competitive hydroelectric producers, but does not consider the dynamic aspects of the strategic behavior of hydro producers. Garcia et al (2001) examine a strategic pricing game between two hydro producers who have a capacity constraint on their reservoirs, demonstrating that the Bertrand paradox of marginal cost pricing is mitigated as firms incorporate the opportunity cost of using water today rather than in a future period. Ambec and Doucet (2003) examine the effects of decentralizing production in a system with only hydro generation in which hydro producers share water resources. They show that there is a trade-off between inefficiency due to market power and inefficiency due to the absence of a market for water. Hansen (2009) analyses a two period model of imperfectly competitive hydro generators, focusing on how the shape of firms' residual demand interacts with uncertain water inflows to generate departures from efficient pricing that do not occur when inflows are deterministic.

A number of papers examine imperfect competition when there are mixed hydro and thermal generation technologies. Bushnell (2003) examines a Cournot oligopoly with fringe producers in which each producer controls both hydro and thermal generation facilities. Both hydro and thermal units face capacity constraints and the producers must decide how to allocate a fixed quantity of water over a number of periods. He solves the model with parameters calibrated to the western United States electricity market and finds that the dynamic allocation of water under imperfectly competitive conditions is not the efficient one. In particular, firms tend to allocate more water to off-peak periods than is efficient. Crampes and Moreaux (2001) model a Cournot duopoly in which a hydro producer uses a fixed stock of water over two periods while facing competition from a thermal producer. They find that hydro production is tilted towards the second period in the closed-loop equilibrium relative to the open-loop,[6] hence there is strategic withholding of water in the first period by the hydro producer. Our model differs from these two in a couple of ways. In both Bushnell (2003) and Crampes and Moreaux (2001), a fixed stock of water is allocated across a finite number of periods as might be the case in a "within–year" model with wet periods followed by dry periods in which hydro reservoirs are exhausted.[7] In contrast, we examine an infinite horizon setting with

[6]Closed-loop strategies depend on the current state as well as calendar time, while open-loop strategies depend on calendar time only. In a closed-loop equilibrium players can manipulate the future state to influence rivals' future actions, whereas this type of behavior is not possible in an open-loop equilibrium. Consequently, comparing the two types of equilibrium provides a measure of the importance of strategic behavior.

[7]Haddad (2011) studies a hydro management problem for a monopoly over two periods who faces alternating low and high water inflows and needs to allocate the available water over the two periods. However, his main focus is on determining the optimal reservoir size.

regular water inflows, which can be viewed as modeling competition at a lower frequency, or as a situation in which there is not a clear dry season during which reservoirs are exhausted. We allow for stochastic inflows, so there are "wet" and "dry" periods in our model, but these occur randomly. By also allowing for stochastic demand and a longer time horizon we are able to examine the implications of market power and water storage for the distribution of electricity prices.

More recently there has been work on the environmental implications of electricity markets with mixed generation technologies. De Villemeur and Pineau (2016) study two different power producing regions: a hydro jurisdiction and a thermal one. They specifically examine a likely integration of these two markets using 2007 data so as to understand how electricity trade would impact social welfare and Green-house Gas emissions in both markets. They find that market integration can substantially benefit the environment as well as total welfare. Genc and Reynolds (2019) investigate market implications of ownership of a new low-cost production technology in a mix of hydro and thermal electricity market. They show that ownership of renewable capacity will matter when there is market power in energy market. They apply their theoretical setting to the Ontario wholesale electricity market to analyze the impact of different ownership structures for wind capacity expansions. Graf and Marcantonini (2017) study the impact of renewable energy on the efficiency of thermal generation in the Italian electricity market. They show that while renewables (solar and wind) displace thermal generators and reduce the total CO2 emissions, average plant emissions relative to output have increased due to inefficient use of thermal plants responding to intermittent renewable generation.

The main novelties of this paper are as follows: i) We formulate an infinite horizon dynamic game-theoretic model involving imperfect competition between hydro and thermal firms taking into account of production capacity constraints (reservoir capacity for hydro and generation capacity for thermal) and uncertain demand and water flow. To our knowledge, this is the first paper in literature combining a long-horizon dynamic imperfect competition along with capacity constraints and uncertain demand and water inflow in an electricity market context. The previous papers have ignored either long-horizon dynamic analysis and/or the imperfect nature of competition and/or capacity constraints and/or uncertainties; ii) We develop numerical solution algorithms to find Markov perfect equilibrium outcomes in electricity markets using collocation methods; iii) In addition to simulations, we apply our dynamic competition setting to Ontario wholesale electricity market to investigate the role of hydro reservoir capacity on market outcomes; iv) To measure inefficiencies stemming from market power and capacity constraints, we compare equilibrium outcomes to the social optimum; v) In contrast to the literature, we examine wholesale price volatility in connection with energy storage and market power in a long-horizon setting.[8]

Determining the equilibrium strategy for hydro generators with exogenous water inflows is a daunting task in a long-horizon model due to the inter-temporal constraints on hydro production, even in the absence of uncertainty. Borenstein and Bushnell (1999) acknowledge the difficulty of computing the equilibrium pattern of hydroelectric generation and instead implement a "peak-shaving" method in which hydro production is allocated to periods of peak demand. While tractable, this approach may understate or overstate the degree of market power depending on how the hydro producer's marginal revenue varies with demand.[9] Instead, we determine the equilibrium allocation of water by a profit-maximizing hydro producer which we can then compare to the optimal allocation.

To examine the question of how the distribution of equilibrium prices is affected by imperfect competition in a mixed hydro/thermal generation context, we develop a model of dynamic quantity competition between a hydro and a thermal generator using a stochastic, dynamic game over an infinite time horizon. The hydro generator is constrained by water availability and reservoir capacity and the thermal generator is constrained by its production capacity. We compute both the Feedback (or Markov perfect) equilibrium as well as the socially optimal solution of the model using a numerical approximation of the value function for the hydro producer and social planner. We demonstrate that the hydro producer engages in strategic withholding of water by comparing the Feedback equilibrium to the myopic Cournot equilibrium. In addition, we compute the socially optimal solution to measure the degree of inefficiency stemming from market power. Our simulations show that, conditional on thermal capacity and water inflow, the equilibrium outcome can be close to the social optimum. This result is interesting in the light of empirical work of Kauppi and Liski (2008) who find only small welfare losses in the Nordic power market even though there is evidence of substantial market power held by hydro producers. In addition, we analyze the higher moments of distribution of equilibrium prices and show that they are different than the ones under the social optimum. Specifically, sub-optimal use of water in equilibrium results in smoother prices than is optimal.

[8]It is common in literature to estimate electricity prices and their volatility using reduced-form statistical models (e.g., Garcia-Martos et al. (2011), Vehvilainen and Pyykkonen (2005). In a recent paper, Cardella et al (2017) investigate price volatility over retail electricity plans (green energy plan based on wind/solar versus conventional plan based on fossil fuel) using an experimental approach.

[9]See Borenstein and Bushnell (1999, pg. 301).

We next turn to a description of the basic aspects of the model for both the non-cooperative game and the social planner's problem. In the third section, we analyze how the level of average mean water inflow affects the equilibrium of the game and the efficient outcome. We then analyze price volatility followed by the effects of thermal capacity. In addition, we calibrate the model's parameters to represent characteristics of the Ontario wholesale electricity market. Finally, we carry out ex-post analysis using actual (hourly generation, demand, export, import, and temperature) data to assess the impact of evolution of Ontario market (deviation from fossil-based to extensive renewable capacity). Specifically, this analysis intends to interlink simulation results to actual Ontario market outcomes.

2 The model

There are two types of technologies used in the industry: a hydroelectric generator uses water held behind dams to generate electricity and N thermal generators use thermal units that burn fossil fuel.[10] Electricity generation for thermal producer i in period t is denoted q_{it} and total thermal generation is $Q_t = \sum_{i=1}^{N} q_{it}$. Thermal generation costs are quadratic in production and each thermal generator has the same costs, $C_i(q_{it}) = c_1 q_{it} + (c_2/2){q_{it}}^2$, $i = 1, ...N$. Thermal generation is also subject to a capacity constraint, $q_{it} \leq K/N$, $i = 1, ...N$, where K represents the aggregate capacity of thermal generation. This results in a linear marginal cost up to capacity which is a commonly used functional form for modeling thermal generation marginal cost.[11]

Assuming that the hydro producer does not have to pay for the water it uses, it has a zero marginal cost of production and we denote the hydro producers profit in period t as $\pi^h(\alpha_t, h_t, Q_t) = (\alpha_t - \beta(h_t + Q_t))h_t$. The hydro producer's reservoir has a capacity denoted W_{max}, and its electricity generation, h_t, is determined by a one-to-one relation with the amount of water it releases from its reservoir.[12] Hydro generation is constrained by the amount of water available for release in the reservoir, W_t. Any water inflow that would result in a water level in excess of W_{max} is spilled at no

[10] Allowing for more than one hydro generation increases the computational complexity of the model substantially as it multiplies the number of state variables. To avoid this "curse of dimensionality" we focus on a single hydro generator only. We expect that additional hydro generates would result in less market power than in the case we present.

[11] Green and Newbery (1992) and Genc and Aydemir (2017) used a similar form for marginal cost in their empirical analyses of the British and the Ontario electricity markets, respectively.

[12] This simplifying assumption is justified in Bushnell (2003) by thinking of water as being measured in equivalent units of energy. However, the actual relationship between energy produced and the level of water in the reservoir depends on a number of factors such as the pressure of the water on the surface of the reservoir, the elevation of the reservoir, the pipe length and size, etc. While these factors would change the transition equation (1), the qualitative nature of our results are unlikely to be affected.

cost.[13] Given reservoir capacity, W_{max}, the transition equation governing the level of water in the reservoir is

$$W_{t+1} = \min[(1-\gamma)(W_t - h_t) + \omega_t, W_{max}], \tag{1}$$

$$\omega_t \sim F(\omega) \tag{2}$$

where W_t is the level of the reservoir at the beginning of period t, γ is a parameter that determines the rate of evaporation/leakage in the reservoir over an interval of time, and ω_t is the rate of inflow into the reservoir over an interval of time. The rate of inflow is considered random due to uncertain rainfall, say, and is modeled as an i.i.d. random variable with distribution function F that occurs after period t decisions are made. We will use $\widehat{W}_t$ to denote the "carryout", or water retained in the reservoir prior to inflow: $\widehat{W}_t = (1-\gamma)(W_t - h_t)$.

The behavior of consumers of electricity in any period $t = 0, 1, 2, ..., \infty$ is summarized by the following inverse demand function:

$$P_t = \alpha_t - \beta(h_t + Q_t), \qquad \beta > 0. \tag{3}$$

$$\alpha_t \sim G(\alpha) \tag{4}$$

The demand intercept, α_t, is an i.i.d. random variable with distribution function G and meant to capture variations in demand conditions, such as weather or economic activity.[14]

Producers choose their outputs simultaneously in each period and all producers discount future payoffs with the common discount factor, $\delta \in (0, 1)$. We next describe the game played by the oligopoly, after which we describe two alternative benchmarks to which we compare the oligopoly solution: the efficient solution and the myopic Cournot equilibrium. The purpose of these comparisons is the following: i) dynamic oligopoly to planner illustrates both the static and dynamic effects of market power; ii) dynamic oligopoly to myopic Cournot illustrates the dynamic effect of market power

[13] It is realistic that the hydro generator may also have a minimum reservoir level constraint. This is why we interpret W_t as water available for use instead of total water in the reservoir. Given this interpretation, $W_t = 0$ corresponds to the minimum reservoir level constraint binding.

[14] While it would be more realistic to allow for serial correlation or seasonality in demand (as well as water inflows), it complicates the solution of the model by adding additional state variables. We do not expect such an extension to substantially affect the average levels of the equilibrium variables. However, it would likely have an effect on the variability of these variables. For example, if α_t is positively serially correlated, the hydro producer will likely reduce output less in the face of a low value of α_t since saving water is less important as α_{t+1} is also likely to be low, and increase output less in the face of a high value of α_t since saving water is more valuable as α_{t+1} is also likely to be high.

alone; and iii) myopic Cournot to myopic planner illustrates the static effect of market power alone.

2.1 Oligopoly

Each producer is assumed to maximize the expected discounted present value of stream of profits. We focus on the case in which producers use Feedback strategies, which are functions of the current state, (α_t, W_t), only. Specifically, we examine Feedback strategies that are i) time-invariant functions of the current state since we have an infinite time horizon, and ii) non-linear functions of the current state to account for potentially binding capacity constraints. Denote the production strategy of the hydro producer by $s^H(\alpha_t, W_t)$ and, since thermal generators have identical generation costs, the identical production strategy of each thermal generator by $s^T(\alpha_t, W_t)$. We assume that all producers observe W_t and α_t before making decisions in period t. We will search for the Feedback equilibrium, which is a Nash equilibrium in the Feedback strategies.

Given the hydro producer's strategy, $s^H(\alpha_t, W_t)$, the problem for thermal producers is to maximize the expected[15] discounted sum of profits:

$$\max_{\{q_{it}\}} E_0 \left[\sum_{t=0}^{\infty} \delta^t \left((\alpha_t - \beta(s^H(\alpha_t, W_t) + \sum_{j=1}^{N} q_{jt}))q_{it} - c_1 q_{it} - (c_2/2){q_{it}}^2 \right) \right] \qquad i = 1, ..., N \tag{5}$$

subject to

$$0 \leq q_{it} \leq K/N, \quad W_{t+1} = \min[(1-\gamma)(W_t - h_t) + \omega_t, W_{max}], \quad t = 0...\infty, \quad W_0 \text{ given.} \tag{6}$$

The problem for a thermal producer is simplified by the fact that thermal producers do not directly influence the future state through their actions. At a given time, all producers choose their strategies simultaneously and independently, given the state, so for a given hydro strategy the generation of a thermal producer does not affect future water availability. Since a thermal producer's choice does not affect its continuation payoff, thermal production is governed by its "static" best response function in the case of an interior solution:

$$q_{it} = \frac{\alpha_t - c_1 - \beta s^H(\alpha_t, W_t) - \beta Q_{-it}}{2\beta + c_2} \tag{7}$$

[15]We use the notation E_t to denote the expectation of a random variable conditional on information known at time t.

where Q_{-it} denotes the output of all thermal firms excluding i. As thermal firms face identical costs they generate identical output, $q_{it} = q_t$, $i = 1, ...N$ and $Q_{-it} = (N-1)q_t$, and solving (7) for q_t and incorporating the capacity and non-negativity constraints, we have the strategy for a thermal producer as

$$s^T(\alpha_t, W_t) = \max\left[0, \min\left[\frac{\alpha_t - c_1 - \beta s^H(\alpha_t, W_t)}{(N+1)\beta + c_2}, K\right]\right] \tag{8}$$

Given thermal producers' strategy, $s^T(\alpha_t, W_t)$, the problem faced by the hydro producer is

$$\max_{\{h_t\}} E_0\left[\sum_{t=0}^{\infty} \delta^t \left[(\alpha_t - \beta(h_t + Ns^T(\alpha_t, W_t)))h_t\right]\right] \tag{9}$$

subject to

$$0 \leq h_t \leq W_t, \quad W_{t+1} = \min[(1-\gamma)(W_t - h_t) + \omega_t, W_{max}], \quad t = 0...\infty, \quad W_0 \text{ given.} \tag{10}$$

The hydro producer's best response to the thermal producers' strategy is determined by the solution to a dynamic optimization problem. The Bellman equation for the hydro producer's problem at any time $t > 0$ is

$$V(\alpha_t, W_t) = \max_{h_t \in [0, W_t]} \left\{(\alpha_t - \beta(h_t + Ns^T(\alpha_t, W_t)))h_t + \delta E_t\left[V(\alpha_{t+1}, W_{t+1})\right]\right\} \tag{11}$$

subject to

$$W_{t+1} = \min[(1-\gamma)(W_t - h_t) + \omega_t, W_{max}]. \tag{12}$$

The solution to this problem yields $s^H(\alpha_t, W_t)$. Define b_{0t} to be the Lagrange multiplier for the constraint $h_t \geq 0$, and b_{Wt} the Lagrange multiplier for the constraint $h_t \leq W_t$.

In order to say more about the solution to the problem in (11), define $\psi(h_t)$ as the derivative of the objective in the maximization problem in (11) with respect to h_t, i.e.,

$$\psi(h_t) = \alpha_t - 2\beta h_t - \beta N s^T(\alpha_t, W_t) - I[h_t < W_t]\delta(1-\gamma)E_t\left[\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right], \tag{13}$$

with W_{t+1} determined by (1) and $I[h_t < W_t]$ the indicator function that equals 1 if $h_t < W_t$ and 0 if

$h_t = W_t$. The necessary conditions for optimal hydro output are then

$$\psi(h_t) + b_{0t} - b_{Wt} = 0 \tag{14}$$

$$b_{Wt}(W_t - h_t) = 0, \quad b_{Wt} \geq 0, \quad (W_t - h_t) \geq 0 \tag{15}$$

and

$$b_{0t}h_t = 0, \quad b_{0t} \geq 0, \quad h_t \geq 0. \tag{16}$$

Solving (14) – (16) for h_t after using (1) for W_{t+1} in (13) yields the hydro producer's equilibrium strategy, $s^H(\alpha_t, W_t)$.

While the solution to this problem requires numerical methods, we can analyse the problem further to identify the strategic effect of the hydro producer's choice. The strategic effect measures the extent to which the hydro producer adjusts its output decision due to consideration of how the future availability of water influences the thermal producers output. In this quantity choice game, we expect that the strategic effect will be to reduce hydro output relative to the case in which the hydro producer did not take this effect into account.[16] We can demonstrate this more formally by expanding the last term in (13). We will assume that the reservoir capacity constraint is non-binding in this analysis (W_{max} is large) in order to keep the expressions simple. In this case, when $h_t < W_t$, $\partial W_{t+1}/\partial h_t = -(1-\gamma)$ is non-random.[17] We will also assume non-binding thermal capacity (large K) for the same reason.

Given the equilibrium strategies, the hydro producer's value function is the Lagrangian function associated with the maximization problem in (11) evaluated at the quantities determined by the equilibrium strategies:

$$\begin{aligned} V(\alpha_t, W_t) = &(\alpha_t - \beta(s^H(\alpha_t, W_t) + Ns^T(\alpha_t, W_t)))s^H(\alpha_t, W_t) + b_{0t}s^H(\alpha_t, W_t) \\ &+ b_{Wt}(W_t - s^H(\alpha_t, W_t)) + \delta E_t\left[V(\alpha_{t+1}, \min[(1-\gamma)(W_t - s^H(\alpha_t, W_t)) + \omega_t, W_{max}])\right] \end{aligned} \tag{17}$$

[16] The quantity choice game is one of strategic substitutability, which means that best responses are downward sloping. This implies that higher hydro output reduces thermal output when the thermal producer plays its best-response. Since the hydro producer's generation is likely to be at least weakly increasing in the level of available water, more water implies higher hydro output and hence lower thermal output.

[17] The analysis goes through if their is a non-zero probability of the reservoir capacity being reached, it is just complicated by having to weight the marginal effect on future water availability by the probability of the reservoir capacity constraint to be non-binding. As this possibility will reduce the hydro producer's concern about future water levels, what is presented here represents the maximal strategic effect.

The value function is kinked at the point where all available water is used, apart from this point we can express the marginal value of water in period t as

$$\begin{aligned}\frac{\partial V(\alpha_t, W_t)}{\partial W_t} = \Bigg[&\alpha_t - \beta(s^H(\alpha_t, W_t) + N s^T(\alpha_t, W_t)) - \beta s^H(\alpha_t, W_t) + b_{0t} - b_{Wt} \\ &+ I[h_t < W_t]\delta(1-\gamma)E_t\left[\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right]\Bigg]\frac{\partial s^H(\alpha_t, W_t)}{\partial W_t} \\ &- \beta N s^H(\alpha_t, W_t)\frac{\partial s^T(\alpha_t, W_t)}{\partial W_t} + b_{Wt} + I[h_t < W_t]\delta(1-\gamma)E_t\left[\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right], \quad (18)\end{aligned}$$

where $I[h_t < W_t]$ is the indicator function that equals 1 if $h_t < W_t$ and 0 otherwise. The first two lines of (18) are simply $(\psi(s^H(\alpha_t, W_t) + b_{0t} - b_{Wt})\frac{\partial s^H(\alpha_t, W_t)}{\partial W_t}$ which is zero in equilibrium by (14), so (18) simplifies to

$$\frac{\partial V(\alpha_t, W_t)}{\partial W_t} = -\beta N s^H(\alpha_t, W_t)\frac{\partial s^T(\alpha_t, W_t)}{\partial W_t} + b_{Wt} + I[h_t < W_t]\delta(1-\gamma)E_t\left[\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right], \quad (19)$$

Applying (19) to period $t+1$ and taking expectations allows us to analyse the $E_t\left[\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right]$ term in (13). We will examine the two cases of a binding/non-binding water availability constraint in turn.

Case 1: $s^H(\alpha_{t+1}, W_{t+1}) = W_{t+1}$. In this case, $b_{Wt+1} > 0$, and $I[h_{t+1} < W_{t+1}] = 0$. In addition, for an unconstrained thermal output, $\frac{\partial s^T(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}} = \frac{-\beta}{(N+1)\beta + c_2}$. Consequently, the marginal value of water in this case is

$$\left.\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right|_{s^H(\alpha_{t+1}, W_{t+1}) = W_{t+1}} = \frac{N\beta^2}{(N+1)\beta + c_2}W_{t+1} + b_{Wt+1}. \quad (20)$$

As both terms in (20) are positive, the value function is increasing in the level of water available, and the positive first term of (20) means that the strategic effect works to increase the marginal value of water in this case, over and above the shadow price of water.

Case 2: $s^H(\alpha_{t+1}, W_{t+1}) < W_{t+1}$. In this case $b_{Wt+1} = 0$ and (19) becomes

$$\left.\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right|_{s^H(\alpha_{t+1}, W_{t+1})<W_{t+1}} = \frac{N\beta^2}{(N+1)\beta + c_2} s^H(\alpha_{t+1}, W_{t+1}) \frac{\partial s^H(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}} + \delta(1-\gamma)E_{t+1}\left[\frac{\partial V(\alpha_{t+2}, W_{t+2})}{\partial W_{t+2}}\right]. \quad (21)$$

We again see a positive strategic effect in the first term of (21) as hydro production is expected to be increasing in available water (at least weakly). In this case as well, the strategic effect works to increase the marginal value of water over and above the expected marginal value of water in the next period.

From the perspective of period t, $E_t\left[\frac{\partial V(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right]$ combines both (20) and (21) as relevant, and since both are increased by the strategic effect of hydro production, we conclude that the strategic effect leads to an increase in the marginal value of future water availability. This results in the hydro producer generating less output in the Feedback equilibrium relative to the situation in which the strategic effect is not taken into account,[18] resulting in more water available in future periods and hence lower thermal output in those future periods.

2.2 Social optimality

We wish to compare the outcome under the oligopoly market structure to what is socially optimal. To this end, we solve the problem faced by a social planner choosing thermal and hydro generation with the objective of maximizing the expected present value of the stream of total surplus, defined to be consumer surplus less generation costs. We will consider the planner as choosing aggregate thermal generation, Q_t, which is optimal to allocate equally among the N thermal generators.

$$\max_{\{h_t, Q_t\}} E_o\left[\sum_{t=0}^{\infty} \delta^t \left(\alpha_t(h_t + Q_t) - \frac{\beta}{2}(h_t + Q_t)^2 - N\left(c_1 Q_t/N + \frac{c_2}{2}(Q_t/N)^2\right)\right)\right] \quad (22)$$

[18] In deterministic dynamic games, the strategic effect is measured by comparing the Feedback equilibrium to the Open Loop equilibrium, in which decisions are not influenced by the current state. In stochastic dynamic games Open Loop equilibria are less well defined, and in our case infeasible as the hydro producer needs to know its current water availability in order to choose its generation in each period. In the results presented below, we compare the Feedback equilibrium outcome to what occurs if the hydro producer behaved myopically: still constrained by water availability, but not taking into account the effects on future play.

subject to

$$0 \leq Q_t \leq K, \quad 0 \leq h_t \leq W_t, \quad W_{t+1} = \min[(1-\gamma)(W_t - h_t) + \omega_t, W_{max}], \quad t = 0...\infty, \quad W_0 \text{ given.} \tag{23}$$

The planner's value function then satisfies the Bellman equation:

$$\begin{aligned} V^P(\alpha_t, W_t) = \max_{h_t, Q_t} \Big\{ & \alpha_t(h_t + Q_t) - \frac{\beta}{2}(h_t + Q_t)^2 - N\left(c_1 Q_t/N + \frac{c_2}{2}(Q_t/N)^2\right) \\ & + \delta E_t\left[V^P(\alpha_{t+1}, W_{t+1})\right] \Big\} \end{aligned} \tag{24}$$

subject to

$$0 \leq Q_t \leq K, \quad 0 \leq h_t \leq W_t, \quad W_{t+1} = \min[(1-\gamma)(W_t - h_t) + \omega_t, W_{max}], \quad t = 0...\infty, \quad W_0 \text{ given.} \tag{25}$$

The necessary conditions for the maximization problem in (24) are

$$\alpha_t - \beta(h_t + Q_t) + \delta E_t\left[\frac{\partial W_{t+1}}{\partial h_t}\frac{\partial V^P(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right] - b^P_{Wt} + b^P_{0t} = 0 \tag{26}$$

and

$$\alpha_t - \beta(h_t + Q_t) - c_1 - (c_2/N)Q_t - a^P_{Kt} + a^P_{0t} = 0 \tag{27}$$

where b^P_{Wt} and b^P_{0t} are the Lagrange multipliers on hydro production's capacity and non-negativity constraints and a^P_{Wt} and a^P_{0t} are the multipliers on thermal production's capacity and non-negativity constraints. Equations (26) and (27) imply

$$a^P_{Kt} - a^P_{0t} + c_1 + (c_2/N)Q_t = \delta E_t\left[\frac{\partial W_{t+1}}{\partial h_t}\frac{\partial V^P(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right] + b^P_{Wt} - b^P_{0t} \tag{28}$$

which for an interior solution simplifies to

$$\delta\left[E_t\frac{\partial W_{t+1}}{\partial h_t}\frac{\partial V^P(\alpha_{t+1}, W_{t+1})}{\partial W_{t+1}}\right] = c_1 + (c_2/N)Q_t, \tag{29}$$

the marginal value of retained water is equated with the marginal cost of thermal production at an optimal solution.

2.3 Numerical solution algorithm

We now describe the algorithm we use to solve the problem under the alternative market structures. As we use the same method for each, we just describe the method in terms of the duopoly problem here.

In order to solve (11), rather than approximate $V(\alpha_t, W_t)$ directly, we solve the problem by approximating $E_t\left[V(\alpha_{t+1}, W_{t+1})\right]$. Recall that the carryout, which is the quantity of water transferred from period t to period $t+1$ prior to the new inflow, is given by $\widehat{W}_{t+1} = (1-\gamma)(W_t - h_t)$. We wish to find a function of $\widehat{W}_{t+1}$ that provides a good approximation to $E_t\left[V(\alpha_{t+1}, W_{t+1})\right]$. This approach has two benefits. First, it allows us to approximate a function of one variable only, $\widehat{W}_{t+1}$, whereas the value function itself is a function of two variables. Second, this approach has the added advantage that the expected value function will likely be a smooth function of $\widehat{W}_{t+1}$, while the value function itself is kinked due to the constraints on both the hydro and thermal producers production.

We approximate the hydro producer's expected value function using the collocation method,[19] which approximates an unknown function with a linear combination of known basis functions at a known set of points. In particular,

$$E_t[V(\alpha_{t+1}, \widehat{W}_{t+1} + \omega_t)] \approx \sum_{i=1}^{n} d_i \phi_i(\widehat{W}_{t+1}) \equiv \widetilde{V}(\widehat{W}_{t+1}) \tag{30}$$

where the ϕ_i are known basis functions. Collocation proceeds by determining the $d_i,\ i = 1, ...n$, in order for the approximation to hold exactly at n collocation nodes, $\widehat{W}_+^1, \widehat{W}_+^2, ..., \widehat{W}_+^n$. For our application, ϕ_i is the i^{th} Chebyshev polynomial and the $\widehat{W}_+^i$ are the Chebyshev nodes. The algorithm we use to find the d_i is described as follows:

0. Choose a starting approximation $\widetilde{V}^0(\widehat{W}_+)$, i.e., starting values $d_i^0,\ i = 1, 2, ..., n$.

1. Given the current approximation, $\widetilde{V}^k(\widehat{W}_+)$, define $V^k(\alpha, W)$ as the solution to the maximization problem in (11) for a given value of α with $\widetilde{V}^k(\widehat{W})$ replacing the expectation of the next period's value function. This solution uses (8) to solve (14). A root-finding algorithm is used to find the optimal hydro production in the case of an interior solution.

[19] Judd (1998), Chapter 11. One popular alternative method would be to discretize the problem and perform a grid search for the optimization. For smooth problems, the collocation method is much more computationally efficient. In addition, the collocation method we employ yields a differentiable value function to aid analysis. Judd (1998) and Rust (1996) provide detailed explanations of alternative methods for the numerical solution of dynamic programming problems.

2. For each of the collocation nodes, $\widehat{W}_+^1, \widehat{W}_+^2, ..., \widehat{W}_+^n$, integrate $V^k(\alpha, \widehat{W}_+^j + \omega)$ numerically over possible demand states and water flows. Solve for the updated values, d_i^{k+1}:

$$\sum_{i=1}^{n} d_i^{k+1}\phi_i(\widehat{W}_+^j) = \iint V^k(\alpha, \widehat{W}_+^j + \omega)dG(\alpha)dF(\omega) \qquad j = 1, 2, ..., n \tag{31}$$

where $G(\alpha)$ is the distribution of α and $F(\omega)$ the distribution of ω. As the ϕ_i are known functions and the $\widehat{W}_+^j$ known values, (31) is linear in the d's and so they are straightforward to compute once the values on the right-hand side of (26) are computed via numerical integration. The updated approximation is then

$$\widetilde{V}^{k+1}(\widehat{W}) = \sum_{i=1}^{n} d_i^{k+1}\phi_i(\widehat{W}). \tag{32}$$

3. If $||d^{k+1} - d^k||$ is sufficiently small, stop. Else, return to step 1.

The parameter n is chosen so that the resulting approximation has relatively low residual error.[20]

3 The effects of water availability

As noted in the Introduction, jurisdictions with available hydroelectric generation differ widely in the share of generation accounted for by hydro. We explore the implications of differing amounts of water availability in this section by developing cases that differ in the rate of water inflow, μ_ω. We focus here on a setting in which the hydro producer has a large reservoir capacity in order to explore what happens when the hydro generator is only constrained by available water and is not much concerned with spillage when water flows result in the reservoir's capacity being exceeded. Later, in Section (6), we develop a case, calibrated to the specific jurisdiction of Ontario, Canada to explore a setting in which available hydro capacity is more limiting. In addition, we focus on the duopoly case, $N = 1$, initially and provide some evidence on the effects of more thermal generators in subsection 3.4.

For each case, to compute the equilibrium strategies we use Chebyshev polynomials for the ϕ_i functions and choose n to provide an acceptable approximation.[21] The value of n is chosen as the

[20] The residual error is a measure of how well the approximation performs at points other than the collocation nodes.

[21] The computations are done with C++ and make use of routines for Chebyshev approximation, numerical integration, and root finding from the Gnu Scientific Library (Galassi et al,2006). Computational time is minimal with the approximation and simulation together taking less than 5 seconds for the cases reported.

smallest value which gives a relative approximation residual on the order 10^{-5}.

In each case, we also present results for the myopic Cournot equilibrium to gauge the effects of the dynamics of water availability on the behavior of the hydro producer. In a myopic Cournot equilibrium, the producers are able to respond to the realization of the demand state (α_t) and the hydro producer may be constrained by water availability, but the hydro producer does not strategically adjust future water levels (essentially, the last term in (13) is suppressed). The purpose of this comparison is to isolate the static effect of market power (as illustrated by the myopic Cournot equilibrium) from the dynamic effects of the control of water availability by the hydro producer. For comparison purposes we also present the myopic planner solution which would correspond to a myopic perfectly competitive equilibrium.[22]

The time-invariant Feedback equilibrium generates a stationary distribution for the variables in the model which we wish to analyse by computing moments (largely means) of this distribution. We do so by applying Monte Carlo methods, drawing sequences of α_t and ω_t from their distributions and applying the equilibrium strategies to generate sequences of equilibrium outputs, price and water levels. Applying this procedure a number of times allows us to estimate the moments of the stationary distribution with the statistics from the sequences generated. The statistics are created by generating 100 sequences of the equilibrium variables of 1,000 periods each.[23] Statistics for the variables of interest are averaged over the 100 sequences of 1000 periods each.

To examine the effects of alternative rates of water inflow, we solve the model for some specific parameter values. This case is designed to be broadly representative of a jurisdiction with roughly equal market share divided between hydro and thermal generation. In addition, we aim to have a relatively small demand elasticity emerge from the parameterization, so the base thermal cost and demand parameters are chosen so that there is a relatively small demand elasticity at the unconstrained (Cournot) equilibrium.

We model both demand and inflow uncertainty as following a normal distribution. While this may not be justified at a high frequency of hours or days, as we are modeling the accumulation of water flows and demand over a longer time horizon, the Central Limit Theorem gives some justification to

[22] As pointed out by a referee that most hydro dominant systems use sophisticated stochastic optimization techniques for hydroelectricity generation. In those models, all realistic hydro production constraints and regulatory constraints as well as demand, generation and water inflow specific uncertainties have been considered. However, those models lack long-horizon imperfect competition under uncertainty allowing strategic firms including hydro producer with the objective of profit maximization subject to dynamic water constraints.

[23] An initial run of 100 periods precedes the 1,000 period sample to minimize any effects of starting values.

treating these as normally distributed.[24] All of our simulations use the same distribution for demand shocks: $\alpha_t \sim N(\mu_\alpha, \sigma_\alpha^2)$, where we set $\mu_\alpha = 200$ and $\sigma_\alpha = 20$ (10% of the mean). The water inflow also follows a normal distribution: $\omega_t \sim N(\mu_\omega, \sigma_\omega^2)$. We will allow different values of μ_ω in our simulations while maintaining the standard deviation to be 10% of the mean in each case. Both the demand shocks and water inflow are assumed to be serially uncorrelated, which could be justified by the relatively long units of time that we are considering.[25]

We choose parameters for the thermal cost function that are roughly consistent with those used in Green and Newbery (1992): $c_1 = 10$ and $c_2 = 0.025$. The thermal generator's capacity, K, is initially set large enough to be non-binding in the duopoly equilibrium ($K = 6.0$, which is twice the average Cournot production level). A relatively high thermal capacity is chosen in order to analyze the effects of the water constraint on the equilibrium in the absence of a binding thermal capacity. After presenting results for this level of capacity we will then demonstrate the effects of varying thermal capacity on equilibrium outcome of the game.

The dynamics of water availability for the hydro producer are governed by the distribution of inflow as well as the reservoir capacity and evaporation parameter. We will examine alternative values of the reservoir capacity and inflow distribution below. We set the rate of water loss due to evaporation etc., γ, to 0.3, which is significant as each time period corresponds to a low frequency such as a month or a season.

Setting the demand parameters $\beta = 20$ and $\mu_\alpha = 200$ gives a demand elasticity of 0.54 at the Cournot solution.[26] The rate at which firms discount future profit, δ, is set at 0.9, which is consistent with an 11% annual rate of return.

We start with a case in which the capacity of the reservoir for the hydro generator is relatively large in order to analyze the situation in which the hydro producer is only constrained by available water and is not much concerned with spillage when water flows result in the reservoir's capacity being exceeded. To this end, we set $W_{max} = 7.0$ which is twice the average hydro production in the unconstrained Cournot game. The parameters used in this subsection are listed in Table 1.

[24] In addition, the assumption of normally distributed shocks is common in the literature. For inflow uncertainty see Pritchard et al (2005), Soliman et al (1986), Christensen and Soliman (1986), Contaxis and Kavatza (1990), and Pritchard (2015). In the case of demand uncertainty see Nolde et al. (2008).

[25] By adding additional state variables, serial correlation in demand and inflow shocks would substantially complicate the simulations.

[26] This is near the upper end of the range of demand elasticity examined by Genc and Aydemir (2017) and Green Newbery (1992).

<< Table 1>>

Three scenarios are examined corresponding to alternative mean water inflow levels and the results are reported in Table 2. The high inflow scenario corresponds to plentiful water, the low inflow scenario corresponds to very scarce water, and the medium inflow scenario corresponds to an average water inflow equal to the hydro output in the unconstrained repeated Cournot game.

<< Table 2>>

3.1 High inflow

The high inflow case represents a benchmark in which neither of the constraints (water or capacity) are binding for the duopoly. For this scenario, we choose a mean inflow of water that is double mean hydro production in the unconstrained game discussed above ($\mu_\omega = 7.0$). In this case we obtain an accurate approximation with an order of $n = 2$, which is expected given that the value function is quadratic if the constraints do not bind.

Not surprisingly, the outcomes in the duopoly equilibrium are the same as in the repeated Cournot game. Neither producer operates at capacity, so we just have an interior solution that replicates the Cournot outcome.[27] This is not socially optimal, since the planner would like to use more of the low cost technology, having the hydro producer at capacity in all periods. This scenario results in the largest welfare loss of the three examined. The duopoly price is roughly seven times the optimal level[28] and there is substantial under-utilization of water, with spillage occurring more than twice as often under the duopoly market structure than is optimal. This results in a shadow price of water that is zero for the duopoly hydro producer but significant (37% of the price level) for the planner. The under-utilization of water in this case can also be seen through the spillage frequency. With the average inflow equal to the reservoir capacity, spillage optimally occurs about 50% of the time whereas it occurs almost 100% of the time under the duopoly market structure, as there is generally more water available than the hydro duopolist wishes to use.

The comparison between the duopoly and socially optimal outcomes in this case represents a measure of the static effect of market power alone as water dynamics do not influence the hydro producer in any way as the water availability constraint does not bind. The myopic Cournot solution

[27] This can be seen by the rows labeled $\%(h = W)$ and $\%(q = K)$ in Table (2), which report the percentage of periods for which the water and capacity constraints bind. A value of zero means that the constraints never bind.

[28] A very high price relative to marginal cost is expected in imperfectly competitive markets with low demand elasticity, a common situation in electricity markets.

is the same as the Feedback one since the constraint on water availability never binds.

3.2 Low inflow

In order to examine the other extreme of water flow that substantially constrains hydro output we examine a scenario with $\mu_\omega = 1.75$, which is half of the average level of hydro production in the high inflow case above. In this scenario, we expect hydro production to be frequently constrained by water availability.

For this low water inflow case, the hydro producer exhausts the available water 74% of the time, which is close to the socially optimal frequency of 76%, and we see that the average hydro output is actually higher than optimal. The reason for this is that the planner places a substantially higher value on water: the average shadow price of water (b_W) is 17.57 for the planner vs. 9.22 for the hydro duopolist. This results in the planner wishing to carry more water over to the next period (0.08 vs. 0.03 on average) for which it needs to produce less hydro electricity on average. Since thermal production is always at capacity in the social optimum, only hydro production can vary to adjust to demand fluctuations. This adds to the value of water to the planner in this scenario as it wishes to use the water not just to reduce the price level, but also to smooth prices.

In order to measure the strategic use of water over time, we compare the myopic Cournot hydro output with that of the Feedback equilibrium. While there is some reduction in output when the hydro producer considers the effects on future water availability, it is rather small. In this scenario, since the water constraint is binding in most periods, hydro output under the two alternatives is almost always the same. However, in the periods in which the hydro producer does use less than the entire stock of available water, it curtails its production more in the Feedback equilibrium than in the myopic Cournot equilibrium.

3.3 Medium inflow

We now examine a case in which the average water inflow is at an intermediate level which we take to be the output level in the unconstrained duopoly game which gives $\mu_\omega = 3.5$. This scenario allows the sharpest view on the extent to which the hydro producer will strategically withhold water since, unlike the high inflow scenario, there is a possibility that the hydro producer will be constrained and, unlike the low inflow scenario, it is likely that the available water will not constrain hydro generation most of the time.

The results are presented in the two central columns of Table 2. The 100% increase in inflow from 1.75 to 3.50 results in a 103% increase in optimal hydro production and a 97% increase in myopic hydro production, whereas the output of the hydro producer in the Feedback equilibrium increases by only 84% (from 1.74 to 3.21). This results in the hydro producer draining its reservoir only 3% of the time, whereas it is optimal to do so 49% of the time, resulting in a substantially higher than optimal average carryout ($\widehat{W}$). This withholding of water is also reflected in the shadow price of water being close to zero for the duopolist while the optimal shadow price of water is 8.09, which represents 44% of the average optimal price level.

The behavior of the hydro producer results in a reduction of output relative to the optimal solution of 0.28 units (3.49-3.21). It is important to note that this output reduction is small relative to the output lost due to market power. This can be seen by examining the output of the thermal producer, who produces 3.13 units of electricity in the Feedback Equilibrium compared to an optimal output of 5.59 units, a reduction of 2.46 units. Hence, while the total electricity generated in the Feedback equilibrium falls short of the optimal level by 2.74 units (30% of the optimal level), only 10% of this shortfall is due to the withholding of water by the hydro producer.

It is notable that in all three situations in Table 2, while there is a welfare loss in the duopoly market structure relative to the efficient one, the distribution of the benefits varies dramatically. Consumer surplus accounts for a larger share of welfare under the optimal solution compared to the duopoly in each case. Consumers benefit relatively more the higher the level of water inflows.

3.4 The effects of the number of thermal generators

The above analysis focused on the duopoly case ($N = 1$), which is the most concentrated market structure in our setting. We now illustrate the outcome with more thermal generators. Taking the parameters from the medium inflow case of subsection 3.3, we solve the model for a range of N. In order to concentrate on the competitive effects of an increase in the number of thermal generators as opposed to effects of capacity, we hold the aggregate capacity of thermal generation constant in the exercise, so each thermal generator has an individual capacity of K/N.

We plot the expected price for both the oligopoly case and the social planner in Figure 1 for $N = 1, 2, .., 15$. We see that the effects of market structure are quite dramatic for relatively low numbers of thermal producers. Even two thermal generators results in a substantial fall in price relative to one generator. However, the effect of increased competition from thermal generators is essentially exhausted after just a few generators are active. In addition, the effect of more thermal generators levels off at a mean price level that is still substantially above that in the socially optimal solution. Increased competition from thermal generators does mitigate market power, but does not eliminate it entirely. This is due to the marginal cost of thermal generation being strictly positive as $c_1 > 0$, and, as this is caused by thermal generators' need to purchase fuel to operate, it is an unavoidable characteristic of thermal generation.

<<Figure 1>>

4 Price volatility

Substantial price volatility has been a common feature in deregulated wholesale electricity markets and is mainly explained as a consequence of inelastic demand and capacity constrained generation combined with the inability to store electricity. In most jurisdictions consumers are shielded from this volatility to the extend that retail pricing is not real-time pricing. However, the effects of price volatility are felt by the utilities that must purchase electricity at a volatile wholesale price and sell at a fixed retail price. This situation is described in Borenstein (2002) for the California crisis in the early days of electricity market deregulation. In that case solvency concerns for the utilities translate into a social concern regarding price volatility as well as skewness.

In terms of our model, the social planner's objective function, (22), is concave in h_t and q_t which are themselves random due to their dependence on the demand and water shocks. Consequently, social welfare depends in a non-linear fashion on the underlying uncertainty, resulting in a social interest in the level of price volatility.

It is well known that an important contributing factor to the volatility of electricity prices is the non-storability of electricity, so understanding the extent to which the storability of water can be used as a substitute for electricity storage is important. We discuss the effects of hydro generation on price volatility and skewness in this section using the simulated results from the previous section.

From the results presented in Table 2, we see that price volatility, as measured by the standard deviation in price, is lower under the duopoly than is socially optimal in the low and medium water inflow cases. However, the reverse occurs in the high inflow case. One force at work is the under-utilization of water by the hydro producer. When it is optimal to be using as much hydro generation as possible, water is not used for price smoothing in a significant way in the optimal solution. When constrained by water availability, there is simply no role for using water to smooth demand fluctuations. In contrast, under the duopoly market structure, the hydro producer is less often constrained by water availability and so reacts more to demand fluctuations, resulting in smoother prices.

Balancing the effect of the water availability constraint on price volatility is the effect of market power on how producers adjust output. As shown in Thille (2006), in a repeated Cournot game with uncertainty, imperfectly competitive firms will not adjust output as much as is optimal in response to demand shocks. This effect dominates the effect of the constraint on price volatility in the high inflow case since the constraint never binds, resulting in the duopoly producing higher price variability than is optimal in that case. Consequently, the effect of market power on price volatility depends on the degree to which the water availability constraint binds for the hydro producer.

The under-utilization of water by the hydro generator results in price skewness substantially lower than occurs in the efficient scenario. This effect is at an extreme in the high inflow case, where the hydro duopolist is unconstrained by water availability resulting in essentially zero skewness. In contrast, it is optimal to use as much water as possible in each period, which results in highly skewed prices as high demand states must be accommodated by increased thermal generation at high marginal cost.

The question remains how the planner can attempt to influence the hydro producer to act efficiently with respect to water use. A common feature of many jurisdictions with substantial hydro capacity is that hydro generation is publicly owned or regulated. We explore this option in Section 6 where we calibrate a case for such a jurisdiction.

5 Effects of Thermal Capacity

In order to analyze how thermal generation capacity, K, affects the equilibrium outcome under the two market structures, we examine the medium average inflow case of Section 3.3 allowing for different levels of K. In order to examine a wide variety of thermal generation capacities, we solve the model for 20 different capacities ranging between zero and five units. For each solution, we simulate the model as above and plot some of the resulting statistics in Figures 2 and 3.

<<Figure 2>>

The top row of Figure 2 plots the average outputs of each producer by market structure. At low levels of thermal capacity, the thermal producer is essentially always operating at capacity, which is socially optimal. At large levels of capacity the thermal producer reduces output below capacity more frequently, resulting in sub-optimal output at higher capacities. In contrast, the hydro producer's average output is below the optimal quantity for any level K, although the magnitude of the difference is not large.

The bottom left graph in Figure 2 demonstrates that price is very close to the optimal level until thermal capacity reaches approximately 2.5. After this point, price levels off and approaches the Cournot price of 70.0, whereas the planner has price falling until thermal capacity is beyond 5.0. The implications for price volatility are demonstrated in the bottom right graph in Figure 2. The duopoly prices are less volatile than is optimal for all capacities depicted in Figure 2.

We plot payoffs and social welfare in Figure 3. Each payoff is very close to the socially optimal one for thermal capacities less than 2.5. From the above discussion we know that this is because the thermal constraint frequently binds under both market structures and the hydro producer does not greatly reduce output relative to the optimal level, so the outcome is not far from optimal.

<<Figure 3>>

Due to the substantial effect that K has on how close the duopoly outcome is to the optimal one, it is interesting to consider what thermal capacity would be chosen if the thermal producer could choose capacity in a previous period. Consider allowing the thermal producer to make a one time investment in capacity before time 0. The slope of the thermal producer's payoff in Figure 3 measures the benefit to the producer of a marginal addition to capacity. The thermal producer would choose a level of capacity that results in a significant departure from social optimality only if the the marginal cost of capacity is relatively low. Notice that for the outcome to diverge significantly from optimality in this case would require a capacity investment that exceeds the "average" Cournot output of the thermal producer (3.0 in this case). Since the region of capacity levels for which the thermal producer's payoff is relatively steep coincides with the region where the equilibrium is nearly optimal, we can suggest that the equilibrium in the dynamic duopoly game allowing for thermal capacity choice will be close to the optimal outcome if the marginal cost of capacity is not too low.

This analysis is consistent with results about the optimality of thermal investment decisions derived from a two-period mixed hydro and thermal generation game in Genc and Thille (2011).

They explain how a thermal producers choice of capacity, facing competition by a hydro player, can be higher or lower than the efficient investment. If the amount of water available for hydro production is very low, so that hydro output is low, the thermal player will choose a capacity lower than efficient so as to exercise market power. On the other hand, if the amount of water is high the thermal player still invests and produces in equilibrium. However, in this case, the level of thermal investment is inefficient because the hydro production can meet all demand contingencies. Clearly, in the former case the thermal player under-invests and in the latter case thermal firm over-invests relative to social optimum: there is no clear prediction regarding the social optimality of the thermal investment decision. Our results are consistent with this in that thermal investment decisions are not clearly inefficient as long as marginal investment cost is not too small.

6 The effects of reservoir capacity: An application to the Ontario wholesale electricity market

The Ontario wholesale electricity market represents a jurisdiction with substantial market shares provided by both hydro and thermal generation. Excluding nuclear generation[29] the generation mix is roughly 40% hydro and 60% fossil fuel thermal.[30] In this section we calibrate our model parameters from data on this market.

Since the Ontario hydro system has a relatively large number of run-of-river generators, a relatively large share of hydro generation occurs with limited reservoir capacity to store water. In the model presented above, (1) implies that the market share of the hydro generator is limited by W_{max}, so it does not allow for significant hydro generation along with limited reservoir capacity since any excess of inflow above W_{max} is spilled. In order to better fit the Ontario system we need to be able to vary W_{max} independently of μ_ω, and so we change the transition equation slightly to

$$W_{t+1} = \min[(1-\gamma)(W_t - h_t), W_{max}] + \omega_t, \tag{33}$$

[29] The Ontario market has a substantial amount of nuclear generation which is used as base-load and whose generation does not vary much over time. Consequently, we can consider demand faced by the hydro and other thermal producers as being net of nuclear generation.

[30] The numbers are from 2007 data, the same used in De Villemeur and Pineau (2016). Since that time, wind and solar generation capacity has increased, which can be accommodated in our model, to the extent that their generation is exogenous, as a change in the mean and standard deviation of demand faced by the hydro and thermal producers.

i.e., the reservoir capacity applies to the carryout before inflows are realized. This change allows us to nest run-of-river scenarios in which water storage is minimal.

A second characteristic of the Ontario market that we wish to address is that, like many other jurisdictions, management of hydro generators is heavily regulated or publicly owned. However, thermal generation capacity is largely private. In order to consider this case we develop a third model which we call a hybrid model.

6.1 Hybrid model: Profit-maximizing Thermal facing Regulated Hydro

As many jurisdictions retain regulatory control over substantial portions of their hydroelectricity generation capacity, we consider a hybrid model where we have the social planner choose the rate of hydro generation while the thermal producer operates in a profit maximizing fashion. This exercise allows us to determine efficient hydro generation while abstracting from the planner's decisions that mitigate inefficiently low thermal generation in the duopoly setting due to the thermal producer's market power.

The solution in this case here is similar to that in the duopoly model explained above, substituting the planner's objective function for the duopoly hydro generators. The planner solves (22) but by choice of h_t only, with q_t determined by (8). The numerical solution method then follows the same steps as were followed for the duopoly model.

6.2 Calibration

In our calibration to the Ontario market we utilize detailed data which is also implemented by Genc and Aydemir (2017), who examined cross border electricity trade, and Genc (2016), who studied wholesale electricity demand elasticity. The detailed description of the data is available in their Appendix A. Our Ontario market model parameters based on these papers are presented in Table 3 and discussed below.

<<Table 3>>

To generate the demand parameters Genc and Aydemir use the actual market price (the Hourly Ontario Energy Price, HOEP) and total market demand (Ontario demand plus export demand) data to obtain the slope coefficient and the intercept distribution reported in Table 3. As explained in Genc and Aydemir, these demand coefficients are computed to generate a demand elasticity of 0.6

in their model. The estimation in Genc and Aydemir is done using hourly data, which is a much higher frequency than we are interested in (e.g. the severe seasonal patterns over the course of a day would complicate our analysis dramatically). For the values in Table 3 we take a weekly average of the hourly estimates, which has the effect of reducing the standard deviation compared to Genc and Aydemir.

The marginal cost function for each thermal generator includes the fuel price as well as technical characteristics of generators (heat and emission rates), permit prices (for NOx and SO2 emissions), and availability of generators and their available production capacities. Total marginal cost for the thermal generator is the summation of the fuel marginal cost, SO2 emission cost, and NOx emission cost. After constructing total marginal cost for each generator, Genc and Aydemir horizontally sum available production capacities of each generator for a given price level to obtain aggregate marginal cost function in the system. The total thermal marginal cost curve includes all active coal, natural gas, and oil-fired generators in the Ontario market in 2007. Because the cost curve is non-smooth, Genc and Aydemir fit it to an affine marginal cost curve to obtain the cost coefficients. The thermal generator's capacity, K, is set large enough to be non-binding in the duopoly equilibrium ($K = 4986$, which is twice the average Cournot production level).

We do not have data on the relevant water flows for the Ontario hydro system, so we calibrate the mean inflow to generate an equilibrium market share for hydro of approximately 40%, which represents hydro's market share in Ontario, excluding nuclear. We do this by setting μ_ω to 40% of the Cournot output that would be produced in a static game between hydro and thermal with the cost and demand parameters described above.[31] We will consider two alternative values for W_{max} in order to compare systems that are predominantly run-of-river to those with more substantial water storage capacity.

6.3 Results

As a baseline model to start from, we examine the case in which reservoir capacity (W_{max}) is limited. This is arguably the case in Ontario in which a large proportion of hydro capacity is run-of-river, so even though hydro represents a significant fraction of generation capacity, the ability to store water is limited. To start with an extreme case, consider $W_{max} = 100$ which is essentially a run-of-river

[31] For the demand and thermal cost parameters described with zero hydro marginal cost the Cournot equilibrium market share for hydro would be 72%, much higher than we observe in Ontario.

scenario. Statistics generated for this scenario are presented in the first three columns of Table 4.

<<Table 4>>

We see a similar pattern in terms of hydro generation to the one in the previous case. As the duopoly hydro generator is quite constrained by water flows its output is not much lower than the social optimum on average, however it does not make use off all available water as it spills 35% of its reservoir capacity on average. Thermal output is again lower than is efficient due to the exercise of market power. It is interesting to note the difference between the hybrid solution and the optimal one. When the planner operates the hydro generator only, it makes no use of the small amount of storage available, running the generator as a run-of-river one. In contrast, in the social optimum, the planner does make use of storage (nearly 30% of the time) since it can adjust thermal output as well when conditions warrant. The planner never spills water, but does make use of the small amount of storage available. Since it cannot adjust thermal output in the hybrid case, the solution is to make maximal use of all available water in each period. This results in a shadow price of water that is substantially higher in the hybrid situation that it is in the fully optimal solution and the hydro generator is run as a (random) baseload generator. The storage capacity is not used for smoothing out the effects of shocks.

A second interesting implication from Table 4 is that, as in the previous case, even though the average outputs of the hydro generator are similar across market structures, the volatility of price does vary substantially. The duopolists sub-optimal use of water results in smoother prices than is efficient. With the hybrid model we now see that this effect is due to a combination of hydro and thermal behavior. Comparing the duopoly to hybrid, managing hydro generation efficiently in a market environment moves the volatility of price some way towards the efficient level. However, the thermal generator also does not react efficiently to shocks.

The last three columns allow a larger reservoir, $W_{max} = 1280$, to see the implications of varying storage capacity. The duopoly hydro generation is now equal to the optimal level, but the general pattern is qualitatively unchanged. The hybrid solution has the available storage unused, as all available water is used each period to mitigate the market power inefficiencies of the thermal generator.

7 Discussion

The results presented above were generated under specific assumptions about model parameters and functional forms and some discussion of the robustness of these results to these assumptions is warranted.

Our assumptions linear demand and quadratic thermal generation costs aided the analysis, but alternative specifications are likely to lead to qualitatively similar results regarding the expected values of the equilibrium prices and quantities. Alternative functions forms that would generate problems are the same ones that would cause trouble in Cournot games generally, i.e., specifications that result in non-concave payoffs and/or non-monotonic or discontinuous best-response functions that may result in the lack of pure strategy equilibria or multiple equilibria.

We assumed normal distributions for the demand and cost shocks which were additive in their effects. Each of these assumptions could be relaxed with our approach and there is in principle no reason why our numerical solution technique could not deal with it. However, our assumption that the demand and water inflow random variable be serially uncorrelated is important. Allowing for serial correlation would require that we consider additional state variables (past demand and water inflow shocks) in the analysis. It is well known that numerical dynamic programming techniques become increasingly difficult to implement as the dimension of the state vector increases (the "curse of dimensionality"). A similar limitation would occur if we were to consider multiple hydro generators, each with its own reservoir.

The results regarding the volatility of prices that we obtained are likely more sensitive to the modeling choices than the mean levels of the variables are. Our result that prices were smoothest under duopoly may change if alternative functions forms or alternative relative volatilities of demand relative to water shocks are implemented. It is known that in models of commodity storage alternative demand specifications (Newbery (1984)) and alternative sources of uncertainty (Thille (2006)) can affect predictions about volatility. Similar findings will most likely occur in our model. However, the result that price volatility differs from the efficient level should be generally true.

Finally, our analysis assumed only one thermal generator in competition with the hydro generator. As long as there are no dynamic elements to the thermal generators decisions, our approach can handle alternative market structures that have more thermal generators, perhaps with different cost structures according to their production technologies, at the cost of some additional complexity.

8 Ex-post Analysis of the Ontario Market

In connection with simulation exercises, we now perform ex-post analysis using hourly actual production data to assess the impact of different Ontario market structures (from more thermal generation to more renewables/hydro) observed over the years. Specifically, this simple analysis intends to explore how different market structures influence hydro and thermal generation and their impact on price.

We examine hourly actual market price (Hourly Ontario Energy Price, HOEP), and actual production (hydro, wind, thermal, and nuclear) as well as imports, exports, and temperature data for 2007 and 2014. While in 2007 the ratio of total thermal output to total hydro generation was 1.18, it was 0.41 in 2014. Clearly, 2007 represents more thermal generation and 2014 corresponds to more hydro generation. Specifically, in 2007 the average total production was 17,819 MWh and the average renewable (mainly hydro) outputs was 3,737 MWh. That is 21% production came from renewables. On the other hand, in 2014 the average total production was 17,388 MWh and the average renewable (hydro plus wind) output was 4,911 MWh (with 4,116 MWh hydro alone). That is, renewable share was 28.2% in 2014 (with 23.7% hydro) in total production. Therefore, these two years represent two different market structures in the Ontario market with higher share of thermal (in 2007) versus higher share of hydro (in 2014). Furthermore, while the price volatility in 2007 was with standard deviation 24.65 and skewness 1.56, the price volatility increased in 2014 with standard deviation 46.74 and skewness 4.92. Therefore, more renewable (hydro plus wind) generation resulted in more price volatility in 2014.

Given the statistics of actual data, we can compare findings in simulations to characteristics of actual data. First, different simulated market structures (duopoly, hybrid, social optimum) lead to different predictions which were also confirmed by the two different market structures analyzed in 2007 and 2014. Year 2014 corresponds to more competitive market as more fringe firms were in play and the largest dominant firm (which is Ontario Power Generation) decommissioned its large thermal assets (it phased off its coal-fired generators) in 2014. Second, simulation results in Table 2 shows that price volatility goes down as more water becomes available for use. On the other hand, the price volatility in 2014 is higher than the one in 2007 because of large proportion of renewables. This is mainly due to large share of intermittent wind generation observed in 2014. However, although share of hydro production increased in 2014 compared to 2007, the volatility in hydro generation went down in 2014. This low volatility in production should translate to low price volatility in the market.

Third, in simulations in Table 4 we observe that larger reservoir capacity does not necessarily imply reduction in price volatility over different market structures (duopoly, hybrid, optimal), although the average prices fall in capacity and reduce from duopoly to hybrid to optimal. We have similar findings when we compare the outcomes in 2007 to the ones in 2014. Although the average price went down under more competitive structure in 2014, the price volatility effect compared to 2007 was opaque. Finally, we run simple regressions to check the impact of hydro generation on price.[32] Using hourly data described above, having price as a dependent variable and the rest of the variables (outputs of hydro, thermal, nuclear, wind, as well as temperature, imports, exports) as independent variables, we find that the coefficient of hydro output in 2014 is larger than the coefficient of hydro generation in 2007. All regression coefficients are significant. This result is conformable with the finding in simulations that more hydro output has larger impact on price than less hydro generation.

9 Conclusion

Dynamic competition between thermal and hydroelectric producers under demand and water inflow uncertainty has interesting implications for the distribution of electricity prices and the efficiency of market outcomes. When both thermal and hydro capacities bind frequently the oligopoly outcome is not far from what is socially optimal conditional on the capacity constraints. Our results illustrate that the worst case scenario occurs when neither thermal nor hydro constraints bind, suggesting that analyses of the costs of market power in electricity markets needs to account for binding capacity constraints or else it will overestimate the degree of market failure.

The hydro producer has an incentive to withhold water for sufficiently high water inflows. However, the reverse can occur when inflows are low: more water may be used than is socially optimal. This pattern of water usage by a hydro producer with market power has significant implications for price variability, since efficient water usage has storage of water driven by the desire to smooth price fluctuations. Prices are smoother than optimal for low to medium water inflows, but more volatile than optimal for high water inflows.

Using a case study calibrated to match the Ontario electricity market, we demonstrate that this sub-optimal use of water persists in this setting even when the hydro technology is nearly run-of-river

[32]We did run OLS, GMM, and two-step GMM regressions. For the sake of brevity, we do not report the regression tables. However, the detailed results are available from the authors upon request.

as well as when significant reservoir storage is possible. By examining a hybrid model for the Ontario case, where hydro generation is publicly controlled, we show it is difficult for the regulator to control the two inefficiencies: a market power distortion and a water storage distortion.

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Table 1: Parameter values for the first scenario

		Value
Thermal:		
	c_1	10.0
	c_2	0.025
	K	6.0
Hydro:		
	W_{max}	7.0
	μ_ω	$\{1.75, 3.5, 7.0\}$
	σ_ω	$0.1\mu_\omega$
	γ	0.3
Demand:		
	b	20
	μ_α	200
	σ_α	20
Discount factor:		
	δ	0.90

Table 2: Descriptive statistics for alternative inflow scenarios

	Low inflow ($\mu_\omega = 1.75$)		Medium inflow ($\mu_\omega = 3.5$)		High Inflow ($\mu_\omega = 7.0$)	
	Duopoly	Optimal	Duopoly	Optimal	Duopoly	Optimal
Quantities:						
h^{a}	1.74	1.72	3.21	3.49	3.50	6.72
q	3.88	5.99	3.13	5.59	3.00	2.77
$\%(h = W)^{\mathrm{b}}$	74.35	76.25	3.00	48.73	0.00	100.00
b_W	9.22	17.57	0.20	8.09	0.00	3.75
$\%(q = K)^{\mathrm{c}}$	0.00	100.00	0.00	50.92	0.00	0.22
a_K	0.00	35.52	0.00	8.15	0.00	0.02
Price:						
p	87.61	45.67	72.84	18.29	69.99	10.09
st.dev.(p)	9.58	17.26	7.19	12.16	6.65	0.39
skew.(p)	0.21	0.66	0.13	1.69	-0.01	22.96
Carryout:						
$\widehat{W}$	0.03	0.08	0.67	0.02	2.45	0.00
$\min(\widehat{W})$	0.00	0.00	0.00	0.00	1.46	0.00
$\max(\widehat{W})$	0.74	2.10	2.62	0.08	3.56	0.00
Spillage:						
Freq.	0.00	0.00	0.00	0.00	99.94	49.76
Payoffs:						
Welfare	7750	8878	8426	9399	8543	9776
Hydro	1526	795	2360	618	2469	678
Thermal	3047	2128	1993	473	1816	2
Consumers	3177	5955	4073	8308	4258	9096
Myopic:						
h	1.75	1.75	3.38	3.50	3.50	7.00
q	3.87	5.98	3.05	5.57	3.00	2.50
n^{d}	7	4	8	4	2	2

[a] Unless otherwise indicated, values given are the mean of the variable over the simulations.

[b] Percentage of time that the water constraint binds.

[c] Percentage of time that the thermal capacity constraint binds.

[d] The approximation order is chosen so that the approximation residual is of the order 10^{-5} or smaller.

Table 3: Parameters: The Case of Ontario

		Value
Thermal cost:		
	c_1	19.886
	c_2	0.0051
	K	4986
Hydro:		
	W_{max}	{100, 1280}
	μ_ω	2559
	σ_ω	256
	γ	0.0
Demand:		
	b	0.00835
	μ_α	127.65
	σ_α	2.071
Discount factor:		
	δ	0.90

Table 4: Descriptive statistics for alternative reservoir capacities: The Case of Ontario

	Low capacity ($W_{max} = 100$)			High capacity ($W_{max} = 1280$)		
	Duopoly	Hybrid	Optimal	Duopoly	Hybrid	Optimal
Quantities:						
h^{a}	2522	2557	2557	2558	2558	2558
q	3977	3963	4986	3963	3963	4986
$\%(h = W)^{\mathrm{b}}$	42.92	100.00	70.71	35.08	100.00	94.32
b_W	1.14	17.01	2.16	0.94	18.60	4.81
Spillage	35.00	0.00	0.00	0.00	0.00	0.00
$\%(q = K)^{\mathrm{c}}$	0.00	0.00	100.00	0.00	0.00	100.00
a_K	0.00	0.00	19.34	0.00	0.00	19.33
Price:						
p	73.37	73.19	64.65	73.19	73.19	64.65
st.dev.(p)	1.45	1.84	2.89	1.44	1.84	2.94
skew.(p)	0.51	-0.01	0.13	0.53	-0.01	0.08
Payoffs:						
Welfare	5339870	5359530	5625070	5361190	5359530	5625150
Hydro	1850770	1869150	1648720	1871940	1869150	1648590
Thermal	1724040	1713540	1597560	1712370	1713540	1597370
Consumers	1765060	1776840	2378790	1776880	1776840	2379170
Myopic:						
h	2557		2558	2557		2558
q	3963		4986	3963		4986
n^{d}	6	4	9	6	8	8

[a] Unless otherwise indicated, values given are the mean of the variable over the simulations. Quantities are measured in MWh and prices in %/MWh.

[b] Percentage of time that the water constraint binds.

[c] Percentage of time that the thermal capacity constraint binds.

[d] The approximation order is chosen so that the approximation residual is of the order 10^{-5} or smaller.

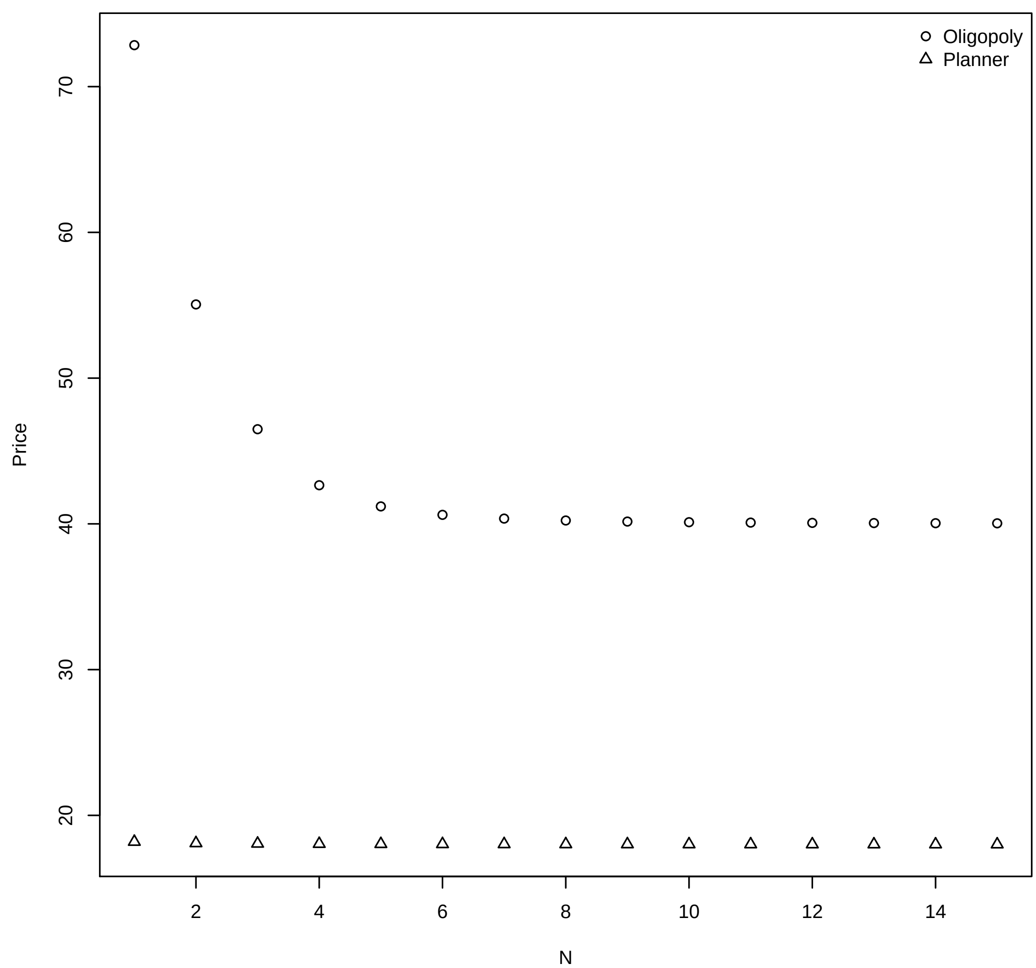

Figure 1: Average price by number of thermal generators

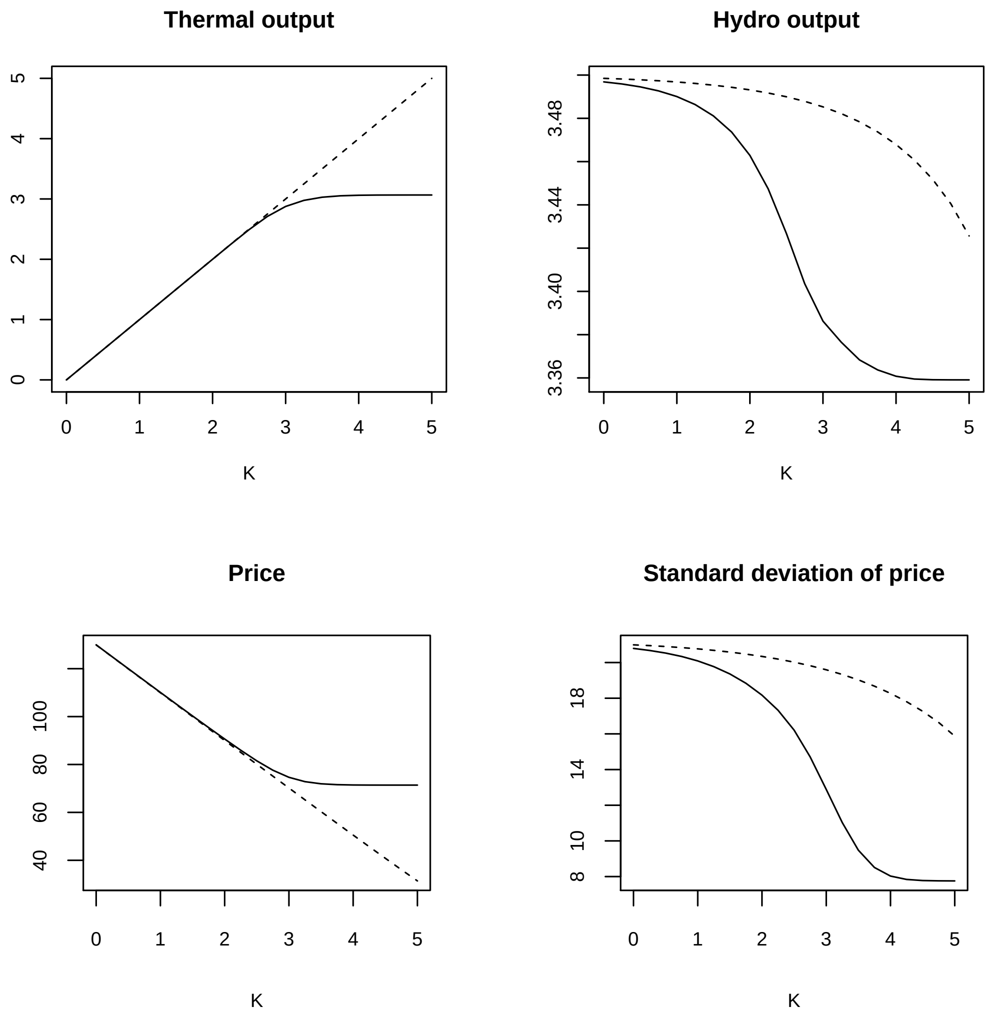

Figure 2: Average model values for alternative thermal capacities: Duopoly (solid line) and Socially Optimal (dashed line)

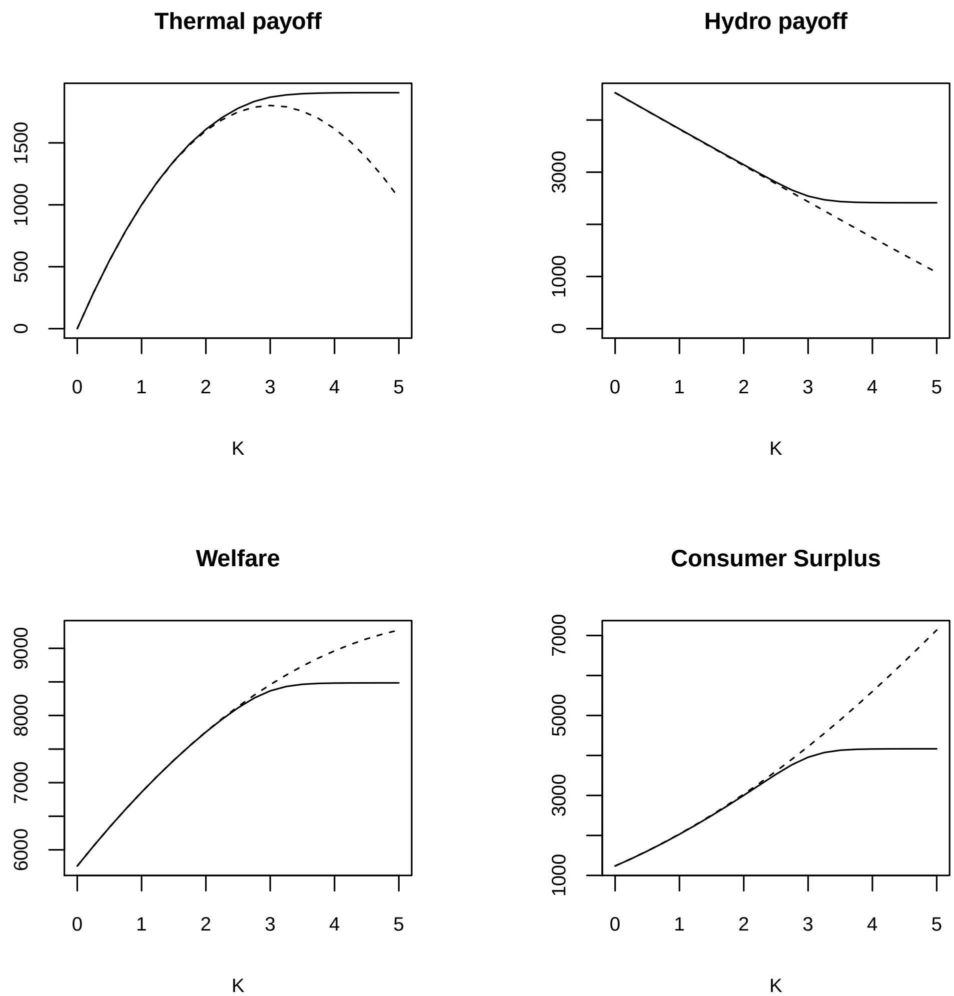

Figure 3: Payoffs for alternative thermal capacities: Duopoly (solid line) and Socially Optimal (dashed line)